Gain Characteristics of Few-Mode EDFA With Different Pump

Wenxuan Xu, Li Pei, Jianshuai Wang, Jingjing Zheng, Zhiqi Li, Tigang Ning, Li Jing, and Guobin Ren

***Abstract*—Few-mode erbium-doped fiber amplifier (FM-EDFA) is a key device to achieve power compensation in long-haul mode division multiplexing (MDM) optical system. The gain characteristics of FM-EDFA are related to the number of modes and multiplexing mode selection. In this paper, the gain characteristics of four modes and six modes FM-EDFA with various mode combinations and different wavelengths are analyzed in single and dual-pump cases. The results show that when the azimuthal-dependent pump is selected, different degenerate mode (odd and even mode) multiplexing will arise DMG. In the replacement of different degenerate modes at the same modal order, the DMG of signal modes $LP_{11o,s}$ and $LP_{21o,s}$ is 1 dB (single-pump: $LP_{01,p}$) while the DMG between signal modes $LP_{11e,s}$ and $LP_{21e,s}$ is 8 dB (dual-pump: $LP_{01,p}+LP_{11e,p}$). Compared with single-wavelength amplification, both the overall gain and the DMG decrease in the multi-wavelength case. The above conclusions can provide a theoretical basis for the selection of FM-EDFA in the wavelength division multiplexing (WDM)-MDM system.**

***Index Terms*—Few-mode erbium-doped fiber amplifier (FM-EDFA), differential modal gain (DMG), mode gain competition.**

I. Introduction

Optical fiber communication is the cornerstone of the information industry. In the past few years, the spectral efficiency has been greatly improved by the technologies—time division multiplexing (TDM), wavelength division multiplexing (WDM), and coherent detection. However, the Shannon formula that shows the capacity of a single channel is $C = B\times\log_2(1+SNR)$ and the capacity is almost reaching its limitation for the existed single-mode optical fiber communication system. Space division multiplexing (SDM), utilizing multi-core fiber (MCF) or few-mode fiber (FMF), is expected to provide a solution for the capacity crunch issue [1]–[4].

In the FMF-based mode-division multiplexing (MDM) system, multiple modes are orthogonal and simultaneously propagate as independent channels, greatly expanding the communication capacity. In favor of long-distance data transmission, the few-mode erbium-doped fiber (EDF) is essentially required. For a practical long-haul MDM system, the FM-EDFA needs to be able to achieve effective and uniform amplification over the C-band [5].

In general, all channels of FM-EDFA should maintain a modal gain of more than 20 dB. Differing from the conventional single-mode EDFA, modal gain competition occurs in FM-EDFA when different mode groups or multiple modes in different wavelengths are multiplexed as the transmission channels. Consequently, the differential modal gain (DMG) arises and a large DMG may lead to a decrease in transmission rate or even fail to work normally. DMG is much more relevant to the multiplexed propagating modes [6]. To reduce DMG, FM-EDFAs with different structures based on cladding-pump or core-pump have been proposed and demonstrated. However, in the cladding pump, there are many difficulties in the experiment of coupling pump light into the optical fiber, while the core pump can be implemented only by a dichroic mirror. The current related core-pump-based FM-EDFA mainly focuses on DMG reduction with parameters optimization, including the erbium ion doping profile, the refractive index of the active FMF, and the pump mode selection [7]–[10].

The transverse mode competition is analyzed in fiber lasers and amplifiers under the different dopant distributions, pump powers, and discriminative loss factors [11]. However, it only discusses the gain competition with one certain multiplexed mode group and does not further discuss the mode competition among different modes or wavelengths, which may attach with totally different DMG. For instance, a low DMG between the mode group LP_{11} and LP_{21} is achieved by tailoring the erbium spatial distribution in a ring-shaped fiber core [12]. However, the modal gain competition is seldom discussed when the mode groups LP_{11} and LP_{21} are multiplexed with different degenerate states (LP_{11o}/LP_{11e} or LP_{21o}/LP_{21e}). Due to the different azimuthal distribution of the certain mode with two spatial degenerate states, it has an obvious effect on the DMG. Additionally, in the WDM-MDM system, several papers related to the WDM system over FM-EDFA are discussed [13], [14], in which it just analyzed the low DMG across the C-band theoretically. Also, there are some works related to the multi-emission in fiber lasers because of the EDF [15], [16]. But the competition in multi-wavelengths is still not revealed.

In this paper, we analyze the modal gain competition with different multiplexed mode groups and wavelengths as signal channels under core pump. Taking a step-index fiber supporting six signal LP_{mn} modes (LP_{01}, LP_{11e}, LP_{11o}, LP_{21e}, LP_{21o}, and LP_{02}) as an example, we analyze the modal gain competition with different multiplexing cases. The multiplexing cases in this

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paper stand for choosing the even or/and odd mode of $m\neq0$ in LP_{mn}. It is found that DMG shows a big difference between the multiplexed spatial degenerate mode when the azimuthal dependent modes are selected. Then, we discuss the competition of modes in multi-wavelengths. The mode competition theory offers good references for FM-EDF optimization and modal gain equalization.

II. Theory

The theory of single-mode EDFA was proposed by C R. Giles [17] and extended to FM-EDFA. Therefore, the following assumptions are made: 1) The model is approximately a quasi-three-level structure under 980 nm pump wavelength, and the excited state absorption (ESA) is ignored; 2) The concentration of erbium ions is uniformly distributed along the longitudinal direction of the fiber, and it does not take into consideration that the influence of erbium ions on the refractive index.

The intensity evolution of signal light, pump light and amplified spontaneous emission (ASE) in the fiber is given by the following power propagation equations:

$$\pm\frac{dP_{p,j}}{dz} = \int\int_{\Omega} \left[N_2\sigma_{e,p,j} - N_1\sigma_{a,p,j}\right] P_{p,j} i_{p,j}\rho d\rho d\varphi - \alpha_{p,j}P_{p,j} - \sum_k d_{p,jk}\left[P_{p,j} - P_{p,k}\right] \tag{1}$$

$$\frac{dP_{s,j}}{dz} = \int\int_{\Omega} \left[N_2\sigma_{e,s,j} - N_1\sigma_{a,s,j}\right] P_{s,j} i_{s,j}\rho d\rho d\varphi - \alpha_{s,j}P_{s,j} - \sum_k d_{s,jk}\left[P_{s,j} - P_{s,k}\right] \tag{2}$$

$$\pm\frac{dP_{ASE,j}}{dz} = \int\int_{\Omega} \left[N_2\sigma_{e,s,j} - N_1\sigma_{a,s,j}\right] P_{ASE,j} i_{s,j}\rho d\rho d\varphi + \int\int_{\Omega} 2\sigma_{e,s,j} h\nu_{s,j}\Delta\nu N_2 i_{s,j}\rho d\rho d\varphi - \alpha_{ASE,j}P_{ASE,j} \tag{3}$$

Where $i_{p,j}$ and $i_{s,j}$ respectively represent the normalized power distribution of the *j*-th pump and signal mode. $P_{p,j}$ and $P_{s,j}$ respectively represent the power of pump and signal. The plus or minus signs in (1)∼(3) indicate the light propagating forward and backward respectively. $P_{ASE,j}$ is the ASE power generated by the signal $P_{s,j}$ in the effective bandwidth $\Delta\nu$ at the center frequency of $\nu_{s,j}$. N_1 and N_2 respectively represent the population densities in the lower and upper energy level, which are not only related to the transverse position of the fiber, but also a function of the longitudinal position. $\sigma_{e,s,j}$ ($\sigma_{a,s,j}$) and $\sigma_{e,p,j}$ ($\sigma_{a,p,j}$) are the emission (absorption) cross-section coefficients at the *j*-th signal light and pump light wavelength, respectively. $d_{p,jk}$ ($d_{s,jk}$) represents the coupling coefficient between the *j*-th and *k*-th pump (signal) light. $\alpha_{p,j}$, $\alpha_{s,j}$ and $\alpha_{ASE,j}$ are the loss coefficients of the pump, signal light and ASE respectively. h is the Planck constant. The integral region Ω is the cross-section of the fiber.

The population in the upper and lower levels is given by the rate equation. The relationship between the population in the upper and lower level is shown as (4) and (5).

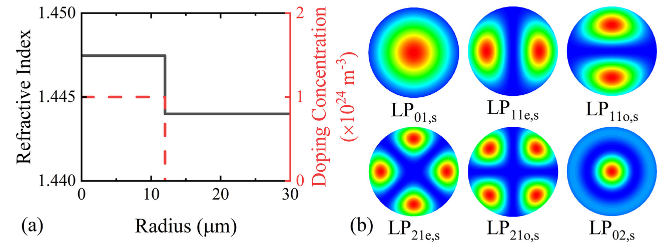

Fig. 1. (a) Refractive index and doping profile for a step-index few-mode EDFA supporting four-mode groups at 1550 nm. (b) Intensity profile of signal modes at 1550 nm.

$$N_2 = \frac{\left(\sum_j \frac{\sigma_{a,p,j}}{h\nu_{p,j}} P_{p,j} i_{p,j} + \sum_k \frac{\sigma_{a,s,k}}{h\nu_{s,k}} (P_{s,k}+P_{ASE,k}) i_{s,k}\right) N_0}{\sum_j \frac{\sigma_{a,p,j}+\sigma_{e,p,j}}{h\nu_{p,j}} P_{p,j} i_{p,j} + \sum_k \frac{\sigma_{a,s,k}+\sigma_{e,s,k}}{h\nu_{s,k}} (P_{s,k}+P_{ASE,k}) i_{s,k} + \frac{1}{\tau}} \tag{4}$$

$$N_0 = N_1 + N_2 \tag{5}$$

It can be seen from (4) that the population in the upper level is affected by the pump mode, signal mode and ASE, which redistributes the population and leads to mode competition among the signal modes. Unlike single-mode EDFA, the different transverse mode power distributions should be considered in FM-EDFA. We analyzed the mode gain and DMG in the conditions as follows: 1) The few-mode fiber is weakly guided, so it can be approximately calculated by linearly polarized (LP) modes; 2) The loss of each mode and the power coupling between various modes are not involved; 3) Without considering the effect caused by mode polarization in the fiber, the gain characteristics of different signal modes under polarization multiplexing are only related to the power distribution themselves.

The above (1)∼(5) are analyzed by the algorithm of the fourth-order Runge-Kutta method cooperated with the shooting method.

III. Different Mode Groups Multiplexing

A. Signal and Pump Mode Profile in FMF

To fully explain the mode competition in the few-mode fiber, the step-index fiber is used as an example and illustrated in Fig. 1(a). The fiber with a 12 μm core radius and the 0.1 effective numerical aperture supports four mode groups ($LP_{01,s}$, $LP_{11,s}$, $LP_{21,s}$, and $LP_{02,s}$) over C-band. The doping concentration is 1×10^{24} m^{-3} and keeps uniform in the whole core. The mode distribution at 1530 nm is shown in Fig. 1(b). At $m=0$, the mode ($LP_{0n,s}$) power distribution is circular symmetry and azimuthal independent; at $m>0$, $LP_{mn,s}$ mode has not only two polarization but also two spatial degenerate modes (named odd and even), which are rotationally symmetric, with the difference angle of $2\pi/m$.

Although the structure used in the discussion is step-index fiber, the analysis applies to other structures or other kinds of fiber amplifiers even to the fiber lasers. In the following

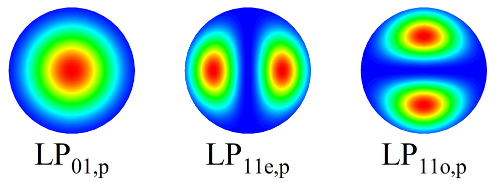

Fig. 2. Intensity profile of pump modes at 980 nm.

TABLE I
FIVE MULTIPLEXING CASES

Case	Signal Mode					
	LP_{01}	LP_{11e}	LP_{11o}	LP_{21e}	LP_{21o}	LP_{02}
1	✓	✓		✓		✓
2	✓	✓			✓	✓
3	✓		✓	✓		✓
4	✓		✓		✓	✓
5	✓	✓	✓	✓	✓	✓

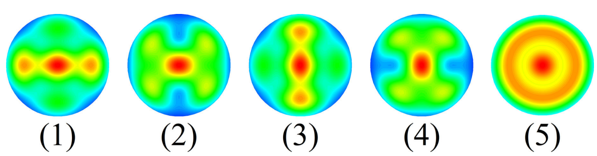

Fig. 3. The mode profile for different multiplexing corresponding to Table I.

discussion, it is assumed that there is equal power (0.1 mW) in each signal spatial mode at 1550 nm. The overlap integral corresponds the amplification effect of pump mode to signal mode, and expressed as below [9]:

$$\eta_{j,k} = \int\int_{\Omega} i_{s,j} i_{p,k} N_0 dS \tag{6}$$

Where Ω is altered to be the doping region. Owing to the uniform doping concentration, (6) can be simplified as below.

$$\eta_{j,k} = \int\int_{\Omega} i_{s,j} i_{p,k} dS \tag{7}$$

Equation (7) indicates the matching degree between the pump and signal intensity profile. Taking the azimuthal symmetry into consideration, $LP_{01,p}$ and $LP_{11,p}$ ($LP_{11e,p}$ and $LP_{11o,p}$) will be selected as the pump mode, although the fiber may support modes at pump wavelength more than that at the signal wavelength. The power distribution of the pump mode selected at 980 nm is displayed in Fig. 2.

Due to the intensity profile of the signal modes in Fig. 1(b), the multiplexed modes can be classified into five cases as shown in Table I. Fig. 3 gives the superposition of signal mode power distribution under five multiplexing cases corresponding to Table I. From Figs. 2 and 3, it can be deduced that the gain of Case 1 and Case 3 are the same when only $LP_{01,p}$ is selected, and that of Case 2 and Case 4 will be the same. When $LP_{11e,p}$ participates in pumping, the gain should be different in all four cases. Case 5 stands for the situation that the six modes are simultaneously injected into the fiber.

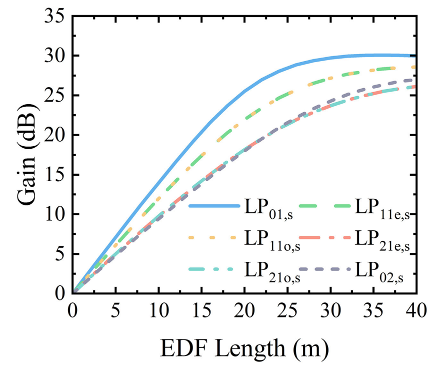

Fig. 4. Modal gain with different EDF lengths at $LP_{01,p}$ pump of 400 mW.

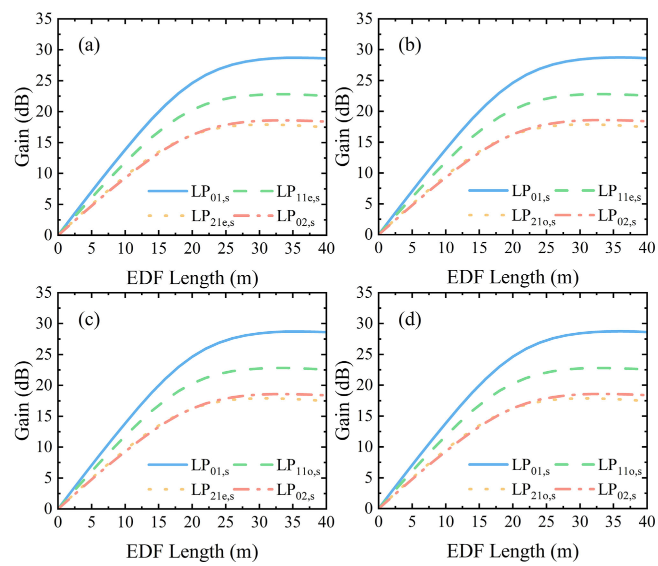

Fig. 5. Modal gain under different EDF length with 400 mW $LP_{01,p}$, when it multiplex $LP_{01,s}$, $LP_{02,s}$ and (a) $LP_{11e,s}$, $LP_{21e,s}$; (b) $LP_{11e,s}$, $LP_{21o,s}$; (c) $LP_{11o,s}$, $LP_{21e,s}$; (d) $LP_{11o,s}$, $LP_{21o,s}$.

B. Single Pump: $LP_{01,p}$

$LP_{01,p}$ pump mode is azimuthal independent.

1) Signal Mode Amplified Separately: To compare with the different multiplexing groups, firstly, the gain and DMG are analyzed in the case of the single signal channel. Owing to the azimuthal independence of the pump mode, the even/odd signal mode can obtain the equal overlap integrals under $LP_{01,p}$ pumping. Fig. 4 shows the relationship between the signal mode gain and the EDF length at the pump power of 400 mW, in the case of the single signal modes transmitting in FMF separately. The gain of each signal mode increases with the growth of the EDF length. At the length of 30 m, $LP_{01,s}$ reaches the maximum gain of 30.0 dB, while that of other modes reaches the maximum when EDF is longer than 30 m. Since each mode enters separately, only the signal mode power distribution affects the amplification effect in this condition. Therefore, due to matching degree with the pump mode, $LP_{01,s}$ obtains the highest gain and $LP_{21e/o,s}$ gets the lowest.

2) Four Signal Mode Multiplexing: Fig. 5(a)~(d) show the mode gain corresponding with the fiber length for four multiplexing cases (Case 1 $\sim$ 4 as shown in Table I). There is no

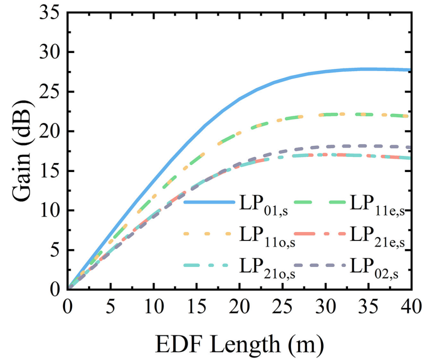

Fig. 6. Modal gain of six multiplexed modes under different EDF length with 400 mW $LP_{01,p}$.

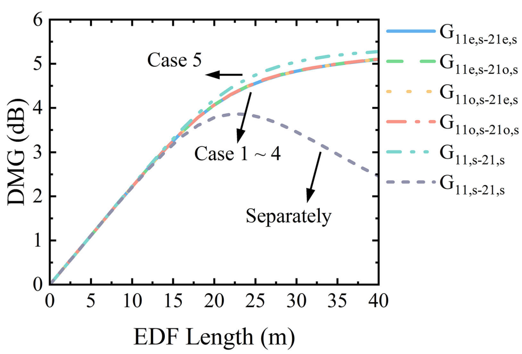

Fig. 7. Under different cases, the maximum DMG between the multiplexed spatial modes of $LP_{11,s}$ and $LP_{21,s}$.

obvious difference among the four multiplexing cases. In the four subgraphs, the gain of $LP_{01,s}$ is the highest and $LP_{02,s}$ is the lowest. There is almost no difference between $LP_{11,s}$ and $LP_{21,s}$. It shows that, under the azimuthal independent pump mode, multiplexing selecting different spatial degenerate modes will not make a remarkable difference.

3) Six Signal Modes Multiplexing: Generally, two spatial degenerate modes are multiplexed simultaneously. Hence, Fig. 6 shows the situation where four mode groups are all injected into the FM-EDFA (Case 5). It is reasonable that all mode gain reduced slightly for the number of modes increased. In addition, it still has no difference between the gain of two spatial degenerate modes in the same mode group.

4) Analysis and Comparison: From Figs. 4∼6, it can be seen that the modal gains decrease with the growth of the multiplexed mode number, because they need to compete for the limited pump power. Among them, the higher modes have a significant decline. In all multiplexing cases, the signal mode $LP_{01,s}$ obtains a higher gain due to the better matching with the pump mode $LP_{01,s}$, while $LP_{21e“o,s}$ gets the lowest gain.

Fig. 7 shows the maximum DMG between the multiplexed $LP_{11,s}$ and $LP_{21,s}$ modes that we are most concerned about in the above amplification cases. Compared with the single signal mode amplified separately, DMG between $LP_{11,s}$ and $LP_{21,s}$ is much worse at longer lengths in Case 1 ∼ Case 5 for the existence of the mode competition. In addition, from the results of Case 1 ∼ Case 5, the multiplexing of six modes or four modes

TABLE II
OVERLAP INTEGRALS OF SIX SIGNAL MODES AND TWO PUMP MODES

Signal\Pump	$LP_{01,p}$ (m^2)	$LP_{11e,p}$ (m^2)	sum (m^2)
$LP_{01,s}$	3.322×10^9	2.369×10^9	5.691×10^9
$LP_{11e,s}$	2.052×10^9	3.491×10^9	5.543×10^9
$LP_{11o,s}$	2.052×10^9	1.164×10^9	3.216×10^9
$LP_{21e/o,s}$	1.316×10^9	1.867×10^9	3.183×10^9
$LP_{02,s}$	2.360×10^9	1.111×10^9	3.471×10^9

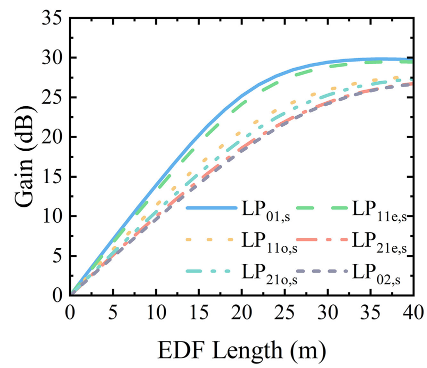

Fig. 8. Modal gain of each signal propagating separately along with EDF length increasing when $LP_{01,p}$ (200 mW) and $LP_{11e,p}$ (200 mW) pumped.

have little impact on DMG under the azimuthal independent pump.

C. Dual-Pump: $LP_{01,p}$+$LP_{11e,p}$

The dual-pump are selected, including the azimuthal independent $LP_{01,p}$ and the azimuthal dependent $LP_{11e,p}$.

1) Single Signal Mode Amplified Separately: Utilizing pump mode selection control is a useful method to improve DMG. The next is going to discuss the mode competition when $LP_{11e,p}$ exists as another pump mode to coordinate with $LP_{01,p}$. Two pump modes have equal power of 200 mW. The overlap integrals of six signal modes and two pump modes have been calculated as Table II. As it presents, $LP_{11e,s}$ prefers to be amplified by $LP_{11e,p}$ than $LP_{11o,s}$.

Similarly, Fig. 8 gives the gain vs. EDF length for each mode amplified singly. From the curves in Fig. 8, the fundamental mode $LP_{01,s}$ still obtains the highest gain. While $LP_{11e,s}$ gets higher gain than $LP_{11o,s}$ for the better match to the pump mode $LP_{11e,p}$.

Different from Fig. 4, the gain of spatial degenerate mode is obviously different in the case of dual-pump. The gain obtained by $LP_{11e,s}$ is much higher than that of $LP_{11o,s}$, Similarly, there is also a gain difference between $LP_{21e,s}$ and $LP_{21o,s}$ mode.

2) Four Signal Mode Multiplexing: Unlikely the results pumped with $LP_{01,p}$, there are great differences in signal gain in Fig. 9. Similarly, the gain of LP_{01} is still the highest. But the gain of LP_{02} is almost the lowest, which is consistent with the results in Table II.

It can be seen from Fig. 9 that the even or odd mode selection of $LP_{11,s}$ has a great impact on their modal gain, but

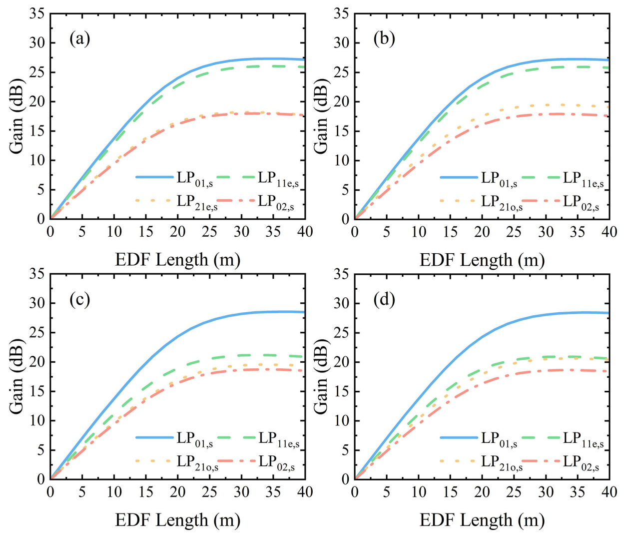

Fig. 9. Modal gain under different EDF length pumped with $LP_{01,p}$ (200 mW) and $LP_{11e,p}$ (200 mW), when multiplex (a) $LP_{11e,s}$, $LP_{21e,s}$; (b) $LP_{11e,s}$, $LP_{21o,s}$; (c) $LP_{11o,s}$, $LP_{21e,s}$; (d) $LP_{11o,s}$, $LP_{21o,s}$.

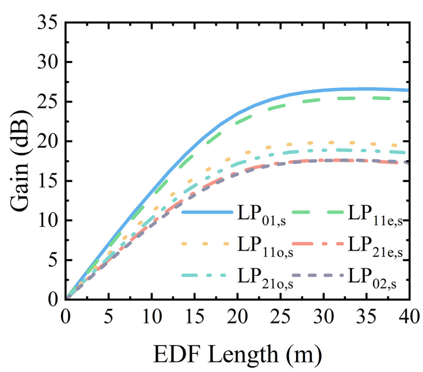

Fig. 10. Modal gain of six multiplexing modes under different EDF length with 200 mW $LP_{01,p}$ and 200 mW $LP_{11e,p}$.

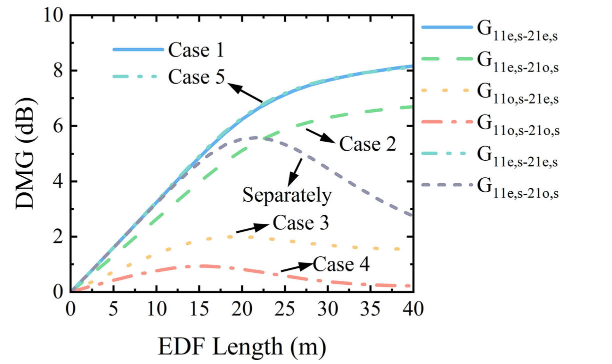

Fig. 11. Gain difference between spatial degenerate mode of $LP_{11,s}$ and $LP_{21,s}$ in different cases.

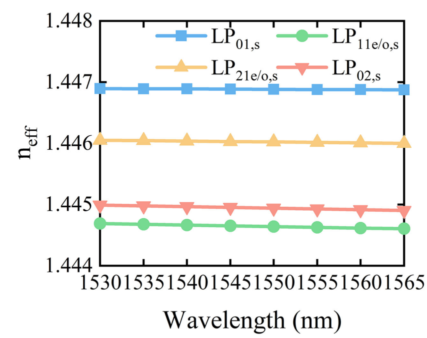

Fig. 12. Effective refractive index of each mode at the wavelength considered.

the selection of $LP_{21,s}$ has little influence. Comparing Fig. 9(a) and (b), their difference is that the former chooses $LP_{21e,s}$ and the latter adopts $LP_{21o,s}$. Obviously, the gain of $LP_{21o,s}$ is a little bit higher than that of $LP_{21e,s}$. with a value of 1.2 dB. We can also find this in the comparison between Fig. 9(c) and (d). In the analysis of $LP_{11,s}$ and $LP_{21,s}$, because of the better match between $LP_{11e,s}$ and the pump modes, $LP_{11e,s}$ is bound to absorb more pump power and it has an obvious impact on $LP_{21,s}$ mode.

3) Six Signal Modes Multiplexing: In Fig. 10, we also simulate the gain and in Case 5. Existing the azimuthal dependent pump, the gain of each spatial degenerate mode is not equal, which is totally different from Fig. 6. The gain of $LP_{01,s}$ and $LP_{11e,s}$ which match well with the two pump mode are much higher than that of the other four modes.

4) Analysis and Comparison: The DMG between the selected degenerate states of $LP_{11,s}$ and $LP_{21,s}$ can be seen from Fig. 11 . In Case 1, 2 and 5, DMG increases with the EDF length growing, and DMG between the selected modes in Case 1 and 5 are almost the same. However, in Case 3 and 4, DMG raises first and then decreases. Regardless of the value or the trend, these five multiplexing cases are very different from the situation where each mode is transmitted separately. This difference is obviously caused by mode competition.

At the length of 30 m, DMG between $LP_{11e,s}$ and $LP_{21o,s}$ (Case 2) is nearly 6.3 dB. However, in Case 3, DMG between $LP_{11o,s}$ and $LP_{21e,s}$ decreases remarkably to around 1.7 dB. Moreover, DMG between $LP_{11o,s}$ and $LP_{21o,s}$ is as low as 0.3 dB at 30 m in Case 4. Combined with the overlap integrals, $LP_{11e,p}$ prefers to amplify $LP_{21o,s}$ than to $LP_{02,s}$, which is contrary to the single pump. It means that the mode competition under different multiplexing cases mainly depends on the azimuthal dependent higher order pump mode.

DMG in Case 3 and 4 is significantly less than Case 1 and 2. If a FM-EDFA is designed to get much higher or lower gain on some of the spatial degenerate modes, for example $LP_{11,s}$ or $LP_{21,s}$, it is a useful way for redundancy some modes to obtain a low DMG.

IV. The Mode Competition in WDM System

In addition to the competition between different modes, there is also gain competition between modes of different wavelengths in WDM which uses different wavelengths as transmission channels. In the following discussions, we use a wavelength interval of 5 nm and select 8 wavelengths starting from 1530 nm. On the other hand, six modes multiplexing, as Case 5 in Table I, is taken as an example to analyze the competition. With the combination of mode and wavelength multiplexing, there

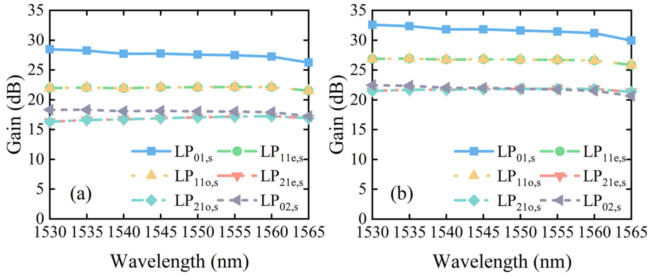

Fig. 13. The spectra under the different pump power, (a) 0.4 W (b) 0.8 W. Six modes under each wavelength signal are amplified by $LP_{01,p}$.

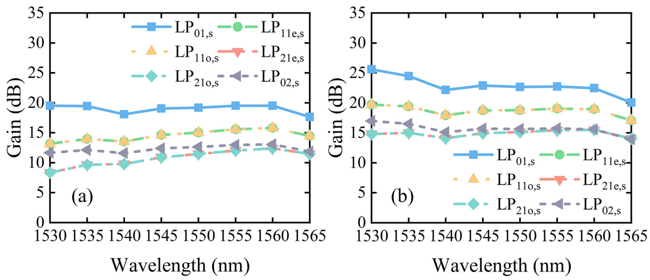

Fig. 14. The spectra under the different pump power, (a) 0.4 W (b) 0.8 W with amplifying 8 wavelength signals by $LP_{01,p}$.

will be 48 signal channels, and each channel power is 0.1 mW. In the following discussion, the EDF length is selected as the optimal length of 30 m.

The effective refractive index of each mode at the wavelength considered is given below. At 1550 nm, the effective refractive index differences between adjacent modes are 0.861×10^{-3}, 1.082×10^{-3} and 0.300×10^{-3}.

A. $LP_{01,p}$ Single Pump

Similarly, we set $LP_{01,p}$ mode as the pump source. Fig. 13 shows the spectra under the different pump power of 0.4 W and 0.8 W. The curve of each wavelength input separately from 1530 nm to 1565 nm is shown in Fig. 13. There is a gradual ascent with the pump power going up. In terms of each mode, it looks flat, although there is a slight decrease at the longer wavelength.

Fig. 14 represents the situation of WDM system where the signals over 8 wavelengths are simultaneously injected into the fiber. It is obvious that there is a remarkable decline when these wavelengths transmit simultaneously. This is caused by the competition generated by the increased numbers of signal channels for occupying the finite pump power.

We can clearly discover that the gains of the short wavelength signals are higher than that of the longer wavelength signals. With the rising of pump power, the modal gain is meant to fluctuate.

B. $LP_{01,p}$ and $LP_{11e,p}$ Dual-Pump

Further, we set $LP_{01,p}$ and $LP_{11e,p}$ as the pump modes. Fig. 15 shows the spectra of six modes simultaneously amplified at a single wavelength separately. From the conclusion drawn in Section III-C, the azimuthal dependent pump mode aggravates the competition, which is still true in this wavelength range. The

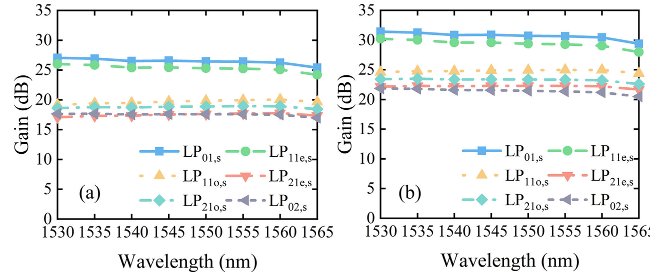

Fig. 15. The spectra under the different pump power, (a) 0.4 W (b) 0.8 W. Six modes under each wavelength signal are amplified by $LP_{01,p}$ and $LP_{11e,p}$ co-pump.

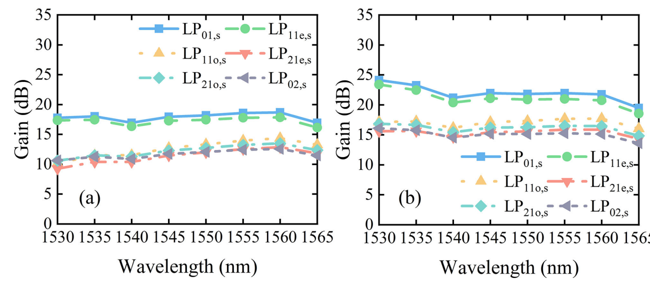

Fig. 16. The spectra under the different pump power, (a) 0.4 W (b) 0.8 W with amplifying 8 wavelength signals by $LP_{01,p}$ and $LP_{11e,p}$ dual-pump.

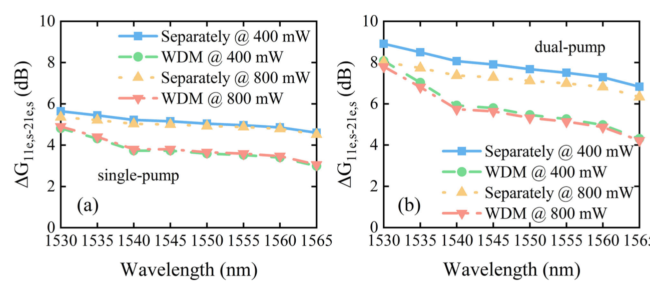

Fig. 17. DMG between $LP_{11e,s}$ and $LP_{21e,s}$ modes under single pump with different power.

difference in the gain of each degenerate mode is also reflected in Fig. 16. That the gain being worse flat, which is caused by multi-wavelength multiplexing, obtained in the discussion of Section IV-A. still exists under the dual pump.

C. Analysis and Comparison

In the above discussion, the mode gain of $LP_{11,s}$ and $LP_{21,s}$ changes most obviously. Therefore, only DMG of $LP_{11e,s}$ and $LP_{21e,s}$ is considered in the next discussion. From Fig. 17, in the case of the same pump, DMG of each wavelength transmitted separately is about 1 dB larger than that of simultaneous transmission. The DMG changes linearly when transmitted separately. This shows that the competition between wavelengths helps to weaken the competition between modes. However, the change of pump power has little effect on DMG.

Comparing the corresponding curves in Fig. 17(a) and (b) under different pump cases, DMG increases by 3 dB. Although there is a large difference in DMG, this is due to the increased competition generated by the existence of azimuthal dependent pumps, not from the competition between wavelengths. The above analysis shows that the wavelength competition mainly affects whether the gain is flat in a range of wavelengths, and has little effect on the competition between various modes.

V. Conclusion

In conclusion, we analyzed the effects of mode competition under different mode multiplexing and different wavelengths on the gain characteristics in the case of single-pump and dual-pump. The results show that when the azimuthal independent pump is selected, different mode combinations have little effect on the mode gain characteristics. However, when the azimuthal dependent pump mode is employed, the gain and DMG fluctuate obviously when multiplexing different odd and even signal modes. Additionally, in a WDM-MDM system, a growing number of signal channels benefit the gain equalization due to the dynamic balance in gain competition. The DMG of all the 48 signal channels is predicted by 3.5 dB only by pump mode selection in a step-index FM-EDF. We believe the DMG can be further reduced by fiber optimization.

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