Entanglement-Based Single-Photon Amplification With High Fidelity of Polarization Feature

Ruitong Zhao, Jianjun Guo, and Lianglun Cheng

Abstract—**The amplification protocol of single-photon is an effective method to overcome the photon loss in the practical quantum communication. Here, we propose an entanglement-based single-photon amplification to nearly eliminate the vacuum state resulting from the photon loss. By the quantum nondemolition detection with cross-Kerr nonlinearity, our proposal has more higher fidelity of the polarization feature. And the total success probability of the amplification can be improved with bigger nonlinearity. This absolute amplification of single-photon is of great value to lengthen the quantum communication distance.**

Index Terms—**Single-photon amplification, entanglement, cross-Kerr nonlinearity.**

I. Introduction

PHOTONS are the best carriers for flying qubits, and single photon transmission is crucial in long-range quantum information, such as qubit transmission [1], quantum key distribution [2]–[7], quantum secure direct communication [8]–[11], multi-party quantum key agreement [12], distributed quantum machine learning [13], [14], and so on. However, the environmental noise may lead to the photon loss during the transmission over a quantum channel, which will reduce the efficiency and security of quantum communication. Single-photon amplification is a fundamental and efficient method to overcome the photon loss [15].

Many researches on single-photon amplification are reported. For example, Gisin *et al.* introduced a heralded qubit amplifier, which provided a realistic solution to overcome the photon loss problem [16]. This method improved the security of device-independent quantum key distribution. Miná *et al.* used this method to realize a quantum repeater [17]. Their repeater allowed the distribution of entanglement with a high fidelity over long distances, without vacuum components and multi-photon errors. Monteiro *et al.* demonstrated the amplification for path entangled states [18]. By exploiting the amplification, the entangled states can maintain high fidelity over loss-equivalent distances of more than 50 km. Kocsis *et al.* demonstrated the first heralded noiseless linear amplification of a photonic qubit encoded in the polarization state [19]. They increased the transmission fidelity of the qubit channel by up to a factor of 5. By optimal quantum cloning, Bartkiewicz *et al.* demonstrated a phase-independent quantum amplifier [20], and they doubled the broadcasting range of quantum information. Bruno *et al.* demonstrated the post-selection free heralded qubit amplification for time-bin qubits and single photon states [21]. They showed that the gain, fidelity and the performance of the amplifier were a function of the loss. The noiseless amplification for single-photon two-mode and three-mode entangled states were also proposed assisted by single-photons [22], [23]. And the polarization feature of the photonic qubit would be remained without reduction. Chen *et al.* put forward a single-photon-assisted noiseless linear amplification protocol to protect the single-photon entanglement in polarization-time-bin qudit state to overcome the photon transmission loss [24]. The polarization–time-bin features of the qudit could be well preserved, and they could not only increase the fidelity of the state, but also overcome the decoherence problem simultaneously. The above amplification proposals use single photons as auxiliary, and entangled photon pairs can also be used to improve the success probability and the fidelity of the amplification. Meyer-Scott *et al.* proposed an efficient amplification of a photonic qubit with a pair of entangled photons as auxiliary [25]. It was shown that the success probability didn't decrease asymptotically to zero even with the increasing gain. Bartkiewicz *et al.* studied the state-dependent qubit amplification with success probability as a function of output qubit fidelity [26]. The qubit amplification procedure could be optimized to obtain better fidelity versus success probability. Wang *et al.* improved the success probability of the amplification by using a four-photon entanglement [27], and illustrated an entanglement distillation protocol for a mixed ensemble composed of four kinds of entangled states and vacuum states [28]. And the inherent channel losses could be well overcome. The linear optical amplification for the multi-mode single-photon W state was proposed by Feng *et al.* [29]. By exploiting the local Bell states, their protocol not only increased the fidelity of the single-photon W state, but also protected the

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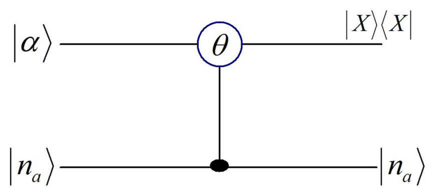

Fig. 1. A schematic drawing of QND with cross-Kerr nonlinearity. $|X\rangle\langle X|$ is the homodyne measurement of the observable $\hat{X}$ to distinguish different phase shifts in the coherent state.

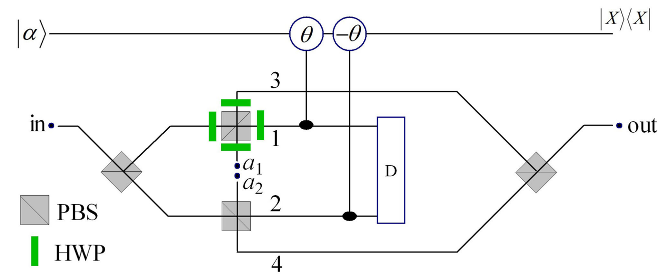

Fig. 2. The schematic diagram of amplification of single-photon with polarization feature. PBS is polarization beam splitter to guide photons with different polarizations into different paths 1-4. θ and $-\theta$ are phase shifts in the coherent state induced by interaction between coherent state and photons in paths 1 and 2. HWP denotes a half-wave plate to implement the conversion $|H\rangle \leftrightarrow |V\rangle$. D represents the measurement of two photons according to the results of homodyne measurement.

information encoded in the photon polarization. Li *et al.* proposed an entanglement-assisted amplification protocol for two-photon polarization–time-bin hyperentanglement [30]. Their protocol could be extended to the N-photon hyperentanglement and realized under current experimental condition. However, these proposals cannot eliminate the vacuum state.

Recently, Zhou *et al.* proposed an entanglement-based linear amplification protocol to solve the photon loss problem [11]. This proposal could completely eliminate the vacuum state. Nevertheless, the bunching effect of photons reduces the total success probability of the amplification proposal and the fidelity of the polarization feature. Cross-Kerr nonlinearity is a feasible approach to realize the quantum nondemolition detection (QND), which can avoid the bunching effect and improve the success probability and efficiency of quantum communication and quantum computation [31]–[36]. The recyclable amplification protocol to protect the single-photon entangled state based on QND has been reported [37]. By repeating the protocol, the success probability and the fidelity could be increased. What's more, this protocol was also useful under high photon loss conditions. However this amplification is not applicable to the polarization feature.

In this paper, QND based on cross-Kerr nonlinearity is introduced to improve the entanglement-based amplification of single-photon with the polarization feature, by converting the bunching items to successful amplification. We firstly introduce the QND with cross-Kerr nonlinearity, and then give the main proposal of the single-photon amplification and evaluate its performance. Finally, discussions and a conclusion are given.

II. QND With Cross-Kerr Nonlinearity

In order to improve the success probability of the single photon amplification, the QND is necessary. So we introduce the QND based on cross-Kerr nonlinearity at first. The cross-Kerr nonlinearity can be described with the Hamiltonian [31]:

$$\hat{H} = \hbar\chi\hat{n}_a\hat{n}_c \tag{1}$$

where $\hbar\chi$ denotes the coupling strength of the nonlinearity, and it is depends on the nonlinear material. $\hat{n}_a$ and $\hat{n}_c$ denote the photon number operators for modes a and c. Suppose that the signal mode a is in a Fock state $|n_a\rangle$, which contain n_a photons, and the probe mode c is in a coherent state $|\alpha\rangle$ with amplitude α. The principle of QND with cross-Kerr nonlinearity can be seen in Fig. 1.

Cross-Kerr nonlinearity will lead the two modes to evolve as follows:

$$|n_a\rangle\,|\alpha\rangle \underset{\rightarrow}{\hat{\mathrm{H}}} |n_a\rangle\,\left|\alpha e^{in_a\theta}\right\rangle \tag{2}$$

where $\theta = \chi t$ and t is the interaction time. It can be found that the phase shift of the coherent state is directly proportional to the photon number in signal mode. If we choose the local oscillator prepared in a large-amplitude coherent state with the same phase as $|\alpha\rangle$ to offset the probe phase, the homodyne measurement can be seen as a measurement of the observable $\hat{X}=\hat{c}+\hat{c}^\dagger$, where $\hat{c}(\hat{c}^\dagger)$ is annihilation (creation) operator for the photons in probe mode. The expectation value of $\hat{X}$ is $\langle\hat{X}\rangle = 2\alpha\cos(n_a\theta)$. So, with the measurement of $\hat{X}$, the phase shift of the coherent state and the photon number in signal mode can be distinguished without destroying the photons in signal mode.

III. Single-Photon Amplification Protocol With Polarization Feature

We suppose that single photon is in an arbitrary polarization state $|\psi\rangle_{in} = \gamma|1\rangle_H + \delta|1\rangle_V$ with $|\gamma|^2 + |\delta|^2 = 1$. For simplicity, we use $|H\rangle_{in}$ and $|V\rangle_{in}$to represent $|1\rangle_H$and $|1\rangle_V$, respectively, and the single state will be:

$$|\psi\rangle_{in} = \gamma|H\rangle_{in} + \delta|V\rangle_{in} \tag{3}$$

After the transmission through the noisy quantum channel with photon loss, the photonic qubit will be converted to a mixed state:

$$\rho = \eta|\psi\rangle_{in}\langle\psi| + (1-\eta)\,|vac\rangle\,\langle vac| \tag{4}$$

It means that the single-photon may be lost with the probability of $1-\eta$, and leads to the vacuum state $|vac\rangle$. The principle of our single-photon amplification protocol is shown in Fig. 2.

To achieve the amplification of single-photon, we introduce a pair of ancillary polarized photons in state :

$$|\psi\rangle_{a_1a_2} = \left(|HH\rangle_{a_1a_2} + |VV\rangle_{a_1a_2}\right)/\sqrt{2} \tag{5}$$

So the state of the three-photon system is $|\psi\rangle_{in} \otimes |\psi\rangle_{a_1a_2}$ with the probability of η or $|vac\rangle_{in} \otimes |\psi\rangle_{a_1a_2}$ with the probability of $1-\eta$. In the first case, the state of the three-photon system will evolve as follows, as a result of PBSs and HWPs:

$$|\psi\rangle_{in} \otimes |\psi\rangle_{a_1a_2} = (\gamma|H\rangle_{in} + \delta|V\rangle_{in}) \otimes \frac{|HH\rangle_{a_1a_2} + |VV\rangle_{a_1a_2}}{\sqrt{2}}$$
$$\to \frac{1}{\sqrt{2}}(\gamma|HHH\rangle_{124} + \gamma|HVV\rangle_{223} + \delta|HVH\rangle_{114} + \delta|VVV\rangle_{123}) \tag{6}$$

Then the three-photon system interacts with coherent state $|\alpha\rangle$ by cross-Kerr medium and the total state will be:

$$|\psi\rangle_f = \frac{1}{\sqrt{2}}(\gamma|HHH\rangle_{124}|\alpha\rangle + \gamma|HVV\rangle_{223}|\alpha e^{-i2\theta}\rangle + \delta|HVH\rangle_{114}|\alpha e^{i2\theta}\rangle + \delta|VVV\rangle_{123}|\alpha\rangle) \tag{7}$$

Now we perform an X homodyne measurement to distinguish different phase shifts of the coherent state. With the value x of the X homodyne measurement obtained, the signal photons will be projected into:

$$\begin{aligned}|\psi_x\rangle_f &= \langle x|\psi\rangle_f \\ &= \frac{1}{\sqrt{2}}[f(x,\alpha)(\gamma|HHH\rangle_{124} + \delta|VVV\rangle_{123}) \\ &\quad + f(x,\alpha\cos 2\theta)(e^{-i\phi(x)}\gamma|HVV\rangle_{223} \\ &\quad + e^{i\phi(x)}\delta|HVH\rangle_{114})]\end{aligned} \tag{8}$$

where

$$f(x,\varepsilon) = \frac{1}{\sqrt[4]{2\pi}}\exp[-\frac{1}{4}(x-2\varepsilon)^2] \tag{9}$$

is the Gaussian probability amplitudes of the outputs with $\varepsilon \in (\alpha, \alpha\cos\theta, \alpha\cos 2\theta)$, and

$$\phi(x) = \alpha\sin 2\theta(x - 2\alpha\cos 2\theta) \bmod 2\pi \tag{10}$$

is the phase factor according to the value of the X homodyne measurement. Obviously, the phase shifts $2\theta$and -2θ cannot be distinguished. If the Gaussian curves with the peaks located at $\alpha$and phase shift 0 of the coherent state are obtained with probability of $\eta/2$, the three-photon system will be in state:

$$|\varphi_0\rangle = \gamma|HHH\rangle_{124} + \delta|VVV\rangle_{123} \tag{11}$$

After passing though the measurement setup shown in Fig. 3, we can get:

$$\begin{aligned}|\varphi_0'\rangle &= \frac{1}{2}[(|HH\rangle_{12} + |VV\rangle_{12})(\gamma|H\rangle_{out} + \delta|V\rangle_{out}) \\ &\quad + (|HV\rangle_{12} + |VH\rangle_{12})(\gamma|H\rangle_{out} - \delta|V\rangle_{out})] \\ &= \frac{1}{2}[(|D_2D_3\rangle + |D_1D_4\rangle)(\gamma|H\rangle_{out} + \delta|V\rangle_{out}) \\ &\quad + (|D_2D_4\rangle + |D_1D_3\rangle)(\gamma|H\rangle_{out} - \delta|V\rangle_{out})]\end{aligned} \tag{12}$$

If the single photon detectors D_2 and D_3 or D_1 and D_4 click, the out state will be $|\varphi\rangle_{out} = \gamma|H\rangle_{out} + \delta|V\rangle_{out}$, which is equivalent to $|\psi\rangle_{in}$. If D_2 and D_4 or D_1 and D_3 click, the out state will be $\gamma|H\rangle_{out} - \delta|V\rangle_{out}$, which can be converted to $|\varphi\rangle_{out}$ by a single-photon operation.

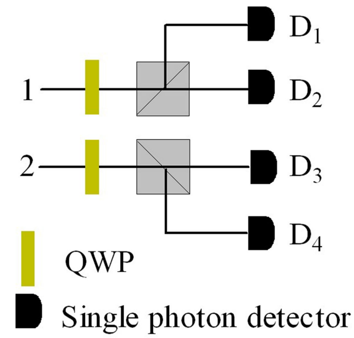

Fig. 3. The schematic diagram of the measurement of two photons for phase shift 0 of the coherent state in X homodyne measurement. QWP denotes a quarter-wave plate to implement $\{|H\rangle \leftrightarrow (|H\rangle + |V\rangle)/\sqrt{2}, |V\rangle \leftrightarrow (|H\rangle - |V\rangle)/\sqrt{2}\}$.

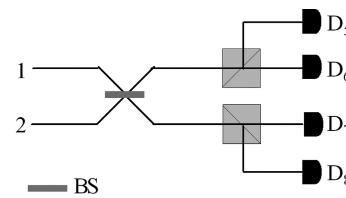

Fig. 4. The schematic diagram of the measurement of two photons for phase shift $\pm 2\theta$ of the coherent state in X homodyne measurement. BS is a 50:50 beam splitter.

If the Gaussian curves with the peaks located at $\alpha\cos(2\theta)$ and phase shift $\pm 2\theta$ of the coherent state are obtained with probability of $\eta/2$, the three-photon system will be in state:

$$|\varphi_{2\theta}\rangle = e^{-i\phi(x)}\gamma|HVV\rangle_{223} + e^{i\phi(x)}\delta|HVH\rangle_{114} \tag{13}$$

Before the single-photon detection, a phase shift should be implemented on photons in path 2 to eliminate the overall phase in equation (13).

After passing though the measurement setup shown in Fig. 4, the state can be described as:

$$\begin{aligned}|\varphi_{2\theta}'\rangle &= \frac{1}{2}[(|HV\rangle_{11} + |HV\rangle_{22})(\gamma|V\rangle_{out} + \delta|H\rangle_{out}) \\ &\quad - (|HV\rangle_{12} + |VH\rangle_{12})(\gamma|V\rangle_{out} - \delta|H\rangle_{out})] \\ &= \frac{1}{2}[(|D_5D_6\rangle + |D_7D_8\rangle)(\gamma|V\rangle_{out} + \delta|H\rangle_{out}) \\ &\quad - (|D_6D_8\rangle + |D_5D_7\rangle)(\gamma|V\rangle_{out} - \delta|H\rangle_{out})]\end{aligned} \tag{14}$$

If the single photon detectors D_5 and D_6 or D_7 and D_8 click, the out state will be $\gamma|V\rangle_{out} + \delta|H\rangle_{out}$. If D_6 and D_8 or D_5 and D_7 click, the out state will be $\gamma|V\rangle_{out} - \delta|H\rangle_{out}$. Both of them can be converted to $|\varphi\rangle_{out}$ by a single-photon operation.

On the other hand, if the photon loss has occurred with the probability of $1-\eta$, the initial state injected into the amplifier is $|vac\rangle_{in}$. Owing to PBSs and HWPs, the state of combined system will evolve as :

$$|vac\rangle_{in} \otimes |\psi\rangle_{a_1a_2} = |vac\rangle_{in} \otimes \frac{|HH\rangle_{a_1a_2} + |VV\rangle_{a_1a_2}}{\sqrt{2}}$$
$$\to |vac\rangle_{in} \otimes \frac{|HH\rangle_{14} + |VV\rangle_{23}}{\sqrt{2}} \tag{15}$$

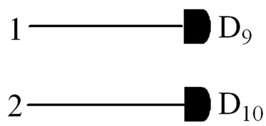

Fig. 5. The schematic diagram of the measurement of two photons for phase shift $\pm\theta$ of the coherent state in *X* homodyne measurement.

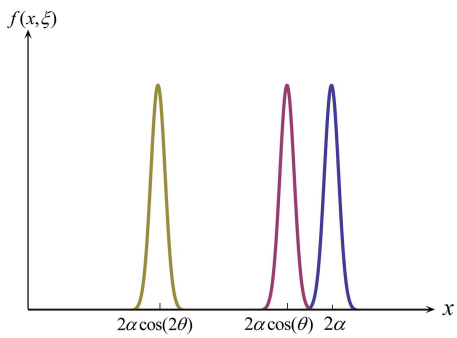

Fig. 6. The Gaussian probability distribution for the result of the *X* homodyne measurement.

Assisted by cross-Kerr medium, the total state will be:

$$|\psi'\rangle_f = \frac{1}{\sqrt{2}}(|HH\rangle_{14}\,|\alpha e^{i\theta}\rangle + |VV\rangle_{23}\,|\alpha e^{-i\theta}\rangle) \tag{16}$$

With the *X* homodyne measurement, the total state will be projected into:

$$\begin{aligned}|\psi'_x\rangle_f &= \langle x\,|\psi'\rangle_f \\ &= \frac{f(x, \alpha\cos\theta)(e^{i\phi'(x)}|HH\rangle_{14} + e^{-i\phi'(x)}|VV\rangle_{23})}{\sqrt{2}}\end{aligned} \tag{17}$$

where $\phi'(x) = \alpha\sin\theta(x - 2\alpha\cos\theta) \bmod 2\pi$. In this case, only phase shift $\pm\theta$ of the coherent state can be obtained. So, the photons state is:

$$|\varphi_\theta\rangle = (e^{i\phi'(x)}|HH\rangle_{14} + e^{-i\phi'(x)}|VV\rangle_{23})/\sqrt{2} \tag{18}$$

Before the single-photon detection, a phase shift should be implemented on photons in path 2 to eliminate the overall phase:

$$|\varphi'_\theta\rangle = \frac{|HH\rangle_{14} + |VV\rangle_{23}}{\sqrt{2}} = \frac{|D_9\rangle\,|H\rangle_4 + |D_{10}\rangle\,|V\rangle_3}{\sqrt{2}} \tag{19}$$

After passing though the measurement setup shown in Fig. 5, if the single photon detector D_9 clicks, the out state will be $|H\rangle_{out}$. If D_{10} clicks, the out state will be $|V\rangle_{out}$. So the total out state of the amplifier is:

$$\rho_{out} = \eta|\varphi\rangle_{out}\,\langle\varphi| + \frac{1}{2}(1-\eta)(|H\rangle_{out}\,\langle H| + |V\rangle_{out}\,\langle V|) \tag{20}$$

The above discussion is based on the ideal conditions. In the experiment, there are small overlaps between the Gaussian curves, which amounts to the error probability $P_{error} = erfc(x_d/2\sqrt{2})/2$, where $x_d$is the distance between two peaks of the Gaussian curves, as shown in Fig. 6. The error probability between Gaussian curves with the peaks located at $2\alpha\cos(2\theta)$ and 2α can be obtained by:

$$P_{error} = \frac{1}{2}erfc[\alpha(1-\cos 2\theta)/\sqrt{2}] \tag{21}$$

For the regime of weak cross-Kerr nonlinearities, we have $\theta << \pi$, so:

$$P_{error} \to \frac{1}{2}erfc[\sqrt{2}\alpha\theta^2] \tag{22}$$

Then, the total out state of the amplifier should be:

$$\begin{aligned}\rho^{\dagger}_{\text{out}} = \frac{1}{P}[&\eta(1-P_{error})|\varphi\rangle_{out}\,\langle\varphi| \\ &+ \frac{1}{2}(1-\eta)(|H\rangle_{out}\,\langle H| + |V\rangle_{out}\,\langle V|)]\end{aligned} \tag{23}$$

where $P = \eta(1-P_{error}) + 1 - \eta = 1 - \eta P_{error}$ is the success probability of the amplification.

If the amplification gain *G* is defined as the fraction between signal probability $P_s$and vacuum probabilityP_{vac}:

$$G = \frac{P_s}{P_{vac}} \tag{24}$$

Our scheme can get infinite gain in ideal condition, which indicates that the vacuum state can be nearly eliminated with ignored P_{error} induced by the loss and spontaneous noises. We estimate the ability of maintaining polarization feature by the fidelity of $\rho^{\dagger}_{\text{out}}$ compared with $|\psi\rangle_{in}$:

$$\begin{aligned}F &= \langle\psi|_{in}\rho^{\dagger}_{\text{out}}|\psi\rangle_{in} \\ &= \frac{1}{P}[\eta(1-P_{error}) + \frac{1}{2}(1-\eta)(|\gamma|^2 + |\delta|^2)] \\ &= \frac{1}{P}[\eta(1-P_{error}) + \frac{1}{2}(1-\eta)]\end{aligned} \tag{25}$$

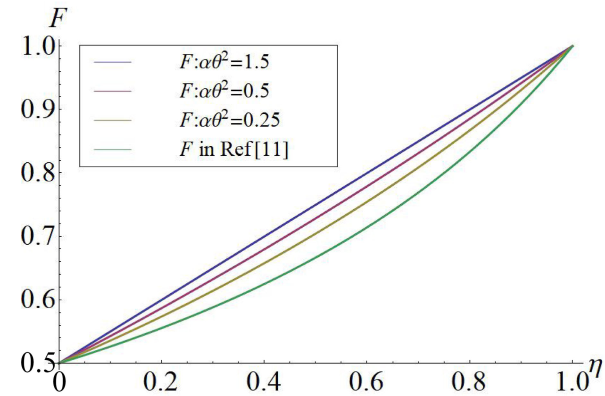

Fig. 7. The schematic diagram of the fidelities of the polarization feature vs η.

Fig. 7 shows the fidelities of the polarization feature in our amplification proposal and ref [11]. It can be found that the fidelities of our amplification are always higher than that in ref [11], and increase with η. Moreover, the error probability of the homodyne measurement reduces the fidelities, that is,

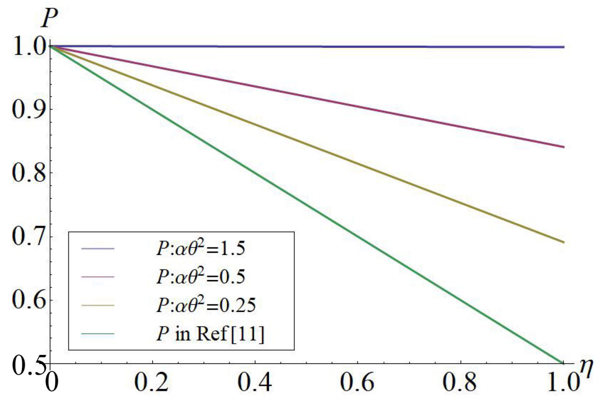

Fig. 8. The total success probabilities of the amplification vs η.

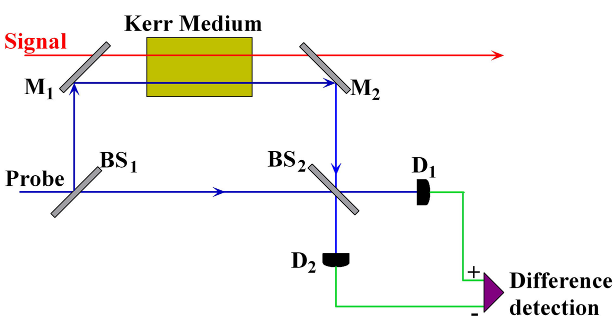

Fig. 9. Schematic of QND based on cross-Kerr effect. BS_1 and BS_2 are 50:50 beam splitters, and M_1 and M_2 are mirrors with unit reflectivity for the probe field and unit transmissivity for the signal light. Signal and a part of probe waves are introduced into the Kerr medium, and the change in the phase of the probe field is detected by balanced homodyne detection using two photon detectors D_1 and D_2.

parameter $\alpha\theta^2$ limits the further improvement of fidelities. When $\alpha\theta^2 > 1.5 (P_{error} < 1.4 \times 10^{-3})$, the error probability can be ignored, and F is optimal in our proposal. And the total success probability of the amplification in our proposal is higher than that in ref [11], as shown in Fig. 8.

IV. Conclusion

Compared with ref [19], [22], [23], [25], [29], our proposal can nearly eliminate the vacuum state in theory, with infinite gain obtained. Ref [11] can also get infinite gain in linear amplification of single-photon, but the fidelity of the polarization feature and the success probability are lower because of the bunching effect. With the QND, our proposal can be achieved with higher success probability, and the fidelity of the polarization feature is more higher than that in ref [11]. Different setups for different states in our proposal are used to improve the polarization fidelity and success probability of the amplification, which is similar to the post-selection. And the fidelity and success probability can also be considerable with only one setup.

In experiment, QND based on cross-Kerr effect can be realized by balanced homodyne detection method, as shown in Fig. 9. The probe light is divided into two beams by BS_1. One beam is transmitted to BS_2, and the other incidents into Kerr medium, and interacts with the signal wave transmitted by M_1 to introduce a phase shift in probe light. Then, the signal export from M_2, and the probe with phase shift is reflected to BS_2. Two beams of probe interfere in BS_2. The interfered waves are converted into electrical signals by D_1 and D_2, and the difference detection is used to check the generated photocurrents which are proportional to the number of incident photons. The phase shift between two beams of probe can be obtained by the results of difference detection, and the number of signal photons is further inferred without destroying the signal photons. The Kerr medium used in the experiment can be nonlinear crystals, such as neodymium glass, KDP crystal, and so on. And there are some methods to make nonlinearities of magnitude 10^{-2}, for example with Electromagnetically Induced Transparencies [38], whispering-gallery micro-resonators [39], and optical fiber systems [40]. Cross-Kerr nonlinearity can be further enhanced in principle with a single two-level atom (with the nonlinear phase shift of π) [41], or atomic gas [42], and atom-assisted optomechanical system [43] as medium with controlled nonlinear coefficient. The nonlinear phase shift required in our proposal can be reduced by increasing the amplitude of the probe beam, because F will be optimal with $\alpha\theta^2 > 1.5$ in our proposal. This makes our proposal more available experimentally in the current technologies with small-but-not-tiny Kerr nonlinearities (magnitude 10^{-2}).

In conclusion, the entanglement-based single-photon amplification protocol is proposed, which can nearly eliminate the vacuum state of the single-photon against photon loss. And the fidelity of the polarization feature and the total success probability of the amplification are higher than the previous schemes. These advantages result from avoiding the bunching effect with the QND based on cross-Kerr nonlinearity. Benefiting from the progress of cross-Kerr nonlinearity made in recent years, our proposal can be realized in experiment and extended to practical application in quantum key distribution and other quantum communication schemes in the near future.

References

[1] W. C. Gao, S. C. Mi, C. Cao, X. Liu, T.-J. Wang, and C. Wang, “Faithful transmission of single-photon qubits using error-rejection coding,” *IEEE Photon. J.*, vol. 11, no. 1, Feb. 2019, Art. no. 7600107.
[2] J. P. Chen *et al.*, “Sending-or-not-sending with independent lasers: Secure twin-field quantum key distribution over 509 km,” *Phys. Rev. Lett.*, vol. 124, no. 7, Feb. 2020, Art. no. 070501.
[3] X. Y. Zhou, C. M Zhang, G. C. Guo, and Q. Wang, “Improved decoy-state measurement-device-independent quantum key distribution with imperfect source encoding,” *IEEE Photon. J.*, vol. 11, no. 3, Jun. 2019, Art. no. 7600207.
[4] X. Q. Zhong *et al.*, “Proof-of-principle experimental demonstration of twin-field quantum key distribution over optical channels with asymmetric losses,” *NPJ Quantum Inf.*, vol. 7, Jan. 2021, Art. no. 8.
[5] D. Wang, X. T. Song, L. J. Zhou, and Y. Zhao, “Real-time phase tracking scheme with mismatched-basis data for phase-coding quantum key distribution,” *IEEE Photon. J.*, vol. 12, no. 3, Jun. 2020, Art. no. 7600307.
[6] C. Cai, Y. M. Sun, and Y. F. Ji, “Simultaneous long-distance transmission of discrete-variable quantum key distribution and classical optical communication,” *IEEE Trans. Commun.*, vol. 69, no. 5, pp. 3222–3234, May 2021.
[7] M. Avesani *et al.*, “Resource-effective quantum key distribution: A field-trial in Padua city center,” *Opt. Lett.*, vol. 46, no. 12, pp. 2848–2851, Jun. 2021.

[8] R. Y Qi *et al.*, "Implementation and security analysis of practical quantum secure direct communication," *Light: Sci. Appl.*, vol. 8, no. 1, Feb. 2019, Art. no. 22.
[9] S. C. Mi, T. J. Wang, G. S. Jin, and C. Wang, "High-capacity quantum secure direct communication with orbital angular momentum of photons," *IEEE Photon. J.*, vol. 7, no. 5, Oct. 2015, Art. no. 7600108.
[10] G. L. Long and H. R. Zhang, "Drastic increase of channel capacity in quantum secure direct communication using masking," *Sci. Bull.*, vol. 66, no. 13, pp. 1267–1269, Jul. 2021.
[11] L. Zhou, Y. B Sheng, and G. L. Long, "Device-independent quantum secure direct communication against collective attacks," *Sci. Bull.*, vol. 65, no. 1, pp. 12–20, Jan. 2020.
[12] B. B. Cai, G. D. Guo, S. Lin, H. Zuo, and C. Yu, "Multipartite quantum key agreement over collective noise channels," *IEEE Photon. J.*, vol. 10, no. 1, Feb. 2018, Art. no. 7600211.
[13] Y. B. Sheng and L. Zhou, "Distributed secure quantum machine learning," *Sci. Bull.*, vol. 62, no. 14, pp. 1025–1029, Jul. 2017.
[14] J. Bang, S. Lee, and H. Jeong, "Protocol for secure quantum machine learning at a distant place," *Quantum Inf. Process.*, vol. 14, no. 10, pp. 3933–3947, Aug. 2015.
[15] G. Y. Xiang *et al.*, "Heralded noiseless linear amplification and distillation of entanglement," *Nature Photon.*, vol. 4, no. 5, pp. 316–319, Mar. 2010.
[16] N. Gisin, S. Pironio, and N. Sangouard, "Proposal for implementing device-independent quantum key distribution based on a heralded qubit amplifier," *Phys. Rev. Lett.*, vol. 150, no. 7, Aug. 2010, Art. no. 070501.
[17] J. Minář, H. De Riedmatten, and N. Sangouard, "Quantum repeaters based on heralded qubit amplifiers," *Phys. Rev. A*, vol. 85, no. 3, Mar. 2012, Art. no. 032313.
[18] F. Monteiro *et al.*, "Heralded amplification of path entangled quantum states," *Quantum Sci. Technol.*, vol. 2, no. 2, 2017, Art. no. 024008.
[19] S. Kocsis *et al.*, "Heralded noiseless amplification of a photon polarization qubit," *Nature Phys.*, vol. 9, no. 1, pp. 23–28, Nov. 2012.
[20] K. Bartkiewicz *et al.*, "Efficient amplification of photonic qubits by optimal quantum cloning," *Phys. Rev. A*, vol. 89, no. 6, Jun. 2014, Art. no. 062322.
[21] N. Bruno *et al.*, "Heralded amplification of photonic qubits," *Opt. Exp.*, vol. 24, no. 1, pp. 125–133, Jan. 2016.
[22] D. D. Wang *et al.*, "Heralded noiseless amplification for single-photon entangled state with polarization feature," *Quantum Inf. Process.*, vol. 17, no. 3, Jan. 2018, Art. no. 56.
[23] Y. Y. Jin *et al.*, "Heralded amplification of single-photon entanglement with polarization feature," *Front. Phys.*, vol. 13, no. 5, Sep. 2018, Art. no. 130321.
[24] L. Q. Chen, Y. B. Sheng, and L. Zhou, "Noiseless linear amplification for the single-photon entanglement of arbitrary polarization–time-bin qudit," *Chin. Phys. B*, vol. 28, no. 1, Jan. 2019, Art. no. 010302.
[25] E. Meyer-Scott *et al.*, "Entanglement-based linear-optical qubit amplifier," *Phys. Rev. A*, vol. 88, no. 1, Jul. 2013, Art. no. 012327.
[26] K. Bartkiewicz, A. Cernoch, and K. Lemr, "State-dependent linear-optical qubit amplifier," *Phys. Rev. A*, vol. 88, no. 6, Dec. 2013, Art. no. 062304.
[27] T. J Wang, C. Cao, and C. Wang, "Linear-optical implementation of hyperdistillation from photon loss," *Phys. Rev. A*, vol. 89, no. 5, May 2014, Art. no. 052303.
[28] T. J Wang and C. Wang, "High-efficient entanglement distillation from photon loss and decoherence," *Opt. Exp.*, vol. 23, no. 24, pp. 31550–31563, Nov. 2015.
[29] Z. F. Feng *et al.*, "Entanglement assisted single-photon w state amplification," *Opt. Commun.*, vol. 340, pp. 80–85, Apr. 2015.
[30] Y. P. Li *et al.*, "Entanglement-assisted noiseless linear amplification for arbitrary two-photon polarization–time-bin hyperentanglement," *Quantum Inf. Process.*, vol. 19, no. 8, Jul. 2020, Art. no. 261.
[31] K. Nemoto and W. J. Munro, "Nearly deterministic linear optical controlled-NOT gate," *Phys. Rev. Lett.*, vol. 95, no. 25, Dec. 2004, Art. no. 250502.
[32] Q. Guo *et al.*, "Simplified optical quantum-information processing via weak cross-Kerr nonlinearities," *Phys. Rev. A*, vol. 83, no. 5, May 2011, Art. no. 054303.
[33] L. Poti, E. Lazzeri, G. Meloni, A. Bogoni, and G. Prati, "All-optical processing by means of cross-phase-modulation-based PM-NOLM interconnected structures," *IEEE J. Sel. Topics Quantum Electron.*, vol. 14, no. 3, pp. 580–586, May/Jun. 2008.
[34] X. Q. Zhang *et al.*, "Greenberger-Horne-Zeilinger states-based blind quantum computation with entanglement concentration," *Sci. Rep.*, vol. 7, no. 1, Sep. 2017, Art. no. 11104.
[35] T. T Song, X. Tan, and T. Wang, "Entanglement concentration for arbitrary four-particle linear cluster states," *Sci. Rep.*, vol. 7, no. 1, May 2017, Art. no. 1982.
[36] F. Wang *et al.*, "Photonic quantum network transmission assisted by the weak cross-Kerr nonlinearity," *Sci. China Phys. Mech.*, vol. 61, no. 6, pp. 12–27, Mar. 2018.
[37] L. Zhou and Y. B Sheng, "Recyclable amplification protocol for the single-photon entangled state," *Laser Phys. Lett.*, vol. 12, no. 4, Mar. 2015, Art. no. 045203.
[38] D. A. Braje, V. Balic, G. Y. Yin, and S. E. Harris, "Low-light-level nonlinear optics with slow light," *Phys. Rev. A*, vol. 68, no. 4, pp. 4343–4349, Oct. 2003.
[39] T. J. Kippenberg, S. M. Spillane, and K. J. Vahala, "Kerr-nonlinearity optical parametric oscillation in an ultrahigh-Q toroid microcavity," *Phys. Rev. Lett.*, vol. 93, no. 8, Aug. 2004, Art. no. 083904.
[40] X. Li, P. L. Voss, J. E. Sharping, and P. Kumar, "Optical-fiber source of polarization-entangled photons in the 1550 nm telecom band," *Phys. Rev. Lett.*, vol. 94, no. 5, Feb. 2005, Art. no. 053601.
[41] H. F. Hofmann *et al.*, "Optimized phase switching using a single-atom nonlinearity," *J. Opt. B*, vol. 5, no. 3, pp. 218–221, 2003.
[42] Y. Mu *et al.*, "Giant Kerr nonlinearities and magneto-optical rotations in a Rydberg-atom gas via double electromagnetically induced transparency," *Phys. Rev. A*, vol. 103, no. 4, Apr. 2021, Art. no. 225501.
[43] Y. Y. Dong *et al.*, "Fluctuation-enhanced Kerr nonlinearity in an atom-assisted optomechanical system with atom-cavity interactions," *Opt. Exp.*, vol. 29, no. 4, pp. 5367–5383, Feb. 2021.