Enhancement of Laser Multiplexing Efficiency for Solid State Lighting Through Photon Recycling

Ching-Cherng Sun, Shih-Kang Lin, Mao-Teng He, Chi-Shou Wu, Tsung-Hsun Yang, and Yeh-Wei Yu

***Abstract*—An optical design of a novel structure has been proposed and demonstrated to perform photon recycling in a transmission-type laser-pumping white light. The structure includes a hemisphere reflector and a phosphor plate located at the center of a hemisphere reflector. The hemisphere reflector is not only used for photon recycling to increase the energy efficiency but also to keep the energy efficiency nearly unchanged when the number of the multiplexing laser increases. The experimental measurement shows that the photon recycling with the hemisphere reflector can increase the energy efficiency from 36.2% to 53.3% for on-axis incidence, and 35.2% to 55.7% for off-axis incidence. Besides, the efficiency keeps the same up to three-laser multiplexing in the experiment, and potentially can be extended to more than 10 in the simulation for various hole structures. The result is not only on the efficiency enhancement but also on the enhanced exitance of the laser white light and the unchanged etendue of the light source. This property is an important advantage for lasers rather than LEDs in a solid-state white light source.**

***Index Terms*—Laser multiplexing, photon recycling, white light lighting, phosphor model.**

I. Introduction

Solid-state lighting (SSL) with light-emitting diode (LED) has been increasing its impact on human life by penetrating general lighting, in both indoor and outdoor applications. The advantages of LED SSL include fast response, high efficiency, wide color range, compact size, robust and environmental benefits [1]–[9]. In general, a white-light LED is a semiconductor light source of blue die covering with a yellow phosphor [6], [10], [11]. Owing to Mie scattering in the phosphor layer or volume, both blue and yellow lose their directionality and the performed white light forms a Lambertian-like light source [6], [12], [13], [14]. Even the intensity between the blue light and yellow light is always different, several effective ways have made the white light with higher angular uniformity [15]–[19]. The inherent Lambertian-like emission characteristic makes the etendue [12], [20], [21] of the light source limited to a certain big, and depends on the light source area which almost equals the phosphor area. One of the shortages of the white light LED is the exitance of the light source is not intense enough to support high-intensity projection applied to a video projector, long-range spotlight, or others. Even with a high-quality thermal dissipation design, the low exitance cannot be compensated by adding more LED dies. As a result, a size-limited optical system cannot effectively collect those rays emitted from the outer area based on the principle of Lagrange invariance [13], [20], [21].

In contrast to an LED, a laser diode (LD) is a light source with high directionality [22], [23]. The etendue of an LD is much smaller, but the exitance is similar to that of an LED. If we apply an LD as the blue light source to pump yellow phosphor [24], [25], the white light is similar to that of an LED die but with a possible speckle pattern [26], [27] and larger angular color deviation [27]. One of the important advantages of using an LD is that the etendue of the LD is much smaller than that of a phosphor plate or LED with the same area. It is, therefore, possible to multiplex LDs to form a white light source with enhanced exitance while keeping the etendue determined by the phosphor layers. If we just illuminate the phosphor plate from one side and collect the white light on the other side, the so-called transmission type will waste the blue and yellow lights which are scattered backward. In contrast, an effective way is to illuminate the phosphor layer on a metal mirror so that all the lights can be forced to the same side [24], [25]. This is the reflective type of white light pumped by LD, and it behaves higher efficiency than that in a transmission type. However, the disadvantage of the reflection type is that the output white light is at the same side of the pumping LD so that the mechanical interference between the white light and the pumping light is not avoidable. In this paper, we propose a novel design of white light pumped by LDs in a transmission type. A hemisphere reflector is proposed to recycle all the backward lights and serves as a retroreflector. The proposed structure can enhance the exitance with unchanged etendue, avoid mechanical interference, and perform more freedom in pumping multiplexing.

II. Design Principle

Light scattering from a phosphor volume pumped by LD can be illustrated in Fig. 1, where intense blue light and yellow light are scattered or radiated forward and backward. The yellow light could have more chance to propagate backward since the yellow

Manuscript received August 26, 2021; accepted September 4, 2021. Date of publication September 8, 2021; date of current version September 20, 2021. This work was supported by the Ministry of Science and Technology of Taiwan under Grants MOST 108-2622-E-008-010-CC2 and 109-2622-E-008-014-CC2. *(Corresponding author: Ching-Cherng Sun.)*

Ching-Cherng Sun is with the Department of Optical and Photonics, National Central University, Chung-Li 32054, Taiwan, and also with the Department of Electrophysics, National Yang Ming ChiaoTung University, HsinChu 30010, Taiwan (e-mail: ccsun@dop.ncu.edu.tw).

Shih-Kang Lin, Mao-Teng He, Chi-Shou Wu, Tsung-Hsun Yang, and Yeh-Wei Yu are with the Department of Electrophysics, National Yang Ming ChiaoTung University, HsinChu 30010, Taiwan (e-mail: seven17241@gmail.com; mark810431@gmail.com; deniswu1024@gmail.com; thyang@dop.ncu.edu.tw; yeh.wei.yu@gmail.com).

Digital Object Identifier 10.1109/JPHOT.2021.3110944

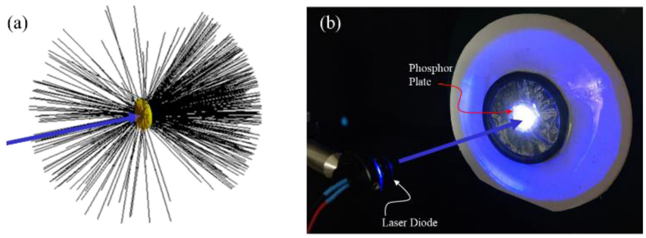


Fig. 1. (a) A schematic diagram for light scattering and radiation forward and backward from a phosphor plate illuminated by a blue laser. (b) The bright spot was observed, which is the strong backward scattering and radiation.

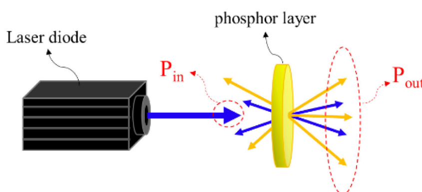


Fig. 2. A schematic diagram for the definition of P_{in} and P_{out}, where P_{in} is the power of incident light by a laser diode and P_{out} is the total output power of the forward light after passing through the phosphor layer.

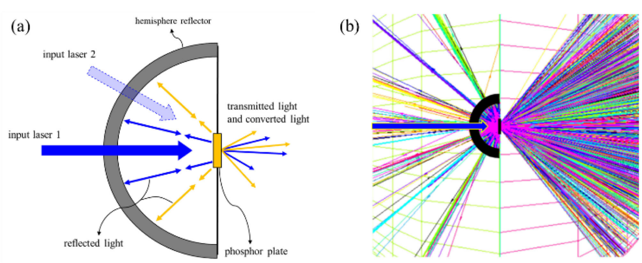


Fig. 3. The structure of the whole system. (a) The hemisphere reflector and the phosphor plate; (b) the ray fans in the simulation.

light is by spontaneous emission so the radiation is isotropic, but the blue light is through Mie scattering. The proposed design is to recruit the backward lights to illuminate the phosphor again. As shown in Fig. 2, the energy efficiency which is defined as the ratio between P_{in} and P_{out} can then be enhanced through photon recycling [28]–[30], where P_{in} is the power of incident blue light and P_{out} is the total output power of the blue and yellow light received after the phosphor layer. As shown in Fig. 3, the recycling is made by using a hemisphere reflector as a retroreflector. The phosphor plate (or layer) must be located at the center of the hemisphere reflector. Then the scattering lights from the phosphor plate are incident on the hemisphere reflector and are reflected reversely to the phosphor plate. The higher the reflectivity of the hemisphere is, the higher the energy efficiency is in the whole system. To enlarge the exitance, there are several holes on the hemisphere reflector so that multiple LDs can illuminate the phosphor plate at the same time and keeps the etendue unchanged.

The optimization of the proposed structure can be done by considering several factors, including the phosphor plate area vs. the radius of the hemisphere reflector, the number and the location of the holes, and the other possible ways to reduce

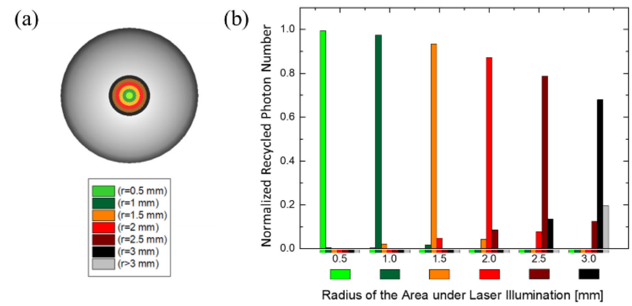


Fig. 4. (a) The cross-section of the hemisphere reflector, where the different ring means the different locations of the phosphor plate. (b) The figure for normalized areal photon number at different area. The horizontal axis corresponds to the locations in (a) for laser illumination, and the vertical axis corresponds to the recycled photon number hitting at the specific region.

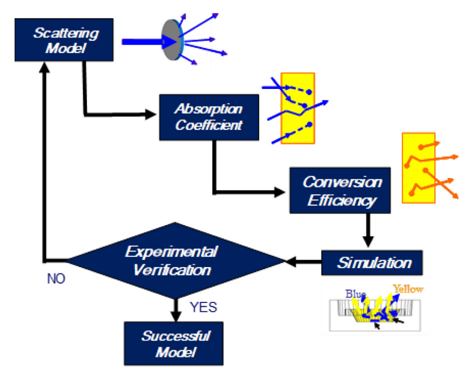


Fig. 5. A flowchart of phosphor model with laser light pumping.

energy loss. The phosphor plate area is a factor not only in etendue but also in the photon recycling effect. A larger phosphor area will cause more recycled light incident on a phosphor location deviated from the center of the hemisphere reflector, and the result is that the etendue is enlarged or the recycling effect is reduced. Fig. 4 shows the simulation of recycling photon number hitting the location of the phosphor plate when the photons are one-time reflected from the hemisphere reflector. When the region on the phosphor for laser incidence is within 0.5mm, more than 90% of recycling photons will hit the central area ($r<5$mm). But when the phosphor plate area increases, the hitting point by the LD could be far away from the central area. Therefore, after several runs of recycling, the reflected photons will be not recyclable. It also means that the larger the phosphor is, the emitting area will be further enlarged or the recycling effect is reduced. From the analysis, the phosphor area needs to be as tightened as possible and to keep small etendue and higher energy efficiency.

The second important optimization is the multiplexing number of the pumping LD. Before the simulation, we have to build up a precise phosphor model with the LD. The phosphor modeling procedure is shown in Fig. 5. The procedure is similar to that for an LED die [31], [32] but easier because the pumping wavelength does not overlap the emission wavelength. However, the difficulty in modeling an LD is the speckle effect, which will cause higher fluctuation of light intensity and obvious error in

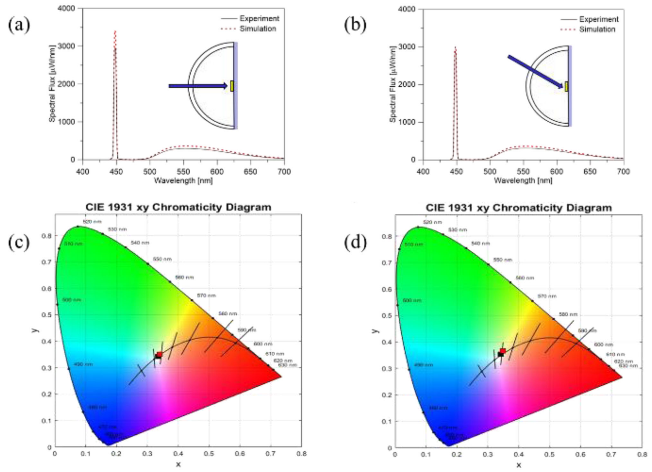


Fig. 6. The simulation of the corresponding measurement of the spectrum when a laser was used to illuminate the phosphor plate with the hemisphere reflector. (a) In the case of on-axis illumination, (b) in the case of off-axis illumination; (c) the color coordinate for simulation and measurement in (a), and (d) the color coordinate for simulation and measurement in (b).

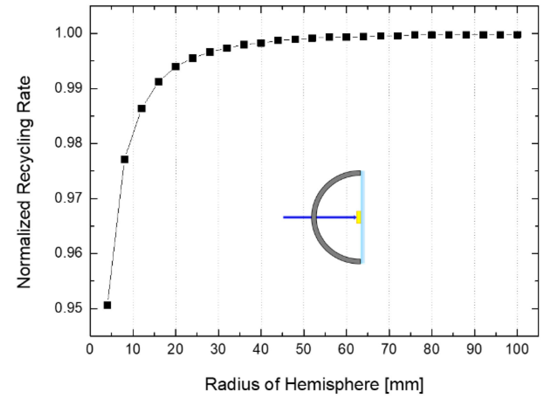


Fig. 7. A simulation of the photon number by recycling as a function of the radius of the hemisphere reflector while the size of the phosphor plate was set 5 mmx5 mm.

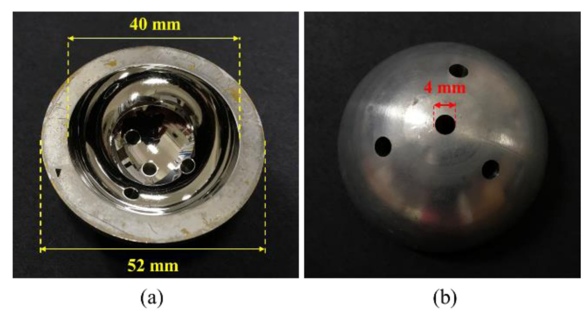


Fig. 8. (a) Front view and (b) back view of the hemisphere reflector.

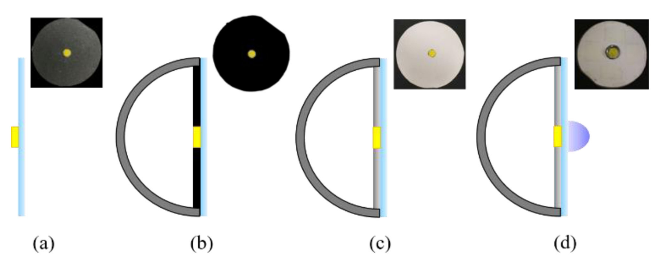


Fig. 9. Four different structures of experimental setup, where the exit faces are different : (a) phosphor is coated on a glass plate. (b) the glass plate is with black coating and a hemisphere reflector is attached, (c) the glass plate is with a reflective film and a hemisphere reflector is attached, (d) a silicone lens is attached on the glass plate shown in (c).

flux measurement. With the degradation of spatial coherence, we can increase the accuracy in an integrating sphere for flux measurement. Fig. 6 shows simulation by the LD-phosphor model and the corresponding measurements. The model is shown accurate enough in optimization and in figuring out the property of the proposed structure. The simulation parameters for the optical system are as follows: the wavelength of the LD is 448 nm; the dimensions of the YAG phosphor plate are $5 \times 5 \times 0.6$ mm, and the concentration of the phosphor was 10%; the radius of the hemisphere reflector was 20 mm; the reflectivity of the reflector was set 80%; the emission flux of the LD was set 50 mW, where the thermal problem was neglected.

Before going to further experiments, we need to optimize the size of the hemisphere reflector. In principle, the larger the radius of the hemisphere reflector is, the more recycled photons will be. However, the phosphor plate size is important to trade off among the exitance, etendue, and thermal problem. The simulation was made by setting fixed the radius of 5 mm for the phosphor plate, and then changed the radius of the hemisphere reflector. The simulation result is shown in Fig. 7, where the recycling rate (the ratio of the one-time recycled photon number on the phosphor plate and the backward scattering photon number) almost reaches the maximum when the radius of the hemisphere reflector is 4 times the phosphor plate size. Therefore, in the experiment, the radius of the phosphor plate was determined 5 mm, while the radius of the hemisphere reflector was 20 mm.

III. Simulation and Experiment

The property of the proposed photon recycling structure can be figured out through the study on two issues: the first one is the photon recycling effect through optimization of light extraction on the phosphor plate, and the other is the benefit by laser multiplexing. Thus we made a sample of the hemisphere reflector, where the radius of the inner hemisphere reflector was 20 mm, and there were four holes with a diameter of 4 mm each, as shown in Fig. 8. One of the holes was normal to the phosphor plate, the other three holes were circular-symmetrically located off-axis, and each location was designed to avoid facing the direction of the specular reflection from the phosphor plate.

In the first part, the key issue is that the phosphor needs to be attached to a transparent plate, which should be in high thermal conductivity and low refractive index to reduce Fresnel loss. In the experiment, we tried a general glass plate and a sapphire plate, but the sapphire plate will induce higher Fresnel loss because of the higher refractive index. Thus we adopted a general glass plate as the support plate rather than a sapphire plate. Fig. 9 shows four different structures that have been established in our experiment. A phosphor plate attaches to the inner side of the glass plate as shown in Fig. 9(a) is the basic structure.

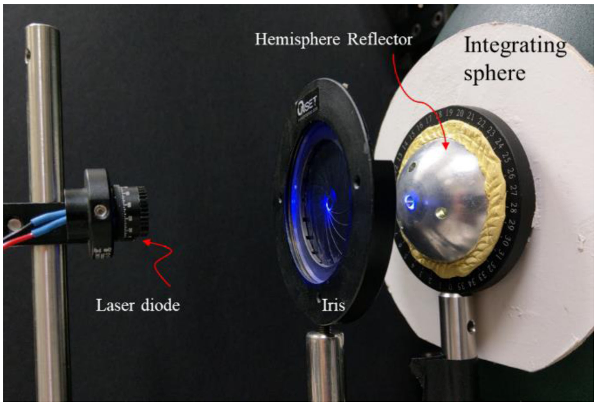


Fig. 10. Experimental setup for blue laser incidence on the hemisphere reflector.

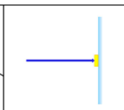
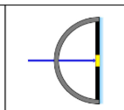
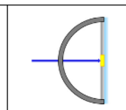
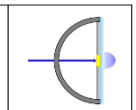
Structure / Measurement / Item				
Blue [W]	0.0129	0.0127	0.0127	0.0126
Yellow [W]	0.0233	0.0363	0.0376	0.0407
Total [W]	0.0362	0.0490	0.0503	0.0533
Lumen [lm]	10.78	16.62	17.21	18.6
Energy Efficiency	36.17%	48.95%	50.30%	53.28%
Enhancement	100%	135.3%	139.1%	147.3%
Luminous efficacy [Lm/W]	31.6	48.7	50.4	54.5
CCT	7918 K	5119 K	5062 K	4915 K
CRI	69.8	65.6	65.3	64.3

Fig. 11. In the case of on-axis illumination, the measurement results for output light flux with/without the recycling structures in different ways.

In order to collect the output flux exactly from the effective phosphor area in the integrating sphere, we need to block the light coming from the other area. Thus, the area other than the phosphor needed to be black, as shown in Fig. 9(b), or coated with a reflection layer as shown in Fig. 9(c) which is aimed to increase the efficiency for multiple-reflection recycling light. Also, we designed a hemisphere lens as shown in Fig. 9(d) to cover the phosphor area to reduce the Fresnel loss of the glass. This approach is similar to the lens encapsulation in the white LED package [11]. The experiment was done by using an LD with a wavelength of 448nm, and the output flux was 100mW. The experimental setup is shown in Fig. 10, where all the experimental data were measured by an integrating sphere with a diameter of twelve inches. The energy efficiency was measured and calculated. The measurement result for normal incidence is shown in Fig. 11. The result shows that the energy efficiency was only 36% without any retroreflector. With using the proposed hemisphere reflector, the energy efficiency was increased to 49% for a black plate and 50% for a reflection plate. The highest energy efficiency was observed when a hemisphere lens was attached to the glass, and the energy efficiency reached 53%. In comparison with the case without photon recycling,

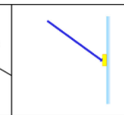
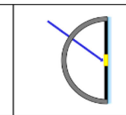
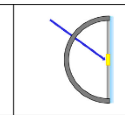
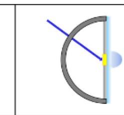
Structure / Measurement / Item				
Blue [W]	0.0119	0.0123	0.0123	0.0123
Yellow [W]	0.0233	0.0390	0.0404	0.0434
Total [W]	0.0352	0.0513	0.0527	0.0557
Lumen [lm]	10.76	17.83	18.46	19.81
Energy Efficiency	35.19%	51.23%	52.66%	55.70%
Enhancement	100%	145.6%	149.7%	158.3%
Luminous efficacy [Lm/W]	31.5	52.2	54.1	58
CCT	7240 K	4956 K	4930 K	4741 K
CRI	69.3	64.6	64.3	64

Fig. 12. In the case of off-axis illumination, the measurement results for output light flux with/without the recycling structures in different ways.

the energy efficiency increases to 147% by the final approach. The luminous efficacy of the white light with photon recycling reached 54.5 lm/W, which was measured in a small injection power with no obvious thermal effect. Also, the associated CRIs were calculated between 60 to 70, which are not as good as that of a white LED owing to the narrow band of the blue light. It should be noted that higher energy efficiency was achieved for recruiting more yellow photons rather than blue photons, which almost kept constant in different arrangements. The yellow light performed more isotropic radiation by down-conversion of the phosphor so that backward yellow photons could be recruited through the proposed structure. Besides, the recruited blue light kept pumping the phosphor and increasing the yellow light output. Therefore, the higher energy efficiency means unavoidable lower CCT, as illustrated in Fig. 11.

In the second experiment, we have to check the property of LD multiplexing by the hemisphere reflector. The simulation shows that the energy efficiency of the laser through the on-axis hole was slightly lower than the off-axis holes because the normal scattering light was more than that in the other direction so that the leakage from the on-axis hole was larger. The experimental measurement result is shown in Fig. 12, where the energy efficiency by the hemisphere reflector for the off-axis laser was higher than that by the on-axis laser, the luminous efficacy of the white light with photon recycling reached 58 lm/W. Also, the higher the energy efficiency was, the more yellow light was observed. Therefore, more photon recycling could decrease the CCT and change the CRI. The experimental measurement of the light source by three LD multiplexing is shown in Fig. 13, where the output flux, as well as the exitance, was almost 300% enhanced through the proposed photon recycling structure. The most important feature is that the etendue was kept the same. Besides, owing to the heavy scattering of the blue light and spontaneous emission of the yellow light, the far-field light patterns are similar to each other.

In order to figure out the effect by the number of the laser-hole pair, we made a simulation based on the LD-phosphor model. The geometry of the hemisphere reflector is shown in Fig. 14,

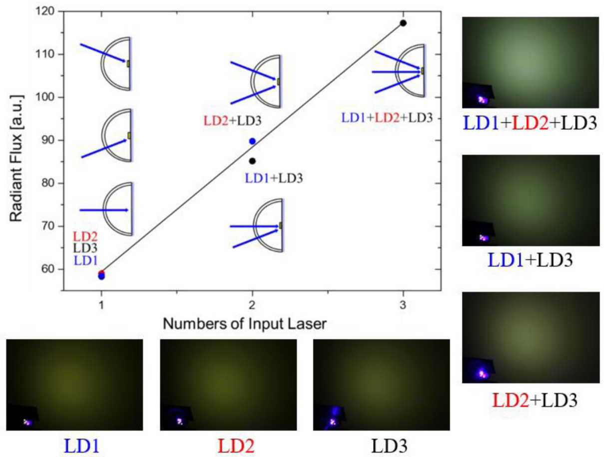


Fig. 13. The output flux of the laser lighting and the corresponding light patterns for different conditions for laser multiplexing.

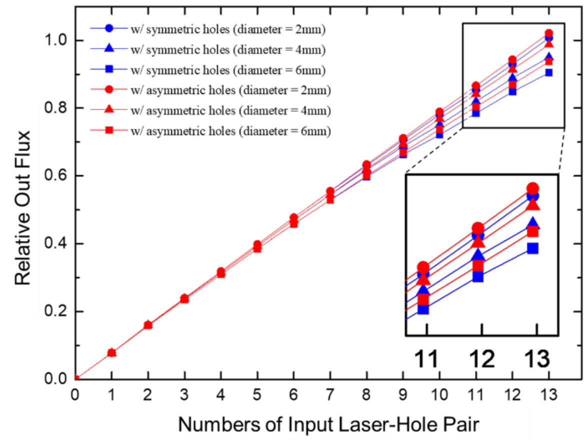


Fig. 15. Simulation for the relative our flux for the hemisphere reflectors with different hole structures.

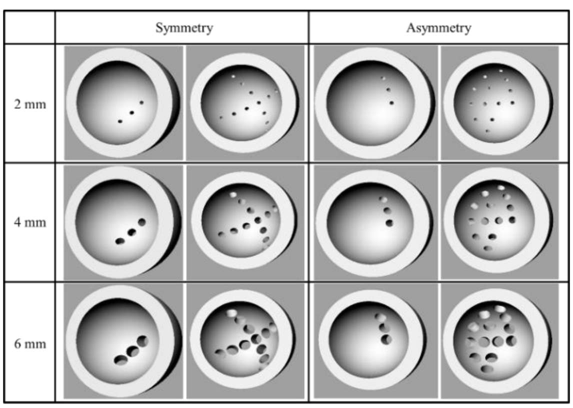


Fig. 14. The structures of the hemisphere reflector with symmetric or asymmetric holes.

where the hole geometries are divided into two groups, one is symmetry and the other is asymmetry. Fig. 15 shows the simulation results with the cases of hole geometry including location, diameter, and number. The simulation indicated that the number of the holes and the hole diameter, or equivalently the total hole area were the factors related to the recycling rate and the energy efficiency. The asymmetry arrangement is better than the symmetry arrangement, but the effect was not obvious.

IV. Conclusion

In this article, we have proposed a novel structure in a transmission type of laser lighting to recruit the backward scattering light. The photon recycling structure is formed by a hemisphere reflector, and the phosphor plate is located at the center of the hemisphere reflector. A precise phosphor model by laser pumping has been made to optimize the whole system. The photon recycling rate achieves stability when the radius of the hemisphere reflector is four-time of the phosphor plate size. Without the hemisphere reflector, the energy efficiency of the white light source was measured 36% in the case at on-axis incidence and 35% at off-axis incidence. With the hemisphere reflector, the energy efficiency can be increased to 150.3% at on-axis incidence and 152.7% at off-axis incidence. If we add a hemisphere lens to extract the light from the phosphor, the energy efficiency can be further increased to 153.3% at on-axis incidence and 155.7% at off-axis incidence. Thus the enhancement is 147.3% at on-axis incidence and 158.3% at off-axis incidence were observed. The luminous efficacy of the light source reached 58 lm/W based on small injection flux. The enhancement is by photon recycling of the yellow light because it is from spontaneous emission by the phosphor and radiates isotropically. Besides, the recycled blue photons keep pumping the phosphor. This conducts a fact that the higher the recycling efficiency is, the lower CCT will be, and the related CRI also changes. Another important benefit of using the proposed hemisphere reflector is the possibility of laser multiplexing. This is the essential point to increase the exitance but to keep the etendue unchanged. Simulation and the corresponding experiment show that the energy efficiency of the system almost keeps the same with multiple laser incidences. In the experiment, three lasers got near 300% enhancement. More enhancement needs a larger hemisphere reflector to contain more holes on the reflector, but heat dissipation will be another problem in the phosphor. However, laser multiplexing is an advantage that LED cannot afford. The further simulation shows that the hole geometry and arrangement do not play an important role in deciding the recycling effect, and it means that the proposed structure is robust in the purpose of laser multiplexing for high-intense light sources. This property is very helpful in high-intensity or long-distance projection with a solid-state light source.

References

[1] N. Narendran, N. Maliyagoda, A. Bierman, R. Pysar, and M. Overington, “Characterizing white LEDs for general illumination application,” in *Proc. SPIE*, San Jose, CA, USA, 2000, pp. 240–248.
[2] A. Zukauskas, M. Shur, and R. Gaska, *Introduction to Solid-State Lighting*. New York, NY, USA: Wiley, 2002.
[3] D. A. Steigerwald *et al.*, “Illumination with solid state lighting technology,” in *IEEE J. Sel. Topics Quantum Electron.*, vol. 8, no. 2, pp. 310–320, Aug. 2002.
[4] E. F. Schubert and J. K. Kim, “Solid-state light sources getting smart,” *Science*, vol. 308, no. 5726, pp. 1274–1278, May 2005.

[5] R. Karlicek, C. C. Sun, G. Zissis, and R. Ma, *Handbook of Advanced Lighting Technology*. Cham, Switzerland: Springer, 2017.
[6] F. Nguyen, B. Terao, and J. Laski, “Realizing LED illumination lighting applications,” in *Proc. SPIE*, San Diego, CA, USA, 2005, Art. no. 594105.
[7] E. F. Schubert, *Light-Emitting Diodes*. Cambridge, U.K.: Cambridge Univ. Press, 2006.
[8] D. Sun, “Challenges and opportunities for high power white LED development,” in *DOE SSL R&D Workshop*, Atlanta, GA, USA, 2012.
[9] “US department of energy,” in *Life-Cycle Assessment of Energy and Environmental Impacts of LED Lighting Products*. WA, DC, USA: USDOE, 2012.
[10] N. G. George, K. A. Denault, and R. Seshadri, “Phosphors for solid-state white lighting,” *Annu. Rev. Mater. Sci.*, vol. 43, pp. 481–501, Jul. 2013.
[11] C. C. Sun *et al.*, “Packaging efficiency in phosphor-converted white LEDs and its impact to the limit of luminous efficacy,” *J. Solid State Lighting*, vol. 1, no. 1, pp. 1–17, Nov. 2014.
[12] C. C. Sun *et al.*, “Review of optical design for vehicle forward lighting based on white LEDs,” *Opt. Eng.*, vol. 60, no. 9, pp. 91501, Aug. 2021.
[13] N. Mahajan, *Optical Imaging and Aberrations: Part I. Ray Geometrical Optics.* Bellingham, WA, USA: SPIE Press, 1998.
[14] W. T. Chien, C. C. Sun, and I. Moreno, “Precise optical model of multi-chip white LEDs,” *Opt. Exp.*, vol. 15, no. 12, pp. 7572–7577, Jun. 2007.
[15] Z. Liu, S. Liu, K. Wang, and X. Luo, “Optical analysis of color distribution in white LEDs with various packaging methods,” *IEEE Photon. Technol. Lett.*, vol. 20, no. 24, pp. 2027–2029, Dec. 2008.
[16] C. Sommer *et al.*, “A detailed study on the requirements for angular homogeneity of phosphor converted high power white LED light sources,” *Opt. Mater.*, vol. 31, no. 6, pp. 837–848, Apr. 2009.
[17] K. Wang, D. Wu, F. Chen, Z. Y. Liu, X. B. Luo, and S. Lu, “Angular color uniformity enhancement of white light-emitting diodes integrated with freeform lenses,” *Opt. Lett.*, vol. 35, no. 11, pp. 1860–1862, Jun. 2010.
[18] Y. Shuai, Y. Z. He, N. T. Tran, and F. G. Shi, “Angular CCT uniformity of phosphor converted white LEDs: Effects of phosphor materials and packaging structures,” *IEEE Photon. Technol. Lett.*, vol. 23, no. 3, pp. 137–139, Mar. 2011.
[19] C. C. Sun *et al.*, “High uniformity in angular correlated-color-temperature distribution of white LEDs from 2800K to 6500K,” *Opt. Exp.*, vol. 20, no. 6, pp. 6622–6630, Mar. 2012.
[20] W. T. Welford and R. Winston, “The generalized etendue or lagrange invariant and the phase space concept,” in *High Collection Nonimaging Optics*, Cambridge, MA, USA: Academic, 1989.
[21] H. Ries, N. Shatz, J. Bortz, and W. Spirkl, “Performance limitations of rotationally symmetric nonimaging devices,” *J. Opt. Soc. Am.*, vol. 14, no. 10, pp. 2855–2862, Oct. 1997.
[22] S. Nakamura *et al.*, “InGaN-based multi-quantum-well-structure laser diodes,” *Jpn. J. Appl. Phys.,* vol. 35, no. 1B, pp. L74–L76, Sep. 1996.
[23] S. Nakamura, S. Pearton, and G. Fasol, *The Blue Laser Diode : The Complete Story*, 2nd ed., Berlin, Germany: Springer-Verlag, 2000.
[24] Y. Xu *et al.*, “Phosphor-conversion white light using InGaN ultraviolet laser diode,” *Appl. Phys. Lett.*, vol. 92, no. 2, Dec. 2008, Art. no. 021129.
[25] K. A. Denault, M. Cantore, S. Nakamura, S. P. DenBaars, and R. Seshadri, “Efficient and stable laser-driven white lighting,” *AIP Adv.,* vol. 3, no. 7, Jul. 2013, Art. no. 072107.
[26] J. Kinoshita *et al.*, “Suppressed speckle contrast of blue light emission out of white lamp with phosphors excited by blue laser diodes for high-brightness lighting applications,” *Opt. Rev.*, vol. 19, no. 6, pp. 427–431, Nov. 2012.
[27] Y. Ikeda *et al.*, “Incoherentized high-brightness white light generated using blue laser diodes and phosphors-effect of multiple scattering,” *J. Light Vis. Environ.*, vol. 37, pp. 95–100, Jan. 2013.
[28] C. C. Sun *et al.*, “Calculating model of light transmission efficiency of diffusers attached to a lighting cavity,” *Opt. Exp.*, vol. 18, no. 6, pp. 6137–6148, Mar. 2010.
[29] C. C. Sun *et al.*, “High-directional light source using photon recycling with a retro-reflective dome incorporated with a textured LED die surface,” *Opt. Exp.*, vol. 21, no. 15, pp. 18414–18423, Jul. 2013.
[30] C. Y. Chen, T. H. Yang, C. H. Hsu, and C. C. Sun, “High-efficiency white LED packaging with reduced phosphor concentration,” *IEEE Photon. Technol. Lett.*, vol. 25, no. 7, pp. 694–696, Mar. 2013.
[31] C. C. Sun *et al.*, “Precise optical modeling for silicate-based white LEDs,” *Opt. Exp.*, vol. 16, no. 24, pp. 20060–20066, Nov. 2008.
[32] T. H. Yang *et al.*, “Precise simulation of spectrum for green emitting phosphors pumped by a blue LED die,” *IEEE Photon. J.*, vol. 6, no. 4, Aug. 2014. Art. no. 8400510.