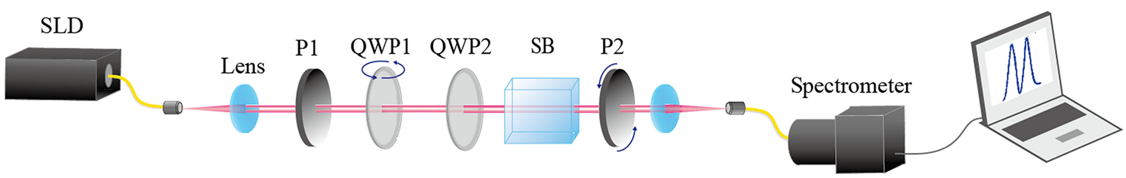
IEEE Photonics Journal
An IEEE Photonics Society Publication

Open Access

Optimization of the Weak Measurement System by Determining the Optimal Total Phase Difference

Volume 13, Number 3, June 2021

Yang Xu
Lixuan Shi
Le Liu
Wenyue Xie
Tian Guan
Xinhui Xing
Cuixia Guo
Dong He
Suyi Zhong
Hailong Zhang
Chongqi Zhou
Zhangyan Li
Yonghong He



DOI: 10.1109/JPHOT.2021.3074631

IEEE photonics SOCIETY
IEEE

Optimization of the Weak Measurement System by Determining the Optimal Total Phase Difference

Yang Xu,[1,2,3] **Lixuan Shi,**[1,4] **Le Liu**,[5] **Wenyue Xie,**[4] **Tian Guan**,[1,6] **Xinhui Xing,**[2] **Cuixia Guo,**[7] **Dong He,**[8] **Suyi Zhong,**[1,2] **Hailong Zhang,**[1,2] **Chongqi Zhou,**[1,4] **Zhangyan Li,**[1,4] **and Yonghong He** [1,2,]

[1]Institute of Optical Imaging and Sensing, Shenzhen Key Laboratory for Minimal Invasive Medical Technologies, Tsinghua Shenzhen International Graduate School, Tsinghua University, Shenzhen 518055, China
[2]Institute of Biopharmaceutical and Health Engineering, Tsinghua Shenzhen International Graduate School, Shenzhen 518055, China
[3]Department of Clinical Laboratory, Peking University, Shenzhen Hospital, Shenzhen 518055, China
[4]Department of Physics, Tsinghua University, Beijing 100084, China
[5]Institute of Materials Research, Shenzhen International Graduate School, Tsinghua University, Shenzhen 518055, China
[6]School of Medicine, Tsinghua University, Beijing 100084, China
[7]School of Mechanical Engineering and Automation, Fuzhou University, Fuzhou, 350108, China
[8]School of Chemical Engineering and Light Industry, Guangdong University of Technology, Panyu District, Guangzhou 510006, China

DOI:10.1109/JPHOT.2021.3074631

Manuscript received March 11, 2021; revised April 7, 2021; accepted April 16, 2021. Date of publication May 7, 2021; date of current version June 11, 2021. This work was supported by the National Science Foundation of China (NSFC) under Grants 61875102 and 61975089, in part by the Tsinghua University Spring Breeze Fund (2020Z99CFZ023), in part by the Science and Technology Research Program of Shenzhen City under Grants JCYJ20200109110606054 and JCYJ20180508152528735, and in part by Oversea Cooperation Foundation, Graduate School at Shenzhen, Tsinghua University under Grant HW2018007. *(Yang Xu and Lixuan Shi contributed equally to this work.)* Corresponding authors: Tian Guan and Yonghong He (e-mail: guantian@sz.tsinghua.edu.cn; heyh@sz.tsinghua.edu.cn).

Abstract: In this study, a method to improve the detection accuracy of a “weak measurement” system of optical rotation was proposed by determining the optimal total phase difference of the system. Analysis of the characteristics of weak measurement regime of the system under different total phase differences provided the total phase difference corresponding to the regime with the smallest ratio of system noise level to sensitivity (that is, the highest system resolution). In this regime, an optical rotation resolution of $5.522 \times 10^{-6\circ}$ was obtained, which is nearly an order of magnitude higher than the original system.

Index Terms: Weak measurement, Optimal total phase difference, Optical rotation resolution.

1. Introduction

Improvement in optical detection technology has played an important role for discovery research in biological sciences. This is mainly attributed to its non-destructive, non-labeling, and highly sensitive nature. Subfields such as drug development, proteomics, cancer research, and enzyme reactions have benefitted the most from these developments. An emerging optical signal amplification technology, widely used in recent years is the so-called weak measurement technology. It is extensively used due to its ability to amplify the measured value to levels much higher than the eigenvalue. The idea behind the weak measurement technique is the selection including appropriate pre-selection and post-selection states; then a parameter called the “weak value” can be obtained in the emitted pointer state [1], [2]. The existence of this weak value makes it possible to achieve a measurement value much larger than the eigenvalue during the measurement process; and therefore, leads to the increase in the detection precision. Examples highlighting the use of weak measurement include single-photon tunneling time [3], techniques to improve surface plasmon resonance resolution [4], sub-pulse width time delays [5], etc. In our previous study [4], we first introduced optical weak measurement technology to the field of biomolecule detection, achieving excellent detection precision [6].

In other previous studies, we combined the weak measurement technology with a total internal reflection system to detect intermolecular interactions, with successful applications in the field of label-free biomolecule sensing [7] and cancer marker detection [8]. Furthermore, we also combined the weak measurement with a polarization detection system to detect the chiral molecule content by using optical rotation. This led to applications such as a chiral sensor based on weak measurement [9], detection of macromolecular content in a mixed solution [10]and detection of amino acid concentration [11]. Application of the weak measurement technology in the above-mentioned two types of systems has increased the detection sensitivity of the original systems by three and two orders of magnitude respectively [12]. However, although high detection sensitivity is very important in the field of biomolecule detection, high detection accuracy is arguably more valuable. The resolution representing the detection accuracy of the system is defined as the ratio of the system’s noise level and sensitivity; therefore, these two quantities need to be controlled at the same time in the same system in order to reach the optimal detection level.

It is well known that in a weak measurement system, with the increase in the system sensitivity, the system light intensity gradually decreases (in parallel, the effect of noise also gradually increases). The definition of the detection system resolution concludes that in a weak measurement system we cannot improve the detection resolution by increasing the detection sensitivity and reducing the system noise level at the same time [13]. Therefore, finding the minimum value of the noise level and sensitivity ratio of the weak measurement system is very important to improve the detection resolution of the system.

In this study, we focused on the weak optical rotation measurement system, and determined the regime of the minimum value of the ratio of the system’s noise level and sensitivity through numerical simulation calculations, thereby significantly improving the detection accuracy of the system. First, the relationship between system noise levels and light intensity was analyzed. Then, by setting different total phase differences (the phase difference between the two vertical polarization components is caused by the wave plates between the pre-selection and the post-selection in the system.) in the weak measurement system, a series of weak measurement regimes was obtained and the properties including sensitivity, light intensity, and standard deviation of each regime were analyzed. Finally, through numerical simulation, the total phase difference corresponding to the regime of the weak measurement system with the best detection accuracy was obtained. Subsequently, a solution of L-proline was used to perform a sample detection test at the calculated regime of the optimal total phase difference of the weak measurement system and an optical rotation detection resolution of $5.522 \times 10^{-6\circ}$, which is nearly one order of magnitude higher than the previous system [11], was obtained.

2. Method

Fig. 1 shows the experimental setup for optical rotation weak measurement system. The light emitted by the super-radiative light-emitting diode (SLD, Thorlabs Inc., SLD830S-A20, 22 mW, Thorlabs Inc.) source is preselected after passing through a pre-polarizer (P1, Thorlabs Inc., LPVIS050-MP, extinction ratio of 100000:1) and two quarter-wave plates (QWP1 and QWP2,

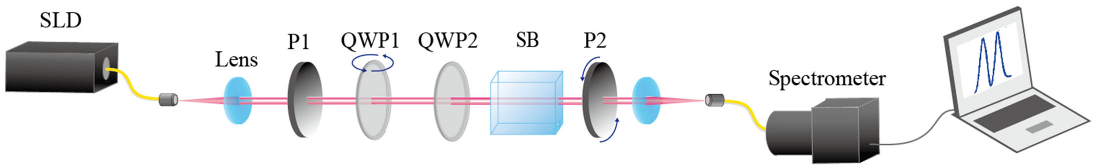


Fig. 1. Structure diagram of optical rotation weak measurement system.

Thorlabs Inc.). Further, the beam passes through a sample box (SB, 2 cm in length) and the post-selective polarizer (P2) in order to achieve weak value amplification, and is received by the Ocean View spectrometer (Ocean Optics, HR4000, Shanghai, China). In this study, digital filtering technology was utilized to add a Gaussian wave with a full width at half maximum of 40nm to the system [14].

The axis of P1 is parallel to the horizontal direction and the axis of P2 has an angle θ to the vertical direction. The axes of QWP1 and QWP2 were initially set at the horizontal and vertical direction, respectively. Based on the assumptions of previous studies [8], [13]–[16], it can be assumed that the pre and post polarization states are approximately represented as follows:

$$|\Psi_i\rangle = \hat{Q}\left(\frac{\sqrt{2}}{2}|+\rangle + \frac{\sqrt{2}}{2}|-\rangle\right) \tag{1}$$

$$\langle\Psi_o| = \langle+|\,\frac{\sqrt{2}}{2}e^{-i(\theta-\pi/2)} + \langle-|\,\frac{\sqrt{2}}{2}e^{i(\theta-\pi/2)} \tag{2}$$

$$\hat{Q} = e^{-i\theta}\ |H\rangle\langle H| + e^{i\theta}\,|V\rangle\langle V|\ = \cos\theta\ |-\rangle\langle-| + \cos\theta\,|+\rangle\langle+|\ + i\sin\theta|-\rangle\langle+| + i\sin\theta|+\rangle\langle-| \tag{3}$$

Where Q represents the operator corresponding to QWP, θ refers to the optical rotation angle to be measured and α represents the tilting angle of QWP.[17]–[19]. $|+\rangle$ and $|-\rangle$ refer to the eigenstates of left circular polarized light and right circular polarized light, respectively. In this system, the measurement operator $\hat{A}$can be represented as $\hat{A} = |+\rangle\langle+| - |-\rangle\langle-|$. Herein, the total phase difference of the weak measurement system can be increased by tilting the QWP vertically by the angle α, which is also the initial tiny phase difference required fo the weak measurement.

By modulating α, the system is adjusted to the weak measurement polarization state to ensure the weak value amplification (WVA) effect. [20].

Assuming a normalized total intensity of the incident light I_0 = 1, the relative light intensity after the measurement of the light path is represented as follows:

$$I = I_0 \cdot |\langle\Psi_o|\Psi_i\rangle|^2 \approx \cos^2(\alpha + \theta + \pi/2) \tag{4}$$

Considering that under the premise of satisfying the weak value amplification, it is $\theta \ll \alpha \ll 1$ [10], [20], [21], and $I \approx \sin^2\alpha$. By adjusting the phase difference of the QWP, the light intensity of the system can be adjusted.

Measurement sensitivity: According to literature studies [10], [17], [22], [23], the expression $A_\omega = \frac{\langle\Psi_o|\hat{A}|\Psi_i\rangle}{\langle\Psi_o|\Psi_i\rangle}$ can be obtained, and within the approximation interval of $kIm(A_\omega) \ll 1/\Delta\lambda$, the relationship between wavelength shift and angle is expected to be near linear, and obtained as follows:

$$\delta\lambda = \lambda - \lambda_0 = \frac{4\pi k(\Delta\lambda)^2\gamma\sin\theta}{\lambda_0\left(1 + \gamma^2 - 2\gamma\cos\theta\right)} \tag{5}$$

Where $\delta\lambda$ represents the shift of the central wavelength, λ_0 represents the central wavelength of the incident light, $\Delta\lambda$ represents the width of the wavelength of the incident Gaussian light, and $\gamma = \frac{\cos(\alpha+\pi/2)-\sin(\alpha+\pi/2)\sin 2\theta}{\cos(\alpha+\pi/2)+\sin(\alpha+\pi/2)\sin 2\theta} = \frac{\sin 2\theta\cos\alpha+\sqrt{I}}{\sin 2\theta\cos\alpha-\sqrt{I}}$ represents the parameters related to the optical path, which have a positive relationship with the light intensity I. The slope of the change in the central wavelength shift with the optical rotation can be obtained as follows:

$$\frac{d\lambda}{d\theta} \approx \frac{4\pi k(\Delta\lambda)^2 \cdot (1-\cos\theta)}{\lambda_0\left[\sin\theta(1-\gamma^{-1})^2 + (1-\cos\theta)^2\right]^2} \quad (6)$$

Where $\frac{d\lambda}{d\theta}$ represents the measurement sensitivity of the optical system. Owing to the following relationship $\theta \ll \alpha \ll 1$, the measurement sensitivity exhibits a negative correlation with the relative light intensity.

The standard deviation of central wavelength: As mentioned in the document [optimizing the signal to noise ratio of a beam definition measurement with interferometric peak values], the measurement noise is mainly attributed to shot noise and technical noise, thus the measurement value of wavelength is

$$\langle\lambda\rangle = \frac{4\pi k(\Delta\lambda)^2 \gamma \sin(\theta-\theta_0)}{\lambda_0\left(1+\gamma^2-2\gamma\cos(\theta-\theta_0)\right)} \pm \frac{\sigma}{\sqrt{I}} \pm \frac{S_\xi}{\sqrt{t}} \quad (7)$$

where σ and S_ξ are constants related to shot noise and technical noise respectively. Therefore, the relationship between the central wavelength noise and the relative light intensity can be approximately expressed as $\sigma_\lambda \propto 1/\sqrt{I}$. The relationship between the central wavelength noise and the relative light intensity is a negative correlation.

Optical rotation resolution: The resolution of or measured by WM can be expressed as follows: $\sigma_\theta = (\frac{d\lambda}{d\theta})^{-1} \cdot \sigma_\lambda \cdot (\frac{d\lambda}{d\theta})^{-1}$ is positively and σ_λ is negatively correlated with the relative light intensity I, respectively. Therefore, herein, it was decided to simulate the measurement system to obtain the best optical resolution point numerically [22].

3. Experimental

In the frequency-domain optical weak measurement system, the change of the quantity to be measured was characterized by the center wavelength offset of the bimodal spectrum unique to weak measurement, following the results of literature studies [6], [11]. Therefore, the sensitivity and noise level of a weak measurement system can be represented by the central wavelength offset and the standard deviation of the central wavelength in a bimodal spectrum, respectively that are caused by a change of 1 unit of the quantity to be measured in the system. In order to control the variables, the integration time for all experiments in this study was set to 10 ms.

3.1 Relationship Between Spectral Intensity and Standard Deviation

First, the weak measurement system shown in Fig. 1 was tuned to a bimodal working state [7]. Then we obtain a series of weak measurement spectra was obtained at different light intensities by adjusting the light source. Herein, an ocean view spectrometer was used to record and calculate the standard deviation of the center wavelength of the bimodal spectrum within a period of 5 min. Fig. 2 exhibits the relationship between the system spectral intensity and the system center wavelength standard deviation. Clearly, the system standard deviation gradually decreases with increasing light intensity, that is, the degree of influence of noise on the system gradually decreases with increasing light intensity.

3.2 Sensitivity and Spectral Intensity of the Weak Measurement System Under Different Total Phase Differences

Furthermore, the system was then set to different regimes by adjusting its total phase difference and the relationship among the sensitivity, light intensity, and total phase difference was analyzed at different regimes.

First, the total phase difference of the weak measurement system was set to 5×10^{-3}rad by adjusting the QWP1 and then the system's front and back selection polarization state were

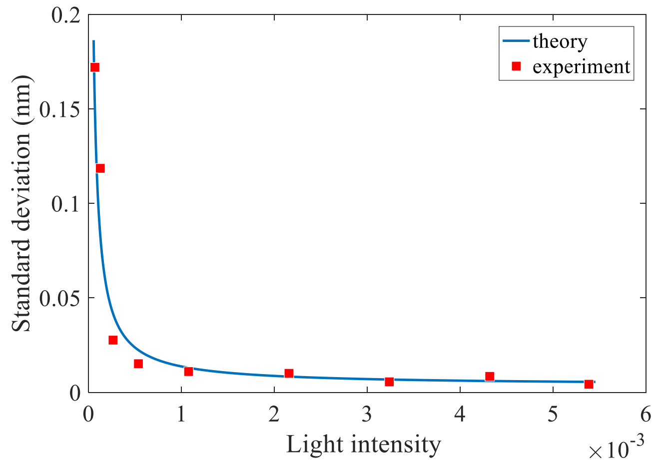


Fig. 2. The relationship between the standard deviation of the center wavelength and the relative light intensity in the weak measurement system.

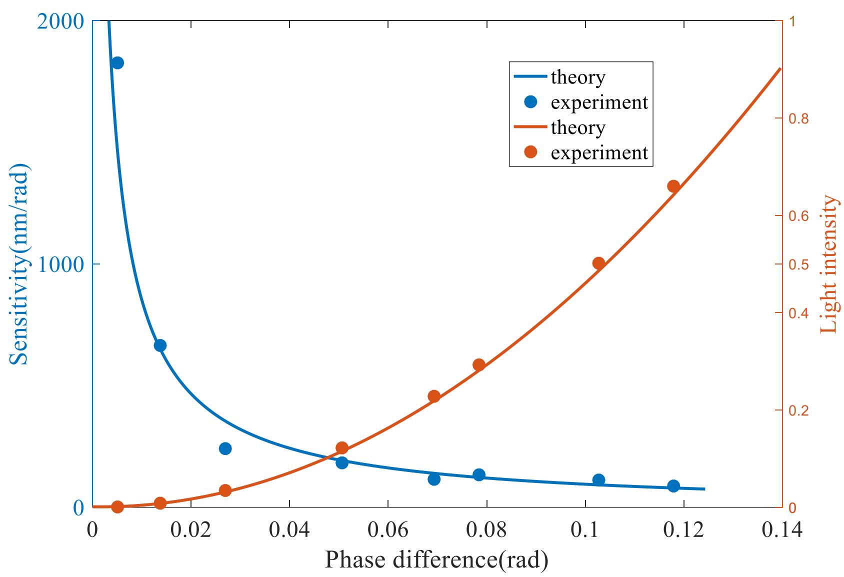


Fig. 3. Variation in sensitivity and relative light intensity of a weak measurement system with total phase difference of the system.

adjusted, in order to bring the system to the double-peak regime. At the same time, the optical rotation changes in the system by rotating P2 were simulated; the structure diagram of the system is shown in Fig. 1. The Ocean View spectrometer was used to collect bimodal spectra corresponding to different optical rotation angles, and use the self-made program to calculate the center wavelength corresponding to the spectrum. The above-described exposed formula (6) was used to calculate the sensitivity of the system in the state of total phase difference. At the same time, the light intensity was recorded (for the convenience of comparison, we used the light intensity of double peaks as the average light intensity in this phase state).

Fig. 1 demonstrates that the system's total phase difference moves through the values 0.0137, 0.0270, 0.0506, 0.0693, 0.0784, 0.1028, and 0.1180 rad by turning the QWP1 vertically. The above-mentioned experiments were repeated to obtain the sensitivity and spectral intensity of the weak measurement system in these different total phase values. Fig. 3 exhibits that the system sensitivity

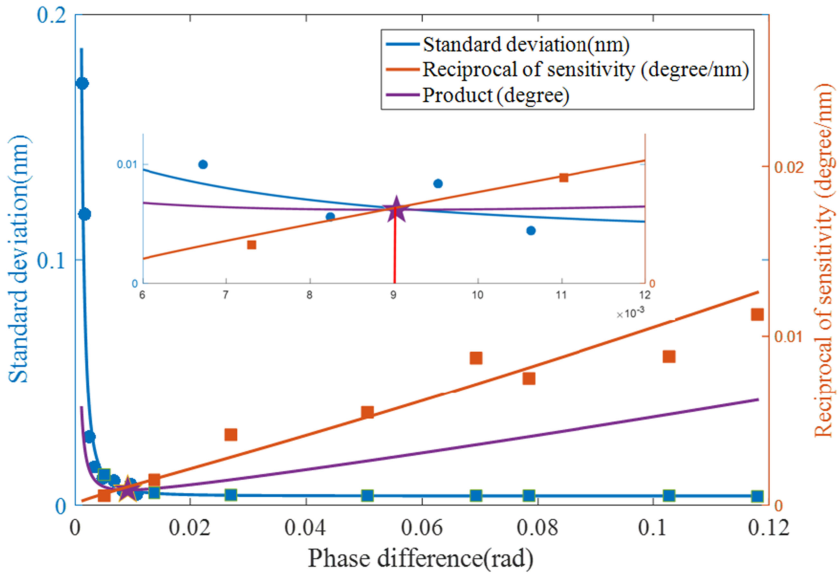


Fig. 4. Variation of the standard deviation, reciprocal of sensitivity and the product of the two quantities with the total phase difference in the weak measurement system.

decreases with the increase in the phase difference and the system light intensity increases as the phase difference increases.

3.3 The Best Resolution Corresponding to the Total Phase Difference

The resolution can be defined as $\sigma = 3\sigma_{so}/S$ [23], where σ_{so} represents the standard deviation of blank measurement, and S denotes the slope of the response curve (that is, the sensitivity of the system). In Fig. 4, the blue and orange lines represent the relationship of the standard deviation and the reciprocal of sensitivity with the total phase difference of the system, respectively; the orange and blue squares represent the reciprocal of sensitivity and the standard deviation at different total phase differences obtained in Section 3.2, respectively. The standard deviations under different light intensities obtained in Section 3.1 correspond to different total phase differences, as shown in blue dots in Fig. 4. By using simulation methods, the product of the reciprocal of the system sensitivity and the standard deviation of the system's central wavelength (that is, the resolution of the system) was obtained as the system's total phase difference changes, and this is also shown in the inset of Fig. 4. Numerical calculations were used to obtain the relationship of the product of the reciprocal of sensitivity and the standard deviation with the total phase difference of system, as shown in purple line in Fig. 4. In this particular setup, the optimal total phase difference is 9×10^{-3} rad, denoted by a star in the inset of Fig. 4.

3.4 Detection Experiments Based on Actual Samples

As a real test of the above-mentioned procedure, the weak measurement system described in this study was set to the two peak regimes with the best detection accuracy, where the total phase difference obtained was 0.0090 rad. Fig. 5(a), demonstrates the use of a concentration gradient of L-proline ([α] 20/D $-85.5°$) solution to 0, 0.5, 1, 1.5, and 2g/L. These solutions were added to the sample box in turn and the corresponding center wavelength shift of the spectrum was recorded. At the same time, as shown in the inset of Fig. 5(a), when the concentration of the L-proline solution was 2g/L, the standard deviation of the center wavelength of the spectrum was calculated and

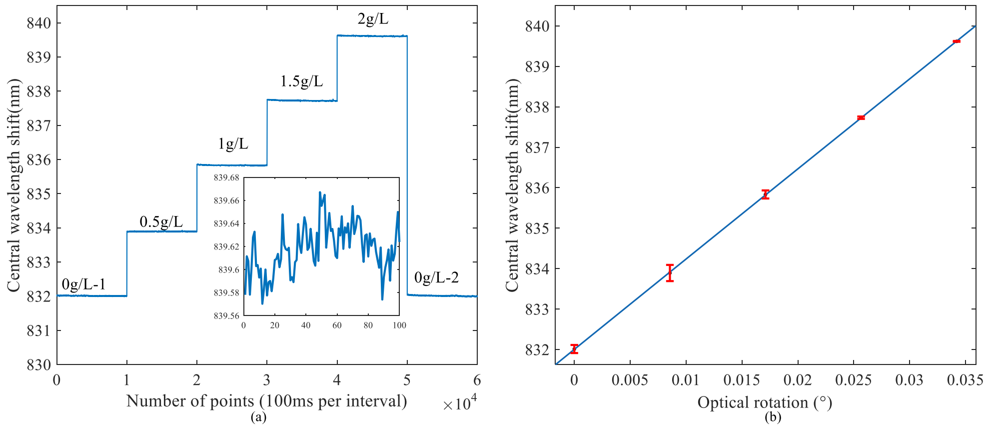


Fig. 5. (a) and (b) the central wavelength and linear fit of the system for different concentrations of L-proline solution in the sample box, respectively.

recorded, which turned out to be 2.102×10^{-4} nm, while the change in optical rotation caused by the same concentration solution was 0.034°. Therefore, the optical rotation detection sensitivity of the system as obtained by using the sensitivity formula $\delta\lambda/\delta n$is 222.361 nm/degree. Finally, the resolution of OR detection of the weak measurement system was calculated as $5.522 \times 10^{-6\circ}$ by using the formula$\sigma = 3\sigma_s/(\delta\lambda/\delta n)$.

4. Conclusion

In this study, the weak optical rotation measurement system was investigated, and the relationship between the system's total phase difference and the system sensitivity was analyzed, as well as the relationship between the system's total phase difference and the standard deviation of the spectral center wavelength was also investigated. The value of the total phase difference corresponding to the weak measurement and the best detection accuracy was obtained by the use of numerical calculation. Furthermore, the sample detection experiments were performed on a solution of L-proline, and an optical rotation resolution of the system was obtained at the optimal regime of $5.522 \times 10^{-6\circ}$. This is almost an order of magnitude better than that in the original system. The light source and spectrometer are different between different frequency domain weak measurement systems; therefore, the total phase difference corresponding to the best resolution regime of the system also differs; however, the best regime of the system can be determined by the method presented in this study. Therefore this study not only improves the detection accuracy of the weak measurement optical rotation detection system, but also provides a feasible idea for improving the detection accuracy of all weak measurement systems.

References

[1] Y. Aharonov, D. Z. Albert, and L. Vaidman, "How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100," *Phys. Rev. Lett.*, vol. 60, no. 14, pp. 1351–1354, Apr. 1988.

[2] N. W. M. Ritchie, J. G. Story, and R. G. Hulet, "Realization of a measurement of a weak value," *Phys. Rev. Lett.*, vol. 66, no. 9, pp. 1107–1110, Mar. 1991.

[3] A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, "Measurement of the single-photon tunneling time," *Phys. Rev. Lett.*, vol. 71, no. 5, pp. 708–711, Aug. 1993.

[4] L. Luo *et al.*, "Precision improvement of surface plasmon resonance sensors based on weak-value amplification," *Opt. Exp.*, vol, vol. 25, no. 18, pp. 21107–21114, Sep. 2017.

[5] L. J. Salazar-Serrano, D. Janner, N. Brunner, V. Pruneri, and J. P. Torres, "Measurement of sub-pulse-width temporal delays via spectral interference induced by weak value amplification," *Phys. Rev. A*, vol. 89, 2014, Art. no. 012126.

[6] D. Li, Z. Shen, Y. He, Y. Zhang, Z. Chen, and H. Ma, “Application of quantum weak measurement for glucose concentration detection,” *Appl. Opt.*, vol. 55, no. 7, pp. 1697–1702, Mar. 2016.

[7] Y. Zhang, D. Li, Y. He, Z. Shen, and Q. He, “Optical weak measurement system with common path implementation for label-free biomolecule sensing,” *Opt. Lett.*, vol. 41, no. 22, pp. 5409–5412, Nov. 2016.

[8] T. Guan *et al.*, “Determination of tumor marker carcinoembryonic antigen with biosensor based on optical quantum weak measurements,” *Sensors*, vol. 18, no. 5, May 2018, Art. no. 1550.

[9] D. Li *et al.*, “A chiral sensor based on weak measurement for the determination of proline enantiomers in diverse measuring circumstances,” *Biosens. Bioelectron.*, vol. 110, pp. 103–109, Jul. 2018.

[10] Y. Xu *et al.*, “Detection of macromolecular content in a mixed solution of protein macromolecules and small molecules using a weak measurement linear differential system,” *Anal. Chem.*, vol. 91, no. 81, pp. 11576–11581, Sep. 2019.

[11] D. Li *et al.*, “Optical rotation based chirality detection of enantiomers via weak measurement in frequency domain,” *Appl. Phys. Lett.*, vol. 112, no. 21, 2018, Art. no. 213701.

[12] R. M. A. Azzam, “Phase shifts that accompany total internal reflection at a dielectric–dielectric interface,” *J. Opt. Soc. Amer. A.*, vol. 21, no. 8, pp. 1559–1563, Aug. 2004.

[13] Y. Xu *et al.*, “Optimization of a quantum weak measurement system with its regimes,” *Opt. Exp.*, vol. 26, no. 16, Aug. 2018, Art. no. 21119.

[14] Y. Xu *et al.*, “Optimization of a quantum weak measurement system with digital filtering technology,” *Appl. Opt.*, vol. 57, no. 27, pp. 7956–7966, Sep. 2018.

[15] Y. Xu *et al.*, “Rapid separation of enantiomeric impurities in chiral molecules by a self-referential weak measurement system,” *Sensors*, vol. 18, no. 11, Nov. 2018, Art. no. 3788.

[16] Y. Xu *et al.*, “Multifunctional weak measurement system that can measure the refractive index and optical rotation of a solution.” *Appl. Phys. Lett.*, vol. 114, no. 18, 2019, Art. no. 181901.

[17] X. Y. Xu, Y. Kedem, K. Sun, L. Vaidman, C. F. Li, and G. C. Guo, “Phase estimation with weak measurement using a white light source,” *Phys. Rev. Lett.*, vol. 111, no. 3, Jul. 2013, Art. no. 033604.

[18] C. Mi, S. Chen, X. Zhou, K. Tian, H. Luo, and S. Wen, “Observation of tiny polarization rotation rate in total internal reflection via weak measurements,” *Photon. Res.*, vol. 5, no. 2, pp. 92–96, 2017.

[19] S. Chen, X. Zhou, C. Mi, H. Luo, and S. Wen, “Modified weak measurements for the detection of the photonic spin hall effect,” *Phys. Rev. A*, vol. 91, 2015, Art. no. 062105.

[20] D. J. Starling, P. B. Dixon, A. N. Jordan, and J. C. Howell, “Optimizing the signal-to-noise ratio of a beam-deflection measurement with interferometric weak values,” *Phys. Rev. A*, vol. 80, 2009, Art. no. 041803.

[21] Z. Qiao *et al.*, “The real-time determination of d- and l-lactate based on optical weak measurement,” *Anal. Methods*, vo. vol. 11, no. 16, pp. 2223–2230, 2019.

[22] V. Thomsen, D. Schatzlein, and D. Mercuro, “Limits of detection in spectroscopy,” *Spectroscopy*, vol. 18, no. 12, pp. 112–114. 2003.

[23] M. Piliarik and J. Homola “Surface plasmon resonance (SPR) sensors: Approaching their limits?,” *Opt. Exp.*, vol. 17, no. 19, pp. 16505–16517, 2009.