

Optical Wireless Communications That Exploits the Human Vision's Tolerance of Light Fluctuations in Intensity and Chromaticity

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Abstract—Color shift keying (CSK) communicates information by emitting light that hops instantaneously from one chromaticity to another. This rapid hopping is too fast to be noticeable by human vision, which senses only the time-averaged color, as the human vision's critical fusion threshold and critical color fusion threshold both lie under 100 Hz. That is, color shift keying exploits the human vision's temporal irresolution of instantaneous fluctuations across chromaticities. Left unexploited, up to now, is the intensity-fluctuation tolerance of the human vision, that humans do not perceive small intensity fluctuations of $\pm 4\%$. This newly proposed constellation-design scheme can raise by 49% the designed color-shift-keying constellation's d_{\min}^c (i.e. the maximally achievable “minimum distance” d_{\min}). Moreover, this enhanced d_{\min}^c can be approximated as a simple mathematical expression in terms of the design parameters to serve as a rule-of-thumb to enhance color-shift-keying systems.

Index Terms—Colorimetry, intensity modulation, light emitting diode, optical communication, visible light communication.

I. THE COLOR-SHIFT-KEYING CONSTELLATION DESIGN PROBLEM AS CUSTOMARILY DEFINE – I.E. WITHOUT EXPLOITING HUMANS' IMPERCEPTION OF SMALL FLUCTUATIONS IN LIGHT INTENSITY

COLOR-SHIFT keying (CSK) is a form of visible light communication, exploiting all three basic chromaticities of red, green, and blue. Each CSK symbol combines these three basic chromaticities in a unique ratio of intensities. Therefore, each CSK symbol is mathematically representable by a triplet that specifies the red/green/blue light-emitting diodes' (LED) emission intensities. Mathematically, the m th symbol (in an M -ary CSK constellation) may be signified by a 3×1 vector of

$$\mathbf{s}_m := \begin{bmatrix} s_m^{(r)} \\ s_m^{(g)} \\ s_m^{(b)} \end{bmatrix}. \quad (1)$$

The constellation's M symbols are subject to a *chromaticity constraint* on the algebraic average of the CSK alphabet set;

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only this averaged chromaticity would be perceivable to humans, whose vision is too slow to discern the underlying instantaneous changes. Mathematically, this chromaticity constraint can be represented by the algebraic average preset in the CSK system to

$$\mathbf{c} := \frac{1}{M} \sum_{m=1}^M \mathbf{s}_m. \quad (2)$$

That is, color shift keying's instantaneous emission may hop rapidly among $\{\mathbf{s}_m, \forall m = 1, 2, \dots, M\}$, but the rapid chromaticity-hopping would be imperceptible to humans, whose vision is too sluggish and who can thus notice only the time-averaged color, which equals the preset \mathbf{c} , on account of the symbol stream's stochastic ergodicity.

To design an M -ary constellation \mathcal{S} of color shift keying, one design criterion is to maximize the “minimum Euclidean distance” d_{\min} among all symbols in the constellation:¹

$$\mathcal{S} := \arg \max_{\{\mathbf{s}_m, m=1, \dots, M\}} \overbrace{\min_{\substack{\forall j \neq k, \text{ with} \\ j, k \in \{1, \dots, M\}}} \|\mathbf{H}(\mathbf{s}_j - \mathbf{s}_k)\|_2^2}^{d_{\min} :=}. \quad (3)$$

The above d_{\min} is defined with regard to a particular 3×3 channel matrix of

$$\mathbf{H} = \begin{bmatrix} h^{(r,r)} & h^{(r,g)} & h^{(r,b)} \\ h^{(g,r)} & h^{(g,g)} & h^{(g,b)} \\ h^{(b,r)} & h^{(b,g)} & h^{(b,b)} \end{bmatrix}, \quad (4)$$

where the off-diagonal entries characterize crosstalk across the three basic chromaticities of red, green, and blue.² Hence, the above-mentioned inter-alphabetic distance is valid at reception (i.e. after the transmission has already propagated through the channel) where it matters to the communication system, instead of at transmission.

Moreover, an *intensity constraint* restricts the objective function optimization in (3). Mathematically,

$$\sum_{i \in \{r, g, b\}} s_m^{(i)} = I, \quad \forall m. \quad (5)$$

¹A constellation's symbol-error rate improves with a larger d_{\min}^c , which denotes the constellation's largest pair-wise inter-symbol minimum distance d_{\min}^c .

²The channel matrix \mathbf{H} relates the emitted color \mathbf{s}_j to the received color $\mathbf{H}\mathbf{s}_j$.

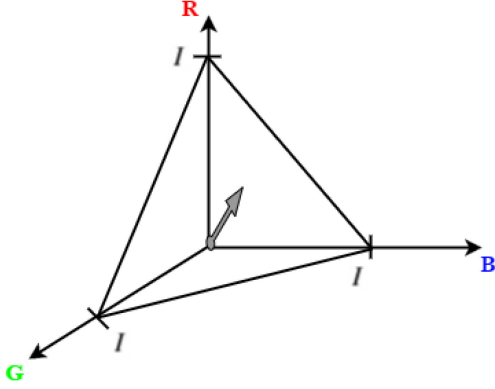


Fig. 1. Color-shift-keying's customary *two*-dimensional color space on which the constellation \mathcal{S} and the present color \mathbf{c} must lie. (This diagram is Fig. 3 in the present authors' [20].)

That is, every symbol's intensity is constricted to an exact preset value of I . Such a constellation's every symbol would lie on the two-dimensional plane shown in Fig. 1.³ This strict equality has been imposed in all current designs [6], [8]–[10], [15]–[20] of the color-shift-keying constellations. Instead, this paper will bring in an unexploited physiological element to this engineering design, by proposing to relax this strict equality constraint, by exploiting the human vision's *imperception* of small intensity fluctuations of up to $\pm 4\%$, in order to increase the constellation's maximum d_{\min} for improved symbol-error rates.

II. TO EXPLOIT THE HUMAN VISION'S IMPERCEPTION OF SMALL VARIATIONS IN LIGHT INTENSITY

The strict constriction of (5) on every symbol's intensity is physiologically *unnecessary*, because that limitation overlooks the human vision's *imperception* of light intensity changes up to a dynamic range of $\pm 4\%$. Please see [1], [3, pp. 90–92], [4, p. 17], [5, p. 462], [11, p. 31], [13, p. 26], and [21].

This paper proposes a relaxation of this unnecessarily precise intensity constriction of (5), to allow each symbol's three intensities to sum to anywhere within a range (not exactly to any

³The humanly perceivable color \mathbf{c} is stated in (2) in terms of the three LEDs' transmission intensity. Fig. 1 shows this 3×1 vector of \mathbf{c} lying in a "color space" that spans only a two-dimensional triangle. Hence, this 3×1 vector \mathbf{c} may be equivalently expressed as a 2×1 vector of $\tilde{\mathbf{c}}$: [20]:

$$\begin{bmatrix} \tilde{\mathbf{c}} \\ \tilde{I} \end{bmatrix} := \mathbf{R}_x \mathbf{R}_z \mathbf{c}, \quad (6)$$

$$\mathbf{R}_x := \begin{bmatrix} 1 & & & 0 \\ 0 & \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) & & -\sin\left(\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) \\ 0 & \sin\left(\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) & & \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) \\ & & & 0 \end{bmatrix}, \quad (7)$$

$$\mathbf{R}_z := \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (8)$$

where \tilde{I} is a nuisance parameter not of interest to the present paper. Incidentally, the white chromaticity may be represented as $\mathbf{c} = \frac{1}{3}[1, 1, 1]^T$ and as $\tilde{\mathbf{c}} = [0, 0]^T$.

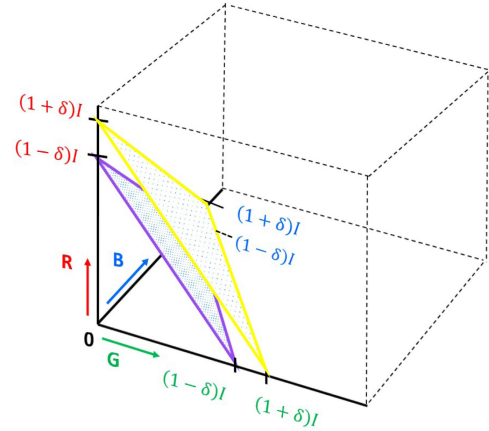


Fig. 2. The proposed *three*-dimensional color space for color-shift-keying constellations.

particular value) of

$$\sum_{i \in \{r, g, b\}} s_m^{(i)} \in [(1 - \delta)I, (1 + \delta)I], \quad (9)$$

with the preset relaxation $\delta > 0$. This new (9) also potentially allows

$$s_m^{(r)}, s_m^{(g)}, s_m^{(b)} \in [0, I + \delta]. \quad (10)$$

A constellation thus designed would have its symbols $\{\mathbf{s}_m, \forall m\}$ (and its preset color \mathbf{c}) lying within a *three*-dimensional slab-like volume shown in Fig. 2.⁴ This relaxation can raise the constellation's maximum d_{\min} . This present work is first in the color-shift-keying literature to exploit the humans' *imperception* of small intensity fluctuations to relax (5) to (9) in order to raise the constellation design's maximally achievable d_{\min}^c without causing any annoyance to the human vision.

III. THE PROPOSED SCHEME'S IMPROVEMENTS OF d_{\min}^c

Color-shift-keying constellation design – as newly proposed here through (1), (3), (2), (4), (9), and (10) – can significantly raise the d_{\min}^c .

Table I illustrates the improvement in d_{\min}^c through a no-cross-talk channel of $\mathbf{H} = \mathbf{I}_3$, for various preset colors (\mathbf{c}), at various constellation sizes (M), and with various extents of relaxation δ . These fractional improvements $\frac{d_{\min}^c(\delta)}{d_{\min}^c(\delta=0)}$ are plotted in Fig. 3, where $d_{\min}^c(\delta)$ symbolizes the d_{\min}^c at a relaxation of δ . Significant increases are obtained in $\frac{d_{\min}^c(\delta)}{d_{\min}^c(\delta=0)}$, e.g. up to 49% of increase at a relaxation of $\delta = 0.04$ for a constellation size of $M = 64$.

Table I and Fig. 3 show the following trends:

- 1) A larger δ (i.e. more relaxation) disperses \mathcal{S} more, hence a larger d_{\min}^c . This is intuitively reasonable, because a larger δ implies a larger support region over which \mathcal{S} can span.

⁴The proposed intensity-constraint-relaxation scheme's "color space" does become *three*-dimensional over $[\tilde{\mathbf{c}}, \delta]$ as shown in Figure 2. However, given $\tilde{\mathbf{c}}$, the perceivable color is unaffected by δ .

TABLE I
THE PROPOSED RELAXATION SCHEME'S IMPROVEMENTS OF d_{\min}°

(a) The pre-set color is white, i.e. $\mathbf{c} = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$. These numerical results are plotted in Figure 3(a).

M	(a) The proposed scheme's d_{\min}° / percentage improvement in d_{\min}°				
	$\delta = 0$ (No Relaxation)	$\delta = 0.01$	$\delta = 0.02$	$\delta = 0.03$	$\delta = 0.04$
4	0.817	0.819 / 0.34%	0.822 / 0.69%	0.825 / 1.04%	0.828 / 1.40%
8	0.471	0.472 / 0.16%	0.476 / 1.05%	0.482 / 2.15%	0.486 / 3.19%
16	0.302	0.305 / 1.05%	0.309 / 2.22%	0.313 / 3.48%	0.317 / 4.85%
32	0.204	0.206 / 1.03%	0.208 / 2.23%	0.211 / 3.56%	0.213 / 4.67%
64	0.141	0.143 / 1.06%	0.144 / 2.20%	0.146 / 3.40%	0.148 / 4.82%

(b) The pre-set color is $\mathbf{c} = [0.6097, 0.3393, 0.0510]$, equivalent to 2700K CCT, which represents a soft/warm light and is common in household lighting [2], [22], [23], [24]. These numerical results are plotted in Figure 3(b). The intensity values are computed from the IEEE color-band-combination #2.

M	The proposed scheme's d_{\min}° / percentage improvement in d_{\min}°				
	$\delta = 0$ (No Relaxation)	$\delta = 0.01$	$\delta = 0.02$	$\delta = 0.03$	$\delta = 0.04$
4	0.414	0.423 / 2.07%	0.431 / 4.17%	0.440 / 6.35%	0.450 / 8.60%
8	0.246	0.250 / 1.64%	0.254 / 3.32%	0.259 / 5.08%	0.263 / 6.92%
16	0.147	0.152 / 3.12%	0.155 / 5.21%	0.158 / 7.50%	0.163 / 10.78%
32	0.0920	0.0956 / 3.91%	0.0985 / 7.07%	0.103 / 11.63%	0.107 / 16.30%
64	0.0580	0.0618 / 6.51%	0.065 / 12.41%	0.069 / 18.74%	0.075 / 28.73%

(c) The pre-set color is $\mathbf{c} = [0.3755, 0.5988, 0.0256]$, as in [12], [14]. These numerical results are plotted in Figure 3(c).

M	The proposed scheme's d_{\min}° / percentage improvement in d_{\min}°				
	$\delta = 0$ (No Relaxation)	$\delta = 0.01$	$\delta = 0.02$	$\delta = 0.03$	$\delta = 0.04$
4	0.384	0.386 / 1.80%	0.398 / 3.74%	0.406 / 5.77%	0.414 / 7.91%
8	0.209	0.213 / 1.69%	0.217 / 3.55%	0.222 / 5.92%	0.227 / 8.41%
16	0.123	0.128 / 3.8%	0.134 / 8.40%	0.137 / 11.51%	0.142 / 15.57%
32	0.0750	0.0778 / 3.78%	0.0824 / 9.82%	0.088 / 17.63%	0.093 / 24.22%
64	0.0445	0.051 / 13.71%	0.054 / 21.80%	0.059 / 33.71%	0.066 / 48.77%

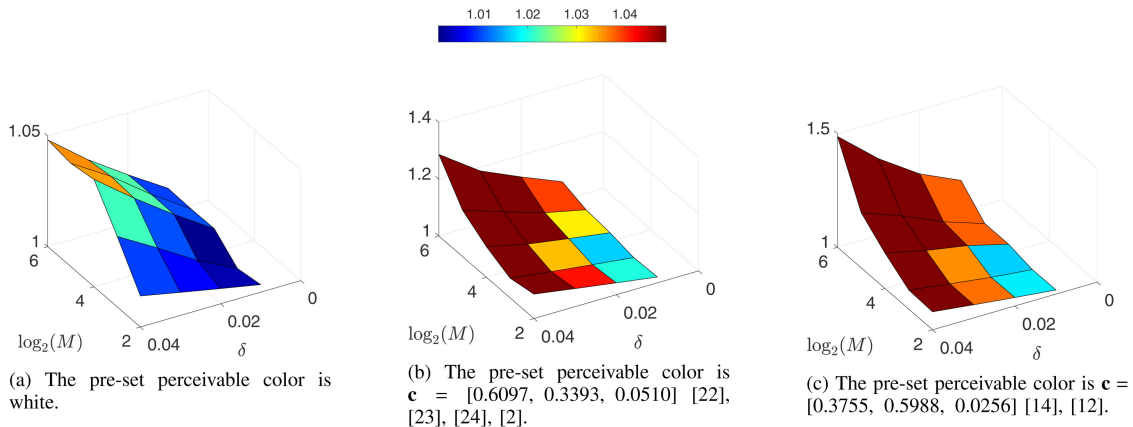


Fig. 3. The proposed intensity-constraint-relaxation scheme's improvements, $\frac{d_{\min}^{\circ}(\delta)}{d_{\min}^{\circ}(\delta=0)}$. These graphs plot the numerical values tabulated in Table I.

- 2) A larger M generally results in more percentage improvement. This may be because of the followings: The M constellation points would be more tightly packed for a larger M ; and any relaxation δ would give a larger $\frac{d_{\min}^{\circ}(\delta)}{d_{\min}^{\circ}(\delta=0)}$ if M is larger.
- 3) A larger $\|\tilde{\mathbf{c}}\|$ (i.e. farther from whiteness on the two-dimensional color space of Fig. 1)⁵ results in more improvement. This may be explained as follows: a larger $\|\tilde{\mathbf{c}}\|$ means farther from the center of the two-dimensional color space, hence the M constellation points are squeezed nearer the triangular color space's edge, so any relaxation would give a larger $\frac{d_{\min}^{\circ}(\delta)}{d_{\min}^{\circ}(\delta=0)}$ if $\|\tilde{\mathbf{c}}\|$ is larger.

IV. TO MATHEMATICALLY RELATE THE PROPOSED SCHEME'S ENHANCED d_{\min}° TO THE DESIGN PARAMETERS OF M , $\tilde{\mathbf{c}}$, AND δ

The proposed scheme's enhanced $d_{\min}^{\circ}(\delta)$ may be expressed explicitly in terms of the constellation's visible chromaticity $\tilde{\mathbf{c}}$, constellation size (M), and the relaxation allowance (δ). The aforementioned design expression is

$$d_{\min}^{\circ} \approx \zeta_1 M^{(\zeta_2\delta + \zeta_3)} + \zeta_4 M^{\zeta_5} (1 + \zeta_6\delta) \|\tilde{\mathbf{c}}\|^{\zeta_7(\log_2 M)^{\zeta_8}}, \quad (11)$$

with $\zeta_1 = 1.9176$, $\zeta_2 = 0.3353$, $\zeta_3 = -0.5945$, $\zeta_4 = -2.9475$, $\zeta_5 = -0.6752$, $\zeta_6 = -0.4899$, $\zeta_7 = 0.9159$, and $\zeta_8 = -0.5891$. This approximate design rule-of-thumb fits 98% of the $\{d_{\min}^{\circ}, \forall \delta, M, \tilde{\mathbf{c}}\}$. This expression is constructed by trial-and-error surface fitting. The above expression's mathematical form degenerates to the one in [20] for $\delta = 0$.^{6,7}

This expression describes the qualitative trends 1)–3) noted above in Section III.

Rounding of the coefficients in (11) gives this more elegant mathematical form:

$$d_{\min}^{\circ} \approx M^{-\frac{1}{2}} \left\{ 2M^{\frac{\delta}{3}} - 3 \left(1 - \frac{\delta}{2} \right) \|\tilde{\mathbf{c}}\|^{(\log_2 M)^{-\frac{3}{4}}} \right\}. \quad (12)$$

This further approximated design rule-of-thumb fits 89% of the $\{d_{\min}^{\circ}, \forall \delta, M, \tilde{\mathbf{c}}\}$. These mathematical expressions can be useful to system engineers in their system development.

V. CONCLUSION

Color Shift Keying (CSK) lightwave transmission embeds digital information in the chromaticity of the emitted light, by distributing the overall emission intensity differently among the red/green/blue light emitting diodes for different information symbols. The transmission's rapidly fluctuating chromaticity is

⁵Because the triangular region is 120° rotationally symmetric, the fitting here would be with respect to $\|\tilde{\mathbf{c}}\|$ instead of $\tilde{\mathbf{c}}$.

⁶More precisely, the coefficient of determination's (R^2) of 98% is a statistical metric whose numerical value (roughly speaking) gives the component of the dataset that is "explained" by the model.

⁷This set $\{d_{\min}^{\circ}, \forall \delta, M, \tilde{\mathbf{c}}\}$ refers to the trivariate dataset of $4 \times 5 \times 2890$ values of d_{\min}° , sampled over four different values of $M \in \{4, 8, 16, 32\}$, unionized with five values of $\delta \in \{0, 0.01, 0.02, 0.03, 0.04\}$, unionized further with 2890 samples of the triangular region in Figure 1.

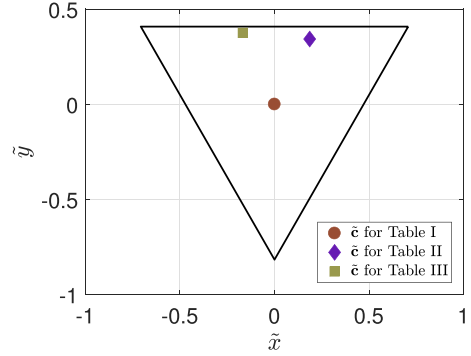


Fig. 4. Chromaticity locations $\tilde{\mathbf{c}}$ for different perceived colors from Tables I(a)–I(c). This triangle is obtained from Fig. 1 by geometrically “rotating” the triangular subregion’s normal vector to coincide with the z -axis, such that the triangle would come to lie on the \tilde{x} - \tilde{y} plane, with the Cartesian origin as the triangle’s new centroid. For a more precise definition of \tilde{x} and \tilde{y} , please consult of Appendix B of [20]. This triangular region is 120°-rotationally symmetric with respect to the white color at the center.

imperceptible to the human eyes, due to their limited resolution in time. Humans can perceive only the temporally averaged chromaticity, which is preset by the color-shift-keying communication system. Overlooked in the open literature on color-shift-keying constellation design is another physiological limitation of the human vision: humans cannot detect intensity fluctuations within $\pm 4\%$. This paper is first to do so in the open literature on the color-shift-keying constellation design. This proposed scheme can improve the maximally achievable d_{\min}° by nearly an half. The fractional improvement $d_{\min}^{\circ}(\delta)/d_{\min}^{\circ}(\delta=0)$ increases monotonically with a larger constellation size (M) and/or with the preset color (\mathbf{c} , and thus the constellation points) moving away from the center of the triangle in Figure 4 (i.e. farther from whiteness). That is, the fractional improvement in d_{\min}° is all the more significant where the constellation is more crowded in the original *unrelaxed* case. This work also advances a simple closed-form expression to characterize approximately how chromaticity, modulation size and intensity sum range affect the maximal d_{\min}° achievable in color-shift-keying constellation design.

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