Dissipation Assisted Frequency Comb and Chaos Generation in the Optomechanics

Yong-Pan Gao, Xin-Chang Liu, Kai Wang, and Yong Zhang

***Abstract*—Dispersion and dissipation are the key parameters of the optical components. Generally, dissipation is an unfavorable factor. In this article, we designed a dispersion-dissipation hybird coupling optomechanical cavity and achieved dissipation-assisted frequency comb and chaos generation. We studied the bistable state and the spectrum of optomechanical cavity. We found that chaos and frequency comb can be effectively adjusted by adjusting the dissipation coupling. We given out the formation mechanism of the two types of frequency combs of the optomechanical cavity. Our result provides an effective way to achieving low threshold, adjustable frequency comb and chaos in an optical microcavity.**

***Index Terms*—Optomechanics, whispering gallery mode, frequency comb.**

I. Introduction

OPTICAL microcavity [1]–[3] is widely studied for its high quality factor and small mode volume. Recently, the study of microcavity non-linearity including kerr nonlinearity [4]–[10], Raman nonlinearity [11]–[13], optomechanically induced nonlinearity [14]–[21], optomagnonics microcavity [22]–[26] become hot points. Combine the nonlinearity and high quality factor of the optical microcavity, various low threshold nonlinear phenomenons such as chaos [27]–[33] and frequency comb [34]–[37] in the optical microcavity [38], [39] can be observed.

Dispersion [40] and dissipation [41] are important parameters of the cavity in the time domain and frequency domain. The dissipation in the optical microcavity directly determines the quality factor of the optical microcavity. Dispersion is the difference in the response of optical materials or structures to light with different frequency. In order to reveal the nonlinearity, the microcavity usually need to have a high quality factor, i.e. low dissipation. On state-of-the-art, the quality factor of the optical microcavity can reach 10^8 or even higher now [1], [42]–[45]. However, it is a great challenge to continue to improve the quality factor of the optical microcavity. Therefore, it has become a new research task to continue to explore the enhancement of the nonlinear expression of the optical cavity under the existing dissipation strength.

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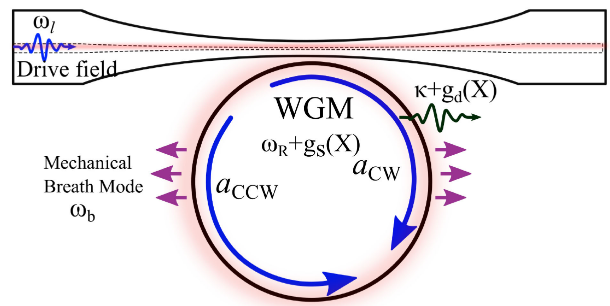

Fig. 1. The dissipation-dispersion hybrid coupling optomechanics in the whispering gallery mode. The perturbations of dispersion and dissipation are very small relative to their inherent values. The WGM cavity is pumped with the drive field with the frequency ω_l through the fiber trapper. The cavity have a mechanical mode with the frequency ω_b. There are both optical $a_{\rm CW}$ and $a_{\rm CCW}$ mode is the WGM. The frequency of the optical modes is $\omega_{\rm R} + g_s(X)$, and its damp is $\kappa + g_d(X)$, the X is the effective mechanical position.

Usually, the dissipation of an optical microcavities is constant. Recently, the emergence of optomechanics has provided a platform for us to study the dispersion and dissipation of dynamics [14], [15], [18]–[21], [46]. In this article, we study the optomechanical nonlinearity of the optical microcavity. In the optomechanical cavity, the mechanical mode and the optical mode are coupled through radiation pressure. Then there will be dynamical dissipation and dispersion in the optical mode. These dynamical dissipation and dispersion is dispersion coupling and dissipation coupling: dispersion coupling corresponding to Kerr nonlinearity; dissipation coupling provides a perturb on intrinsic dissipation of the cavity. We find the joint effect of dispersion coupling and dissipation coupling will bring controllable nonlinear threshold value feature. Our research provides a platform for realizing low-threshold nonlinear optical microcavity.

II. Model and the Hamiltonian

The schematic diagram is shown in Fig. 1. This whispering gallery mode scheme also valid for the Fabry-Perot cavity and photonic crystal cavity [47]–[49]. The optical whispering gallery mode (WGM) $a_{\rm CW}$ and $a_{\rm CCW}$ is pumped by the drive field in the fiber tapper. Because of radiation pressure, the mechanical mode will oscillate. The periodically change of the geometric boundary disturb both the dissipation and dispersion of the optical mode. So, there are two interactions in this process: the dispersion coupling, which will change the eigen-frequency of the cavity; the dissipation coupling which will disturb the spectrum width of the cavity. The hybrid of these two coupling can introduce a remarkable reduce of the threshold of nonlinearity.

The Hamiltonian of optomechanics in a rotating frame with the drive frequency ω_l is:

$$H/\hbar = -\Delta\hat{a}^\dagger\hat{a} + \omega_b\hat{b}^\dagger\hat{b} - g\hat{a}^\dagger\hat{a}(\hat{b}^\dagger + \hat{b}) + H_\mathrm{d}, \tag{1}$$

here $\Delta = \omega_l - \omega_c$ is the drive-cavity(ω_c) frequency detuning. The frequency of the mechanical mode is ω_b. The $\hat{a}$ and $\hat{a}^\dagger$ is the annihilation and creation operator of the optical mode; $\hat{b}$ and $\hat{b}^\dagger$ is the annihilation and creation operator of the mechanical mode. The H_d shows the external drive which can be written as $H_\mathrm{d} = \Omega(\hat{a} + \hat{a}^\dagger)$. Ω is the pumping strength.

We assume the optical dissipation of the cavity is mechanical resonator position related, i.e. $\gamma_\mathrm{opt} = \gamma + g_\gamma(\hat{b}^\dagger + \hat{b})$. The mechanical dissipation is Γ. We take the mechanical frequency ω_b as one unit of the frequency domain, then the time scale is $\tau = \omega_b t$. We set the ratio of the dissipation coupling g_γ to the dispersion coupling is $g_\gamma/g = r$. For simplify, we rescaled the optical and mechanical field as $\alpha = [\omega_b/(2\Omega)]\langle\hat{a}\rangle$ and $\beta = [g/\omega_b]\langle\hat{b}\rangle$. To discuss the system in the real number region we write $o = o_r + io_i$ (o can be α or β). The power $P = 8\Omega^2 g^2/\omega_b^4$ and $\Omega = \sqrt{\kappa P_l/(\hbar\omega_l)}$. Then we have:

$$\dot{\alpha}_r = -\frac{\Delta}{\omega_b}\alpha_i - \frac{\gamma}{2\omega_b}\alpha_r + 2\beta_r\alpha_i + 2r\alpha_r\beta_r, \tag{2a}$$

$$\dot{\alpha}_i = \frac{\Delta}{\omega_b}\alpha_r - \frac{\gamma}{2\omega_b}\alpha_i - 2\beta_r\alpha_r + 2r\alpha_i\beta_r + 1/2, \tag{2b}$$

$$\dot{\beta}_r = \beta_i - \frac{\Gamma}{2\omega_b}\beta_r, \tag{2c}$$

$$\dot{\beta}_i = -\beta_r - \frac{\Gamma}{2\omega_b}\beta_i + \frac{P}{2}(\alpha_r^2 + \alpha_i^2). \tag{2d}$$

The derivation operator $\mathcal{D}$ of above equation is

$$\begin{bmatrix} -\frac{\gamma}{2\omega_b} + 2r\beta_r & -\frac{\Delta}{\omega_b} + 2\beta_r & 2\alpha_i + 2r\alpha_r & 0 \\ \frac{\Delta}{\omega_b} - 2\beta_r & -\frac{\gamma}{2\omega_b} - 2r\beta_r & -2r\alpha_i + 2\alpha_r & 0 \\ 0 & 0 & -\frac{\Gamma}{2\omega_b} & 1 \\ P\alpha_r & P\alpha_i & -1 & -\frac{\Gamma}{2\omega_b} \end{bmatrix}, \tag{3}$$

the features of point $x = (\alpha_r, \alpha_i, \beta_r, \beta_i)$ follows this derivation operator. Based on this operator, we can discussion the stability of the system on some points, especially in the following article, we will discuss the stability of the system when $do/dt = 0$. The first order perturbation at certain point is

$$\dot{\mathbf{x}} = \mathcal{D}\cdot\mathbf{x} + \mathcal{O}(\delta\mathbf{x}) \tag{4}$$

When we studied the steady state with $dx/dt = 0$, we can get the relationship between the mechanical resonator and optical mode.

$$\beta_r = \frac{4\omega_b^2}{4\omega_b^2 - \Gamma^2}\frac{P}{2}(\alpha_r^2 + \alpha_i^2) \approx \frac{P}{2}(\alpha_r^2 + \alpha_i^2). \tag{5}$$

Because mechanical frequency is much smaller than the optical frequency, the position of the mechanical resonator can be taken as the steady state for optical oscillation. We eliminate the influence of the mechanical mode, and reduce the dimension of the matrix as:

$$\begin{bmatrix} -\frac{\gamma}{2\omega_b} + 2r\beta_r & -\frac{\Delta}{\omega_b} + 2\beta_r, \\ \frac{\Delta}{\omega_b} + 2\beta_r & -\frac{\gamma}{2\omega_b} - 2r\beta_r \end{bmatrix}. \tag{6}$$

The duration equation is:

$$\begin{vmatrix} -\frac{\gamma}{2\omega_b} + 2r\beta_r - \lambda & -\frac{\Delta}{\omega_b} + 2\beta_r, \\ \frac{\Delta}{\omega_b} + 2\beta_r & -\frac{\gamma}{2\omega_b} - 2r\beta_r - \lambda. \end{vmatrix} = 0 \tag{7}$$

We set the mechanical frequency is 1 unit, i.e. $\omega_b = 1$. The the duration equation can be write as

$$\begin{vmatrix} -\gamma/2 + 2r\beta_r - \lambda & -\Delta + 2\beta_r \\ \Delta + 2\beta_r & -\gamma/2 - 2r\beta_r - \lambda \end{vmatrix} = 0. \tag{8}$$

When $dx/dt = 0$, we neglect the optomechanical coupling, and get $\alpha_r^2 + \alpha_i^2 \approx 1/4$. Then, P determines the system stability. The corresponding quadratic equation is

$$(\gamma/2 + \lambda)^2 - 4(r^2 + 1)\beta_r^2 + \Delta^2 = 0. \tag{9}$$

The root of the equation is

$$\lambda_\pm = -\frac{\gamma}{2} \pm \sqrt{4(r^2 + 1)\beta_r^2 - \Delta^2}. \tag{10}$$

The sign of the real part is determined by the r or the dissipation coupling strength. So, the dissipation coupling can tuning the features of the system.

To the features of the studied system, we studied its bistable condition. The bistable state of the system is determined by the following equation:

$$\begin{aligned} 2P = \beta_r \left(4 + \Gamma^2\right) \big[16\left(1 + r^2\right)\beta_r^2 + \gamma^2 \\ + 4\Delta^2 - 8\beta_r(r\gamma + 2\Delta)\big) \end{aligned} \tag{11}$$

III. The Bistable and Stability of the Hybrid Coupling System

We show the bistable feature in Fig. 2 under different dissipation coupling strength in (a) and quality factor in (b). The dissipation coupling strength is 0.2, 0.1, 0, -0.1, and -0.2 times of the dispersion coupling strength in (a), the dissipation is 1, 0.7, 0.5, 0.1 and 0 times of the mechanical frequency in (b). These figures shows that the bistable be tuned both by the dissipation coupling and cavity quality factor. In these two figures, the black line shows the zero dissipation coupling strength and 0.5 dissipation coupling strength condition. Fig. 2(b) shows that the nonlinearity threshold increase with the dissipation. We can reduce the threshold by improve the cavity quality factor. In Fig. 2(a), the bistable situation change under different dissipation coupling. When the dissipation coupling is 0.2 dispersion coupling strengrh, the threshold is near zero which is the same as the zero dissipation condition under quality factor tuning method in (b). It is impossible to get a zero dissipation cavity in practice. The dissipation coupling scheme is of great value for ultra low threshold nonlinearity.

Bistable analysis can't fully reveal the nonlinear features of the system. We show the bifurcation and stability under first

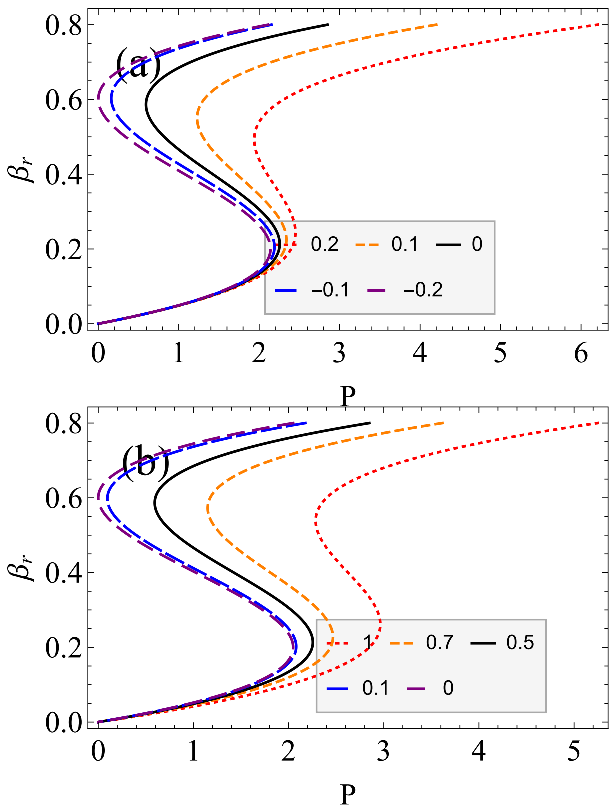

Fig. 2. The stable line when we change the dissipation coupling and directly change the dissipation. (a) The bistable line with different dissipation coupling strength (g_γ) in the unit of dispersion coupling strength. (b) The bistable line with different optical dissipation (γ). The $\Gamma = 0.0012$, $\Delta = 1$, and (a) $\gamma = 0.5$, (b) $g_\gamma = 0$.

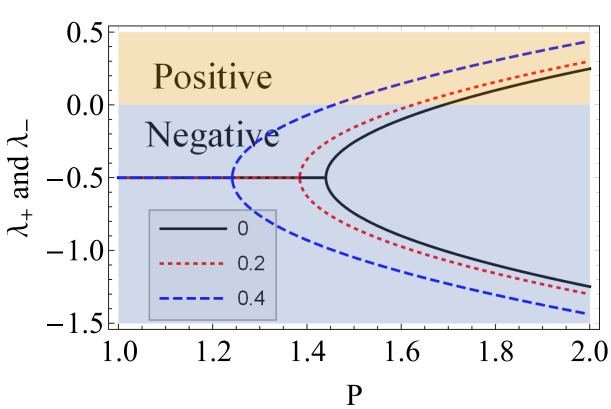

Fig. 3. The eigenvalue bifurcation and negative positive conversion under different drive power. The black, red, and blue line corresponding to the dissipation is 0, 0.2, and 0.4 times the dispersion coupling strength. The dissipation of the optical mode is $\gamma = 1$, the drive detuning is $\Delta = -1.1$.

order disturbance in Fig. 3. The black line shows the zero dissipation coupling conditon, the dissipation coupling strength with 0.2(0.4) dispersion coupling is shown in red(blue) line. We can find that with the increase of dissipation coupling, the bifurcation point reduce. Even more, the positive root which corresponding to the unstable fix point will appear with lower pumping power. The above results all show that the nonlinearity of the system under dissipation coupling has a lower power threshold.

IV. Numerical Simulation of the Hybrid Coupling System

Qualitative analysis shows the properties of the system, but can't give high precision properties of a specific parameter. We need quantitative calculations to study the precise properties.

Frequency comb is an important symbol of optomechanical nonlinearity. To study the frequency comb of our scheme, we numerical studied the (2), and performed the Fourier transform of the time list of the optical intensity($I = |\alpha|^2$). We got the frequency spectrum of the hybrid optomechanical system. The spectrum is shown in Fig. 4. The (a), (b), (c) symbol different dissipation coupling strength. The (1)–(6) symbols different drive power. In Fig. 4(a)(1), the pump power is 0.8, the system is below the nonlinearity threshold, the system is single frequency. When the drive strength is $P = 1.0$, four weak sidebands appear in Fig. 4(a)(2). When the drive strength is increased to $P = 1.2$, the number of sidebands increase to 10 in Fig. 4(a)(3), the amplitude of sidebands also increase. It is because that the system work in the strong nonlinearity region. When we increases the drive strength to 1.4 and 1.6, the numbers of sidebands go on increase and the spacing of the sidebands is reduced in Fig. 4(a)(4,5). However, when the drive strength is 2.2, the features of the sidebands changes in Fig. 4(a)(6), the distance between adjacent comb teeth becomes larger. We call this condition as wide teeth region(WTR)

Fig. 4(a) shows the zero dissipation coupling strength condition. To show the effect of dissipation coupling, we studied the spectrum when dissipation coupling strength is 0.1 dispersion coupling in Fig. 4(b). Then, we can find that the chaos generate threshold value is reduced to P=1.0, while the threshold of WTR reduce to $P = 1.6$. When compare Fig. 4(a)(5) with Fig. 4(b)(5), we can judge that the dissipation coupling changer the nature of frequency comb. There are two stable point when there is not dissipation coupling as shown in Fig. 3 with the pumping power $P = 1.6$. Four-wave mixing generate a new frequency tooth, so the sidebands number increase. However, when dissipation coupling exist, one root become positive as shown in Fig. 3, there is one stable fixed point in the system, the four wave mixing disappear, the WTR phenomenon appear.

Fig. 3 show dissipation coupling can significantly reduce the threshold value of the stages of nonlinearity. The chaos threshold value without hybrid coupling is well above P=1.6. However, the chaos threshold will reduce to P=1.0 hybrid coupling when dissipation coupling strength is only 0.1 times the dispersion coupling. For WTR, the threshold is P = 2.2, P = 1.6, and P = 1.4 for the non-hybrid coupling, dissipation coupling strength is 0.1 dispersion coupling, and dissipation coupling strength is 0.2 dispersion coupling. These results show the dissipation coupling can reduce the threshold without high quality factor.

Studying the properties of the mechanical mode is an important way to explore the nonlinearity of the optomechanical cavity. We show the mechanical oscillation amplitude $1/\sqrt{2}(\beta + \beta^*)$ in Fig. 5, the system have different nonlinearity

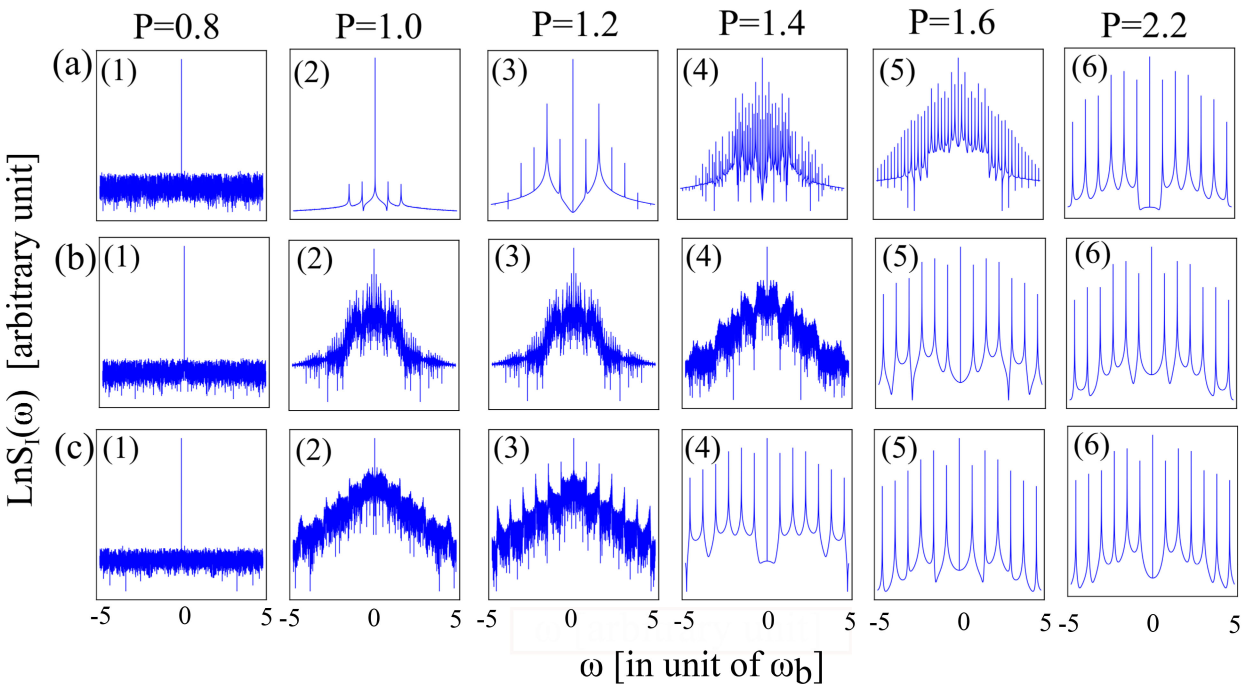

Fig. 4. The spectrum under different drive. (a) Without dissipation coupling assisted. (b) The dissipation coupling strength is 0.1 times of dispersion coupling strength. (c) The dissipation coupling strength is 0.2 times of dispersion coupling strength. The optical dissipation is $\gamma = 1$ in these figures, and the drive detuning is $\Delta = -1$, the mechanical frequency $\omega_b = 1$ and its dissipation is $\Gamma = 1.1 \times 10^{-3}$.

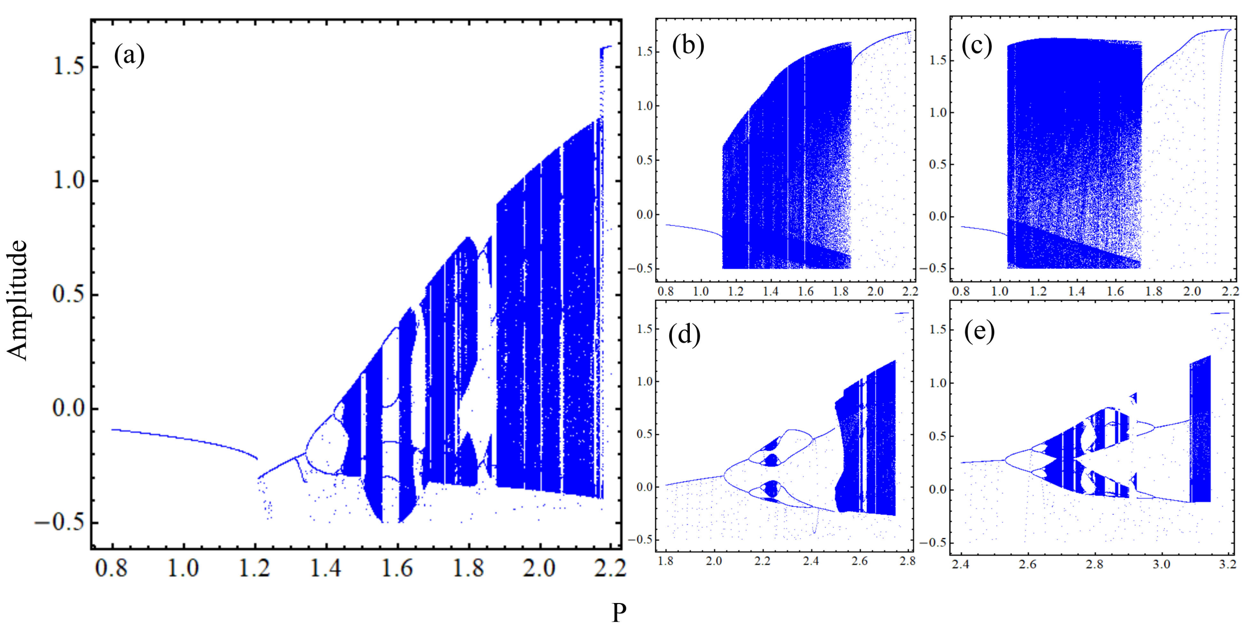

Fig. 5. Bifurcation diagram of the amplitude of the mechanical limit cycle under different dissipation coupling. (a) The dissipation coupling is 0. (b) The dissipation coupling is 0.1 time the dispersion coupling. (c) The dissipation coupling is 0.2 time the dispersion coupling. (d) The dissipation coupling is -0.1 time the dispersion coupling. (e) The dissipation coupling is -0.2 time the dispersion coupling. The optical dissipation is $\gamma = 1$ in these figures, and the drive detuning is $\Delta = -1$, the mechanical frequency $\omega_b = 1$ and its dissipation is $\Gamma = 1.1 \times 10^{-3}$.

region under different dissipation coupling. Even more, the nonlinearity class is also different. The result well meet with the two qualitative method in Fig. 2 and Fig. 3. For comparison, we show the bifurcation with zero dissipation coupling in Fig. 5(a), the dissipation coupling strength is positive 0.1 dispersion coupling in (b) and 0.2 dispersion coupling in (c), and the dissipation coupling strength is negative 0.1 dispersion coupling in (e) and 0.2 dispersion coupling in (f). The bifurcation diagram are completely different in these figures. When the dissipation coupling is zero, the nonlinearity region is range from 1.35 to 2.15. When the dissipation coupling is positive, the thershold value will reduce with the increase of the coupling strength. Even more, the mechanical resonator is chaotic in all nonlinearity region. When the dispersion coupling is negative, the throshold value is increase, the chaotic regions decrease significantly. This indicate the nonlinearity can be modified by the dissipation coupling strength.

V. Feasibility

We choose the whispering gallery mode cavity to demonstrate the feasibility of previous study. The frequency of the mechanical mode is $51.8{*}2\pi$ MHz, the mechanical dissipation is $41{*}2\pi$ kHz. When we set $\gamma = 1/2$ is previous text, the corresponding dissipation is $25.9{*}2\pi$ MHz. The dissipation coupling strength is $100{*}2\pi$ MHz/nm which can be achieved in a whispering gallery cavity with radius 5 μm [50]), while the dissipation coupling strength is $1{*}2\pi$ GHz/nm. Then the effective mechanical mass is 24.16 fg [51]. We will have $g = 5 \times 10^{-5}$. When the drive power is 10 mW, the P will be 2.3 in this paper. There parameters consist with the state-of-the-art.

VI. Summary

Based on the dissipation-dispersion hybrid coupling optomechanics, we studied the threshold reduction without the improving cavity quality factor.We combined the methods of bistable analysis and fixed point analysis to study the generation of frequency comb in optomechanics. We found when the two fixed points are both stable, two adjacent frequency combs will generate a new frequency through four-wave mixing. When there is only one stable point, four-wave mixing disappear, the interval between the frequency teeth is fixed. We believe our scheme will benefit the achievement of low threshold frequency combs and chaos generation and provide a controllable on-chip nonlinearity optical device.

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