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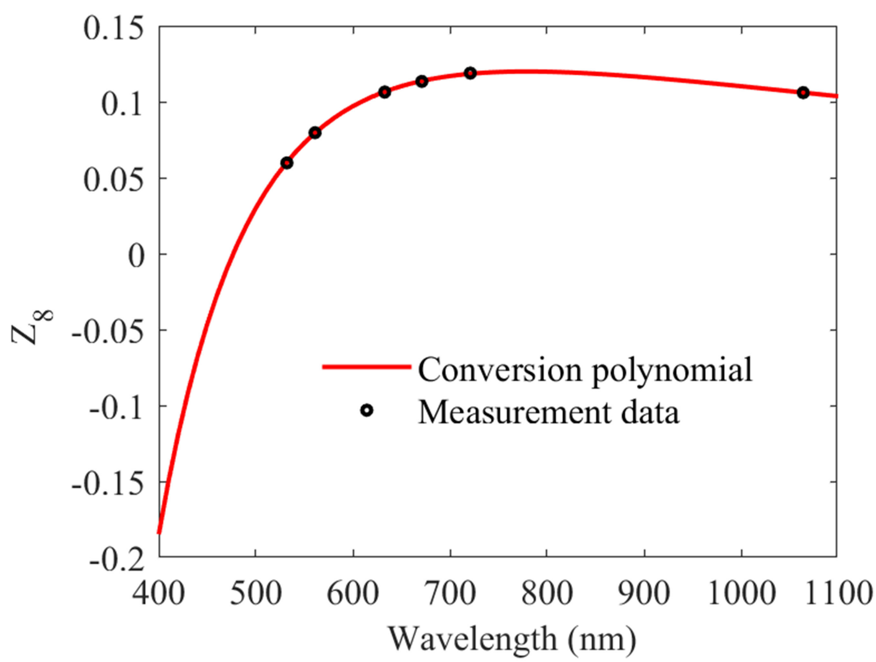

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An Improved Fitting Method for Predicting the Zernike Coefficient–Wavelength Curves

Fang Wang,[1] Qiyuan Zhang,[2] Haoyu Wang,[1] Dayong Zhu,[1] Huaikang Zhu,[1] Wenxin Jia,[3] and Sen Han[1,2]

[1]School of Optical-Electrical and Computer Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China
[2]Suzhou H&L Instruments LLC, Suzhou 215123, China
[3]Suzhou University of Science and Technology, Suzhou 215009, China

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Abstract: Broadband transmitted wavefront measurement is a detection method that uses transmitted wavefronts at a few specific wavelengths though a function of Zernike coefficients and wavelength to predict other transmitted wavefronts at any wavelength within a certain range. This method is used when the designed wavelength of the interferometer and the detection wavelength are different. The function of the Zernike coefficients and wavelength can be expressed by the Conrady formula which can be used to describe a monotonic function curve or a function curve with an inflection point. Two methods, i.e., solving and fitting, can be used to determine the coefficients of the Conrady formula. Owing to the inevitable errors in the actual measurements, the fitting method is preferred to determine the coefficients of the Conrady formula. However, by analyzing the Conrady formula, it was found that the curve obtained by fitting the Conrady formula is a monotonic curve. This method cannot be used to obtain a curve with an inflection point or a slow-changing curve in the long waveband. In this paper, we propose an improved data fitting approach that transforms the Conrady formula into a polynomial formula to fit a given set of data for determining the Conrady formula coefficients. The correctness of the method was verified via simulations. Finally, the feasibility of the proposed method was verified by changing the light source and using the Fizeau interferometer to measure the transmitted wavefront of the doublet lens. To ascertain the feasibility of using visible light to predict the invisible light band, a near-infrared 1064 nm light source was added to the experimental set-up to broaden the range of the transmitted wavefront prediction waveband. Experimental results showed that the conversion polynomial fitting method can be used to obtain monotonic curves and the curve with an inflection point and a slow-changing curve in the long waveband, thereby mitigating the limitation of directly fitting the Conrady formula using a set of data points. Furthermore, using this method, we verified that the visible light can be used to predict the transmitted wavefront at any wavelength in the invisible light band. This observation is highly significant for the practical applications of the broadband transmitted wavefront measurement technique.

Index Terms: Broadband transmitted wavefront measurement, conversion polynomial fitting, near-infrared.

1. Introduction

Broadband transmitted wavefront measurement is a method that uses transmitted wavefronts at a few specific wavelengths to predict the transmitted wavefront at any wavelength within a certain range through a function of wavefront and wavelength. Since the Zernike coefficients are orthogonal in the unit circle and similar to the Seidel aberration, the characteristics of the transmitted wavefront are usually described by a combination of Zernike polynomials [1], [2], [7]. Owing to the wavelength limitation of the instrument in practical measurements, the specific wavelength that can be used for the measurement is very limited [3]–[7], [10]. The broadband transmitted wavefront measurement method can overcome the limitation of the waveband range in the traditional measurement techniques. The relationship between the Zernike coefficients and wavelength is the key factor in predicting the transmitted wavefront at other wavelengths.

In our previous studies [7]–[12], the transmitted wavefront measurement of an optical system for different wavelengths of light is essentially similar to dispersion phenomenon. Therefore, the dispersion formula can be used to describe the function of the Zernike coefficient and wavelength. Through the simulation and analysis of various types of optical systems, two formulas were obtained that could be used to describe the relationship between the Zernike coefficients of the transmitted wavefront and wavelength, namely the Conrady formula and the apochromatic characteristic formula (ACF). The ACF can be considered as a set of formulas. By modifying the power series of the wavelength term, the calculation accuracy of the Zernike coefficient-wavelength curve (the curve describes the function of the Zernike coefficients and wavelength) can be improved. The ACF is suitable for describing different types of optical systems. The main feature of the two formulas is that only a few coefficients are required to be calculated, which means that only a small number of data points are necessary to predict the other data in a large band. Two methods can be used to predict the curve. One method is to calculate the curve by solving. The Conrady formula has three unknown quantities, i.e., A, B, and C. Thus, three sets of data are required to determine the coefficients of the Conrady formula. The other method is to calculate the curve by fitting the Conrady formula using several data points (greater than or equal to the number of coefficients to be solved) by following a certain algorithm (the most commonly used algorithm is the least squares method) Through MATLAB [7]–[9]. In theory, if the values of wavelength and the Zernike coefficients are accurate, then choosing the solving method to determine the coefficients of the Conrady formula is a more accurate choice. However, in actual measurements, errors are inevitable. Even if the measurement data has small errors, it may cause major changes in the solved curve. Moreover, these errors may even change the shape (trend) of the curve, a phenomenon that has been discussed in [7]. For example, some curves are monotonous Zernike coefficients–wavelength curves. While using data with errors, a curve with an inflection point is obtained. In this case, the error in predicting the Zernike coefficients of other wavelengths is huge, especially for the far-band data, such as the near-infrared band (1064 nm), in the prediction curve. The fitting method can have a certain inhibitory effect on the error. However, directly fitting the Conrady formula using the tool provided by MATLAB can only generate a monotonic curve, which has been discussed in [9]. The curve that represents the relationship between the Zernike coefficients and wavelength of a monochromatic optical system is mostly a monotonic curve, and sometimes has an inflection point. Therefore, the fitting method is better for this type of system. However, owing to the wide variety of optical systems, there are many shapes of Zernike coefficient–wavelength curves, especially the curves with an inflection point, which are observed in many optical systems. Hence, the method by directly fitting the Conrady formula can effectively predict the Zernike coefficients–wavelength curves of only a few systems. To use the fitting method to predict more Zernike coefficients–wavelength curves, we require an improved fitting formula. In fact, in many applications, polynomial formulas are used to fit the curves, since polynomials are general formulas, which can increase the number of fitting items to improve the fitting accuracy [13]–[16]. A larger number of fitting data increases the fitting accuracy. The broadband transmitted wavefront measurement method uses different formulas and less data. Thus, this method is more susceptible to the influence of measurement errors in practical applications. This drawback

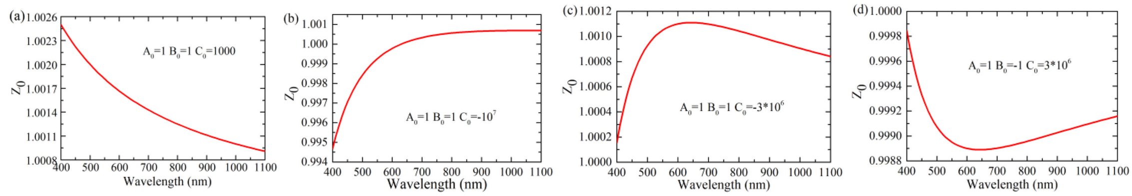

Fig. 1. Curves described by the (1). (a) $A_0 = 1$, the signs of B_0 and C_0 is the same. (b) $A_0 = 1$, the signs of B_0 and C_0 is the different. (c) $A_0 = 1$, the signs of B_0 and C_0 is the different. (d) $A_0 = 1$, the signs of B_0 and C_0 is the different.

ultimately impedes the practical applications and promotion of broadband transmitted wavefront methods.

In continuation of our previous investigations, in this study, we focus on the methods to reduce the effect of errors on the prediction curve in actual measurements, improve the prediction accuracy, and broaden the waveband of the broadband transmitted wavefront measurement technique. This paper focuses on: (i) analyzing the characteristics of the Conrady formula, (ii) proposes an improved fitting method to determine the coefficients of the Conrady formula, each term in Conrady formula is multiplied by λ^n to transform the Conrady formula into a polynomial formula, then the transformed polynomial formula is used to determine the coefficients of the Conrady formula, and the function of Zernike coefficients and wavelength is not changed, and (iii) proposes different calculation methods to determine the coefficients of the Conrady formula for different types of Zernike coefficient–wavelength curves. For the improved fitting method, we evaluate the numerical value, number, and location of the chosen data points on the predicted curve. Simulations and experiments have proved that the proposed method can predict the curve with high accuracy without the requirement of large number of data points. This advantage is highly significant for the practical applications of broadband transmitted wavefront measurement technique.

2. Principle

2.1 Conrady Formula

The dispersion formula reflects the deflection of the incident light of different wavelengths after passing through a medium. The transmitted wavefront measurement of an optical system for different wavelengths of light is essentially similar to dispersion phenomenon. Therefore, the Conrady formula can be used to describe the function of the Zernike coefficient and wavelength. The Conrady formula is defined as [8]:

$$Z_i = A_i + \frac{B_i}{\lambda} + \frac{C_i}{\lambda^{3.5}} \tag{1}$$

Where Z_i is the Zernike coefficient; A_i, B_i and C_i are constants; λ is the wavelength; $1/\lambda$ and $1/\lambda^{3.5}$ are monotonic functions; and i varies from 0 to 36.

From (1), we can infer that when the signs of the coefficients of the $1/\lambda$ and $1/\lambda^{3.5}$ are same, (1) remains a monotonic function, as shown in Fig. 1(a); On the contrary, when the signs are opposite, (1) yields a curve with an inflection point, as shown in Fig. 1(b), Fig. 1(c), and Fig. 1(d). We assume that the Zernike coefficient is the first term, that is, $i = 0$.

When A_i, B_i, and C_i have different combinations, (1) describes curves of different shapes. However, it can be used to describe curves with one inflection point at most, as shown in Fig. 1. In fact, the Zernike coefficient–wavelength curves of many optical systems have one inflection point, except for the monotonic curve and the curve with two inflection points. The curves with two inflection points need to be represented by the ACF. Therefore, the Conrady formula can describe most of the Zernike coefficient–wavelength curves. When the Conrady formula is used to describe

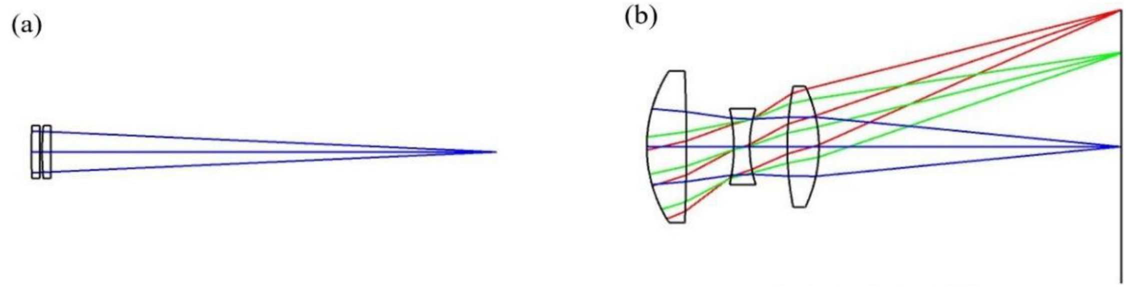

Fig. 2. The monochromatic optical systems used in simulation. (a) F/11 (F-number) (b) Cooke.

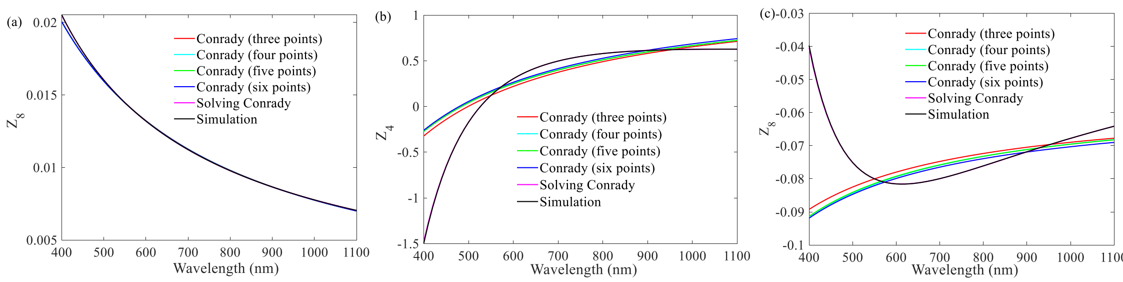

Fig. 3. Results of solving and fitting the Conrady formula. (a). Z_8 from Fig. 2(a) (b). Z_4 and Z_8 from Fig. 2(b).

the Zernike coefficient–wavelength curves, it can describe both monotonic curve and a curve with an inflection point. This is the most significant difference between the Conrady formula describing the Zernike coefficient–wavelength curves and the refractive index curves.

2.2 Applying the Conrady Formula

The Conrady formula can be used to describe the Zernike coefficient–wavelength curves of many optical systems. The coefficients of the Conrady formula can be determined by solving or fitting (1). To compare the two methods, we used two monochromatic optical systems for analysis. One is with F/11 (F-number) and the field of view (FOV) is designed as 0°, as shown in Fig. 2(a). The other is Cooke lens with F/5 (F-number) and the maximum FOV is designed as 20°, as shown in Fig. 2(b). The materials of two optical systems are BK7 glass. We selected three sample curves that can be described using the Conrady formula: (i) Z_8 curve from Fig. 2(a), as shown in Fig. 3(a); (ii) Z_4 curve from Fig. 2(b), the FOV is selected as 20°, as shown in Fig. 3(b), and (iii) Z_8 curve from Fig. 2(b), the FOV is selected as 20°, as shown in Fig. 3(c).

To determine the coefficients of the Conrady formula, we selected three (530 nm, 720 nm, 1060 nm), four (530 nm, 630 nm, 720 nm, 1060 nm), five (530 nm, 630 nm, 670 nm, 720 nm, 1060 nm) and six points (530 nm, 630 nm, 670 nm, 720 nm, 1060 nm) to directly fit the Conrady formula. And we select three points (530 nm, 720 nm, 1060 nm) to solve the Conrady formula. It is evident from Fig. 3(a) that the curves that are obtained by fitting as well as solving the Conrady formula are consistent with the simulated curve. This consistency shows that the two methods can be used to obtain curves similar to those shown in Fig. 3(a). Fig. 3(b) and 3(c) show that the method of solving the Conrady formula can be used to predict the Zernike coefficient–wavelength curves, which are similar to the curves shown in Fig. 3(b) and 3(c). However, directly fitting the Conrady formula is not suitable for these curves, irrespective of the number of data points used for the fitting process.

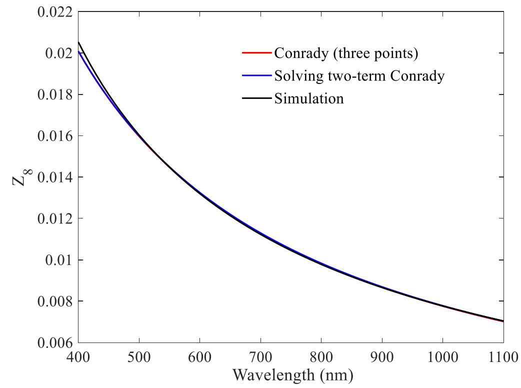

Fig. 4. Results of solving the two-term Conrady formula and fitting the Conrady formula.

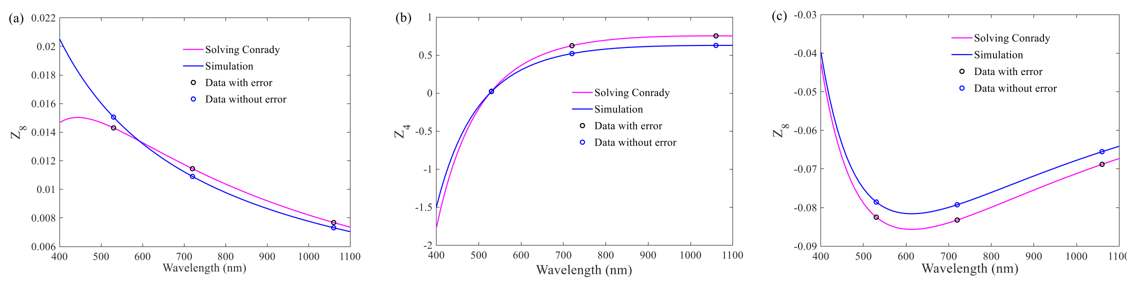

Fig. 5. Influence of error on solving the Conrady formula for predicting the curve.

The reason for observing this phenomenon is explained as: For the three simulated Zernike coefficient–wavelength curves, the ones determined by the fitting method are all monotonic, regardless of their shape. The coefficient C_i in (1) that is obtained by fitting the Conrady equation using MATLAB is relatively small compared to the denominator $\lambda^{3.5}$. Therefore, the third term in (1) can be ignored, thereby modifying (1) as:

$$Z_i = A_i + \frac{B_i}{\lambda} \tag{2}$$

(2) is referred to as the two-term Conrady formula, which plays an important role in the fitting process [9]. Hence, the curves that are determined by directly fitting the Conrady equation are monotonic. As shown in Fig. 4, the curves that are determined by solving the two-term Conrady formula with two data points (530 nm and 1060 nm) and directly fitting the Conrady formula with three data points (530 nm, 720 nm, and 1060 nm) are basically the same. This further indicates that the first two terms of the Conrady formula dominate the fitting process. Hence, we can choose to obtain some of the Zernike coefficient–wavelength curves by solving the two-term Conrady equation. In this case, the solving method is relatively simple and uses lesser number of data points.

As shown in the preceding analysis, the direct fitting method is suitable only for certain types of Zernike coefficient–wavelength curves. However, many other curves that are similar to those shown in Fig. 3(b) and 3(c) can be obtained specifically by solving the Conrady formula. However, in actual measurements, calculation of the Zernike coefficient–wavelength curves by solving the Conrady equation is severely affected by measurement errors. As shown in Fig. 5, the Zernike coefficient–wavelength curves that are obtained by solving the Conrady equation using three data points are marred by errors.

Hence, both the Conrady formula solving and fitting methods have considerable limitations in actual measurements, which in turn impedes the effectiveness and applicability of the broadband transmitted wavefront measurement technique.

2.3 An Improved Fitting Method

To improve the applicability of broadband transmitted wavefront measurement technique, we need to find an improved fitting approach.

We found that the most common formula in the fitting method is a polynomial formula. As long as the number of data points and polynomial terms are sufficiently large, we can fit most types of curves. Following this, we develop an improved approach of transforming the Conrady formula into a polynomial formula. To obtain this new polynomial formula, each term in (1) is multiplied by λ^n as:

$$Z_i \cdot \lambda^{\mathrm{n}} = A_i \cdot \lambda^{\mathrm{n}} + B_i \cdot \lambda^{(n-1)} + C_i \cdot \lambda^{(n-3.5)} \tag{3}$$

When $n \geq 3.5$, we obtain the coefficients A_i, B_i and C_i by fitting the data points with (3). Then, A_i, B_i and C_i are substituted back into (1) to predict the Zernike coefficient–wavelength curves. Thus, the accuracy of the fitting coefficients is reflected in the predicted Zernike coefficient–wavelength curve, as more accurate fitting coefficients lead to a more accurate Zernike coefficient–wavelength curve. In this way, the disadvantages of directly fitting or solving the Conrady formula are mitigated, thereby improving the prediction accuracy of the broadband transmitted wavefront measurement technique. Here, we refer to this improved method as the conversion polynomial method. In the next section, we verify the feasibility and accuracy of the proposed conversion polynomial method through various simulations.

3. Simulation

Since the curve in Fig. 3(a) can be obtained by directly fitting the Conrady formula, the conversion polynomial method is first used to determine the coefficients of the Conrady formula of the curves in Fig. 3(b) and 3(c).

For a better comparison with the actual experiment, the data points that are used to fit the conversion polynomial formula should be consistent with those used in the actual experiments. Hence, we selected the data points at 530 nm, 560 nm, 630 nm, 670 nm, and 720 nm [8]. Although the Zernike coefficient–wavelength curve can be predicted using five wavelengths, the range covered by these five wavelengths is relatively small. In the previous experiment, it was impossible to judge the accuracy of the predicted curve in a wider data range. Hence, in our experiment, we added the 1064 nm laser source to use up to six wavelengths in the simulation (1060 nm added) and expand the band range of the simulated curve to 400–1100 nm.

3.1 Fitting the Conversion Polynomial Formula

To predict the curve by fitting the conversion polynomial formula to the data points, we should consider the three main factors that affect the accuracy of the prediction: error, number, and position of the fitted data points.

In the simulation, first, we randomly generated 1%, 2%, 5%, 10%, and 20% errors in the data, and then fitted the conversion polynomial formula to this data. At least three known data points are required to determine the coefficients of the Conrady formula. Thus, the number of data points that are required for fitting can be selected as 3, 4, 5, and 6. For a better analysis and comparison, the interval between the selected data points should be as large as possible. Table 1 illustrates the selection of the data points for fitting.

To verify the applicability of the conversion polynomial formula fitting method, we analyzed the three influencing factors separately.

TABLE 1
Selected Data Points for Fitting

Points	530 nm	560 nm	630 nm	670 nm	720 nm	1060 nm
Three	●		●		●	
Three	●				●	●
Four	●		●		●	●
Five	●	●	●	●	●	
Five	●		●	●	●	●
Six	●	●	●	●	●	●

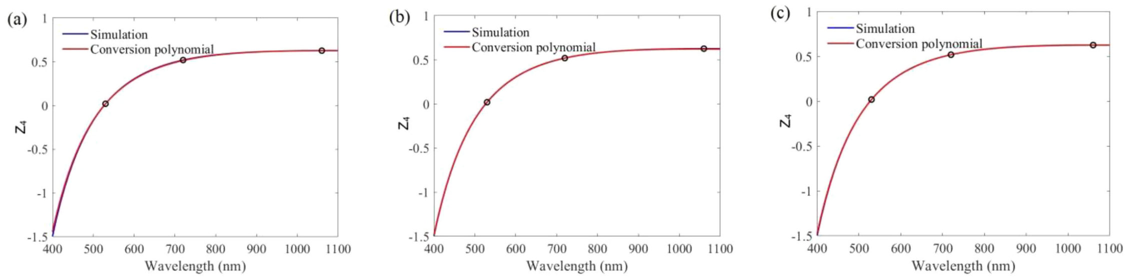

Fig. 6. Fitting the error data with the conversion polynomial formula. (a), (b) and (c) are form the 200 simulated results. The three small black circles in (a), (b) and (c) represent three data points at 530 nm, 720 nm and 1060 nm.

3.1.1 Effect of Error on the Prediction Accuracy

Initially, 1%, 2%, 5%, 10%, and 20% errors were generated in the data points randomly to produce a new set of data points in the simulation. Next, these new data points were fitted with the conversion polynomial formula. First, we selected three data points at 530 nm, 720 nm, and 1060 nm to analyze the Zernike coefficient–wavelength curve shown in Fig. 3(b). We randomly generated errors between -1% and $+1\%$ for the selected fitting data, and then compared the simulated data with the results obtained by fitting the conversion polynomial formula to the error included data. The measurements in actual experiments should be repeatable. Hence, in the simulation, we randomly generated 200 errors for each set of data. Fig. 6 shows three out of the 200 results.

It can be seen from Fig. 6 that the curve predicted by fitting the conversion polynomial formula to the error data is consistent with the simulation curve, which shows the effectiveness of the proposed novel method. Generally, if the prediction accuracy is not good, the error is reflected at the two ends of the curve. Therefore, the error percentages at 400 nm and 1100 nm are calculated to compare the influence of error on the accuracy of predicting the curve using the error data. Fig. 7 shows the error percentage at 400 nm and 1100 nm for 200 simulations.

Using the same method, we analyzed the other four situations. Fig. 8 shows the error percentage at 400 nm using data containing 1%, 2%, 5%, 10%, and 20% error for fitting the conversion polynomial formula. The simulation results indicated that when the random error value reached 10%, the method of fitting the conversion polynomial formula influenced the prediction accuracy considerably. When the random error was within 5%, only an acceptable prediction accuracy was obtained. Hence, as long as the error was within a certain range, this method could effectively predict the Zernike coefficient at any wavelength in a larger band.

3.1.2 Effect of the Position of Fitted Data Points on the Prediction Accuracy

In the next step, we analyzed the influence of the position of the fitted data points on the prediction accuracy by fitting the conversion polynomial formula to the error included data. To analyze the

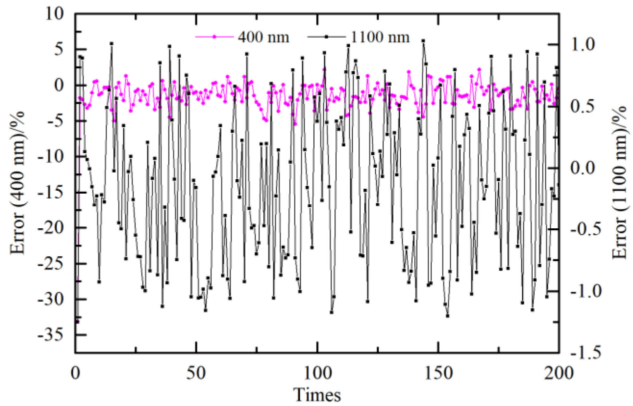

Fig. 7. Error percentages at 400 nm and 1100 nm for 200 simulations.

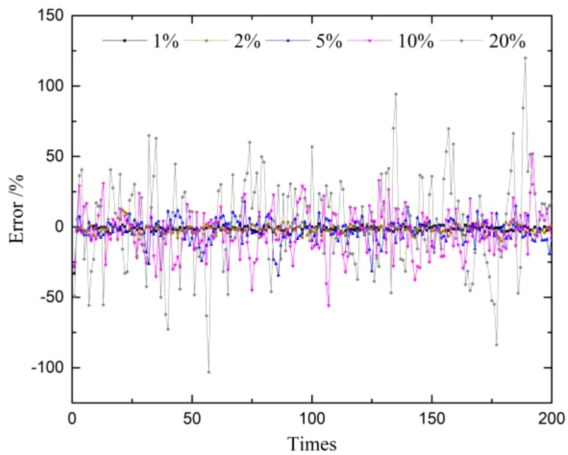

Fig. 8. Error percentages at 400 nm using data containing 1%, 2%, 5%, 10%, and 20% error.

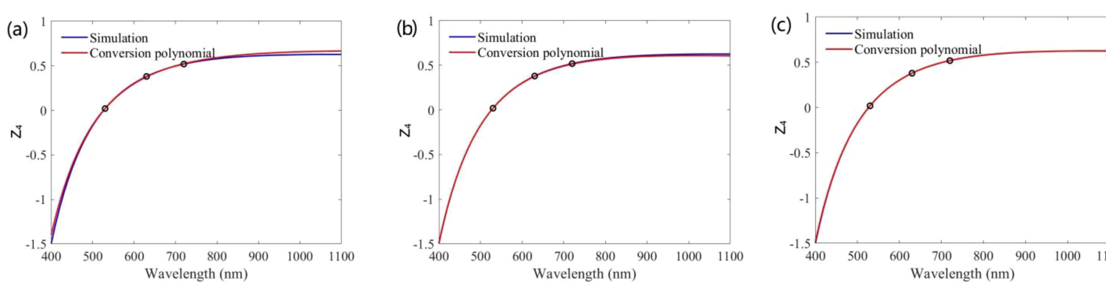

Fig. 9. Fitting results using data points at 530 nm, 630 nm and 720 nm. (a), (b) and (c) are form the 200 simulated results. The three small black circles in (a), (b) and (c) represent three data points at 530 nm, 630 nm and 720 nm.

Zernike coefficient–wavelength curve shown in Fig. 2(b), we selected three data points at 530 nm, 630 nm, and 720 nm and randomly generated errors between −1% and +1% into these selected points. Next, we compared the simulated curves with the results obtained by fitting the conversion polynomial formula. Fig. 9 shows three simulation results out of the total 200 simulations that were

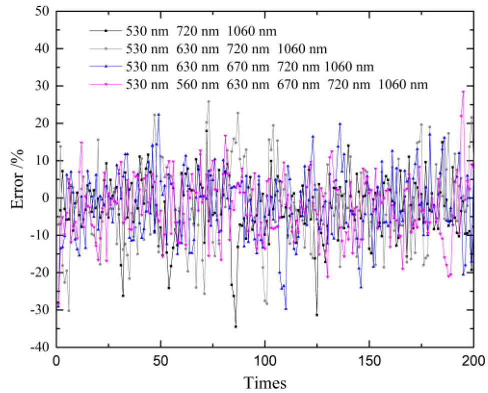

Fig. 10. Error percentage at 400 nm using data containing 5% error at various positions for 200 simulations.

TABLE 2
RMS for Fitted Error At 400 nm for 200 Simulations

Position	1%	2%	5%	10%	20%
530-630-720	3.167188	5.627372	13.877764	22.746928	55.081367
530-720-1060	2.674133	3.670753	8.435449	16.430812	32.724322
530-630-720-1060	2.733516	3.798517	10.878597	18.198971	37.305789
530-560-630-670-720	3.606036	6.696723	19.243473	35.976356	68.869670
530-630-670-720-1060	2.671897	4.064069	9.371227	19.718565	35.715442
530-560-630-670-720-1060	2.511909	3.576408	8.494731	18.657072	34.907311

performed. From Fig. 9, it can be seen that the prediction accuracy was not as good as that shown in Fig. 6.

3.2.3 Effect of the Number of Fitted Data Points on the Predicting Accuracy

Next, we analyzed the influence of the number of data points that were used for fitting the conversion polynomial formula on the prediction accuracy. Using the same method, we compared the fitted results of four data points at 530 nm, 630 nm, 720 nm, and 1060 nm with the fitted results of three data points at 530 nm, 720 nm, and 1060 nm. The comparison showed that the results of using four points and three points were basically the same. Similarly, the prediction accuracy that was obtained by using five points and six points for the fitting did not show significant improvement as compared to that obtained with three points. Fig. 10 shows the percentage of error at 400 nm for the different numbers of used data points.

Table 2 and Table 3 show the Root Mean Square error for the fitted error percentage at 400 nm/1100 nm for 200 simulations for the different cases illustrated in Table 1.

From the results listed in Table 2 and Table 3, we can infer that the conversion polynomial formula fitting method to determine the coefficients of the Conrady formula can predict the curve, shown in Fig. 3(b), with a high accuracy. The prediction accuracy for the curve is affected mainly by the measurement error and position of the used data points for the fitting. When the error is very small, the prediction accuracy is very good. However, when the error is more than 10%, the prediction accuracy becomes very poor. If the used data points cover a wider band range, then a relatively good prediction accuracy is obtained. When other conditions remain the same, increasing only the number of used data points for fitting does not improve the prediction accuracy, instead it may become worse. We analyzed the curve shown in Fig. 3(c) (the curve with inflection point), the

TABLE 3
RMS for Fitted Error At 1100 nm for 200 Simulations

Position	1%	2%	5%	10%	20%
530-630-720	3.497186	7.078747	15.915635	29.429014	64.085468
530-720-1060	0.641171	1.267156	3.349392	6.235561	12.000362
530-630-720-1060	0.65333	1.211675	3.236745	6.433151	11.715178
530-560-630-670-720	3.799569	7.345181	20.722472	37.141405	74.818407
530-630-670-720-1060	0.633198	1.264076	3.047559	6.287129	12.551979
530-560-630-670-720-1060	0.656009	1.313786	3.129831	6.374374	12.189359

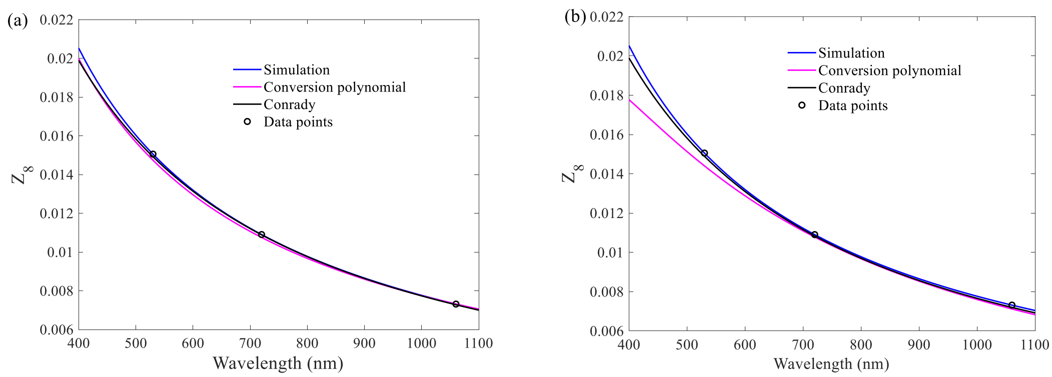

Fig. 11. Prediction results for the monotonic curve by fitting the conversion polynomial and Conrady formulas with error data at 530 nm, 720 nm and 1060 nm. (a) 1% error (b) 5% error.

coefficients of the Conrady formula of the curves in Fig. 3(c) can also be determined by using the conversion polynomial method. Therefore, in order to improve the prediction accuracy of this method in practical applications, it is not necessary to measure a large number of sample data, as required in many other curve-fitting applications. However, to fit the curve, it is important to reduce the measurement error as much as possible, and choose data points that cover a wider band range.

3.2 Fitting the Conversion Polynomial Formula to a Monotonic Curve

To study the versatility of the conversion polynomial formula fitting method, we selected three data points at 530 nm, 720 nm, and 1060 nm, and generated randomly 1%, 5% errors into the selected data points respectively. Then, we fit the conversion polynomial and Conrady formulas to the error data for obtaining the curve shown in Fig. 3(a). Then, we compared these fitting results with the simulated curve. In Fig. 11, it can be seen that the three curves are relatively consistent when the error is small. However, when the error is relative high, the accuracy of fitting the conversion polynomial is poor. Thus, the conversion polynomial fitting method can also predict the monotonic curve shown in Fig. 3(a) when the error is small. Otherwise, fitting the Conrady formula is a better choice for predicting for the curve shown in the Fig. 3(a).

4. Experiment

4.1 Longitudinal Chromatic Aberration

The experimental set-up for broadband transmitted wavefront measurement was the same as that used in a previous study with some modifications [7]. The main body of the experimental device was the Fizeau interferometer. To perform broadband transmitted wavefront measurement, we selected

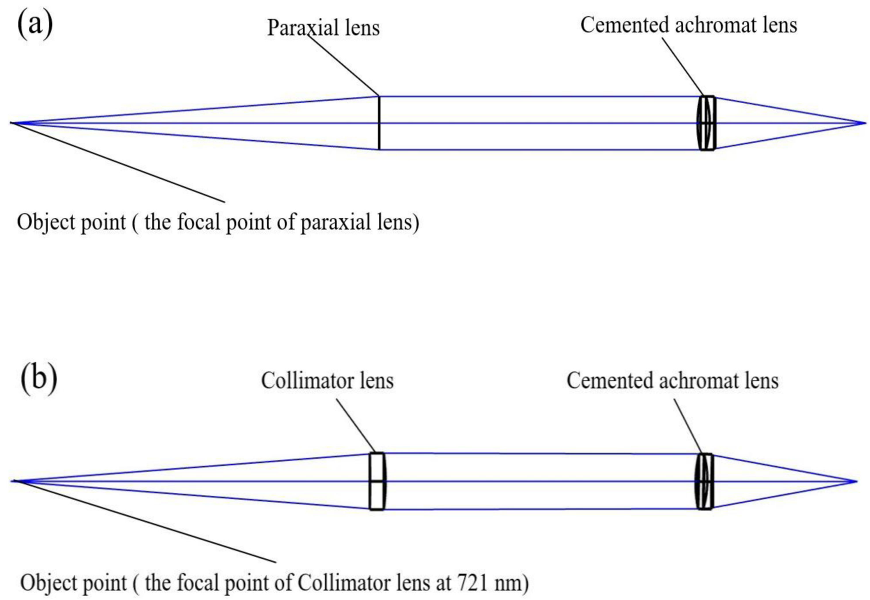

Fig. 12. Two optical systems in the interferometer. (a) an ideal paraxial lens that did not produce aberration (b) a practical collimator.

six laser sources with wavelengths of 532 nm, 561 nm, 632.8 nm, 671 nm, 721 nm, and 1064 nm. During the experiment, adjusting the collimation system of the interferometer for each light source impacts the final measurements. To reduce the influence of the change in beam collimation that was caused by changing the light sources on the measurement, we adjusted and fixed the position of the optical system of the interferometer with respect to the 721 nm laser light source (721 nm was in the middle of the six wavelengths used in the experiment and the beam collimation showed the smallest variation when the light sources were changed). Thereafter, the collimator position was not changed while changing the light sources for different wavelengths [7], [9]. Contrary to the experimental device used in a previous study, in the present case we added a 1064 nm laser source and adjusted the position of the interferometer optical system with respect to the 721 nm laser source. Because the waveband of the wavelengths that were used in this experiment was broader, we first analyzed the influence of the chromatic aberration of the optical system on the predicted results in the simulation.

The Fizeau interferometer contains a collimating system and an imaging system. The optical system is a monochromatic system; therefore, chromatic aberration occurs when the wavelength changes. As the interferometer detects only the difference between the test beam and the reference beam, the longitudinal chromatic aberration influence of the imaging system is strongly suppressed because of the common-path configuration of the Fizeau interferometer. In addition, the focal length of the imaging lens is usually small; thus, the longitudinal chromatic aberration influence of the imaging system is very small. The main problem arises from the chromatic aberration of the collimating system. The focal length of the collimator changes as we change the lasers with different wavelengths. This changes the collimation of the outgoing beam. Because the focal length of the collimator in the small-diameter laser interferometer is small, the focal length of the collimator changes slightly as the lasers are changed.

Based on the aforementioned reasons, a 3/4″ caliber interferometer was used in the experiment. For a better comparison with the experimental results, we chose a cemented achromatic lens with known parameters for the simulation. The lens was placed in the optical path with the x-axis and y-axis tilted by 2°. We developed two systems in Zemax optics software: one with an ideal paraxial lens that did not produce aberration and the other with a practical collimator (as shown in Fig. 12).

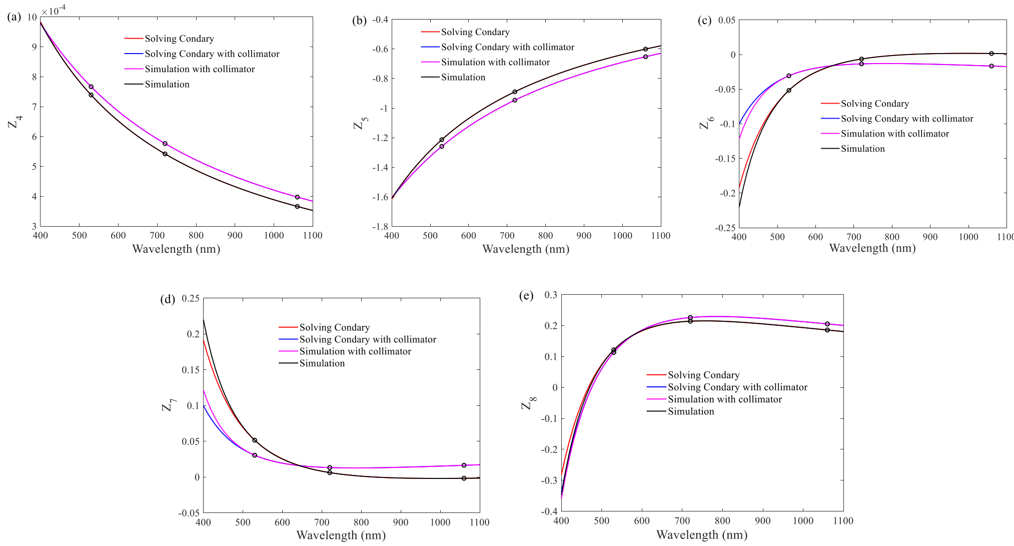

Fig. 13. Zernike coefficient–wavelength curves of the cemented achromatic lens in two cases. (a) The black line shows the Z_4 simulation result for an ideal paraxial lens. The red line shows the Z_4 result obtained by solving Conrady formula for an ideal paraxial lens. The blue line shows the Z_4 result obtained by solving Conrady formula for a practical collimator. The pink line shows the Z_4 simulation result for a practical collimator. The selected data points is at 530 nm, 720 nm and 1060 nm. (b) Z_5 (c) Z_6 (d) Z_7 (e) Z_8.

The material of the two lenses in the collimator were same, and the design wavelength was 721 nm. Using this, we compared the transmitted wavefront Zernike coefficient of the two systems at these wavelengths from 400–1100 nm when the defocus of the system was at zero. When we changed the wavelength, the distance between the object point and the collimator remained unchanged, which was consistent with the physical experiment.

Fig. 13 shows the Zernike coefficient–wavelength curves ($Z_4 - Z_8$) of the cemented achromatic lens in the two cases. From these curves, it can be seen that the longitudinal chromatic aberration influence of the collimation system affects the prediction accuracy of the Zernike coefficient–wavelength curves, especially the Zernike coefficients Z_6 and Z_7. However, it can also be seen that the overall trend of the predicted curves in the two cases is consistent. All these curves ($Z_4 - Z_8$) can be predicted with good accuracy by solving the Conrady formula using three data points at 530 nm, 720 nm, and 1060 nm. In fact, the cemented achromatic lens and collimator lens can be combined to form a new optical system, which is equivalent to the simulated optical system. The simulation results show that the chromatic aberration of the collimation system of the interferometer influence the prediction of the Zernike coefficient–wavelength curves. However, this influence is small. Moreover, the Zernike coefficient–wavelength function does not change. Hence, through the experiment, we can still verify whether the Conrady formula can be used to predict the Zernike coefficients at any wavelength.

4.2 Analysis and Discussion

In the actual experiment, the test lens was a doublet lens. We placed it in a tilted state to measure the Zernike coefficients in six cases. However, the degree of tilt was unknown. We performed repeated measurements for each laser source. The averaged Zernike coefficient measurement results ($Z_4 - Z_8$) are shown in Table 4.

To evaluate the error in the Zernike coefficients measurements, we define the maximum measurement error as:

$$\mathrm{Errmax}_i = \left|\frac{\max(Z_i) - \min(Z_i)}{\min(|Z_i|)}\right| * 100\% \tag{4}$$

TABLE 4
Measured Zernike Coefficients ($Z_4 - Z_8$)

Zernike coefficients	532 nm	561 nm	632.8 nm	671 nm	721 nm	1064 nm
Z_4	-1.646580	-1.553830	-1.370150	-1.293410	-1.201110	-0.813720
Z_5	2.049189	1.930470	1.712785	1.621429	1.511657	1.035484
Z_6	-0.020010	-0.031320	-0.04476	-0.046690	-0.048360	-0.037530
Z_7	-0.023620	-0.029240	-0.03869	-0.041550	-0.045510	-0.041520
Z_8	0.059736	0.079644	0.106430	0.113402	0.118895	0.106041

TABLE 5
Maximum Measurement Error ($Z_4 - Z_8$)

Zernike coefficients	532 nm	561 nm	632.8 nm	671 nm	721 nm	1064 nm
Z_4	0.131%	0.038%	0.284%	0.098%	0.094%	0.095%
Z_5	0.083%	0.039%	0.071%	0.056%	0.045%	0.108%
Z_6	10.640%	2.360%	1.303%	1.037%	1.659%	2.946%
Z_7	8.493%	1.198%	4.512%	3.681%	3.608%	1.960%
Z_8	2.303%	0.309%	0.768%	0.319%	0.307%	1.884%

Where i is from 0 to 36, Z_i is the 37 terms Zernike coefficients of the Zernike polynomial.

Thus, the maximum error in the measurement of every Zernike coefficient in the six cases can be calculated using (4). The corresponding results are shown in Table 5.

From Table 4, it can be seen that the values of Z_6 and Z_7 are negligible. Therefore, they are susceptible to external factors. This is evident in Table 5 where the maximum errors in the measurement of Z_6 and Z_7 are relatively high. From Table 5, the values of Z_6 and Z_7 at 532 nm are higher. The reason of the phenomenon is that there are several sharp points among the Z_6 and Z_7, however, the average value maintains a stable state. If the several sharp points are removed, Z_6 is 4.65%, Z_7 is 6.23%. And the measurement error of the data is used to explain the fluctuation of the data.

Based on the simulation results for the doublet lens in Fig. 13 and the simulation analysis described in Sections 2 and 3, the Z_4- and Z_5-wavelength curves can be predicted by directly fitting the Conrady formula (or solving the two-term Conrady equation) and the conversion polynomial formula to the data points. The Z_6-, Z_7- and Z_8-wavelength curves can be predicted by fitting only the conversion polynomial formula to the data points.

To measure Z_4 and Z_5 by fitting the Conrady formula, we selected three data points at 532 nm, 721 nm, and 1064 nm; four data points at 532 nm, 632.8 nm, 721 nm, and 1064 nm; five data points at 532 nm, 632.8 nm, 671 nm, 721 nm, and 1064 nm; six data points at 532 nm, 561 nm, 632.8 nm, 671 nm, 721 nm, and 1064 nm. To solve the two-term Conrady equation for predicting the Zernike coefficient–wavelength curves, we selected two data points at 532 nm and 1064 nm. From Fig. 15(a) and 15(b), it can be seen that the measured Zernike coefficients not selected in six cases are consistent with the predicted results. The black curve in Fig. 15(a) represents the outcome of fitting the Conrady formula to three selected data points at 532 nm, 632.8 nm, and 721 nm. This shows that the Zernike coefficient Z_4, experimentally measured at 1064 nm, is well predicted by the fitting method. Similarly, Fig. 15(b) shows the same outcome. These results implied that we can use the Zernike coefficients in the visible band to predict the Zernike coefficients in the near-infrared band.

In the simulation in the section 3, in order to achieve multiple loops to generate multiple sets of simulation results, we directly call the Lsqcurvefit function provided by the MATLAB tool to fit the data. However, there are sometimes wrong results during the fitting process. For the actual measurement, there is only one set of data results. In order to avoid fitting errors, we can also

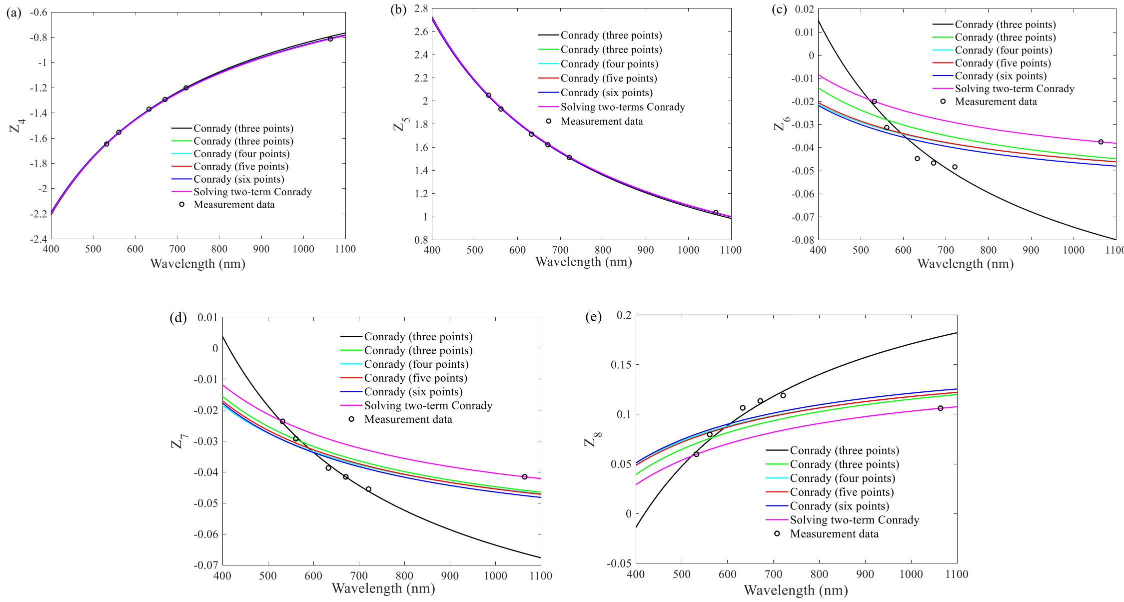

Fig. 15. Predicted curves by fitting Conrady and solving two-terms Conrady. (a) Z_4 (b) Z_5 (c) Z_6 (d) Z_7 (e) Z_8.

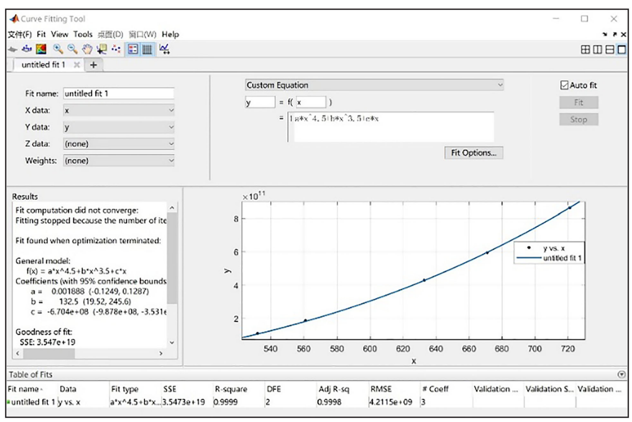

Fig. 14. Fitting the conversion polynomial using the MATLAB's curve fitting.

choose MATLAB's curve fitting tool to intuitively fit the data. As shown in the figure below, choose different fitting options to fit the curve so that the fitted data is close to the fitted curve, thereby obtaining a more accurate Zernike-wavelength curve.

To predict the curves for Z_6, Z_7 and Z_8, we used the same method as that for Z_4 and Z_5. Fig. 15(c)–(e) show that direct fitting of the Conrady formula is not suitable for predicting the curves (Z_6–Z_8). Hence, the fitting results in Fig. 15(c)–(e) are not accurate, deviating from measured data points. These observations are consistent with the simulation results described in Section 2.2.

Hence, to predict the curves for $Z_4 - Z_8$, we used the conversion polynomial fitting method. According to (3), the value of n was set to 4.5 and six data points (532 nm, 561 nm, 632.8 nm, 671 nm, 721 nm, and 1064 nm) were used. To verify the universality of the conversion polynomial fitting method for predicting the curves, we selected $Z_4 - Z_8$ curves. Fig. 16(a) and 16(b) indicate that the conversion polynomial fitting method can be used to predict Z_4 and Z_5 curves as well. The applicability of the conversion polynomial fitting method to predict Z_6, Z_7, and Z_8 is shown by

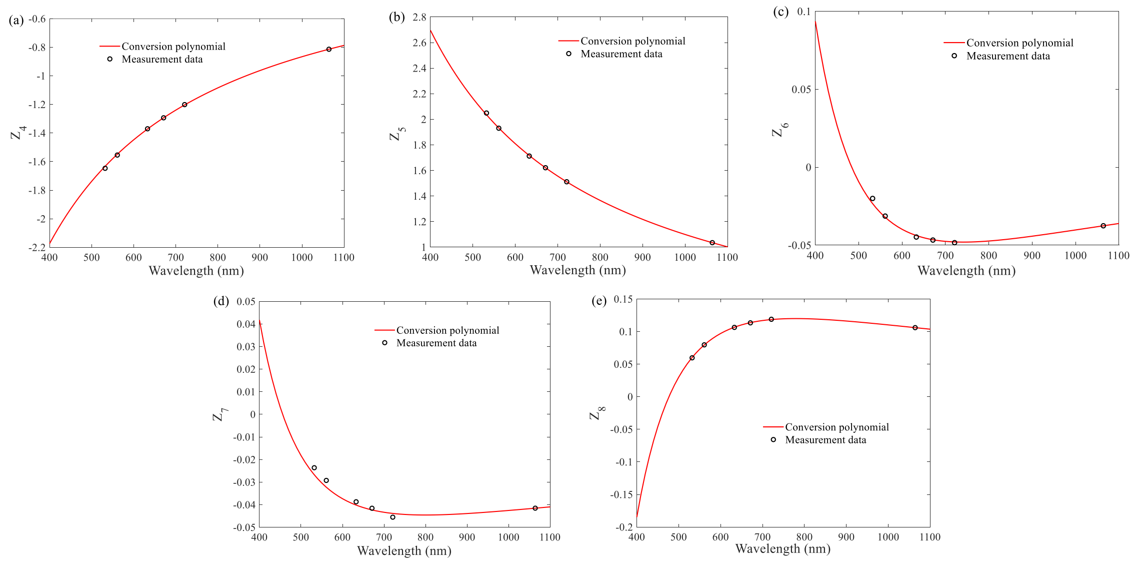

Fig. 16. Curves predicted by fitting the conversion polynomial formula. (a) Z_4 (b) Z_5 (c) Z_6 (d) Z_7 (e) Z_8.

TABLE 6
The Predicting Error ($Z_4 - Z_8$)

Zernike coefficients	532 nm	561 nm	632.8 nm	671 nm	721 nm	1064 nm
Z_4	0.837%	0.368%	0.123%	0.00169%	0.187%	0.00476%
Z_5	0.658%	0.095%	0.249%	0.00735%	0.022%	0.0000689%
Z_6	16.59%	2.428%	2.167%	0.436%	0.949%	0.0458%
Z_7	13.231%	10.304%	3.808%	1.824%	3.647%	0.021%
Z_8	0.844%	0.148%	0.230%	0.284%	0.365%	0.0048%

Fig. 16(c)–(e). To evaluate quantitatively the error in the predicted Zernike coefficients, we define the predicting error as:

$$\mathrm{Err}_i = \left|\frac{Z_{i-predicting} - Z_{i-measuring}}{Z_{i-measuring}}\right| * 100\% \tag{5}$$

Where i is from 0 to 36, $Z_{i-predicting}$ is the predicted 37 terms Zernike coefficients of the Zernike polynomial, $Z_{i-measuring}$ is the measured 37 terms Zernike coefficients of the Zernike polynomial.

Hence, the predicting error of the Zernike coefficients can be calculated using (5). The corresponding results are shown in Table 6. From Table 4, it can be seen that the values of Z_6 and Z_7 are negligible. Therefore, they are susceptible to external factors. So it can be seen that in Table 6 where the predicting errors of Z_6 and Z_7 are relatively high. However, from Fig. 16 and Table 6, the measured results are consistent with the predicted curves.

To further verify whether the Zernike coefficient–wavelength curve of the test lens can be expressed by the Conrady formula, we fit the conversion polynomial formula to the selected five data points at 532 nm, 561 nm, 632.8 nm, 671 nm, and 721 nm and used the data point at 1064 nm to ascertain the validity of the predicted curve obtained by the conversion polynomial fitting method.

The results in Fig. 17 imply that Z_4, Z_5, Z_6, and Z_8 are well predicted, and the measurement results are consistent with the predicted curve. The Zernike coefficient of the infrared wavelength is still well predicted using the visible light measurement data in a limited bandwidth. However, the predicted result of Z_7 is slightly poor, because the values Z_7 is negligible and it is susceptible to

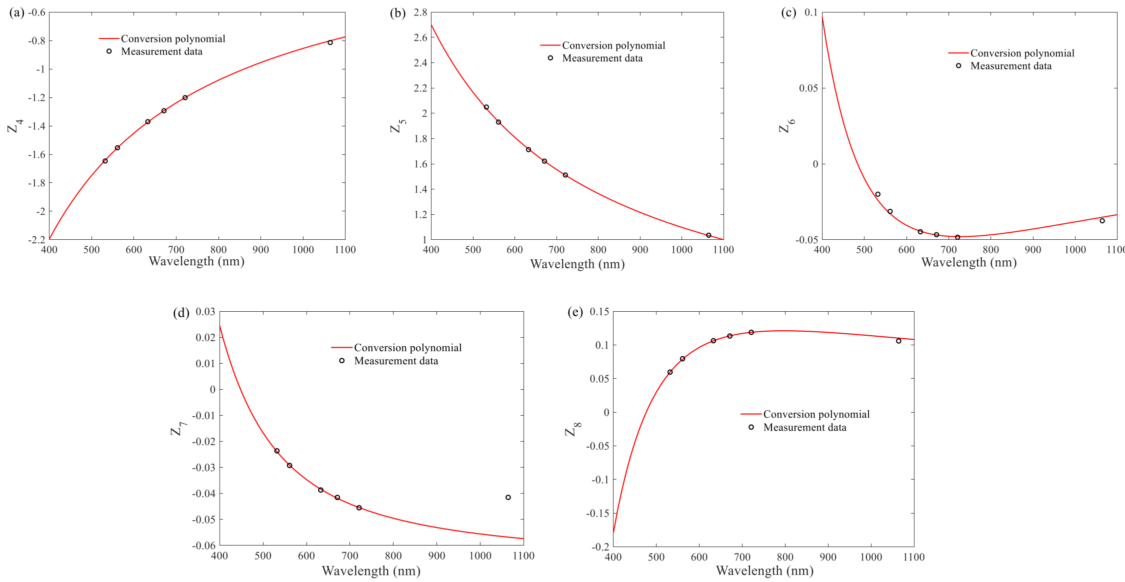

Fig. 17. Curves predicted by fitting the conversion polynomial formula. (a) Z_4 (b) Z_5 (c) Z_6 (d) Z_7 (e) Z_8.

external factors. It can be seen in Fig. 17 that the predicted results at 1064 *nm* are not as good as those in Fig. 16, which proves that the position of the used data points is very important for the accuracy of the curve in the case of errors in measurement.

5. Conclusion

In this paper, we propose an improved fitting method to overcome the drawback of direct fitting of the Conrady formula, which can predict only the monotonic Zernike coefficient–wavelength curves. In the proposed improved fitting method, we convert the Conrady formula into a polynomial formula, and then directly fit the converted polynomial formula to the data points. Simulation results show that this method can predict the Zernike coefficient–wavelength curves with better accuracy. The main factors that affect the accuracy of the predicted curve include the error size of the measured data and position of the data points to be fitted. Therefore, to predict the Zernike coefficients with better accuracy in practical applications, we should minimize the transmission wavefront measurement error at each wavelength, and choose data points that cover a wider band for fitting the converted polynomial formula with better statistics. The experimental results show that monotonic curves with small changes can be calculated by solving the two-term Conrady formula or by directly fitting the Conrady formula. For monotonic curves with a slow-changing in the long waveband and curves with inflection points, it is necessary to apply the conversion polynomial fitting method to calculate the required curve. If the measurement error is small, our proposed improved method can predict all types of curves with good accuracy. Hence, applying the improved method to predict the Zernike coefficient–wavelength curves is highly significant in the practical applications of the broadband transmitted wavefront measurement technique. The experimental results also verified that we can use the Zernike coefficients in the visible waveband to predict those at the invisible waveband, which is of considerable practical significance. In addition, it was observed via the simulated measurement system that while changing the light source, longitudinal chromatic aberration that is generated by the collimation system of the laser interferometer also affects the prediction accuracy of the curves. Hence, for a better prediction accuracy, a more precise interferometric measurement system needs to be developed, in which especially the collimation system can be adjusted. At the same time, a short-wave laser light source should be added to the measuring device. In addition, the improved fitting method can be used to predict the apochromatic transmission characteristic curve theoretically, which needs to be verified by a more complex measurement system.

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