Efficient Switchable Common Path Interferometer for Transmission Matrix Characterization of Scattering Medium

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***Abstract*—Transmission matrix can be used as an elegant tool to represent the light transmission characteristics of a scattering medium. In this paper, we propose a novel common path interferometer technique to measure the transmission matrix utilizing the polarization rotation of incident beam. Specifically, the horizontally polarized and vertically polarized components of the incident beam are used as signal light and reference light, respectively. By rotating the polarization of the incident beam, the ratio between the powers of signal and reference light can be freely adjusted and balanced. The phase shift method is employed to retrieve the transmission matrix of scattering medium. Once the transmission matrix is known, the polarization of the incident light beam can be rotated to switch off the reference and only the signal will remain. Focusing and manipulation of light through a ground glass has been experimentally demonstrated with the measured transmission matrix. An optical focus enhancement reaches 83% of the theoretical maximal value is obtained, which is about 30% improvement compared to the previous co-propagation method. The transmission matrix characterization method has the potential to become a general tool to harness the multiple scattering of light through complex medium.**

***Index Terms*—Beam control, transmission matrix, wavefront shaping.**

I. Introduction

When light propagates through scattering media, multiple scattering effects lead to complex speckle patterns [1], which often time represent as obstacles for many applications. How to circumvent this obstacle is of fundamental interests to many research fields, such as spatiotemporal localization [2], quantum encryption [3], random Raman laser [4], and biological tissue imaging [5]. The compensation of multiple scattered light can be traced back to the scattering field focusing first achieved by I. M. Vellekoop *et al.* in 2007 [6], where a spatial light modulator (SLM) and feedback shaping algorithm are used to shape the wavefront of the optical field illuminating the scattering medium so that the scattered field can be focused into a bright optical focal spot. Afterwards, researchers proposed a variety of multiple scattering light compensation schemes to improve their effectiveness. The first way to compensate the scattering medium is the digital optical phase conjugation method [7], [8]. This method can be roughly divided into two steps: phase recording and conjugate phase reverse propagation. In the phase recording stage, the phase of the scattered light field is obtained through the holographic technique, and then the conjugate phase is calculated. The conjugate phase reverse propagation stage is to utilize the time reversal characteristic of the conjugate phase to realize imaging or focusing. The digital optical phase conjugation method has the advantages of high precision and easy implementation, and has been successfully applied in endoscopy [9], [10] and photoacoustic imaging [11]. However, this method requires prior knowledge and alignment correction, and only one output mode can be calibrated at a time. The second type of compensation method for multiple scattered light is the wavefront shaping technology based on feedback optimization [12]–[15]. This method uses various optimization algorithms to iteratively obtain the optimal wavefront corresponding to the target light field. The wavefront shaping technology based on feedback optimization treats the modulation of the light field by the scattering medium as a "black box", and continuously adjusts the wavefront to approach the optimal value according to a certain optimization strategy. Feedback optimization method has the advantages of simple optical system and high precision, and has been proved to be able to accurately compensate the scattered light field in recent work [16]–[18]. Unfortunately, this method also compensates one output mode at a time, and the calibration time is proportional to the complexity of the target, which is not conducive to practical applications. The third method of manipulating multiple scattered light is the transmission matrix (TM) method [19], [20]. The TM method describes a definitive relationship between the input light field and the multiple scattered light. Due to the conjugate characteristics and time reversal characteristics of the TM, focusing [21]–[23] or imaging [24]–[26] of multiple scattered light can be realized conveniently once the TM characterizing the scattering medium is known. Compared with the other two methods, TM can be used

Manuscript received February 27, 2022; revised April 11, 2022; accepted April 12, 2022. Date of publication April 19, 2022; date of current version April 28, 2022. This work was supported by the National Natural Science Foundation of China under Grants 92050202, 62075132, and 11874226. *(Corresponding author: Qiwen Zhan.)*

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Digital Object Identifier 10.1109/JPHOT.2022.3167689

to calculate the required incident field to produce arbitrary linear combination of output modes. Thus, accurate determination of the TM for the specific scattering medium is the key.

The characterization of the scattering medium TM requires the measurement of the amplitude and phase information of the multiple scattered light. However, limited by the fast oscillation characteristics of light, one cannot directly obtain the phase information of the light field. To solve this problem, researchers have proposed a variety of TM measurement methods and achieved outstanding results. The first reliable TM measurement scheme is based on the phase retrieval algorithm [19], [27]–[30]. Specifically, various iterative optimization algorithms are used to retrieve the phase information of the scattered light field from the pure intensity measurement, and then the TM of the scattering medium can be calculated. The TM measurement scheme based on the phase retrieval algorithm has high accuracy and the optical measurement system is simple. However, limited by the principle, this method is easy to fall into the local optimal value. The second TM measurement method is based on holographic interferometry [31]–[35]. Through introducing a reference field to interfere with the scattering field, the wavefront of scattering field can be reconstructed from the interference patterns. However, the holographic method needs to introduce an additional reference light, which reduces the stability and limits application range of the measurement system. To overcome this shortcoming, the co-propagation phase shift method is proposed and demonstrated by S. M. Popoff *et al.* in 2010 [20], [24], [36], [37]. In this method, the signal beam and the reference beam propagate along the common path, which enhances the stability of the system. The phase of the signal beam can be retrieved by using a four-step phase shift method. However, this co-propagation method needs to spatially divide the incident beam into a signal area and a reference area, which sacrifices part of the modulation freedom of the SLM and cannot eliminate the disturbance introduced by the reference light during the following focusing and imaging processes.

In this work, we propose and demonstrate a novel common path interference method utilizing the polarization characteristics of liquid crystal SLM to measure the scattering medium TM in an efficient way. Specifically, the horizontally polarized component of the incident light beam that is modulated by the liquid crystal SLM is used as the signal light, while the vertically polarized component that is not modulated is used as the reference light. These two polarization components are brought into interference by projecting both components with a linear polarizer. This method strictly ensures the common path similar to those reported in Ref. [20], [24], [36]. Different from the traditional method, the reference is introduced to characterize the TM of the scattering medium only, and can be switched off afterwards during the imaging and focusing of the signal, avoiding the negative effect from the reference beam after the TM characterization. We experimentally verify the superiority of the proposed method by focusing the scattered light and controlling the transmission energy of the focal spot. Moreover, the influence of the incident light wavelength, the power ratio of signal light to reference light, and the number of control segments on the experimental results are also discussed. The TM characterization method demonstrated in this work has the potential to be applied in broad imaging and communication applications.

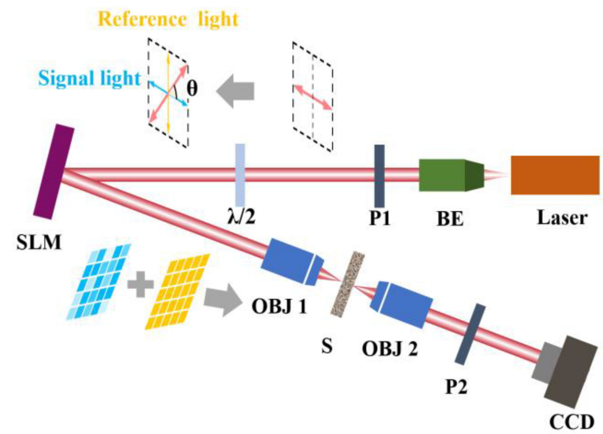

Fig. 1. Schematic diagram of the TM measurement system. BE, beam expander; P1, P2, polarizer; SLM, spatial light modulator; OBJ1, OBJ2, objective lens; S, scattering medium; CCD, charge-coupled device; λ/2, half-wave plate.

II. Experimental Methods

The experimental setup used to characterize the TM of a scattering medium is illustrated in Fig. 1. The light source is a tunable laser (Santec TSL-550, wavelength range: 1480 nm–1630 nm). The incident beam from the laser is expanded by a beam expander (BE, Nikon, CFI 10X, 10×, NA = 0.25). A half-wave plate is used to rotate the polarization direction of incident beam. Before passing through the scattering medium (S, ground glass diffuser, LBTEK, DW110-220, 220 grits), the phase distribution of incident beam is manipulated by a spatial light modulator (SLM, HOLOEYE, GAEA-2, pixel size: 3.74 μm × 3.74 μm, resolution: 4160 × 2464). The incident beam can be divided into signal and reference parts according to the polarization directions. The horizontal polarization component of the incident beam is used as signal light after being modulated by the liquid crystal SLM, and the vertical polarization component that is not modulated by the liquid crystal SLM is used as reference light. Reflected light from the SLM is focused on the surface of scattering medium by an objective lens (OBJ 1, Nikon, CFI 20X, 20×, NA = 0.40). The light transmitted through the scattering medium is collected by an objective lens (OBJ 2, Nikon, CFI 100X, 100×, NA = 0.90). The intensity pattern is measured by using a CCD camera (Xenics, Bobcat 640 GigE, 16-bit, spectral range: 900 nm–1700 nm).

In the previous works, the incident beam is spatially divided into signal and reference parts to form a common path interferometer. Typically, the pixels in the peripheral area are used as the reference and the pixels in the main central area of the SLM are used for the signal. This method suffers several drawbacks. First of all, the setup is not strictly common path, as the signal and reference come from different spatial location of the SLM. Secondly, the area that serves as the reference still reflect light after the TM is determined, thus also involves in the imaging and focusing process later and introduce disturbance that degrade the quality of imaging and focusing. In this work, the reference and signal are strictly common path as they share exactly the same area on the SLM. In addition, the reference can be easily switched off by rotating the incident polarization after the TM is determined. With the TM of scattering media, the conjugate and time reversal characteristics of the TM can be used to manipulate the light after the scattering medium to realize various imaging and focusing functions.

The TM determination process can be basically divided into the following steps. First of all, the power ratio of the signal light to the reference light is controlled by rotating the half-wave plate. For instance, when the incident light beam is polarized along $\theta = 45°$, the power ratio is fixed at 1:1. Then, the column vectors of the Hadamard matrix generated in advance are sequentially loaded on the SLM to modulate the signal beam [38]. Specifically, the modulation area of the SLM is subdivided into $\boldsymbol{N}$ independent control segments, and the elements in the Hadamard matrix are loaded onto the corresponding segments. The independent control segment is defined as the superpixel composed of $\boldsymbol{R_1}\times\boldsymbol{R_2}$ adjacent pixels in the SLM modulation area. Assuming that the size of the SLM modulation area is $\boldsymbol{k_1}\times\boldsymbol{f_1}$ pixels, and the dimension of the Hadamard orthogonal basis is $\boldsymbol{k_2}\times\boldsymbol{f_2}$, then $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ can be expressed as $\boldsymbol{R_1} = \boldsymbol{k_1}\,/\,\boldsymbol{k_2}$ and $\boldsymbol{R_2} = \boldsymbol{f_1}\,/\,\boldsymbol{f_2}$, respectively. The linear polarizer P2 with a transmission axis at 45° is used to project the signal and reference polarizations to form interference patterns. The CCD camera is used to collect the speckle pattern of the interference field between the signal and reference beam. The TM of the scattering medium can be retrieved by using the phase shift method [20]. Finally, after the TM of the scattering medium is retrieved, the polarization of the incident beam will be rotated to the horizontal direction again to switch the reference light off. The disturbance of reference beam is now eliminated, and the light field of transmission beam through the scattering media can be modulated with high quality.

To quantitatively evaluate the accuracy of the measured TM, light focusing experiments are performed. Specifically, it takes the m-th column of the conjugate transposed matrix of the TM as the incident wavefront [37], and then observes whether the focus spot is formed at the m-th pixel in the target plane. If the focus spot is observed, it proves that the measured TM is effective, and the accuracy of the TM is proportional to the relative intensity of focus spot. Furthermore, the optimal wavefront for multipoint focusing can be obtained by summing the columns of the conjugate transpose matrix of the TM. The focusing effect of the optical focus can be evaluated by the enhancement factor η defined in Eq. (1) of Ref. [6], [19] as:

$$\eta = \frac{I_{foc}}{I_{avg}} \tag{1}$$

where I_{foc} is the peak focal intensity and I_{avg} is the ensemble averaged intensity generated from input without modulation. For phase-only modulation, the theoretical enhancement factor can be expressed as [6], [19]:

$$\eta_{\max} = \pi(N-1)/4 + 1 \tag{2}$$

where $\boldsymbol{N}$ is the number of independent control segments of the liquid crystal SLM.

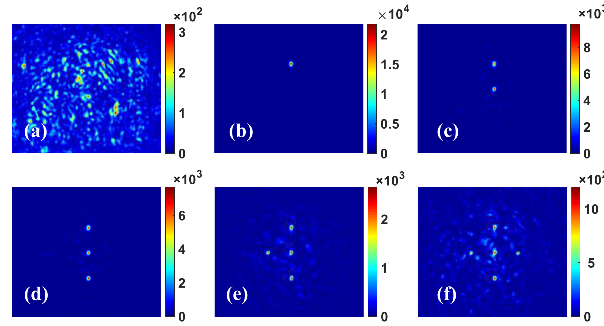

Fig. 2. Focusing through scattering medium using the calibrated TM. (a) Scattering field corresponding to random wavefront; (b)-(f) Focal spot corresponding to the optimized wavefront.

III. Experimental Results

Using the methods and optical measurement systems described in Sections 2, we experimentally measured the TM of the scattering medium. The retrieved TM is used to control the multiple scattered light after the scattering medium and the experimental results are shown in Fig. 2. When no SLM phase pattern is applied to the incident wavefront, typical speckle pattern can be observed by the CCD camera as shown in Fig. 2(a). By loading the compensation phase pattern to the SLM according to the measured TM, a nicely focused spot can be obtained, which verify the effectiveness of the TM. The accuracy of the measured TM can be described by the enhancement factor in Eq. (1). When the number of independent control segments N = 1024, the corresponding enhancement factor η is 667, which reaches 83% of the theoretical enhancement factor ($\eta_{\max}$ $\approx$ 804). This difference may be caused by the environmental noise and fluctuations in the intensity of incident beam. [37] Compared with the previous co-propagation method (achieving about 50% of the theoretical enhancement), [37] the retrieval method proposed in this paper improves the enhancement effect by about 30% under the same experimental conditions. This enhancement is attributed to the fact that by using the scheme proposed in this paper, the reference beam can be switched off after the TM is obtained, thereby eliminating its disturbance on the transmission field. To further evaluate the performance of TM, multiple-spot focusing experiments are carried out, and the experimental results are shown in Fig. 2(c)–(f). It can be found that multiple focal spots can be easily synthesized according to the retrieved TM without additional optimization or shaping, which further illustrates the effectiveness of the measured TM.

Normally, the number of independent control segments N will affect the control effect of multiple scattered light, so it is necessary to conduct a quantitative study. To study the influence of N on the accuracy of TM, a set of comparative experiments was carried out. The power ratio of signal light to reference light is fixed at 1:1, and the enhancement factor of the TM is measured when N = 256, 512, 1024, 2048 and 4096, respectively. The experimental results are shown in Fig. 3, and it can be found that the enhancement factor is proportional to N. Moreover, it can also be observed that the upward trend of the enhancement factor to flatten off gradually, which can be explained by the speckle decorrelation and the measurement noise during the experiment [37].

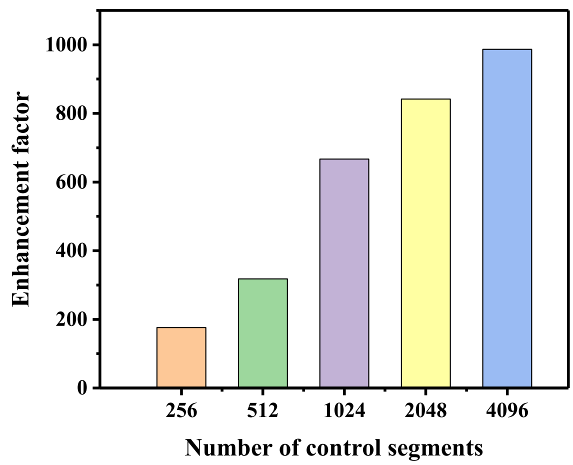

Fig. 3. The enhancement factor as a function of the number of control segments.

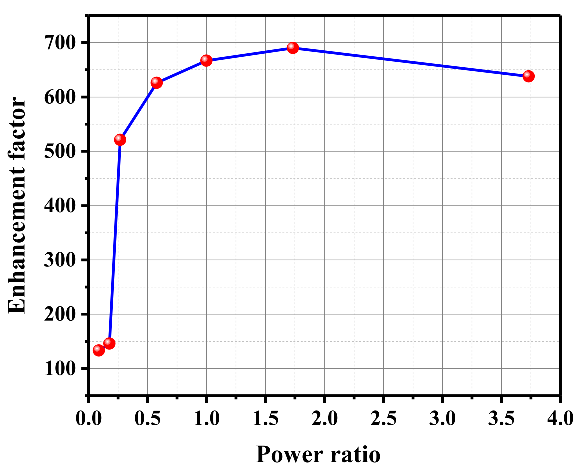

Fig. 4. The enhancement factor as a function of the power ratio of signal light to reference light.

The TM measurement method proposed in this paper allows easy adjustment of the power ratio between the signal land the reference through rotating the polarization of the incident light, which can be very helpful in searching for the optimal TM. We study the relationship between the enhancement factor and the power ratio, and the experimental results are shown in Fig. 4. Ideally, the power ratio between signal and reference beam should be 1:1 for the best interference fringe contrast. However, Fig. 4 shows that the best enhancement factor is achieved with the power ratio of 1.73:1 rather than 1:1. For the signal beam, the incident power is reduced during the phase modulation process, while the power of reference beam remains unchanged. It can be explained by the power loss of the signal beam during the phase modulation process. Therefore, it is necessary to properly increase the power ratio between the signal beam to the reference beam in the incident beam to retrieve the optimal TM.

Generally, wavelength fluctuation of the light source will adversely affect the control of multiple scattered light. To assess this effect quantitatively, we experimentally demonstrate the tolerance of TM to wavelength variation of incident light. Firstly, when $N = 1024$ and the wavelength of incident light is 1550 nm, the TM of the scattering medium is measured and single-spot focusing is achieved on the detection plane. The wavelength of the incident light is then changed while the corresponding light field on the detection plane is recorded. The experimental results are shown in Fig. 5, where the inserted pictures are the output light fields on the detection plane corresponding to different wavelengths of 1480 nm, 1550 nm and 1630 nm. It can be seen from Fig. 5 the TM has a high tolerance to wavelength fluctuations that in the wavelength range of 1480 nm to 1630 nm. Excellent focus spot can be maintained across the bandwidth despite of the fluctuation of the enhancement factor.

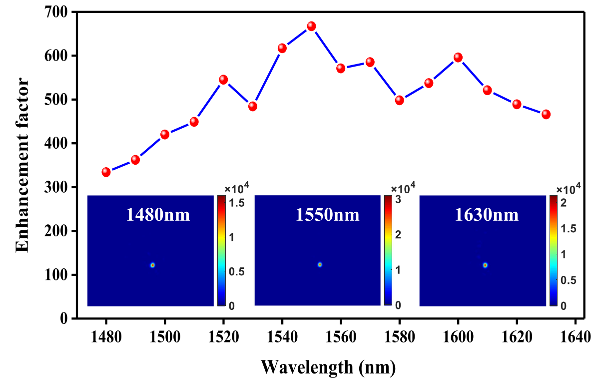

Fig. 5. The enhancement factor as a function of the incident light wavelength. The inserted pictures show the focusing effect corresponding to different incident wavelengths.

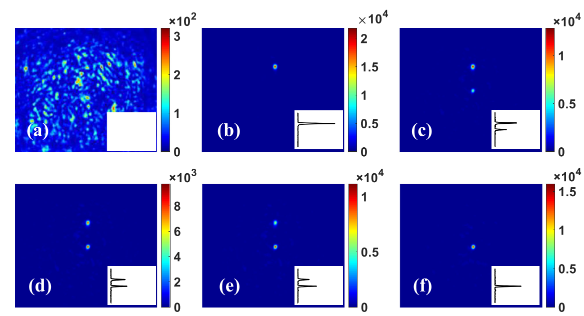

Fig. 6. Experimental results of intensity modulation for multiple focal spots. (a) Scattering field corresponding to random wavefront. (b)-(f) Controlling the intensities of multiple focal spots. (The insets show intensity profiles along vertical direction.).

To further prove the scalability of the method, we demonstrate the control of the intensities of two focus spot based on the measured TM. Specifically, by assigning different weighting factors [39], the intensities of multiple focal spots can be individually and simultaneously controlled. The experimental results of controlling the intensities of multiple focal spots are shown in Fig. 6. In Fig. 6(a), the speckle pattern of the transmission beam can be observed when no compensation phase pattern is applied to the incident wavefront. Based on the measured TM, the intensity of the two spots can be tuned independently as demonstrated in Fig. 6(b)–(f).

IV. Conclusion

In summary, we demonstrate a common path interference method based on the polarization response of liquid crystal SLM to retrieve the TM of the scattering medium. Specifically, the horizontally polarized and vertically polarized components of the incident beam are used as signal light and reference light, respectively. The phase shift method is used to retrieve the TM of the scattering medium. By rotating the polarization of the incident beam, the power ratio between signal and reference can be readily adjusted to achieve optimized condition for the TM characterization. To control the transmitted field through scattering media, the signal field can be designed based on TM. The reference beam can be switched off, to eliminate its disturbance on the transmission field. The accuracy and validity of the retrieved TM are experimentally verified through the focusing and control of multiple scattered light. Moreover, the expandability of the scheme is further explored by controlling the intensities of the multiple focal spots independently. The proposed method to measure TM for scattering media has the advantages of compact structure, high stability, excellent versatility and high precision that may find wide application in the fields of optical communication and imaging.

Acknowledgment

The authors declare that there are no conflicts of interest related to this paper.

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