Hypergraph Based Resource Allocation Algorithms for VLC Network With Heterogeneous LEDs and Varied Users

Bo Bai, Jiajun Deng, Xianqing Jin, Bo Su, and Tao Shang

Abstract—**With distinguished advantages over radio frequency communication, visible light communication (VLC) has been considered as one of the potential and promising wireless communication technologies. However, the densely deployed light emitting diode (LED) lamps introduce severe inter channel interference (ICI), which drastically deteriorates the VLC network performance. In addition, the limited wireless access capabilities of heterogeneous LED lamps, as well as the different data transmission requirements of varied user equipments (UEs), will bring new challenges to the VLC network in real-life applications. To mitigate the ICI and improve the VLC network performance, the hypergraph theory is introduced in detail. Effective data links between one UE and all of its available LED access points (APs) are taken as a hyperedge to form an amorphous cell for each UE. Considering the UE satisfaction performance, as well as the spectrum efficiency and service fairness, the non-layered resource allocation (RA) algorithm based on carrier aggregation and the directly overlapped RA algorithm based on carrier aggregation and cooperative transmission are proposed. Numeric results show that the proposed RA algorithms outperform the classic Dsatur RA algorithm in terms of the UE satisfaction, spectrum efficiency and service fairness.**

Index Terms—**Visible light communication, resource allocation, hypergraph theory, heterogeneous.**

I. Introduction

WITH distinguished features like ultra-high bandwidth, immunity to electromagnetic interference and free license, visible light communication (VLC) has become one of the potential and promising wireless communication technologies [1]–[4]. With the development of light emitting diode illumination device, the VLC technology has also inherited the advantages like environmentally friendly, ease implementation, low cost and wide coverage. These make the VLC technology drawing much attention from both the academia and industry, and also being recommended as one of the most important parts of the 6th generation communication technologies [5]. Considering the relatively small coverage or power of a single LED, multiple LEDs (-lamps) are always densely deployed on the ceiling to meet the illumination requirement. Simultaneously these LED lamps can be utilized as access points (APs) to satisfy the different data transmission requirements of varied user equipments (UEs). In a dense VLC network, a UE may capture light signals emitted from different LED lamps, and the inter channel interference (ICI) can frequently occurs. This sharply deteriorates the received signal quality, and dramatically decreases the VLC system performance.

Many works have been done to solve the ICI problem resulted from the densely deployed LED APs. Unity frequency reuse and fractional frequency reuse planning are studied in the traditional fixed-shape cell and merged cell [6], and combined transmission and vectored transmission among different LED APs are investigated to improve the VLC network performance. An amorphous cell and a novel user-centric (UC) VLC network were firstly introduced in [7], and a UC cluster formation method as well as a low-complexity greedy algorithm relying on LED AP to UE distance-based weight were further proposed to mitigate the interference in a UC VLC network [8],[9]. As the optical code division multiple access (OCDMA) mechanism has been largely exploited for dimming control, two resource allocation (RA) algorithms based on direct sequence OCDMA are proposed in [10] for a multi-color LED VLC network, which can maximize the network transmission rate or achieve UE fairness, respectively. To support the changing UE count in a red-green-blue (RGB) LED based VLC network, the LED APs are divided in to sectors with different color value, and an optical beamforming-based RA algorithm was proposed to increase the data rate without consideration of the UE fairness performance [11]. As multiple colors can offer more light bands to increase the transmission speed, VLC network employing the 3-color and 4-color synthetic white LEDs are introduced in [12], and a new greedy RA algorithm is designed to optimize the VLC network transmission speed with considering the illumination quality. However, this may involve a power optimization problem of different colors. A novel interference graph model with small scale is designed to analyze the inter-user interference, and a greedy RA algorithm based on graph coloring theory is proposed in [13] to maximize the sum capacity while ensuring a certain degree of fairness. By

Manuscript received June 28, 2021; revised July 29, 2021; accepted August 6, 2021. Date of publication August 10, 2021; date of current version August 25, 2021. This work was supported in part by the National Natural Science Foundation of China under Grants 61801356, 61971394, and 61631018, in part by Key Research and Development Projects of Shaanxi Province (2020ZDLGY05-02), and in part by Key Research Program of Frontier Science, Chinese Academy of Sciences (QYZDY-SSW-JSC003). *(Corresponding author: Bo Bai.)*

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Digital Object Identifier 10.1109/JPHOT.2021.3103660

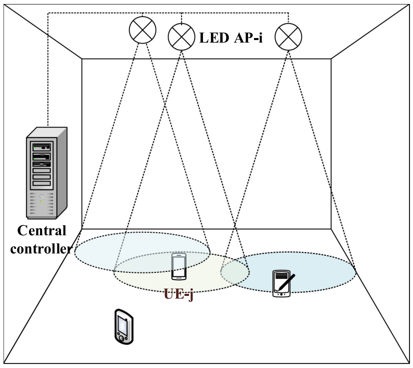

Fig. 1. A typical VLC network with dense LED lamps and randomly located UEs.

giving higher priority to UE with good channel conditions, a new three-term priority factor model taking VLC network downlink throughput, user fairness and time latency into consideration is introduced in [14], and an interference-graph-based scheduling algorithm was further proposed to improve the VLC network performance. Considering the transmission power and the orientation of LED AP and UE, a heuristic AP grouping algorithm was proposed based on the interference graph to preselect candidate APs from the huge number of feasible APs, and an interference management (IM) was further proposed to achieve the joint optimization of AP grouping, AP-user association and AP power allocation [15]. To effectively mitigate ICI and maximize benefit of the cooperative transmission, the cell-centric (CC) and UC clustering were discussed in [16]. Considering the random orientations of UEs, a rotation model was employed, and a modified interference graph based greedy algorithm and a heuristic RA algorithm based on bipartite graph theory were proposed to maximize spectral efficiency while ensuring user fairness.

As the primary function of a wireless access network is to satisfy the different data transmission requirements of varied UEs, the UE satisfaction should be assigned a higher priority rather than performance like service fairness, system throughput under the constrain of interference mitigation. However, the UE satisfaction performance is rarely discussed in the existing literature. When considering the heterogeneity of the commercial LED lamps like illumination power, response time, deployed location and direction, as well as coverage area, the access capability of each LED AP is different for varied UEs with different pointing angles or locations. In this paper, the heterogeneities of LED APs and different requirements of varied UEs are carefully taken into account, and two RA algorithms based on hypergraph theory are also proposed. It is proved that the proposed RA algorithms can largely satisfy the UE requirement under the interference mitigation constrain, and maximize the spectrum efficiency and service fairness performance for the VLC network with heterogeneous LED APs and varied user requirements.

II. System Model and Problem Formulation

As shown in Fig. 1, a typical VLC network with densely deployed LED lamps on the ceiling is considered. The LED lamps are employed to illuminate the indoor environment, and employed as LED APs to simultaneously satisfy the wireless data transmission requirement. It is assumed that all the LED lamps are connected to a central controller to form a cooperative VLC network. Data transmission requirements of varied UEs, resource allocation status of heterogeneous LED APs, as well as each LED AP to UE channel quality, are collected by the central controller. A RA schedule is generated by the central controller, and several LED APs are selected to serve a given UE according to this schedule.

A. System Model

According to the impulse response characteristic of visible light signal in the indoor environment [17], only the line-of-sight (LoS) light signal and the primary reflected light signal are considered. The channel gain G_{ij} between the LED AP-*i* and the UE-*j* is

$$G_{ij} = G_{ij\_LoS} + G_{ij\_NLoS}, \tag{1}$$

where $i \in \mathcal{I} = \{1, 2, \ldots, I\}$ and $j \in \mathcal{J} = \{1, 2, \ldots, J\}$ are the index of the LED AP and the UE, respectively. I and J are the total number of the LED APs and UEs in the VLC network. $G_{ij\cdot LoS}$ is the channel gain of the LoS light signal,

$$G_{ij\_LoS} = \frac{(m+1)\,A}{2\pi D_{ij}^2}\cos^m(\phi)\,T_s(\psi)\,g(\psi)\cos(\psi), \tag{2}$$

where m is the Lambertian emission order, and is given by the LED semi-angle at half illuminance $\Phi_{1/2}$ as $m = -\ln 2 / \ln(\cos \Phi_{1/2})$. A is the physical area of the detector in the UE, D_{ij} is the distance between the LED AP-*i* and the UE-*j*. ϕ and ψ are the irradiant angle and incident angle of the light signal at the LED lamp and the detector, respectively. $T_s(\psi)$ is the gain of the optical filter, and $g(\psi)$ is the gain of an optical concentrator mounted in front of the detector,

$$g(\psi) = \begin{cases} \frac{n^2}{\sin^2(\Psi_c)}, & 0 \le \psi \le \Psi_c \\ 0, & \psi \ge \Psi_c \end{cases}, \tag{3}$$

where n is the refractive index, and Ψ_c is the field of view (FoV) angle of the detector.

G_{ij_NLoS} is the non-light-of-sight (NLoS) channel gain of the primary reflected light signal,

$$\begin{aligned} G_{ij\_NLoS} &= \tfrac{(m+1)A}{2\pi^2 D_{ij,1}^2 D_{ij,2}^2} \\ &\rho\cos^m(\phi)\cos(\alpha)\cos(\beta)\,T_s(\psi)\,g(\psi)\cos(\psi) \end{aligned}, \tag{4}$$

where $D_{ij,1}$ is the distance between the LED lamp and reflected point on the wall, and $D_{ij,2}$ is the distance between the reflected point on the wall and the detector. ρ is the reflectance factor of the wall. α and β are the incident angle and the irradiant angle of the light signal at the reflected point on the wall.

The noise received by the UE consists of three parts, the shot noise, the thermal noise and the ICI from non-assigned LED APs. For simplicity, the optical power P_{ot} of each LED lamp is assumed to be the same, thus the signal to interference and noise ratio (SINR) for UE-j can be represented as,

$$\gamma_j = \frac{(R_e \cdot P_{sig,j})^2}{\sigma_{shot}^2+\sigma_{thermal}^2+(R_e \cdot P_{ICI,j})^2} = \frac{\sum_{i\in\mathcal{S}_j}(R_e \cdot G_{ij}P_{ot})^2}{\sigma_{shot}^2+\sigma_{thermal}^2+\sum_{i\notin\mathcal{S}_j}(R_e \cdot G_{ij}P_{ot})^2}, \quad (5)$$

where R_e is the detector responsibility. $P_{sig,j}$ and $P_{ICI,j}$ are the optical power of light signal and ICI received by UE-j, respectively. $\mathcal{S}_j$ is the LED AP set scheduled by the central controller to serve UE-j. σ_{shot}^2 and $\sigma_{thermal}^2$ are the variance of the shot noise and the thermal noise,

$$\sigma_{shot}^2 = 2qR_e\left(P_{sig,j}+P_{ICI,j}\right)B + 2qI_{bg}I_2B, \quad (6)$$

$$\sigma_{thermal}^2 = \frac{8\pi kT_K}{G}\eta AI_2B^2 + \frac{16\pi^2 kT_K\Gamma}{g_m}\eta^2A^2I_3B^3, \quad (7)$$

where q is the electronic charge, B is equivalent noise bandwidth, I_{bg} is background current, $I_2 = 0.562$ and $I_3 = 0.0868$ are the noise bandwidth factors [17]. k is the Boltzmann's constant, T_K is absolute temperature, G is the open-loop voltage gain, η is the fixed capacitance of photo detector per unit area, Γ is the field effect transistor (FET) channel noise factor, g_m is the FET transconductance.

To further investigate the spectrum efficiency of the VLC network, the tight capacity bound [18] is adopted. The spectrum efficiency for UE-j can be represented as,

$$R_j = 0.5 \cdot \log_2\left(1+\frac{e\cdot\gamma_j}{2\pi}\right), \quad (8)$$

where $e = 2.718$ is the Euler's number, and the factor 0.5 is resulted from the non-negative constrain of the visible light signal.

B. Problem Formulation

Considering the relatively small coverage of each LED AP, the limited FoV angle of the detector on each UE, as well as the randomly distributed locations and directions of the LED APs and UEs, the UE can only capture light signals emitted from some specified LED APs, and obtain an effective data transmission link when the channel SINR exceeds a given threshold $\gamma_{\rm th}$. The binary matrix $\mathbf{R}_{I\times J}$ represents the system status of the VLC network, where $r_{ij} = 1$ represents there exists an effective data transmission link between LED AP-i and UE-j, and $r_{ij} = 0$ represents none. Let set $\mathcal{X}_j$ represents the available LED AP set for UE-j, and set $\mathcal{Z}_i$ represents the available UE set among LED AP-i's covered area, then

$$\bigcup\nolimits_{j\in\mathcal{J}}\mathcal{X}_j \subseteq \mathcal{I}, \quad (9)$$

as there may exist some LED APs serving no UEs in their covered areas, that is $\mathcal{X}_j = \emptyset$.

$$\bigcup\nolimits_{i\in\mathcal{I}}\mathcal{Z}_i \subseteq \mathcal{J}, \quad (10)$$

as there may also exist some UEs with no effective data transmission link from any LED AP, that is $\mathcal{Z}_i = \emptyset$.

Resulted from the LED lamps' heterogeneity, as well as the aging effect over time, the effective transmission bandwidth of each LED AP is different. This leads to different access capabilities. Considering the different transmission requirements of varied UEs, and to fully utilize the LED AP's access capabilities, the LED AP's access capability is divided into and represented by the different resource block numbers.

Let the matrix $\mathbf{A}_{I\times K}$ represent the RA schedule generated by the VLC network central controller, where $a_{i,k} \in$\{-1, 0, 1, 2, …, J\}, $k \in \mathcal{K} = \{1, 2, \ldots, K\}$ is the resource block index of the LED APs. $K = \max(K_1, K_2, \ldots, K_I)$ is the maximum resource block number of all the LED APs, and K_I is the total resource block number of LED AP-I. $a_{i,k} = j$ represents the k^{th} resource block of LED AP-i is assigned to UE-j, $a_{i,k} = 0$ represents the k^{th} resource block of LED AP-i is not assigned to any UE, and $a_{i,k} = -1$ represents the k^{th} resource block of LED AP-i is not existed at all.

The data transmission requirements of varied UEs mainly depend on the UEs' data service type. In order to satisfy the varied UEs requirements, and fully develop the VLC network's capability, intensity modulation and direct detection (IM/DD) method with different modulation schemes or modulation orders are selected for each data transmission link. The modulation scheme and order closely depends on the SINR characteristic of the UE on different resource blocks. And this will promise different data transmission rates on each assigned resource block for varied UEs.

The main goal of the RA is to satisfy the varied UE requirements under the ICI mitigation constrain. The VLC network spectrum efficiency and the service fairness performance should also be taken into account.

$$(\mathbf{P1}): \max_{\mathbf{A}} U(a_{i,k}), \quad (11)$$

$$st. \sum_{k\in\mathcal{K}}\delta\left(a_{i,k}\right) \le K_i, \forall i\in\mathcal{I}, \quad (12)$$

$$a_{i,k} \in \left\{0, j\right\}, \forall i\in\mathcal{X}_j, \forall k\in\mathcal{K}, \quad (13)$$

where $U(a_{i,k})$ is the utility function related to allocation matrices, and $\delta(\cdot)$ is the step function

$$\delta(x) = \begin{cases} 1 & x > 0 \\ 0 & otherwise \end{cases}. \quad (14)$$

It can be found that the illuminance constrains are not considered in **P1**, this is because the illuminance mainly depends on the LED lamps distribution on the ceiling, and the RA procedure may hardly reduce the illuminance performance.

Therefore, (12) means each LED AP can only serve one UE at most on each resource block. Constrain (13) means each LED AP in the set $\mathcal{X}_j$ can only serve UE-j or no UE at all on one resource block. This is designed to mitigate the ICI among different LED APs.

For calculation simplicity, it is assumed that the detector on each UE is the same, and is vertically pointed up to the ceiling. In addition, each LED AP is assumed to has the same semi-angle at half illuminance $\Phi_{1/2}$, and is vertically pointed down to the floor. For the randomly pointed detectors or LED APs, the rotation model in [17] can be employed to transfer them back into the equivalent scene with vertically pointed up detectors and pointed down LED APs.

III. Hypergraph Based RA Algorithms

A. Basic Hypergraph Theory

In mathematics, a hypergraph is a generalization of a graph in which an edge can connect any number of vertices [19]. A hypergraph $\mathcal{H}$ can be represented by

$$\mathcal{H} = (\mathcal{V}, \mathcal{E}), \tag{15}$$

where $\mathcal{V} = \{v_1, v_2, \ldots, v\,|\mathcal{V}|\}$ is the vertex set, and $\mathcal{E} = \{e_1, e_2, \ldots, e_{|\mathcal{E}\,|}\}$ is the edge set. Operator $|\cdot|$ is the modulo operation. Different from the edge in a non-hypergraph, the edges in a hypergraph can connect any number of vertices instead of merely two. The hypergraph $\mathcal{H}$ can also be represented by a $|\mathcal{E}\,| \times |\mathcal{V}|$ incidence matrix $\mathbf{M}_{|\varepsilon| \times |\mathcal{V}|}$ in algebra, where

$$m_{ij} = \begin{cases} 1 \text{ if } v_i \in e_j \\ 0 \text{ otherwise} \end{cases}. \tag{16}$$

A hypergraph $\mathcal{H} = (\mathcal{V}\,, \mathcal{E})$ is an bipartite hypergraph if

$$\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2 \text{ with } \mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset, \tag{17}$$

and each edge in $\mathcal{E}$ joins at least one vertex in $\mathcal{V}_1$ to at least one vertex in $\mathcal{V}_2$.

Let the vertex set $\mathcal{V}$ represent all the LED APs and UEs, and let the edge set $\mathcal{E}$ represents the effective data transmission links between each UE and all of its available LED APs on one resource block. The VLC network can be represented by a hypergraph set $\mathcal{H} = \{\mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_K\}$, where $\mathcal{H}_k = (\mathcal{V}, \mathcal{E}_k)$ is the generated hypergraph on resource block *k*, and $\mathcal{E}_k$ are the effective data transmission links on resource block *k*. Furthermore, divide the vertex set $\mathcal{V}$ into two parts, and let the vertex set $\mathcal{V}_1$ and the vertex set $\mathcal{V}_2$ represent all the LED APs and UEs, respectively. When not considering the LED APs $\mathcal{Z}_i = \emptyset$ having no UEs in their covered areas, or the UEs $\mathcal{X}_j = \emptyset$ having no effective data transmission link from any LED AP, each hypergraph $\mathcal{H}_k$ of the VLC network can be transferred to a bipartite hypergraph $\mathcal{H}_k = (\{\mathcal{V}_1, \mathcal{V}_2\}, \mathcal{E}_k)$. As each edge in $\mathcal{E}_k$ joins only one UE in $\mathcal{V}_2$ and at least one LED AP in $\mathcal{V}_1$, the edge in the bipartite hypergraph forms an amorphous cell for each UE. Therefore, the RA problem can be converted into a hypergraph coloring problem.

A typical graph coloring problem requires to find an assignment of colors to all the vertices, such that no two adjacent vertices share the same color while minimizing the number of colors. In the conventional graph coloring based RA works, either the LED APs [20] or the UEs [21] are solely chosen to form an interference graph. While for the VLC network with heterogeneous LED APs and varied UEs, an edge in the generated hypergraph merely represents an effective data transmission link on one resource block, and may join more than two vertices. To fully develop the VLC network access capability and mitigate the ICI, the non-layered RA algorithm and the directly overlapped RA algorithm are proposed in this paper.

B. Non-Layered RA Algorithm

Considering the different access capabilities of LED APs, the number of assigned resource block for each LED AP to UE data transmission link is strictly limited by its total resource block number, as well as the data transmission requirements of its served UEs. Separate the resource block of each LED AP into different layers, assign only one resource block to the UEs at a time and layer by layer, the RA problem of the VLC network with heterogeneous LEDs and varied users can be easily fixed with a classic Dsatur RA algorithm [22]. Therefore, the Dsatur RA algorithm is chosen as a benchmark to test the following RA algorithms proposed in this section.

However, in the classic Dsatur RA algorithm, one UE need to merely capture the light signal from its assigned VLC AP on one resource block at a time, and frequently the available data transmission rate on one resource block cannot satisfy the UE when a high bandwidth service is required.

With the consideration of mismatch between the LED AP's available transmission rate on one resource block and the UE requirement, the non-layer RA algorithm based on the carrier aggregation method is proposed to improve the VLC network performance. In the non-layer RA algorithm, all the resource blocks, instead of merely one, of a selected LED AP can be assigned until its served UE is satisfied or no more resource block is left. Define the interfered neighbor as the UEs which can capture the light signal from the same LED AP. In order to mitigate the ICI and fully develop the VLC access capability, the UE with least interfered neighbors is preferentially chosen, and the LED AP with the least covered UEs among the chosen UE's available LED AP set is firstly selected to serve it. The non-layered RA algorithm described in Algorithm 1 can also be summarized in 3 steps.

1) Hypergraph Construction: Generate the system status matrix $\mathbf{A}_{I\times J}$ according to the channel SINR γ between each VLC AP and UE and the effective data link SINR threshold γ_{th}. Construct the system hypergraph $\mathcal{H}_1$ on the 1st resource block by edges joining each UE to all of its available LED APs.

2) Hypergraph Simplification: Let sets $\mathcal{X}_{\text{done}}$ and $\mathcal{Z}_{\text{done}}$ represent the LED AP set with all resource blocks assigned and UE set with all requirement satisfied, respectively. Simplify $\mathcal{H}_1$ by moving the VLC APs covered no UEs $\mathcal{X}_j = \emptyset$ from $\mathcal{V}_1(\mathcal{H}_1)$ to the LED AP set $\mathcal{X}_{\text{done}}$, and moving the UEs having no effective data transmission link $\mathcal{Z}_i = \emptyset$ from $\mathcal{V}_2(\mathcal{H}_1)$ to the UE set $\mathcal{Z}_{\text{done}}$.

3) Resource Allocation: Choose the UE-*j* with least interfered neighbors, sort the chosen UE-*j*'s available LED APs in $\mathcal{X}_j$ with covered UE number in increasing order, and assign the resource blocks of the APs to UE-*j* until UE-*j* is satisfied or all the available LED APs in $\mathcal{X}_j$ are checked. Check whether UE-*j* is satisfied, if yes, move UE-*j* from $\mathcal{V}_2(\mathcal{H}_1)$ to $\mathcal{Z}_{\text{done}}$. Then check whether the resource blocks of each LED APs in $\mathcal{X}_j$ are all assigned, if yes, move these LED APs from $\mathcal{V}_1(\mathcal{H}_1)$ to $\mathcal{X}_{\text{done}}$. Rerun the resource allocation until either $|\mathcal{X}_{\text{done}}| = I$, $|\mathcal{Z}_{\text{done}}| = J$, or all the UEs are checked.

Algorithm 1: Non-Layered RA Algorithm.

Input: Channel SINR between each VLC AP and UE γ, effective data link SINR threshold γ_{th}, resource block number of each VLC AP K_i, required data transmission rate for each UE
Output: Resource allocation schedule $\mathbf{A}_{I\times K}$

Generate the hypergraph $\mathcal{H}_1$ on the 1st resource block with the channel SINR γ and SINR threshold γ_{th};
Initialize LED AP set $\mathcal{X}_{\mathrm{done}} = \emptyset$, UE set $\mathcal{Z}_{\mathrm{done}} = \emptyset$ and UE set $\mathcal{Z}_{\mathrm{checked}} = \emptyset$;
while $|\mathcal{X}_{\mathrm{done}}| \neq I$, $|\mathcal{Z}_{\mathrm{done}}| \neq J$ or $|\mathcal{Z}_{\mathrm{checked}}| \neq J$
  $\mathcal{X}_{\mathrm{done}} = \mathcal{X}_{\mathrm{done}} \cup (\mathcal{X}_j = \emptyset)$, $\mathcal{V}_1(\mathcal{H}_1) = \mathcal{V}_1(\mathcal{H}_1) \setminus \mathcal{X}_{\mathrm{done}}$;
  $\mathcal{Z}_{\mathrm{done}} = \mathcal{Z}_{\mathrm{done}} \cup (\mathcal{Z}_i = \emptyset)$, $\mathcal{V}_2(\mathcal{H}_1) = \mathcal{V}_2(\mathcal{H}_1) \setminus \mathcal{Z}_{\mathrm{done}}$;
  Choose UE-*j* $\in \mathcal{E}_1$, where UE-*j* has the least neighbors;
  Sort APs $\in \mathcal{X}_j$ by the covered UE number in increasing order;
  for each AP-*i* $\in \mathcal{X}_j$ **do**
    for each available resource block $k \in \mathcal{K}$ **do**
      if UE-*j* is NOT satisfied **then**
        $a_{j,k} = j$;
      end if
    end for
  end for
  $\mathcal{Z}_{\mathrm{checked}} = \mathcal{Z}_{\mathrm{checked}} \cup$UE-*j*, $\mathcal{V}_2(\mathcal{H}_1) = \mathcal{V}_2(\mathcal{H}_1) \setminus$ UE-*j*;
  if UE-*j* is satisfied **then**
    $\mathcal{Z}_{\mathrm{done}} = \mathcal{Z}_{\mathrm{done}} \cup$UE-*j*;
  end if
  if all resource blocks of AP-*i* $\in \mathcal{X}_j$ are assigned **then**
    $\mathcal{X}_{\mathrm{done}} = \mathcal{X}_{\mathrm{done}} \cup$AP-*i*, $\mathcal{V}_1(\mathcal{H}_1) = \mathcal{V}_1(\mathcal{H}_1) \setminus$ AP-*i*;
  end if
end while

C. Directly Overlapped RA Algorithm

As discussed above, each edge in the generated hypergraph for the VLC network forms an amorphous cell for the UE joining this edge. According to the RA schedule generated by the central controller, some specified LED APs among the amorphous cell are selected to serve this UE. Therefore, the valid signal light received by one UE would be a superposition of the LoS light signal emitted from the selected LED APs, as well as their primary reflected NLoS light signal. However, either in the classic Dsatur RA algorithm or in the non-layered RA algorithm, one UE need to merely deal with the light signal from only one of its assigned VLC APs at a time, and treat the received light signal from other LED APs as the ICI.

In a typical indoor environment, the time delay between the received light signal from different LED APs, as well as the time delay between the LoS light signal and its primary reflected NLoS signal, is relatively small when compared to the symbol time duration, or can be easily compensated with a channel coding scheme like cyclic redundant prefix/suffix. With the help of cooperative transmission, all the selected LED APs broadcast the identical light signals on the assigned resource block, and the served UE could take all the received light signals from these LED APs as the valid signals, either the LoS or NLoS signals. This will transfer part of the ICI back to the valid signals, and improve the served UE's SINR performance, as the valid signal increases and both the ICI and the shot noise decrease. Therefore, a better modulation scheme or a higher modulation order could be employed, and a higher transmission rate could be further achieved.

In order to take the cooperative transmission advantage and further improve the VLC network performance, the directly overlapped RA algorithm based on the cooperative transmission and carrier aggregation methods is proposed. In the directly overlapped RA algorithm, more than one LED APs are simultaneously selected to serve one UE, instead of only one LED AP at a time when compared with the algorithms proposed above. All the available combinations of the LED APs among each UE's amorphous cell are sequentially checked, and the LED AP combination with higher transmission bandwidth are chosen first to serve this UE. In addition, all the resource blocks, instead of merely one, of a selected LED AP combination can be assigned until their served UE is satisfied or no more resource block is left. Similar with the non-layered RA algorithm, the UE with least interfered neighbors in the VLC network is preferentially chosen in order the mitigate the ICI and fully develop the VLC network's access capability.

The directly overlapped RA algorithm described in Algorithm 2 can be summarized in the following 3 steps.

1) Hypergraph Construction.: Generate the system status matrix $\mathbf{A}_{I\times J}$ according to the channel SINR γ between each VLC AP and UE and the effective data link SINR threshold γ_{th}. Construct the system hypergraph $\mathcal{H}_1$ on the 1st resource block by edges joining each UE to all of its available LED APs.

2) Hypergraph Simplification: Let sets $\mathcal{X}_{\mathrm{done}}$ and $\mathcal{Z}_{\mathrm{done}}$ represent the LED AP set with all resource blocks assigned and UE set with all requirement satisfied, respectively. Simplify $\mathcal{H}_1$ by moving the VLC APs covered no UEs $\mathcal{X}_i = \emptyset$ from $\mathcal{V}_1(\mathcal{H}_1)$ to the LED AP set $\mathcal{X}_{\mathrm{done}}$, and the UEs having no effective data transmission link $\mathcal{Z}_j = \emptyset$ from $\mathcal{V}_2(\mathcal{H}_1)$ to the UE set $\mathcal{Z}_{\mathrm{done}}$.

3) Resource Allocation: Choose the UE-*j* with least interfered neighbors. Generate the available combination set $\mathcal{C}_j$ with the LED APs among the chosen UE-*j*'s amorphous cell $\in \mathcal{X}_j$, and sort the LED AP combinations $\mathcal{C}_j$ by the transmission bandwidth in decreasing order. Assign the resource blocks of all the LED APs in the first LED AP combination C_1 to UE-*j*, until it is satisfied or all the available LED AP combinations $\mathcal{C}_j$ are checked. If UE-*j* is satisfied, move UE-*j* from $\mathcal{V}_2(\mathcal{H}_1)$ to $\mathcal{Z}_{\mathrm{done}}$. Check whether the resource blocks of each LED APs in $\mathcal{X}_j$ are all assigned, if yes, move the LED APs from $\mathcal{V}_1(\mathcal{H}_1)$ to $\mathcal{X}_{\mathrm{done}}$. Rerun the resource allocation until either $|\mathcal{X}_{\mathrm{done}}| = I$, $|\mathcal{Z}_{\mathrm{done}}| = J$, or all the UEs are checked.

Algorithm 2: Directly Overlapped RA Algorithm.

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Input: Channel SINR between each VLC AP and UE γ, effective data link SINR threshold γ_th, resource block number of each VLC AP K_i, required data transmission rate for each UE
Output: Resource allocation schedule A_{I×K}
Generate the hypergraph H_1 on the 1st resource block with the channel SINR γ and SINR threshold γ_th;
Initialize LED AP set X_done = Ø, UE set Z_done = Ø and UE set Z_checked = Ø;
while |X_done| ≠ I, |Z_done| ≠ J or |Z_checked| ≠ J
    X_done = X_done ∪(X_j = Ø), V_1(H_1) = V_1(H_1) \ X_done;
    Z_done = Z_done ∪(Z_i = Ø), V_2(H_1) = V_2(H_1) \ Z_done;
    Choose UE-j ∈ E_1, where UE-j has the least neighbors;
    Generate the available combination set C_j with APs ∈ X_j;
    Calculate the SINR and its corresponding data transmission rate for each LED AP combination Cj ∈ C_j;
    Sort combination set C_j by the corresponding data transmission rate in decreasing order;
    for each Cj ∈ C_j do
        Generate the available resource block set K for Cj
        for each available resource block k ∈ K do
            if UE-j is NOT satisfied then
                Assign the resource block k of AP combination C_j to UE-j;
            end if
        end for
        Z_checked = Z_checked ∪UE-j, V_2(H_1) = V_2(H_1) \ UE-j;
    end for
    if UE-j is satisfied then
        Z_done = Z_done ∪UE-j;
    end if
    for each LED AP-i ∈ C_j do
        if all resource blocks of AP-i are assigned then
            X_done = X_done ∪AP-i, V_1(H_1) = V_1(H_1) \ AP-i;
        end if
    end for
end while
```

IV. Performance Analysis

In this section, a typical indoor circumstance is set up to analyze the performance of the proposed RA algorithms. VLC network performance including the user satisfaction, the spectrum efficiency and the user fairness are carefully analyzed in this section. Let I_S represents the UE service satisfaction index (SSI)

$$I_S = \frac{1}{|\mathcal{J}|} \cdot \sum\nolimits_{j\in\mathcal{J}} \left[\sigma\left(\frac{\sum_{k\in\mathcal{K}} R_j^k}{\mathcal{R}_j}\right)\right]^2, \tag{18}$$

which is used to indicate the satisfaction performance of all UEs in the VLC network. $R_j^k = \frac{1}{2} \cdot \log_2(1 + \frac{e\cdot\gamma_j^k}{2\pi})$ and γ_j^k are the spectrum efficiency and SINR of UE-*j* on resource block *k*, respectively. $\mathcal{R}_j$ is the data transmission rate requirement of UE-*j*, and $\sigma(\cdot)$ is the top function defined as

$$\sigma(x) = \begin{cases} 1 & x > 1 \\ x & otherwise \end{cases}. \tag{19}$$

Let E_S presents the spectrum efficiency of VLC network

$$E_S = \frac{1}{|\mathcal{K}|} \cdot \sum\nolimits_{k\in\mathcal{K}} \sum\nolimits_{j\in\mathcal{J}} R_j^k. \tag{20}$$

Let I_F presents the service fairness index (SFI) [23]

$$I_F = \frac{\left[\sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} R_j^k\right]^2}{|\mathcal{J}| \cdot \sum_{j\in\mathcal{J}} \left[\sum_{k\in\mathcal{K}} R_j^k\right]^2} = \frac{[|\mathcal{K}| \cdot E_S]^2}{|\mathcal{J}| \cdot \sum_{j\in\mathcal{J}} \left[\sum_{k\in\mathcal{K}} R_j^k\right]^2}, \tag{21}$$

which is used to indicate the UE fairness performance of all UEs in the VLC network.

A. System Configuration

An 8 m × 8 m × 3 m (width×length×height) room with uniformly distributed 9 and hexagonally distributed 7 LED lamps deployed in the ceiling is considered, shown in Fig. 2. Varied UEs with different data transmission requirements are randomly located in this room. To simplify the computation, all the detectors on the UEs are assumed to vertically pointed to the ceiling and mounted on the same height of 0.7 m. And the UE locations are modeled to obey a uniform distribution ranging from 0 m to 8 m in both x and y axes. All the relevant system parameters are listed in Table I.

The data transmission requirements of varied UEs mainly depend on the UEs' data service types. Based on the service benchmarks like data rate and delay time, the typical data service of a UE can be classified into three types, the short message service, the voice service and the video service. And a UE may randomly take merely one data service or any combination of them. The typical date transmission rates are 1 kbps, 8 kbps and 2 Mbps for the short message service, voice service and video service (720 P@30fps), respectively. And the UE data transmission requirements are also modeled to obey a uniform distribution among all the data service combinations. While the bit error rate upper limit of the forward error correction (FEC) coding with 7% coding overhead is 3.8×10^{-3}, the SINR thresholds γ_{th} for different modulation schemes and orders are listed in Table II.

B. Numerical Results

Fig. 3 shows the UE SSI, SFI and spectrum efficiency of the proposed non-layered and the directly overlapped RA algorithms against the UE number, while the detector FoV, the LED

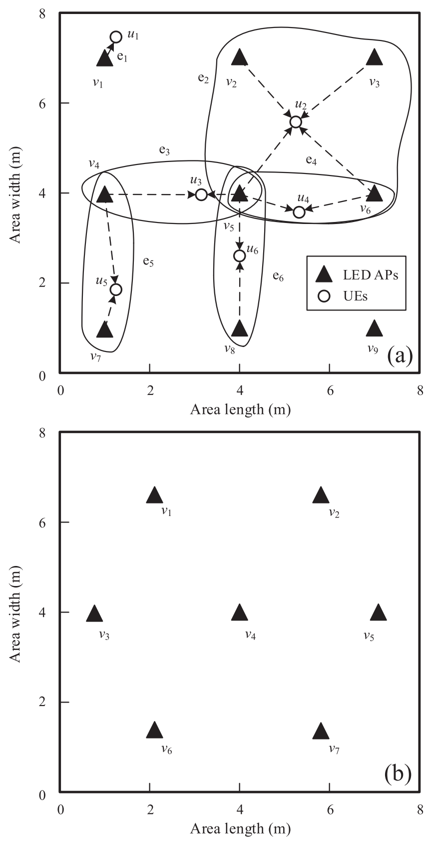

Fig. 2. Layout of LED APs and UEs projected on the horizontal plane, the dotted lines with arrows are the effective data transmission link. (a) Uniformly distributed LED lamps. (b) Hexagonally distributed LED lamps.

TABLE I
RELEVANT SYSTEM PARAMETERS

Parameter	Value
LED power, P_{ot}	10 W
Semi angle at half power, $\Phi_{1/2}$	45°
PD area, A	4 mm^2
Refractive index n	1.5
Optical filter gain	1
Reflectance factor of the wall ρ	0.7
Background current I_{bg}	5.1×10^{-3}A
Open-loop voltage gain G	10
FET channel noise factor Γ	1.5
FET transconductance g_m	3×10^{-3} mS
Fixed capacitance of photo detector η	112 pF/cm^2
Equivalent noise bandwidth b	10 MHz
Boltzmann's constant k	1.28×10^{-23}
Detector responsibility R_e	0.54
Noise bandwidth factors I_2	0.562
Noise bandwidth factors I_3	0.0868

TABLE II
MODULATION SCHEMES AND ORDERS

Modulation scheme	Modulation order	SINR threshold
OOK/BPSK	2	5.6 dB
QPSK	4	8.61 dB
PSK	8	13.47 dB
QAM	16	15.22 dB
QAM	32	19.19 dB
QAM	64	21.18 dB
QAM	128	24.95 dB
QAM	256	26.93 dB

semi-angle at half illuminance and the subchannel bandwidth are fixed at 40°, 45° and 1000 kHz, respectively. The solid and dotted curve are for the uniformly and hexagonally distributed LED lamps, respectively. And it is the same for the following figure in this section. In Fig. 3(a), the UE SSI for all the three RA algorithms gradually decline when the UE number grows. This mainly results from mismatch between the increase of the total data transmission requirements and limited access capability of the LED APs. The non-layered and directly overlapped RA algorithms greatly outperforms the classic Dsatur RA algorithm, as they both allocate as many resource blocks as possible at a time to satisfy the UE data transmission requirements. And the directly overlapped RA algorithm slightly outperforms the non-layered as it further takes the SINR improvement advantage resulted from the cooperation of available LED APs, and increases the data transmission rate by employing an advanced modulation scheme or a higher modulation order.

For the UE SFI of the three algorithms, they all start from 1 when there only exist one UE in the VLC network, and decline drastically then gradually when the UE number grows. This is because the UE SFI benchmark is primarily designed to measure the allocated resource block difference among all the UEs, and more UEs in the VLC network will introduce more divergence. Also the directly overlapped RA algorithm outperforms the former two algorithms for the similar reasons listed above.

The spectrum efficiency of all the three algorithms increase rapidly when the UE number grows. This is mainly resulted from increase of the resource block allocation rate of each LED AP, as more data transmission requirement will appear in the VLC network when the UE number grows. And the directly overlapped RA algorithm outperforms the former two algorithms for the similar reasons in Fig. 3(a).

It can be found from Fig. 3 that the spectrum efficiency is improved when the UE number grows, but at the cost of sacrificing the UE satisfaction and fairness performance. The proposed non-layered and directly overlapped RA algorithms outperform the classic Dsatur either with the uniformly or the hexagonally distributed LED lamps. This proves the applicability of the proposed RA algorithms against different LED lamp distributions in the ceiling. Finally, all the three RA algorithms with the uniformly distributed LED lamps outperform the hexagonally distributed LED lamps. This is mainly because there exist 9 instead of 7 LED lamps, and more LED APs usually means more access capability for the UEs.

Fig. 4 shows the performance of the proposed RA algorithms against the detector FoV angle, while the UE number, the LED semi-angle and the subchannel bandwidth are fixed at 20, 45° and 1000 kHz, respectively. The UE SSI, spectrum efficiency and SFI of all the three RA algorithms firstly increase when the detector FoV grows from 10°, and then decrease when the

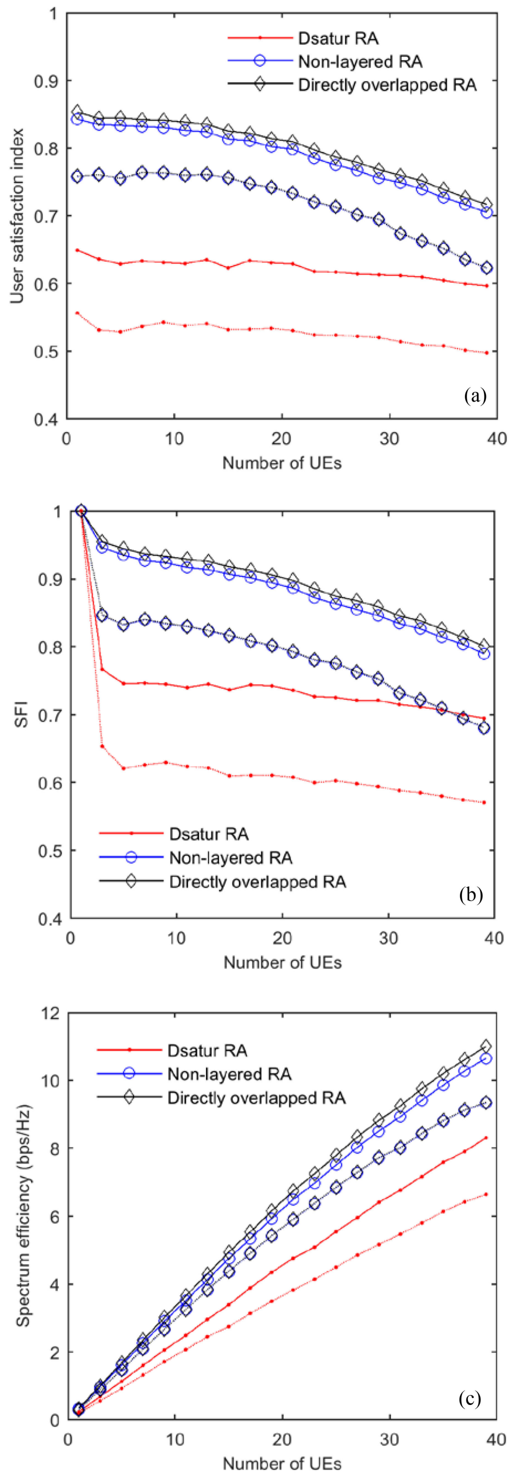

Fig. 3. (a) UE SSI against UE number. (b) SFI against UE number. (c) Spectrum efficiency against UE number.

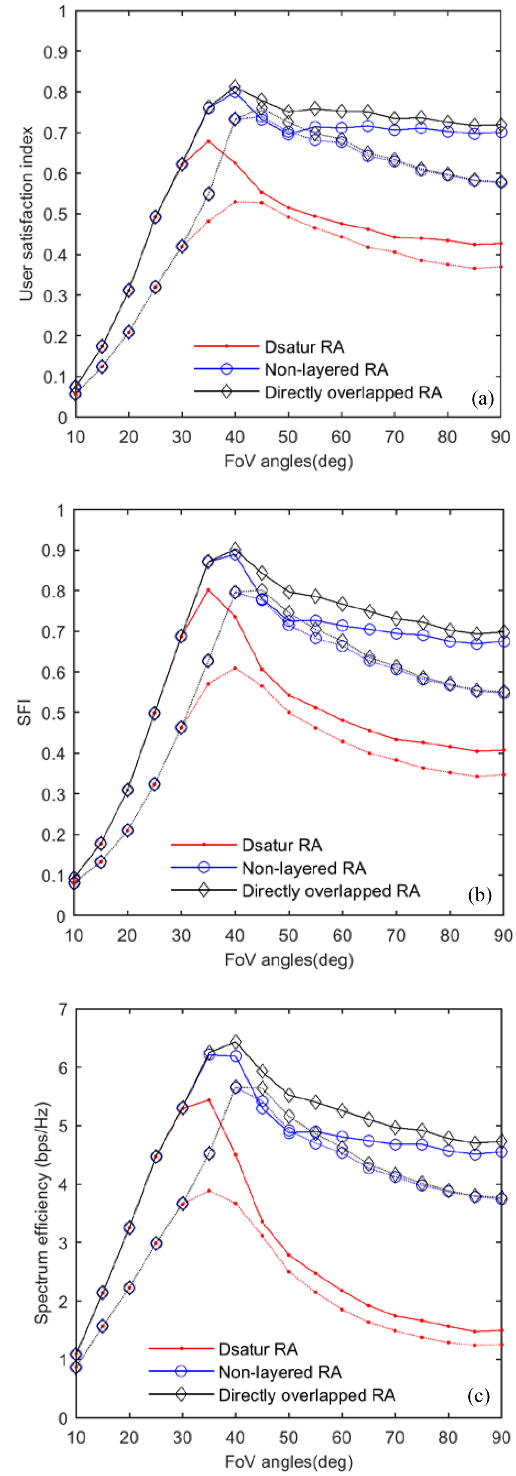

Fig. 4. (a) UE SSI against detector FoV. (b) SFI against detector FoV. (c) Spectrum efficiency against detector FoV.

detector FoV keeps growing up to 90°. This is mainly because most UEs may not capture any effective light signal from any LED APs and fail the VLC data link when the FoV is too small, but too many available LED APs for one UE may also introduce severe ICI on the VLC data link when the FoV is too large. Not surprisingly, the directly overlapped RA algorithm is superior to the former two RA algorithms for the similar reason discussed above.

As seen from Fig. 4, it can be observed that the detector FoV angle will significantly limit the VLC network performance. A properly selected FoV helps improve the VLC network performance, and the LED lamp distribution form, as well as the

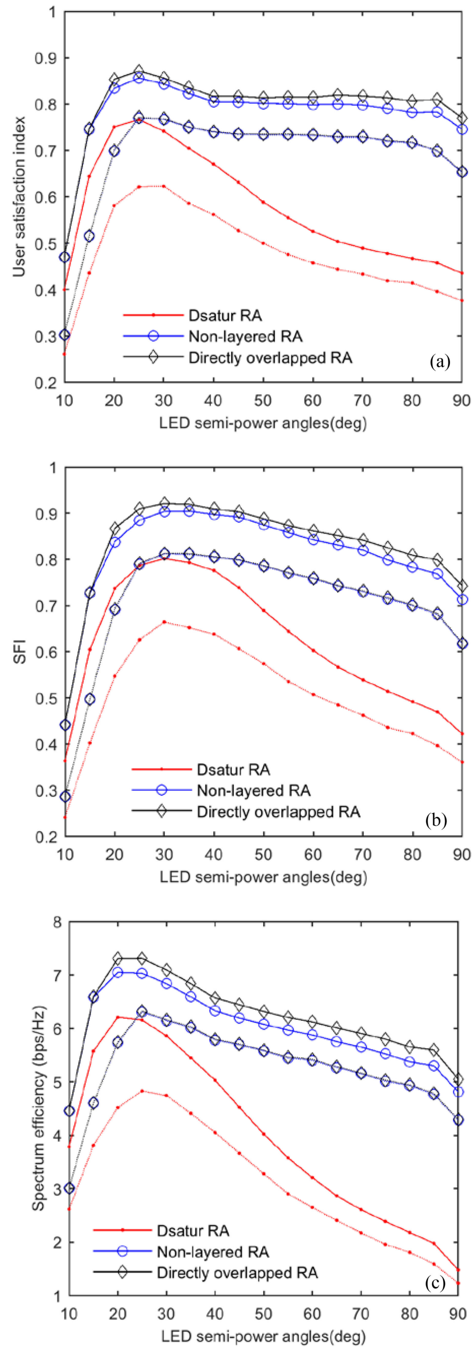

Fig. 5. (a) UE SSI against LED semi-angle. (b) SFI against LED semi-angle. (c) Spectrum efficiency against LED semi-angle.

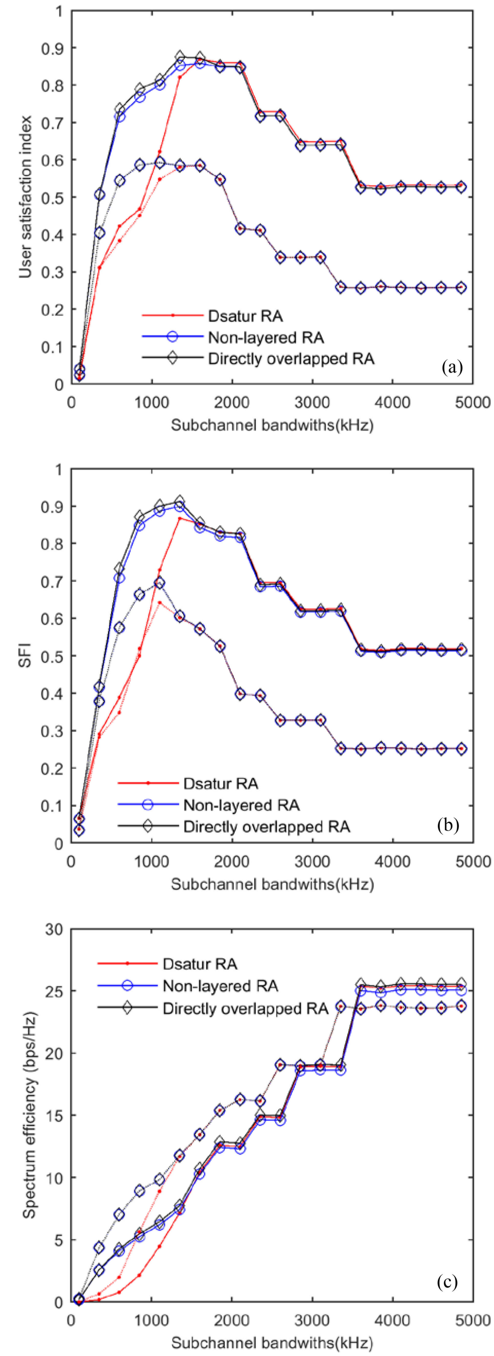

Fig. 6. (a) UE SSI against subchannel bandwidth. (b) SFI against subchannel bandwidth. (c) Spectrum efficiency against subchannel bandwidth.

vertical distance between the LED lamps and the detector, also closely related to the VLC network performance.

Fig. 5 shows the performance of the proposed RA algorithms against the LED semi-angle, while the UE number, the detector FoV angle and the subchannel bandwidth are fixed at 20, 40^{o} and 1000 kHz, respectively. Similar with the performance against detector FoV angle given in Fig. 4, the UE SSI, spectrum efficiency and SFI of all the three RA algorithms firstly increase when the LED semi- angle grows from 10^{o}, and then decrease when the semi-power angle keeps growing up to 90^{o}. This is mainly because the Lambertian emission order declines sharply when the LED semi- angle grows, and makes the channel gain between LED AP and UE firstly grows dramatically then declines gradually. Not surprisingly, the directly overlapped RA algorithm is superior to the former two RA algorithms for the similar reason discussed above.

It can be observed that the LED semi-angle will significantly limit the VLC network performance, and a properly selected LED semi-angle will help to improve the VLC network performance. As the LED semi-angle is closely related the illumination uniformity performance in the room, there also exists a tradeoff between the VLC network and the indoor illumination performance.

Fig. 6 shows the performance of the proposed RA algorithms against the subchannel bandwidth, while the UE number, the detector FoV angle and the LED semi-angle are fixed at 20, 40^o and 45^o, respectively. The UE SSI and SFI of all the three RA algorithms firstly increase when the bandwidth grows from 100 kHz, and then decrease when the bandwidth keeps growing up to 2500 kHz. This is mainly because higher bandwidth promises higher data transmission rate on one resource block, but excessive bandwidth will decrease not only the resource block number but the served UE number of each LED AP. The spectrum efficiency of all the three RA algorithms increases rapidly when the subchannel bandwidth grows, which is resulted from the decrease of the available resource block number of each LED AP, and thus the increase of resource block allocated rate of the VLC network.

The directly overlapped RA algorithm slightly outperforms the non-layered RA algorithm, and greatly outperforms the classic Dsatur RA algorithm for the similar reason discussed above, when the subchannel bandwidth is relatively small. But surprisingly, the performance of the classic Dsatur RA algorithm gets better and even slightly superior to the non-layered and the directly overlapped RA algorithm when the subchannel bandwidth is more than 1500 kHz. This is mainly because one resource block can satisfy most of the UEs' data service types or combinations, and carrier aggregation method in the non-layered and directly overlapped RA algorithms can satisfy high bandwidth requirement, but waste the chance to serve more UEs.

V. Conclusion

For a typical indoor VLC network with densely deployed heterogeneous LED lamps and a central controller, the RA problem for varied UEs (or service requirements) has been investigated. To satisfy the different data transmission requirements of varied UEs under the constrain of limited access capabilities of heterogeneous LED APs, both the LoS and the primary reflected NLoS light signals are considered. To mitigate the ICI resulted from densely deployed LED lamps and improve the VLC network performance, the hypergraph theory has been introduced in detail in this paper. The non-layered RA algorithm based on the carrier aggregation method, and the directly overlapped RA algorithm based on both the carrier aggregation and cooperative transmission methods were proposed. Numeric results have shown that the proposed RA algorithms are superior to the classic Dsatur RA algorithm in terms of the UE satisfaction, spectrum efficiency and SPI, and is appropriate for different LED lamp distributions. The directly overlapped RA algorithm has the best performance, as it can fully take the data link SINR improvement advantage resulted from the cooperative transmission of the LED APs. It was also observed from the numeric results that the detector FoV angle strictly limits the VLC network performance, as small FoV may fail the VLC data link, but large FoV introduces severe ICI. In addition, a properly selected LED semi-angle can maximize the data link channel gain and improve the VLC network performance at the sacrifice of the indoor illumination performance.

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