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Numerical and Experimental Analysis of Yb:YAG Thin Disk Regenerative Amplifier

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Abstract: We present an numerical and experimental analysis of a Yb:YAG thin disk regenerative amplifier. Group velocity dispersion, third order dispersion, and self-phase modulation (SPM) effects are considered in the simulations of the amplification process. By virtue of the simulations, a compact femtosecond Yb:YAG thin-disk chirped-pulse regenerative amplifier delivering an average power of 50 W at a central wavelength of 1030 nm with a propagation factor of $M^2 < 1.4$ at a repetition rate of 200 kHz and a pulse duration of 500 fs was constructed. Numerical simulation based on this Yb:YAG thin-disk was carried out to investigate the pulse parameters evolution in each round trip. The measured experimental results in pulse width, spectrum, and pulse energy together fitted quite well with the numerical simulation. This plays an important role in the design of high-peak-power regenerative amplifier and will facilitate further power scaling and suppression of gain narrowing of the whole system.

Index Terms: Yb:YAG thin disk regenerative amplifier, high power, numerical simulation.

1. Introduction

Nowadays, high-power femtosecond lasers have become a powerful tool for many industrial and scientific applications. Industrial applications of femtosecond lasers involved of micromachining process such as shaping, drilling, and cutting. Popular scientific application fields are high harmonic generation or ultrafast spectroscopy. The high peak power is also beneficial to the nonlinear frequency conversion. The traditional method to generate such high energy ultrashort pulsed lasers is based on Ti: sapphire. This broadband gain medium could directly produce intense pulses with a duration of a few femtoseconds. Li *et al.* proposed a Ti: sapphire amplifier with output energy of 216.96 J with a pulse duration of 21 fs, implying a peak power of 10.3 PW [1]. However, the main drawback of Ti: sapphire is the large quantum defect related to its absorption band in the green spectral region, making it necessary to take elaborate ways for thermal management and application of relatively complicated pump lasers. By contrast, with brilliant thermal properties,

relatively broad emission bands, and especially high pumping efficiencies, Yb-doped materials make it possible to conduct efficient, robust, and compact femtosecond laser systems due to their reduced quantum defect between laser and pump photons and simple electronic-level structure. Yb-doped fiber lasers have been proved to be one of the best choices due to its capability of generating short pulse durations (usually between 300 and 500 fs) with high average power. Müller et al. conducted a 3.5 kW Yb all-fiber ultrafast laser system with a pulse duration of 430 fs with output energy of 43.75 μ J [2]. However, one of the main limitations is the output pulse energy available from this gain medium due to the strong confinement of the laser within the core of the fiber. Yb-doped bulk crystals are now widely recognized as excellent gain medium in diode-pumped ultrafast and high-power lasers. There are plenty of Yb-doped gain mediums such as Yb:KYW, Yb:KGW, Yb:CaF₂, Yb:CALGO, and Yb:YAG. Pouysegur *et al.* reported on a Yb:KYW regenerative amplifier which generated 0.01 mJ pulses with a duration of 145 fs [3]. Average output energy of 1.2 mJ based on Yb:KGW dual-crystal regenerative amplifier at a repetition rate of 1 kHz with a pulse width of 227 fs has been reported by He et al. [4]. P. Sevillano et al. presented a diode pumped ultrafast amplifier based on Yb:CaF₂ which delivered 32 mJ energy at a repetition rate of 200 Hz with a pulse width of 300 fs [5]. Caracciolo et al. demonstrated a 28-W, solid-state Yb:CALGO regenerative amplifiers with a pulse duration of 214 fs [6].

But Yb-doped-crystals mentioned above usually suffer from other deleterious effects. In general, the low gain and poor thermal parameters originates from the disorder of the host matrix although a disorder matrix implies the broader gain bandwidth. Among all of the Yb-doped materials, Yb:YAG is now the most widely investigated and developed, and exhibits the highest emission cross-section and excellent thermal properties (with a thermal conductivity of 10 W/m/K for an undoped crystal). Combining the balanced performances in all kinds of field with a thin-disk geometry, Yb:YAG thin disk lasers could generate high-energy, high-power, and ultrashort pulses while keeping the repetition rate above several hundred kilohertz. Among various setups, regenerative amplifiers (RAs) are routinely used to boost the output of mode-locked oscillators due to their high gains of several orders of magnitude and a resonator structure to support good spatial quality [7]-[10]. In 2016, Krausz. et al. have already presented a 2 mJ Yb:YAG thin disk regenerative amplifier with a pulse width of 210 fs at the repetition rate of 100 kHz [11]. Later in 2017, above 200 mJ output at the repetition rate of 5 kHz was obtained by the same group [12]. Such high peak power will potentially be harmful for the optical elements in the amplifier without elaborate design. For the safety of those optical elements in the regenerative amplifier, the specific pulse evolutions such as the variations of pulse width, spectrum, and pulse energy in each round trip should be studied and calculated carefully. Not only for the safety of the optical elements in the regenerative amplifier cavity, numerical analysis of the pulse evolution is guite valuable for the design of the whole chirped-pulse amplification (CPA) system. If the seed available is given, it helps to decide the necessity of pre-amplifier, which will reduce the number of round trips to maintain a broader bandwidth. The beforehand numerical analysis about the evolvement of bandwidth, pulse duration and peak power will also facilitate the arrangement of both pulse stretcher and compressor, which are high-priced components. For example, the pulse bandwidth, peak power and the amounts of dispersions finally determines the size of the gratings and related optics. Additionally, the preliminary configuration of eigen mode in the resonator needs to take into account of peak power before compression. In which the beam sizes on the gain medium and Pockels cell (PC) should be particularly considered. Limited by the applicable high voltage at high repetition rates, the aperture size of pricy Pockels cell (PC) should be the right fit, that is, to avoid damage and reduce insertion loss on the one hand and relieve requirements of voltage on the other hand. Furthermore, numerical simulation helps to select a filter, pulse shaper or implement a method to suppress the gain narrowing effect [13]-[16]. One scheme to overcome the gain narrowing in high-power regenerative amplifier needs to control the amount of pre-chirp before seeding which should be compensated just before strong self-phase modulation (SPM) effects occurs, and this also needs information about how the characteristics of pulses evolve [15].

In this letter, we carried out a high-power femtosecond chirped pulse regenerative amplifier numerical simulation based on a Yb:YAG thin-disk to investigate the pulse parameters evolution



Fig. 1. Schematic of the experimental setup: TFP, thin-film polarizer; HWP, half-wave plate; QWP, quarter-wave plate; CAV, concave mirror; VEX, convex mirror.

in each round trip. An output energy of 0.23 mJ at the repetition rate of 200 kHz with pulse width of 500 fs at a central wavelength of 1030 nm was realized experimentally at the same time. Simulation analysis fitted quite well with measured experimental results about pulse width, spectrum, and pulse energy, which plays an important role in the design of high-peak-power regenerative amplifier. The numerical code will facilitate the suppression of gain narrowing phenomenon to achieve high power and broader bandwidth simultaneously, which will be conducted in our next step.

2. Setup of the Regenerative Amplifier

The fundamental layout of the Yb:YAG regenerative amplifier, which was used for obtaining the experimental results to be reference for the simulations, is schematically illustrated in Fig. 1. The front end for the system generated a 25 μ J pulse with a bandwidth of about 8 nm at a repetition rate of 200 kHz and a central wavelength of 1032 nm. The stretched pulse duration was about 750 ps to guarantee the safety of the optical elements of the regenerative amplifier system. An optical isolator consisting of a thin-film-polarizer (TFP), Faraday rotator, and a half-wave plate (HWP) was plug into the system after the seed, to protect the seed source from the output high peak-power pulse. One more same construction was inserted after that to separate the output and input pulses. The seed pulses were coupled into the cavity by our homemade telescope system so that the waist beam size on the end mirror of the regenerative amplifier was about 1 mm in diameter, just to match the diameter to the resonator eigenmode of regenerative amplifier. The Pockels cell used a single BBO crystal with a length of 25 mm and a clear aperture of 6.5.mm \times 6.5 mm $(7 \text{ mm} \times 7 \text{ mm} \text{ total area})$. It was switched with a high-voltage driver, which applied 6 kV voltage at 200 kHz to BBO, corresponding to the quarter-wave voltage of such a BBO crystal. The pulse remained propagating inside the amplifier during the switch-on time of the Pockels cell. During those round trips, it was amplified by a 0.13 mm, 11%-doped Yb:YAG thin-disk with a radius of curvature of 4 m without pump and a diameter of 12 mm. Radius of curvature of the disk increase slightly, but no larger than 5 m as the increased pump power up to 150 W while keeping the chiller temperature at 20 °C. Those measured radius of curvature results were measured with the help of

a commercial wavefront detector [17]. One side of the disk is high-reflective (HR) coated for both laser and pump wavelength and the other is antireflective coated (AR). The thin-disk was pumped by a fiber-coupled laser diode and a pump optics with 24 passes through the water-cooled thin disk was used. The fiber had a numerical aperture of 0.22 and a core diameter of 600 μ m. The pumping central wavelength was fixed at 969 nm, which is the zero-phonon line of Yb:YAG. The pump spot diameter on the disk was 3.6 mm. The thin-disk was tested in a short V-shaped cavity containing a flat HR mirror and a flat outcoupling mirror for continuous wave (CW) output. The output coupler with 2% output coupling efficiency was proved the best coupling efficiency, and an unpolarized average power of 78 W at pump power of 150 W was obtained in CW operation. During the CW operation, the temperature at the center of the thin disk was measured to be 33.8 °C by a infrared thermometer at a pump power of 150 W, corresponding to a pump power density of 1.5 kW/cm².

The amplifier cavity is designed to match the pump spot diameter and the aperture of the BBO for fundamental mode operation. One concave mirror whose radius of curvature was 6 m and one convex mirror whose radius of curvature was 4 m, were used in the amplifier cavity, and the rest mirrors were all flat HR mirrors. The total length of the cavity is 1.7 m. Continuous wave (CW) operation was first conducted to confirm the optimum cavity performance, in which the combination of TFP3 and QWP served as the output coupler and the PC driver was switched off. At the pump power of 150W, a s-polarized CW output power of 50 W is achieved by adjusting the QWP for the optimum outcoupling. The reduction of the power compared with the short V-shaped operation was due to the larger cavity length and the losses in additional cavity elements (Pockels cell, QWP, and TFP3).

3. Experimental Verification of The Simulations

To investigate the pulse propagation process in the regenerative amplifier, we used the split-step Fourier method (SSFM) to solve the nonlinear Schrodinger equation (NLSE) [16], [18]–[19]. Group velocity dispersion, third order dispersion, and SPM effects are considered in the simulations. It has been proved that the SPM effects could be ignored in our system since the injected pulse width is 750 ps, which is relatively long. However, in order to suit the simulations for other situations, we recommend considering the SPM effects when necessary. The simulations of the last part in this section also have showed the necessity. The modeled elements were mainly the Pockels cell and laser crystal. Effects of other elements like TFP and cavity mirrors were negligible, since their value or thickness were quite smaller compared with BBO. Here we listed the parameters used in the simulation including dispersion and nonlinearity of BBO and Yb:YAG: the second-order and third-order dispersion coefficients of BBO are 4e-26 s²/m and 6.65e-41 s³/m, respectively; the second-order and third-order dispersion coefficients of Yb:YAG are 7e-26 s²/m and 6.7e-41 s³/m, respectively; the nonlinear refractive index of BBO and Yb:YAG are 5.6e-20 m²/W and 6.9e-20 m²/W, respectively. The temporal pulse shape in this simulation was assumed to be a Gaussian one. The emission cross-section and absorption cross-section at the temperature of 30 °C was referred to investigate the gain narrowing effects [20]. Those results were applied to pump power beyond 90 W, while the cross-section data at the temperature of 25 °C were more suitable for pump power below 90 W according to the former temperature measurements at the center of the disk. As for the pump power of 150 W, cross-section data at the temperature of 35 °C were more suitable since the temperature was measured to be 33.8 °C. The numerical model reported in [21], was used to calculate the small-signal gain G and the single-pass loss L. At the same time, Q-switch operation and small seeding experimental results further confirmed the calculated $G = 1.08 \pm 0.01$ and $L = 2\% \pm 0.1\%$.

When seeded with 25 μ J pulse energy, an output energy of 0.25 mJ at 200 kHz was generated at the pump power of 150 W after propagating for 21 round trips, corresponding to an overall optical to optical efficiency of 33%. The output spectrum bandwidth was 2.8 nm compared with the wider spectrum of the seed source with 8nm, which revealed a strong gain-narrowing phenomenon. Both of the measured spectrum of seed source and amplified output were shown in Fig. 2.



Fig. 2. Spectra of the seed source and the amplifier pulse.



Fig. 3. (a) simulated and experimental pulse widths and energy for the same round trips of 21 at different pump power; (b) simulated and experimental spectra widths and pulse peak power for the same round trips of 21 at different pump power.



Fig. 4. (a) simulated and experimental pulse widths and energy at the same pump power of 90 W for different round trips; (b) simulated and experimental spectra widths and pulse peak power at the same pump power of 90W for different round trips.



Fig. 5. (a) output beam quality of the seed; Far-field beam profile (left) and near-field beam profile (right); (b) output beam quality of the amplified output; Far-field beam profile (left) and near-field beam profile (right).



Fig. 6. Intensity autocorrelation trace and the sech²-fitted trace.



Fig. 7. Simulation of the pulse spectrum at different output peak power intensity of the BBO in PC.

With the round trips set to 21, several results of the pulse duration, spectrum width and peak power were simulated using the same parameters except the pump power. Then we carried out another set of simulation using the same parameters except round trips at the same pump power of 90 W. In order to verify those calculated simulations above, two sets of experiments based on our Yb:YAG regenerative amplifier were carefully carried out. Both of the two experiments used the same seed source of 25 μ J, 200kHz, and 750 ps, just the same as the experimental setup. Related

to that, we keep the round trips constant just to change the pump power in the first experiment. Then in the second experiment, only round trips were changed to measure the pulse characteristics at the same pump power of 90 W. Fig. 3 and Fig. 4 revealed that our simulation agreed quite well with measured experimental results

Spatial characteristics of the output beam were also measured by a charge-coupled device camera (CCD). The beam quality data were illustrated in Fig. 5.(b) with insets of the near-field beam profile and the far-field beam profile, presenting a beam quality with $M_x^2 = 1.248$ and $M_y^2 = 1.206$ in the orthogonal directions. At the same time, we also presented the beam quality of the seed with $M_x^2 = 1.239$ and $M_y^2 = 1.196$ in the orthogonal directions to compare with the amplified output beam quality, which were illustrated in Fig. 5.(a). Those results were measured by one commercial M^2 measuring system.

The amplified pulse with a pulse width of about 270 ps was then sent into the compressor consisting of a pair of gratings and some folding mirrors. The pulse of 500 fs duration was finally obtained after optimizing both of the incident angle and the distance between the gratings. The Intensity autocorrelation traces of the output pulses as well as the sech²-fitted traces were showed in Fig. 6. The total efficiency of the compressor was 85%, and the final output power after the compressor was 46 W.

We further investigated the SPM-induced spectrum-broaden effects by reducing the pulse duration of seed source into 196 fs (corresponding to the transform limited pulse duration of the seed) and pulse energy of seed source into 1 μ J in our simulation while keeping other parameters the same as the former simulation and the experiment. Wider spectrum was founded in the simulation results as increased output power. Since BBO crystal plays the most important role in SPM effects for its length and relatively high nonlinearity, we focused on the BBO's peak power intensity's influence on the spectrum width. Fig. 7 showed three different spectrum widths at different peakpower intensities of BBO, respectively. As shown in Fig. 7, there was no spectrum-broadening phenomenon when the peak power intensity of the BBO was below 2 GW/cm². Significant double peaks in the spectrum would not occur until the peak power intensity of the BBO reached around 6.2 GW/cm². As the peak power intensity of BBO increased up to around 10 GW/cm², even the triple peaks in the spectrum would be observed. However, limited by the damage threshold of BBO, peak power intensity of 10 GW/cm² would be potentially harmful for the crystal, so that the experimental condition to observe the tripe peaks in the spectrum would be guite tough to achieve. Post-compression techniques should be alternative methods to obtain wider spectra width [22]. And our simulation model was also suitable for calculating the spectra and pulse width in these experiments combined with beam propagation simulations.

4. Conclusion

In conclusion, we have demonstrated a high-power Yb:YAG thin disk regenerative amplifier at the repetition rate of 200 kHz with pulse duration of 500 fs at the central wavelength of 1030 nm. The numerical analysis of the system prior to the experiment facilitated configuration of the parameters in our experiment. At the pump power of 150 W after propagating for 21 round trips, an output energy of 0.25 mJ was generated, corresponding to an overall optical to optical efficiency of 33%. The output beam quality was measured as $M_x^2 = 1.248$ and $M_y^2 = 1.206$ in the orthogonal directions. During each round trip of the regenerative amplifier, the measured output including pulse energy, duration, spectrum width together agreed quite well with the corresponding simulation results. This was quite helpful in the design of higher peak-power regenerative amplifier system in the future. At the same time, the further simulation of a 196 fs seed pulse proved that high peak-power intensity induced strong SPM effects can broaden the spectrum significantly. With higher damage threshold of BBO available, higher energy and wider spectrum with shorter pulse width laser sources were expected to be built for applications such as attosecond pulse generation and spectroscopy.

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