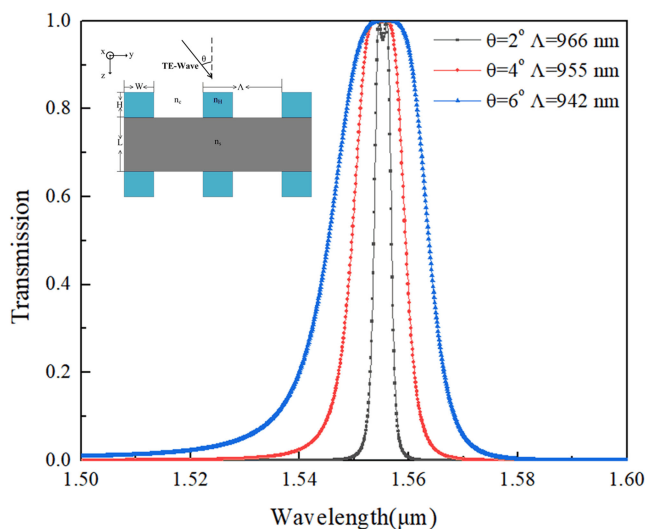


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# Tunable Flat-Top Filtering Response in Cascaded Resonant Waveguide Gratings

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**Abstract:** The applicability of resonant waveguide grating (RWG)-based structures as filters to control the spectral response in an optical communication system is investigated. A new physical model for the structure is established on the basis of the Fabry–Pérot (FP) etalon model and coupled leaky mode theory (CLMT). It is found that the flat-top spectral response of the filter is achieved by the combined effect of the guided-mode resonance of an RWG and its Fabry–Pérot resonance (FPR). The bandwidth-tunable spectral response of the filter can be varied according to the change in the eigenvalues of the RWG by changing the structural parameters such as strip width of the grating and incident angle. The flat-top and bandwidth-tunable RWG-based resonant filter is a promising application for high-performance optical communication systems.

**Index Terms:** Tunable filters, gratings, subwavelength structures.

## 1. Introduction

Resonant waveguide gratings (RWGs) based on guided-mode resonance have been widely studied owing to their simple structure and excellent performance [1]. They are utilized in many functional devices such as optical filters [2], optical sensors [3], absorbers [4], and ultra-broadband reflectors [5]. A single-layered RWG is applied as an optical filter because of the resonant coupling of an incident light wave to a leaky waveguide mode [6]; however, it was found that the Lorentzian-type spectral response in a single RWG hindered its further application in optical communications. It is desirable for a resonant filter to exhibit a flat-top spectral response for application in high-performance optical communication systems; this is because a flattened response allows for some wavelength shift of the source with high signal fidelity. In recent years, two types of cascaded RWGs were proposed to achieve a flat spectral response: In the first, RWGs were used as wideband reflectors without the guided resonance mode, and the Fabry–Pérot resonant mode was excited in the cavity of the cascaded RWGs. Cascaded identical resonant gratings were reported as multicavity structures to achieve a narrow-band and flat-top reflection spectrum in [7]. Chang

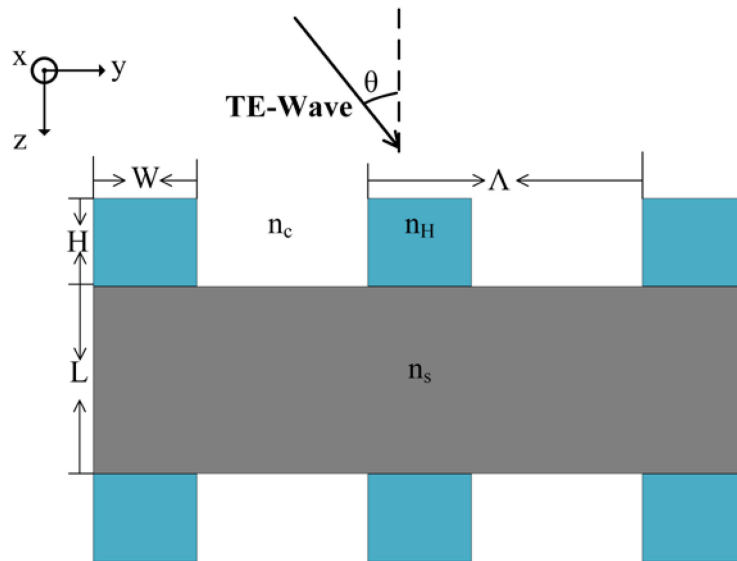


Fig. 1. (Color online) Schematic configuration of a double-RWG-based filter.

*et al.* experimentally demonstrated a flat-top spectral response, for the first time, using cascaded subwavelength resonant gratings [8]. Song *et al.* investigated the tunable characteristics of guided-mode resonances in two-coupled resonant gratings based on the temporal coupled-mode theory [9], [10]. The second type was based on a resonant filter of cascaded RWGs. The guided-mode resonance and Fabry–Pérot mode resonance of a resonant structure were combined to achieve a high-performance, flat-top filter. Ko *et al.* proposed a new approach to design a flat-top filter based on cascaded identical RWGs [11]. Kawanishi *et al.* also reported cross-stacked guided-mode resonant gratings, which resulted in a narrow-band flat-top spectrum with a full width at half maximum (FWHM) of 7 nm [12]. Recently, Ferraro *et al.* demonstrated a novel flat-top filter for terahertz telecom applications based on cascading two identical RWGs [13].

A tunable spectrum response is an important characteristic of RWG-based filters. In previous studies, a conventional method was used to control the spectral linewidth of periodic RWGs, namely, to modulate the structural parameters such as the grating thickness [14], filling factor [15], and modulated index [16]. In addition, other grating structures, such as compound waveguide gratings, were provided [17]. However, it is not easy to control the bandwidth of an RWG by merely adjusting its structural parameters. In this study, we focused on tuning the filtering response in cascaded RWGs by changing the angle of incidence of the incident light wave. The analytical expressions of the spectral response were then derived as a theoretical model by combining the F-P etalon model [18] and coupled leaky mode theory (CLMT). The flat-top filters with tunable bandwidth were designed by the theoretical FP-CLMT model and identified by employing the rigorous coupled-wave analysis (RCWA) method [19].

## 2. Structure and Theoretical Model

The schematic of the double-RWG-based filter extending infinitely in the  $x$ -direction and periodically in the  $y$ -direction is shown in Fig. 1. It consists of two identical silicon RWGs ( $n_H = 3.47$ ) and a spaced silica interlayer substrate ( $n_s = 1.45$ ). In this case, one RWG with an air cladding ( $n_c = 1$ ) is on top of the other, and the two are separated by a homogeneous layer of the silica interlayer. The structural parameters of the device are as follows: the thicknesses of the top/bottom RWGs and silica interlayer are  $H$  and  $L$ , respectively; the period and strip width of the top or bottom RWG are  $\Lambda$  and  $W$ , respectively; and the incident angle is denoted by  $\theta$ . The incident light wave with

transverse electric (TE) polarization propagates through the silicon grating layer of the resonant structure at an oblique angle of incidence  $\theta$ . The guided mode is excited by the incident light wave, and it then leaks from the double-layered RWG-based structure into the silica region; this enables the coupling of the guiding mode resonance to that of the Fabry–Pérot resonant cavity structure. The optical resonance is tuned by altering the incident angle to achieve filtering of the wavelength.

Let us briefly review the coupled leaky mode theory model to compute the transmission spectrum in a single RWG. In accordance with previous studies, we considered a single RWG composed of silicon grating layer and silica substrate to be a single-mode lossless resonator. The silicon grating layer is considered to be a uniform slab, and its effective refractive index  $n_{\text{eff}}$  is given by [20]

$$n_{\text{eff}} = \sqrt{F n_{\text{H}}^2 + (1 - F) n_{\text{C}}^2} \quad (1)$$

where  $F$  is the fill factor of the RWG satisfying the equation  $F = W/\Lambda$ . The background scattering matrix  $C$  of the resonator is given by [21]

$$C = e^{j\varphi} \begin{pmatrix} r & jt \\ jt & r \end{pmatrix} \quad (2)$$

where  $\varphi$  is a phase factor;  $r$  and  $t$  are the amplitude reflection and transmission coefficients in the direct process. They are both real and calculated by the slab waveguide theory, satisfying the equation  $r^2 + t^2 = 1$  [10]. We chose a period unit of a single RWG to calculate its transmission spectrum [22]. In a unit cell, the Floquet periodic boundary conditions are used for lateral boundaries, whereas scattering boundary conditions are used for the upper and lower boundaries. The eigenvalues of the leaky mode of the single-layered RWG are then obtained by using the finite element method (FEM) [23]. As the single RWG is an asymmetric resonator, a small correction is applied to the eigenvalues of the form  $N = N_r - i * N_i$ , where  $N_r$  and  $N_i$  are the real and imaginary parts of  $N$ , respectively. Thus, the transmission spectrum of the RWG is described by the CLMT model as follows [21], [23]:

$$T_0 = \frac{t^2(\omega - N_r)^2 + r^2 N_i^2 + 2rt(\omega - N_r) N_i}{(\omega - N_r)^2 + N_i^2} \quad (3)$$

where the central resonant frequency  $\omega_0 = N_r$ . The total radiative quality factor of a RWG is calculated using the expression  $Q = N_r/2N_i$ . According to [Ref. 11, Fabry–Pérot cavity plays a import role in the cascaded RWGs filter. Therefore, a new formula for the transmission spectrum, based on the FP etalon model, is given by [18]

$$T = \frac{T_0^2}{1 + R_0^2 - 2R_0 \cos \delta} \quad (4)$$

where  $T_0$  and  $R_0$  are the transmission and reflection coefficients of the single RWG, respectively. The round-trip phase of the F-P etalon cavity is expressed as

$$\delta = \frac{4\pi n_s L}{\lambda} \cos \varphi - 2\phi_R \quad (5)$$

where the angle of transmission  $\varphi$  is determined by the Snell's law,  $n_{\text{eff}} \sin \theta = n_c \sin \varphi$ , and the round-trip phase  $\phi_R$  is the reflective phase of the single RWG calculated by the RCWA method.

### 3. Results and Discussion

First, we compared the spectral responses of single and double RWGs using the RCWA method as shown in Fig. 2. We assumed the structural parameters of the present structure to be as follows:  $\Lambda = 955$  nm,  $W = 314$  nm,  $H = 220$  nm, and  $L = 2.235$   $\mu\text{m}$ .  $L$  was large enough to keep the two RWGs uncoupled. The input light wave was TE polarized at an incident angle of  $3^\circ$ . Fig. 2 shows that the resonant wavelengths of the two simulations approach a value of 1552.4 nm. The double RWG exhibited a flat-top and steep-edge spectral response, and its bandwidth was 3 nm less than

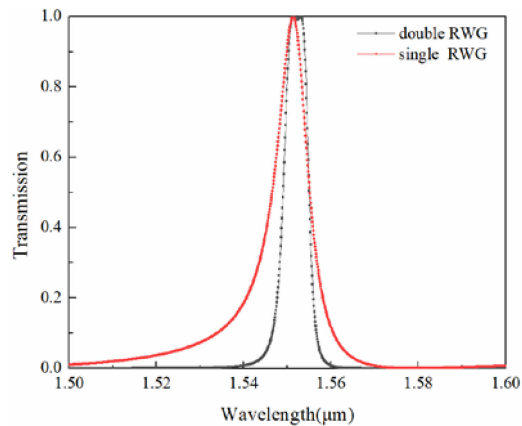


Fig. 2. (Color online) Transmission spectra of the double and single RWGs at an incident angle of  $3^\circ$ .

TABLE 1

Eigenvalues of the TE Eigenmodes of the Single RWG At Different Incident Angles

$\theta$ (deg)	$N = N_{\text{real}} - i * N_{\text{imag}}$
2	$1.2160 \times 10^{15} - 1.2891 \times 10^{12} i$
4	$1.2128 \times 10^{15} - 4.9943 \times 10^{12} i$
6	$1.2063 \times 10^{15} - 1.1754 \times 10^{13} i$

that of the single RWG. According to [11], the main physical mechanism was cavity-based FPR that was combined with the leaky mode resonance of the double RWG.

To control the bandwidth of the optical spectrum using the incident angle, we investigated the eigenvalues of the TE eigenmodes of the single RWG for different incident angles using the FEM method. It was found that the real and imaginary parts of the eigenvalues  $N$  were both tuned by changing the incident angle  $\theta$  as listed in Table 1.

where  $N_{\text{real}}$  and  $N_{\text{imag}}$  are the real and imaginary parts of complex eigenfrequency  $N$ , respectively. The real part of the eigenvalue  $N_{\text{real}}$  corresponds to the resonant frequency of the single RWG while the imaginary part  $N_{\text{imag}}$  represents the radiative decay rate. Previous studies have demonstrated that the optical properties (i.e., scattering, absorption, reflection and transmission) are mainly governed by the leaky mode supported by the grating structure [24], [25]. Also, the quality-factor  $Q$  can be obtained by  $Q = N_{\text{real}}/2N_{\text{imag}}$ . From the definition of Q-factor, it can be found that the imaginary part  $N_{\text{imag}}$  is half of full width half maximum (or linewidth) of optical resonance. According to the calculated results in Table 1, the central resonant frequency showed a red shift, whereas the radiative quality factor decreased with increase in the angle  $\theta$ . We calculated the transmission  $T_0$  of the single RWG for different incident angles  $\theta$  by using the coupled leaky mode model, as shown in Fig. 3. We observed that the spectral linewidth and resonant wavelength of  $T_0$  were simultaneously tuned by the change in the incident angle. Physically, the loss of a leaky mode of the RWG is increased with the increase angle. Therefore, to control spectral linewidth, one must control the loss of the leaky mode [26], [27]. Interestingly, one can understand that the tunable spectral response occurred owing to the changed information of a leaky mode of the single RWG.

Fig. 4 shows the transmission spectrum for different incident angles using the FP-CLMT model for TE-polarized light in the wavelength range of 1.53–1.58  $\mu\text{m}$ . We also calculated the same using

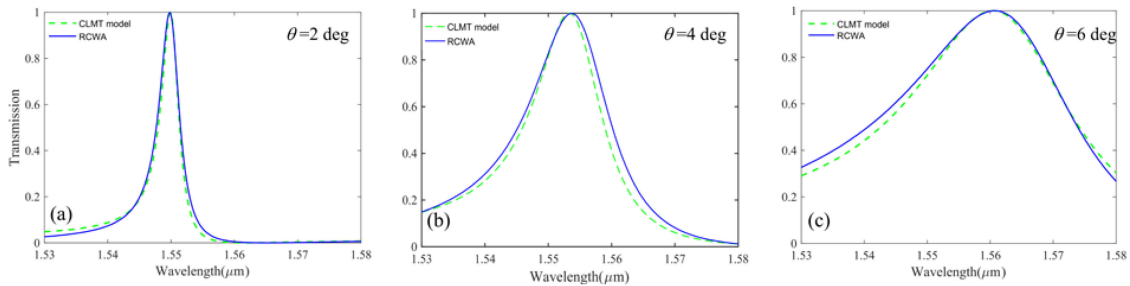


Fig. 3. (Color online) Spectra of a single RWG for TE-polarization plane wave using the RCWA simulation and CLMT model for different incident angles, namely, (a)  $\theta = 2^\circ$ , (b)  $\theta = 4^\circ$ , (c)  $\theta = 6^\circ$ .

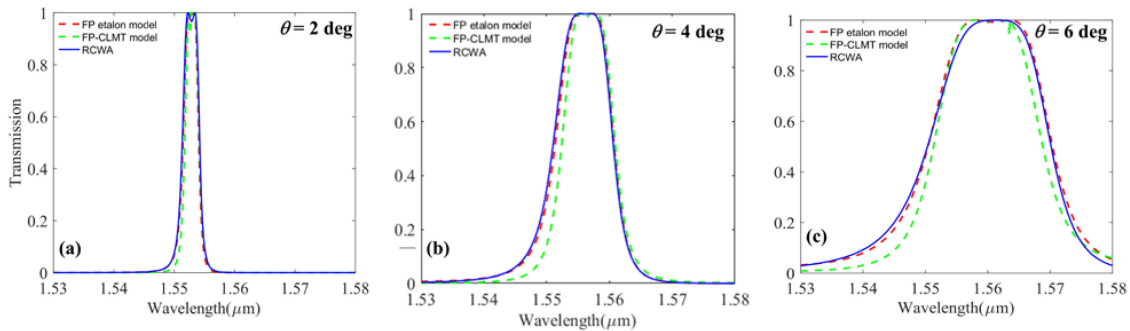


Fig. 4. (Color online) Spectra of a double-RWG-based filter for TE-polarized plane wave using RCWA method, F-P etalon model, and FP-CLMT model for different incident angles, namely, (a)  $\theta = 2^\circ$ , (b)  $\theta = 4^\circ$ , (c)  $\theta = 6^\circ$ .

the F-P etalon model and RCWA simulation methods. It was found that the resonant wavelength showed a red shift and the bandwidth of the spectrum increased with increasing angle of incidence. The optical resonance of the double-layered RWG-based structure as well as that of the proposed filter can be controlled by varying the incident angle that is determined by the eigenmodes of the single RWG. A comparison of the theoretical results obtained from the FP-CLMT model and the results of the RCWA method and F-P etalon model are shown in Fig. 4. There is a discrepancy between the RCWA simulated results and those calculated by using the FP-CLMT model. This may be because the single-layered RWG was assumed to be a single mode resonator in the FP-CLMT model, and the calculations were made by including the contribution of the fundamental leaky mode; whereas the total light fields of the double-layered RWGs were represented by all the Fourier components of the incident light wave and the transmission spectrum was expressed as a sum over its coupled waves in the RCWA method.

Fig. 5 shows the transmission spectrum of the resonant structure with respect to the wavelength and incident angle  $\theta$  for TE-polarized light using FP-CLMT model method. It is seen that the linewidth of the transmission spectrum increases with increase in the incident angle  $\theta$ . Further, the bandwidth can be tuned by changing the incident angle. Therefore, the tunable range can be easily controlled by changing the incident angle without altering the other structural parameters. It is also observed that the resonant wavelength can be changed by varying the incident angle. Here, it is easy to understand that the RCWA simulation results using the calculated results based on FP-CLMT model. Thus, the bandwidth tunable filter can be designed and analyzed with the information about the TE eigenmodes of the top or bottom RWG at different incident angles.

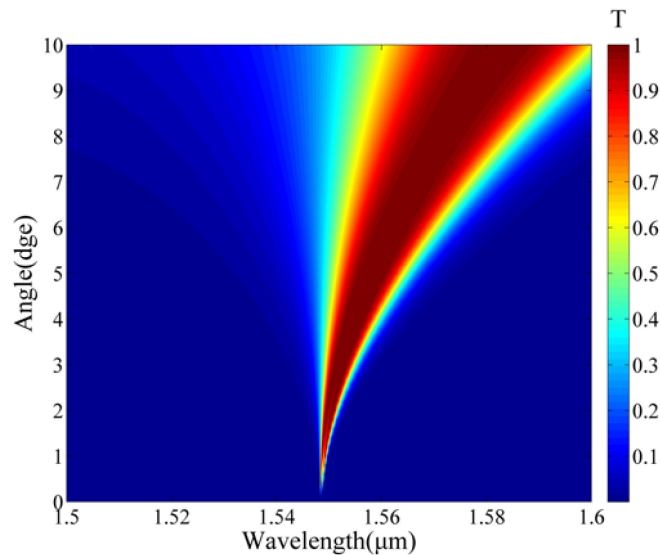


Fig. 5. (Color online) Spectra of the double-RWG-based filter with respect to wavelength and incident angles.

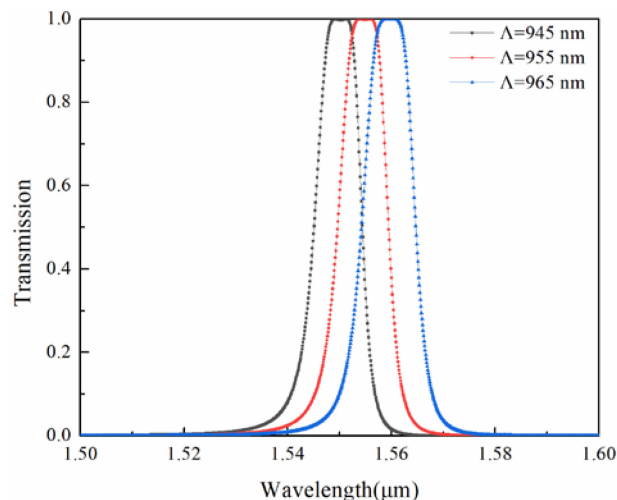


Fig. 6. (Color online) Spectra of the double-RWG-based filter with different grating widths for TE-polarization plane wave.

Fig. 6 demonstrates that the optical resonance of the double-RWG-based filter can be tuned by adjusting the grating period  $\Lambda$  for TE-polarized light at an incident angle of  $4^\circ$ . The remaining structural parameters were the same as those for obtaining Fig. 4(b). We verified the eigenvalues of the single RWG with different  $\Lambda$  using the coupled leaky mode theory. The variation in radiative quality factors was not evident with increase in the period of the grating. It was also seen that the resonant peak shifted with change in the grating period whereas the variation in the spectral bandwidth was slow. This is different from the previous results shown in Fig. 4. It is possible because the external leakage rate of the leaky mode is relatively stable with respect to the change of period in the communication band. Physically, the coupled leaky mode resonance can occur while the mode matching between the incident light wave and a leaky mode of the RWG. Therefore,

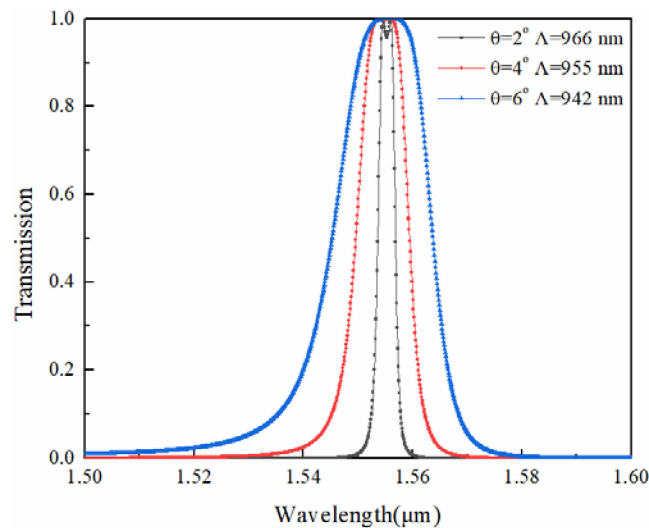


Fig. 7. (Color online) Spectra of the double-RWG-based filter for different grating periods and incident angles.

one can understand that the spectral response exhibits redshift regularly owing to the real part change of the eigenvalue of the leaky mode in a RWG.

Fig. 7 shows that the bandwidth of the spectrum of the proposed structure can be tuned by varying the grating period as well as the incident angle. It is seen that the resonant wavelength was maintained at the wavelength of 1555.8 nm and the bandwidth ranged from 2.9–18.7 nm for a significant change in the incident angle and grating width. Therefore, the spectral bandwidth of the proposed resonant filter can be controlled by the incident angle and its resonant wavelength can be altered by varying the grating period. In this study, we presented a flat-top bandwidth-tunable filter by adjusting the angle of incidence and focusing on controlling the information about the eigenmodes of the resonant structure. Thus, the spectral width of the flat-top filters can be tuned by simply changing the angle of incidence satisfying the conditions of the leaky mode resonance. In order to realize the design, one can choose the appropriate incident angle in accordance with the chirping grating period using the results from FP-CLMT model method.

#### 4. Conclusion

In summary, we investigated the spectrum of a resonant structure based on cascaded resonant gratings. The bandwidth and resonance peak of a flat-top spectrum of the resonant structure were tuned by varying the incident angles and structural parameters in the telecommunication band by the combined effects of the guided mode and F-P resonances. The proposed resonant filter exhibits potential in practical applications such as WDM systems.

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