Design of Ring-Core Few-Mode-EDFA With the Enhanced Saturation Input Signal Power and Low Differential Modal Gain

Yihong Fang, Yan Zeng, Yuwen Qin, Ou Xu, Jianping Li, and Songnian Fu

***Abstract*—We propose a design of ring-core few-mode fiber (RC-FMF) with the erbium doping at the cladding region near the outer ring edge. After the parameter optimization of the RC-FM-EDF, the intensity overlapping difference between LP_{01} and LP_{11} modes can be minimized, leading to a DMG of 0.22 dB for the corresponding ring-core few-mode erbium-doped fiber amplifier (RC-FM-EDFA). Meanwhile, in comparison with uniform core doping design, the saturation input signal power of the RC-FM-EDFA can be enhanced from −17.7 dBm and −16 dBm to −8.5 dBm for LP_{01} and LP_{11} modes, respectively. Consequently, our proposed RC-FM-EDFA is characterized by both low differential modal gain (DMG) and the enhanced saturation input signal power. Those new characteristics of the RC-FM-EDFA is ideally desired for future agile networks.**

***Index Terms*—Few-mode erbium-doped fiber amplifier (FM-EDFA), ring-core fiber, differential modal gain (DMG), saturation input signal power.**

I. Introduction

With the rapidly increasing requirement of transmission bandwidth, space-division-multiplexing (SDM) technique based on either few-mode fiber (FMF) or multi-core fiber (MCF) have become a hot research topic around the world, in order to overcome the capacity crunch of traditional single-core single-mode fiber (SMF) [1]–[4]. To realize mode-division multiplexing (MDM) transmission over the long-haul FMF, inline optical amplifiers are indispensable. It has been reported that few-mode erbium-doped fiber amplifiers (FM-EDFAs) can be applied for the long-haul FMF transmission with good performances [5]–[21]. One of the most important FM-EDFA characteristics is the differential modal gain (DMG), which needs to be carefully mitigated, for the successful implementation of multiple-input multiple-output (MIMO) signal processing at the receiver side. Since it is revealed that the DMG is determined by the intensity overlapping between the guided mode and the pump mode, together with the erbium doping distribution [16], the existing schemes to mitigate the DMG can be generally divided into three categories. First, the mode profile of the pump laser can be optimized, and the cladding pumping becomes a popular choice because it can provide more uniform pump power distribution over the optical fiber cross section [5]. Secondly, the optimization of the erbium doping distribution is helpful [10]–[12], including the convenient step-index FMF with the ring doping [13] and the doping combination at both ring and central areas [14]. Finally, the corresponding refractive index (RI) profile can be designed in order to manipulate various intensity profiles of guided modes [21]. Since various mode profiles are concentrated for the ring-core erbium-doped fiber (RC-EDF), DMG is expected to reduce to a low level. A RC-FM-EDFA has been numerically investigated, by comparing the performance between the core and the cladding pumping [17]. Moreover, small DMG of 1.6 dB has been achieved for the two-mode RC-EDFA [18], and a procedure of the RC-FM-EDFA design has been reported to achieve a DMG of around 1 dB [19]. Moreover, a RC-FM-EDFA is numerically investigated to obtain an DMG of less than 0.6 dB by the use of extra annulus doping [20], and the DMG suppression can be realized via additional trench configuration of the RC-FM-EDF [21]. Meanwhile, there occurs another headache for the gain saturation effect arising in the EDFA, due to the small saturation input signal power. Therefore, the total input power must set less than the saturation input signal power, in order to secure the linear optical amplification. Researchers are focused on optimizing the erbium doping profile to reduce the gain compression in a single-mode EDFA, and a single-mode EDFA with 4 dB gain compression is demonstrated under a cladding pumping power of 25W [22].

In this paper, we design a RC-FM-EDF and investigate the characteristic of corresponding optical amplifier, under the condition of the cladding pumping. The DMG between LP_{01} and LP_{11} modes can be reduced to 0.22 dB, due to the parameter

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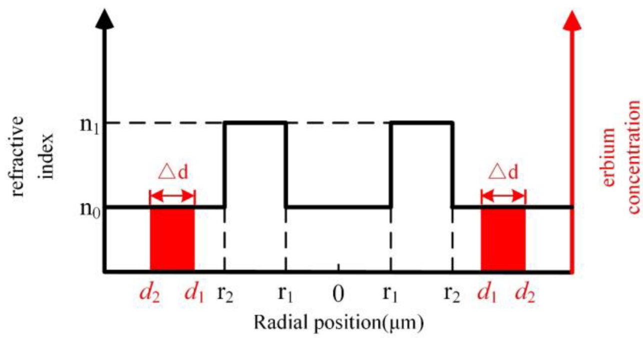

Fig. 1. Schematic of RC-FM-EDF, and the red area is denoted as the erbium doping region.

optimization of the RC-FM-EDF. Meanwhile, the saturation input signal power can be enhanced with the optimization of distribution of the erbium ions doping. We believe that, the corresponding RC-FM-EDFA with low DMG and large saturation input signal power is highly desired for future agile optical networks.

II. Design of the RC-FM-EDF

The schematic of the RC-FM-EDF is shown in Fig. 1, where r_1 and r_2 are the inner and outer radii, and the RI of the core and the cladding are represented as n_1 and n_0, respectively. First, we investigate the guided mode arising in the RC-FM-EDF, in order to specify the range of r_1 and r_2, for the ease of only LP_{01} and LP_{11} mode propagation at 1550nm. We calculate the cutoffs of LP_{01} and LP_{11} modes with respect to the inner and outer radii of RC-FM-EDF, by the use of the transfer matrix method [23]. When the relative RI difference is $\Delta\mathrm{n} = 0.4\%$, the cutoffs of each mode at 1550 nm can be obtained, as shown in Fig. 2. Here, the blue line represents the cutoff line of LP_{11} mode while the green one is the cutoff line of LP_{21}, and the region between two cutoff lines only allows LP_{01} and LP_{11} modes to co-propagate at 1550 nm. Since the gain characteristics of the odd and even modes are the same [24], only LP_{11} mode is discussed in the following.

Next, we consider setting the erbium doping region at the low RI region, in order to mitigate the gain saturation effect, as shown in Fig. 1. When the input signal power is more than the saturation input signal power, the pump can no longer replenish the inversion as fast as the signal depletes it, and gain saturation effect is caused. As for the EDFA, the population inversion is determined by the intensity overlapping of signal and pump at the erbium doping region, and therefore the mitigation of the gain saturation effect becomes possible by reducing the intensity profile of signal in relevant to the erbium doping region, under the condition of the cladding pumping. Thus, the erbium doping near the core region is helpful to only cover the evanescent tails of the intensity profile for all guided modes, thus leading to the successful mitigation of the gain saturation effect.

Because minimizing the difference of overlapping factor is effective to mitigate the DMG, the difference of the overlapping factor between LP_{01} and LP_{11} modes in the erbium doping region is also considered. The overlapping factor of the *i*-th signal mode Γ_i can be described as [25],

$$\Gamma_i = \int_0^{2\pi}\int_{d_1}^{d_2} I_{s,i}(r,\varphi) I_{p,j}(r,\varphi) N_0(r,\varphi) r dr d\varphi \quad (1)$$

where $I_{s,i}$ and $I_{p,j}$ are the normalized intensity profiles of the *i*-th signal mode and the *j*-th pump mode, respectively, $N_0(r,\varphi)$ is the total number of erbium ions per unit volume. Under the condition of the cladding pumping, the normalized intensity of the pump can be treated as a uniform one over the erbium doping area [26]–[28]. The difference of the overlapping factor $\Delta\Gamma$ for LP_{01} and LP_{11} modes is derived

$$\begin{aligned}\Delta\Gamma &= |\Gamma_{01} - \Gamma_{11}| \\ &= I_p N_0 \left| \int_0^{2\pi}\int_{d_1}^{d_2} I_{s,\mathrm{LP}_{01}} r dr d\varphi - \int_0^{2\pi}\int_{d_1}^{d_2} I_{s,\mathrm{LP}_{11}} r dr d\varphi \right|\end{aligned} \quad (2)$$

where $I_{s,\mathrm{LP}_{01}}$ and $I_{s,\mathrm{LP}_{11}}$ represent the normalized intensity profiles of LP_{01} and LP_{11} modes, respectively, the range of erbium doping area is from d_1 to d_2, leading to an erbium doping width of $\Delta d = d_2 - d_1$. The overlapping factor difference is determined by the field distribution difference ΔI between LP_{01} and LP_{11} modes, which is

$$\Delta I = \left| \int_0^{2\pi}\int_{d_1}^{d_2} I_{s,\mathrm{LP}_{01}} r dr d\varphi - \int_0^{2\pi}\int_{d_1}^{d_2} I_{s,\mathrm{LP}_{11}} r dr d\varphi \right| \quad (3)$$

Hence, DMG can be evaluated by calculating the difference of the field distribution ΔI between LP_{01} and LP_{11} modes.

In order to maintain a relatively larger gain, the erbium doping region must be chosen as near as possible to the core, and thus the inner radius of the doped range is chosen the same as r_2, i.e., $d_1 = \mathrm{r}_2$. Considering the erbium doping processing during the EDF fabrication, Δd should set to be larger than 0.5 μm. Meanwhile, the mode profile of LP_{01} arising in the RC-FMF is similar to a doughnut shape, while that of LP_{11} is similar to two petals. Therefore, when the parameter combination of r_1 and r_2 is closed to the cutoff line of LP_{11}, the more energy of LP_{11} mode may leak into the cladding area, resulting in that the energy of LP_{11} closer to that of LP_{01} mode at the cladding region. Consequently, the parameter combination is chosen near to the cutoff line of LP_{11}, in order to achieve a low ΔI at the erbium doping region. Meanwhile, in order to ensure the steady propagation of LP_{01} and LP_{11} modes with low modal crosstalk, the effective RI Δn_{eff} between LP_{01} and LP_{11} modes should be larger than 1×10^{-3}. After the numerical simulation, Δn_{eff} is gradually reduced to smaller than 1×10^{-3}, when r_1 is more than 4 μm, and thereby the inner radii should not be more than 4 μm. The field distribution difference between LP_{01} and LP_{11} are calculated by considering next four scenarios, including (1) the cutoff line of LP_{11}+0.8 μm, (2) the cutoff line of LP_{11}+0.6 μm, (3) the cutoff line of LP_{11}+0.4 μm, and (4) the cutoff line of LP_{11}+0.2 μm, which are denoted as the red circle, star, square and angle line, respectively, as shown in Fig. 1. The difference of the field distribution ΔI is shown in

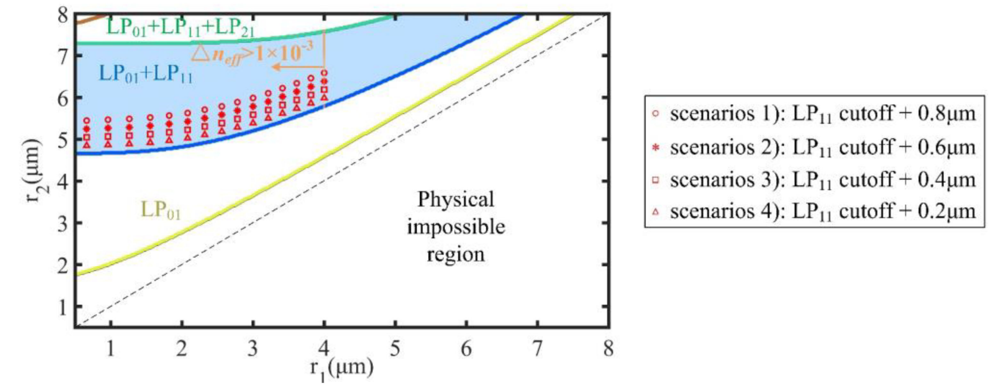

Fig. 2. Map of existing LP modes for various combinations of r_1 and r_2.

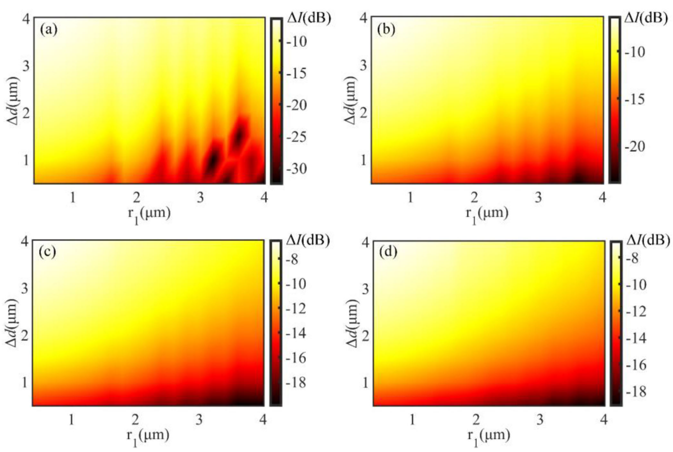

Fig. 3. ΔI between LP_{01} and LP_{11} modes with various erbium doping widths when scenarios are (a) the cutoff line of $LP_{11} + 0.2$ μm, (b) the cutoff line of $LP_{11} + 0.4$ μm, (c) the cutoff line of $LP_{11} + 0.6$ μm and (d) the cutoff line of $LP_{11} + 0.8$ μm, respectively.

Fig. 3, corresponding to various Δd and the above-mentioned four scenarios. When the inner radius is $r_1 = 4\,\mu$m, small ΔI can be obtained in Figs 3(a)–(d). Therefore, the inner radius of RC-FM-EDF is chosen as $r_1 = 4$ μm.

Next, the relationship between ΔI and various parameter combinations of the erbium doping width $r_2 = 6\,\mu$m and outer radii r_2 is investigated under the condition of $r_1 = 4\,\mu m$, as shown in Fig. 4(a). We can observe that the lowest ΔI of -73.8 dB can be obtained when $r_2 = 6\,\mu$m and $\Delta d = 0.5\,\mu$m are chosen. Finally, the parameters of RC-FM-EDF are $r_1 = 4\,\mu$m, $r_2 = d_1 = 6\,\mu$m, and $d_2 = 6.5\,\mu$m. The corresponding normalized intensity profiles of LP_{01} and LP_{11} modes are indicated in Fig. 4(b).

Alternatively, we also take into account of another possible erbium doping region at the low RI region from 0 μm to 4 μm, as shown in Fig. 5(a). Although the erbium doping region is still within the low RI area, the smallest ΔI of -25 dB is obtained, as shown in Fig. 5(b), leading to a large DMG. Therefore, the optimal scheme to obtain both low DMG and high saturation input signal power is to realize the erbium doping at the cladding region closed to the outer radius with an erbium doping width of 0.5 μm.

III. Characteristics of RC-FM-EDFA

The operation of the FM-EDFA can be described by the coupled differential equations for the signal powers $P_{s,i}$, amplified spontaneous emission (ASE) powers $P_{ase,i}$, and cladding pump powers P_p [15], [29], [30], as shown in (4).

$$\begin{cases} \dfrac{dP_{s,i}(z)}{dz} = P_{s,i}(z)\int_0^{2\pi}\int_{d_1}^{d_2} I_{s,i}\left(\sigma_{es}N_2(z) - \sigma_{as}N_1(z)\right) rdrd\varphi \\ \dfrac{dP_{\pm ase,i}(z)}{dz} = P_{\pm ase,i}(z)\int_0^{2\pi}\int_{d_1}^{d_2} I_{s,i}\left(\sigma_{es}N_2(z)\right. \\ \qquad \left. - \sigma_{as}N_1(z)\right) rdrd\varphi \\ +2\int_0^{2\pi}\int_{d_1}^{d_2} \sigma_{es} h v_s \Delta v N_2(z) I_{s,i} rdrd\varphi \\ \dfrac{dP_p(z)}{dz} = -P_p(z)\int_0^{2\pi}\int_{d_1}^{d_2} \sigma_{ap} N_1(z) I_{p,j} rdrd\varphi \end{cases} \tag{4}$$

where $P_{s,i}(z)$, $P_{ase,i}(z)$ and P_p represent the power of *i*-th signal mode, ASE and pump, respectively. σ_{es} and σ_{as} denote the emission and absorption cross-sections for the signal mode, and σ_{ap} is the absorption cross-sections for the pump [16], respectively. Moreover, Δv and v_s are the amplifier bandwidth and the signal frequency, respectively, and h is the Planck constant. The EDF is modeled with a quasi-three-level system under the 980 nm cladding pumping. Propagation loss of EDF is ignored during the numerical simulation. The populations at the low and upper energy levels N_1 and N_2 are

$$\begin{cases} N_2(z) = \frac{\Omega_a N_0}{\Omega_a + \Omega_e}, N_1(z) = N_0 - N_2(z) \\ \Omega_a = \sum\limits_{i=1}^{m} \frac{P_{s,i}(z) + P_{ase,i}(z)}{h v_s}\sigma_{as} I_{s,i} + \sum\limits_{j=1}^{n} \frac{P_{p,j}(z)\sigma_{ap} I_p}{h v_p} \\ \Omega_e = \sum\limits_{i=1}^{m} \frac{P_{s,i}(z) + P_{ase,i}(z)}{h v_s}\sigma_{es} I_{s,i} + \frac{1}{\tau} \end{cases} \tag{5}$$

where N_0 is the total number of erbium ions per unit volume, Ω_e and Ω_a denote the emission and absorption rates, respectively, m refers to the number of the signal guided modes and n is the total number for pump modes. Because of the cladding pumping, $n = 1$ is satisfied. v_p is pump frequency and the spontaneous emission lifetime is τ. Using the fourth-order Runge-Kutta method [27],

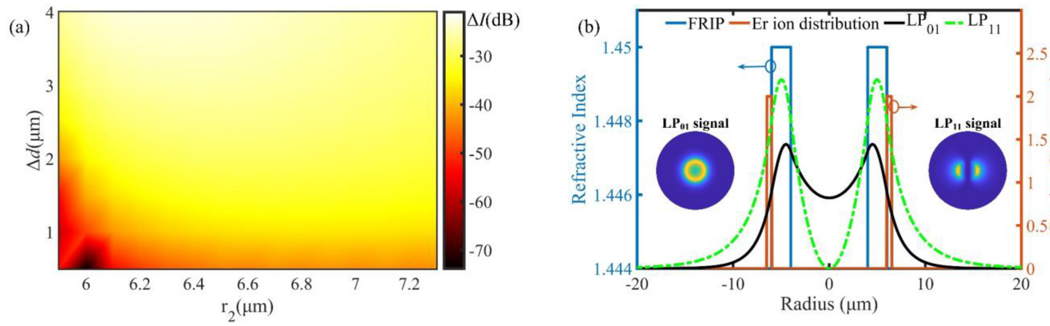

Fig. 4. (a) ΔI between LP_{01} and LP_{11} with various doping widths Δd and outer radii r_2 under the condition of $\mathrm{r}_1 = 4\,\mu\mathrm{m}$. (b) The schematic diagram of RC-FM-EDF and the corresponding normalized intensity profiles of LP_{01} and LP_{11} modes.

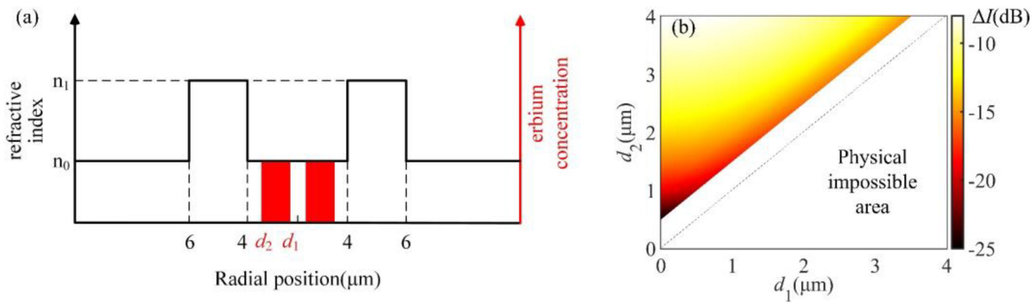

Fig. 5. (a) Schematic diagram of erbium doping at the low RI region near the inner cladding, and (b) corresponding ΔI for different d_1 and d_2.

TABLE I
PARAMETERS OF THE RC-EDFA

Parameters	Value	Unit
pump power P_p	14.0	W
small signal input power P_s	30.0	μW/mode
fiber length L	10	m
Er^{3+} concentration N_0	2×10^{25}	ions/m^3
τ	10	ms

[31], the simultaneous differential (4) and (5) can be solved numerically with the boundary condition of input and output of the RC-FM EDFA, i.e., $z = 0$ and $z = L$. The signal gain is defined as

$$\mathrm{G}_i(\mathrm{dB}) = 10\log_{10}\frac{P_{s,i}(z = L)}{P_{s,i}(z = 0)} \quad (6)$$

In order to achieve more than 20 dB small-signal gain (SSG) for each mode, the parameters of the RC-FM-EDFA are set as shown in Table I.

The impact of erbium doping on the fiber refractive index can be reasonably ignored, because the mole fraction for the Er-doped SiO_2 with Er doping concentration at the order of 10^{25} m^{-3} is below 1‰ [32]. The gain variations of LP_{01} and LP_{11} modes with respect to the RC-FM-EDF length is shown in Fig. 6(a). In order to evaluate the amplifier characteristic of the RC-FM-EDFA, the DMG is defined as [23]

$$\mathrm{DMG}(\lambda) = \max\{\,|\mathrm{G}(i,\lambda) - \mathrm{G}(j,\lambda)|\}\ \ (i \neq j) \quad (7)$$

where i and j denote the different signal modes arising in the RC-FM-EDF, and G is the modal gain. As shown in Fig. 6(a), not only can all modal SSGs exceed 20 dB but also low DMG can be obtained as 0.22 dB, when the erbium doping region is near the outer radius of the core. Meanwhile, the effect of the erbium doping region located both at core and outer cladding on the DMG performance are compared. When the erbium ions are assumed to be doped uniformly at the ring-core, under the condition of $d_1 = 4\,\mu\mathrm{m}$ and $d_2 = 6\,\mu\mathrm{m}$, the pump power should be tuned to 1.51W, in order to realize the SSG more than 20 dB for LP_{01} mode by the use of 10m RC-FM-EDF. In comparison with the cladding doping scheme, owing to the obvious difference of field distribution ΔI between LP_{01} and LP_{11} modes, the DMG is worsen to 4 dB.

Meanwhile, the relationship between the SSG of the RC-FM-EDFA and the input signal power is investigated, as shown in Fig. 6(b). The input signal power of each mode increases gradually from -40 dBm to 5 dBm, leading to the occurrence of gain saturation effect. Generally, the saturation input signal power is defined as an input signal power where its gain is compressed from its SSG value by 3 dB [22]. When the erbium doping range is at the core, the saturation input signal powers are -17.7 dBm for LP_{01} and -16 dBm for LP_{11} modes, respectively, as shown in Fig. 6(b). The overlap factor of the LP_{11} mode $\Gamma_{\mathrm{LP}_{11}}$ is smaller than that of LP_{01} mode $\Gamma_{\mathrm{LP}_{01}}$, leading to a weaker population inversion for LP_{11} than LP_{01}. Consequently, the saturation input power of LP_{11} mode is larger than that of LP_{01} mode. However, as the erbium doping region is at the cladding near to the outer radius of the core, owing to lower intensity at the erbium doping

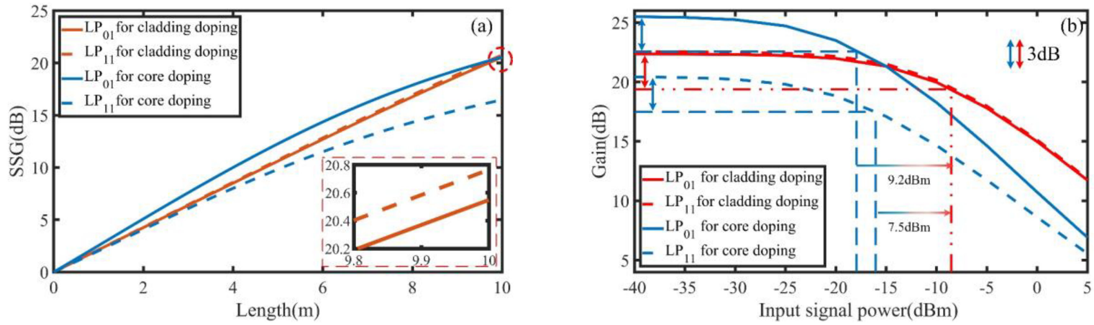

Fig. 6. (a) Small-signal gain with respect to the RC-FM-EDF length and (b) gain with respect input signal powers for different doping scheme.

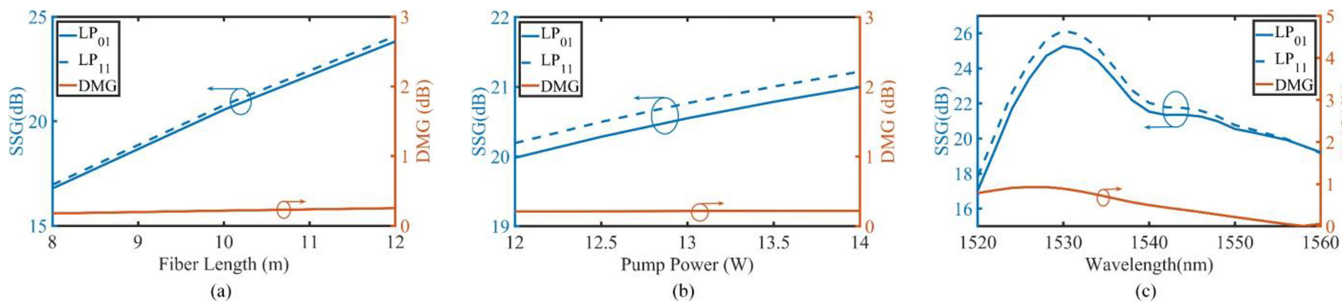

Fig. 7. Variations of SSG and DMG with respect to (a) RC-FM-EDF length, (b) pump power, and (c) operation wavelength.

area, less population inversions happen and the gain saturation effect can be mitigated. The saturation input signal power is about −8.5 dBm for each mode. The saturation input signal powers of LP_{01} and LP_{11} modes are enhanced by 9.2 dB and 7.5 dB, respectively, and the output signal can be enhanced from 7.63 dBm for the FM-EDFA with the core doping to 15.575 dBm for the proposed RC-FM-EDFA. Therefore, the linear optical amplification range can be substantially extended by our proposed RC-FM-EDFA.

Furthermore, relationships between modal gains versus the RC-FM-EDF length and the pump power are investigated. Fig. 7(a) shows the variation of gain and DMG with respect to the RC-FM-EDF length, when the pump power is 14W. In particular, when the RC-FM-EDF length is changed from 8 m to 12 m, the SSG of LP_{01} mode is increased from ∼17.5 dB to ∼24.6 dB, while the one for LP_{11} is increased from ∼17.6 dB to ∼24.8 dB simultaneously. The corresponding DMG is still below 0.25 dB. On the other hand, the variation of the SSG and DMG for 2-LP modes with the growing pump power is evaluated, as shown in Fig. 7(b). The modal SSG is enhanced with the gradually increasing pump power when the fiber length is 10m, while the DMG is stable. Both the SSG and the DMG versus the operation wavelength at C-band are evaluated. As shown in Fig. 7(c), the SSG of each mode is more than 19 dB over C-band, while the DMG can be obtained below 1 dB. When the input power is relatively large, the gain variation is almost the same as that of the SSG, and the gain compression effect occurs.

Finally, the fabrication tolerances of the proposed RC-FM-EDF and the deviation of the erbium doping region are evaluated. At the cladding region, the intensity profile of LP_{01} is rapidly decreased than that of LP_{11}. As shown in Fig. 8(a), when the erbium doping width is enlarged, the intensity of each mode is increased so that the SSG can be increased accordingly, and the SSG enhancement of LP_{01} mode is smaller than that of LP_{11} mode. However, ΔI between LP_{01} and LP_{11} modes is also enlarged with the increasing erbium doping width, according to the calculated result shown in Fig. 4(a). Thus, the DMG is worsen. What's more, when there occurs a deviation $\Delta\eta$ between the erbium doping area and the outer radius r_2 of the core, i.e., $\Delta\eta = d_1 - \mathrm{r}_2$, the variations of the SSG and the DMG are analyzed as well. Here, the erbium doping width is still set as $\Delta d = 0.5\,\mu\mathrm{m}$. As shown in Fig. 8(b), SSG of two LP modes are decreased, because the intensity of all guided modes gradually becomes weak when the erbium doping region moves outwards along the radial direction, and the SSG reduction of LP_{01} mode is faster than that of LP_{11} mode. Simultaneously, as the deviation $\Delta\eta$ is enlarged, the intensity profile difference ΔI is gradually increased, resulting in a worsen DMG, but the DMG is still smaller than 2 dB. From the above analyzing, the RC-FM-EDF has a high tolerance to the variation of structural parameters.

IV. Conclusion

We propose a RC-FM-EDFA design to simultaneously achieve both low DMG and higher saturation input signal power, under the condition of the cladding pumping. During the optimization procedure, we first specify the combinations of inner and outer radii, in order to maintain LP_{01} and LP_{11} modes propagation. Since the saturation input signal power can be

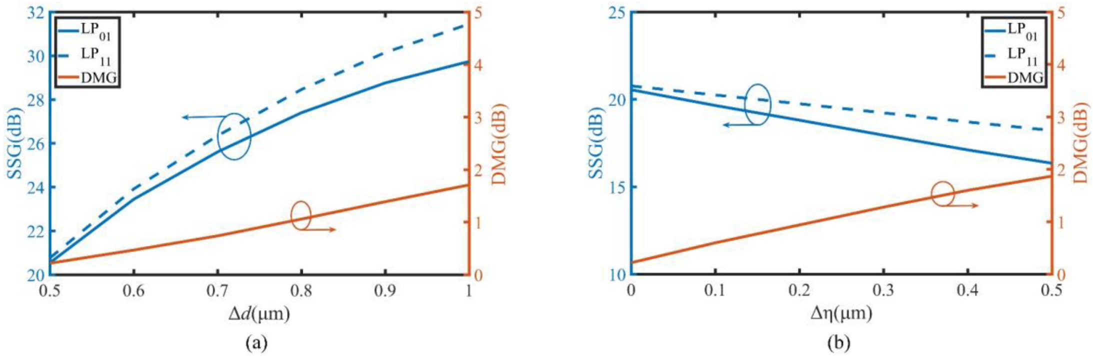

Fig. 8. Variations of SSG and DMG with respect to (a) the erbium doping width and (b) the deviation $\Delta\eta$ of the erbium doping region under the condition of 0.5 μm erbium doping width.

enhanced by properly choosing the erbium doping area at the cladding region, the optimal combination of inner and outer radii is obtained by comparing the intensity profile difference between LP_{01} and LP_{11} modes at the cladding erbium doping region. The proposed RC-FM-EDFA can achieve low DMG of 0.22 dB and enhance the saturated input power from -17.7 dBm for LP_{01} mode and -16 dBm for LP_{11} mode to -8.5 dBm simultaneously. Since the RC-FM-EDF design procedure can be extended to more guided modes, the corresponding RC-FM-EDFA with enhanced saturation input signal power and small MDG is ideally desired for future agile optical networks

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