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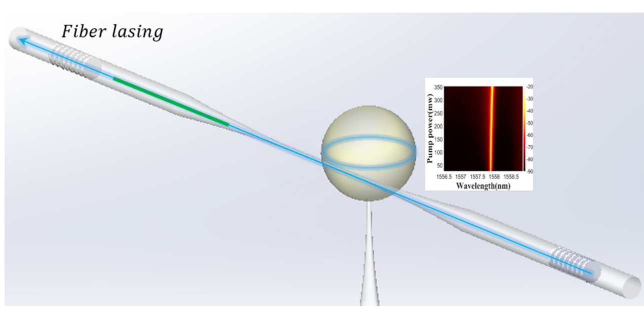

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Whispering-Gallery-Mode Resonator-Based Hybrid Cavity for Mode Stabilization of a Fiber Laser

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Abstract: The performances of a fiber laser stabilized by a whispering-gallery-mode(WGM) resonator depends heavily on the coupling of the resonator and fiber system, and is labile due to the lasing mode competitions among excitable WGMs. Here, we propose a hybrid cavity where a linear fiber cavity is coupled by a WGM resonator for dual stabilizations of the laser mode, and experimentally demonstrate single-mode lasing emission with stable and high signal to noise ratio (up to 61 dB), narrow linewidth (low to ~5 kHz) operations. By adjusting the coupling of the linear fiber cavity and the WGM resonator, the fiber laser can operate in two- to four- mode lasing without mode hopping as well.

Index Terms: WGM resonator, Fiber laser, Coupled cavities.

1. Introduction

Whispering-gallery-mode (WGM) resonators with high quality (Q) factor resonance and small mode volume (V_m) enable significant enhancement of optical interaction between light and matter [1], and have been intensively studied in the past three decades for the applications of ultralow-threshold microlaser [2], [3], high sensitive sensor [4], and optical wave conversion through nonlinear effect [5], [6]. The rare-earth ion doped WGM glass or crystal microresonators, with high Q resonances and small V_m, allow a significant increasement of Purcell factor (proportional to a factor of Q/V_m) and are promising for lasing emission with an ultra-low threshold,and a wide range of lasing bands have been demonstrated [7]. In addition to the direct lasing emission in active resonators, passive WGM resonators are employed as a mode stabilization element for fiber laser in near infrared band [8]–[14], while the narrow linewidth fiber lasers have found a wide range of applications [15]–[18], such as optical communication, microwave wave generation, sensor, and high resolution range measurement, for instances. So far, two mode stabilization schemes have been proposed. One is based on the light feedback from the counter propagating mode which is internally coupled with the forward propagating mode via backward scattering in WGM resonator [8]. Another scheme is based on the add-drop system with two waveguides coupling [9], where the dropped resonance

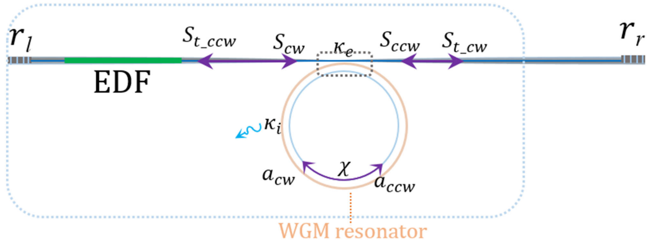

Fig. 1. Theoretical module of the erbium-doped fiber laser. r_l and r_r are the reflecting coefficients of left and right mirrors of the linear fiber cavity, EDF: erbium-doped fiber, S_{cw} and S_{ccw} are the light waves propagating in the directions labelled by arrows, S_{t_cw} and S_{t_ccw} denote the transmitted waves. κ_e and κ_i are the field loss rate associated with the fiber coupler and the intrinsic loss rate of WGM, respectively. $|a_{cw}|^2$ and $|a_{ccw}|^2$ denote the CW and CCW WGM energies stored in the resonator as χ denotes the coupling rate of them.

signal performs as the feedback of a fiber-ring laser as well. Both schemes can provide the narrow linewidth optical feedback to the fiber system and either single- or multi-mode, narrow linewidth fiber lasers have been reported. Except directly based on the narrow linewidth feedback of the resonance, the peak-like asymmetric resonances in microcavity can also be applied for selecting the lasing modes of a fiber laser [19]. By thermally tuning the resonances via optical absorption, continuously all-optical tunable fiber lasers were demonstrated as well [20]. However, the operation of fiber laser stabilized by a WGM resonator is heavily dependent on the resonator coupling with the fiber system. Several WGMs can be simultaneously excited and the laser operation is labile due to the competition for gain of the excitable modes. In order to quell the lasing competition for a stable operation, the matching of the waveguide mode with a particular WGM should be finely tuned.

Here, we propose a hybrid cavity, consisting of a linear fiber cavity which is coupled by a WGM resonator, for mode stabilization of a fiber laser. Our analysis reveals that the hybrid cavity enables the mode-hopping free laser operation as the lasing mode is dually stabilized by the WGM resonator and the linear fiber cavity. With this cavity, wherein a silica-microsphere is employed as the WGM resonator and the linear fiber cavity is constructed by a chirped fiber Bragg grating (CFBG) and a fiber mirror (FM), we experimentally demonstrate a fiber laser with stable, single-mode operation, By tuning the coupling of the linear fiber cavity and the microsphere, stable laser operations with two- to four-mode lasing are also demonstrated.

2. Mode Resonances in the Hybrid Cavity

The fiber laser module with a hybrid cavity is shown in Fig. 1. The linear cavity is constructed by two reflecting mirrors with reflection coefficients r_l and r_r, and coupled by a WGM resonator. A length of erbium-doped fiber (EDF) provides gain for laser emission. The coupling between the waveguide and WGM resonator can be described by a group of dynamic coupled mode equations [22]:

$$\begin{pmatrix} \frac{da_{cw}}{dt} = (i\Delta\omega - \kappa)a_{cw} + i\chi a_{ccw} + \sqrt{2\kappa_e}S_{cw} \\ \frac{da_{ccw}}{dt} = (i\Delta\omega - \kappa)a_{ccw} + i\chi a_{cw} + \sqrt{2\kappa_e}S_{ccw} \end{pmatrix} \tag{1}$$

where $\Delta\omega$ is the angular frequency detuning between the light wave in resonator and WGM resonance, κ_e describes the field loss rate associated with the waveguide coupling as $\kappa = \kappa_e + \kappa_i$ denotes the total field loss rate of waveguide coupled WGM resonance(κ_i is the intrinsic field loss rate of WGM resonance). The coupling rate between clockwise(CW) and counterclockwise(CCW) WGMs is described by χ. κ_i relates to the optical quality of the fabricated WGM resonator, such as the smoothness of light reflecting surface, optical absorption coefficient of resonator material. κ_e is determined by many factors, including the matchings of group velocities and polarizations, and fields overlap between waveguide mode and WGM mode, and can be optimized by choosing suitable size of waveguide, adjusting coupling position and tuning the polarization state of

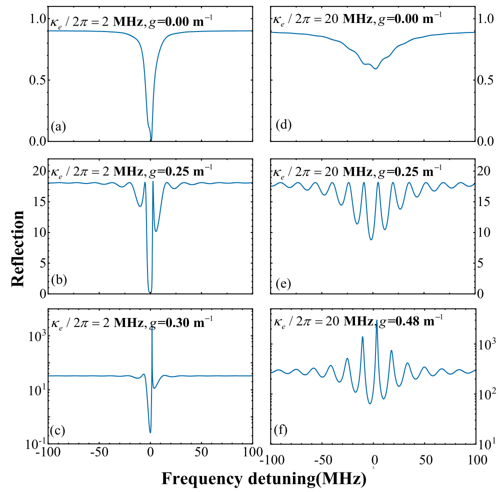

Fig. 2. Reflection spectra of the optical system enclosed in the dashed rectangle in Fig. 1 versus frequency detuning around the resonance of WGM with $\kappa_e/2\pi = 2$ MHz (a)–(c), $\kappa_e/2\pi = 20$ MHz (d)–(f) with an increasing gain g.

waveguide mode through a polarization controller (PC). S_{cw} and S_{ccw} denote the light waves of the waveguide modes propagating in directions labelled by arrows as $|S_{cw}|^2$ and $|S_{ccw}|^2$ are the propagating powers. $|a_{cw}|^2$ and $|a_{ccw}|^2$ denote the CW and CCW WGM energies stored in the resonator. The transmitted waves $S_{t\cdot cw}$ and $S_{t\cdot ccw}$ are described as

$$\begin{pmatrix} S_{t\_cw} = -S_{cw} + \sqrt{2\kappa_e} a_{cw} \\ S_{t\_ccw} = -S_{ccw} + \sqrt{2\kappa_e} a_{ccw} \end{pmatrix} \tag{2}$$

Thus, the input and output waves of a waveguide coupled WGM resonator are related by a transfer matrix S for the stable regime at which the power loss in WGM resonator equals the power coupled into the resonator and the left side of (1) equals 0:

$$\begin{pmatrix} S_{cw} \\ S_{t\_ccw} \end{pmatrix} = S \begin{pmatrix} S_{t\_cw} \\ S_{ccw} \end{pmatrix} \tag{3}$$

and

$$S_{cw} = r_l e^{i\theta_l + 2gL_g} S_{t\_ccw} \tag{4}$$

wherein g and Lg denote the gain coefficient and the length of EDF, $\theta l = 2\omega Llne/c$ is the phase shift that the waveguide mode acquires as it propagates by a distance of $2Ll$ at the left side of the WGM resonator with a group velocity c/ne. Therefore, we can find the reflection coefficient r of the wave S_{ccw} coupled into optical system enclosed by the dashed rectangle:

$$r = \frac{S_{t\_cw}}{S_{ccw}} \tag{5}$$

In Fig. 2, we show the reflection spectra for two coupling states without or with gain: $\kappa_e/2\pi = 2$ MHz, and $\kappa_e/2\pi = 20$ MHz. Here, due to that κ_i and χ are intrinsically associated with the quality of WGM resonator, we keep them as constants $\kappa_i/2\pi = 1$ MHz, $\chi/2\pi = 0.5$ MHz, and Ll and r_l are set as 6 m and 0.95. For the first state at which the linewidth of WGM $2\kappa/2\pi$ is less than the free spectrum range (FSR) $\sim$17 MHz of the linear fiber cavity, as presented by (a) to (c), we can find that only one Fabry Pérot (FP) mode at position of WGM evolves into lasing as a sufficient gain $g = 0.30$ m^{-1} is given. However, as shown in (d) to (f), several FP modes can be excited and have a chance to become a lasing mode and compete for the gain with each other as $2\kappa/2\pi$ $>$ FSR due to the space hole burning effect of the fiber laser with a standing-wave resonator. Namely, several FP modes amidst the spectral band of WGM linewidth may be exited for lasing.

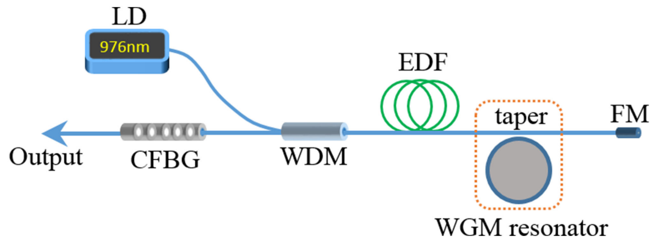

Fig. 3. (a) Schematic diagram of the fiber laser. LD: laser diode, CFBG: chirped fiber Bragg grating, WDM: wave division multiplexer, FM: fiber mirror.

In addition, many WGMs in the spectrum band of amplified spontaneous emission (ASE) of EDF (more than 60 nm) have the opportunity to be excited for the fixed coupling position of waveguide at the same time. Thus, once a sufficient gain is provided by EDF, several WGMs can involve into lasing of single- or multi-FP modes. The envelop of the 'inverted' peaks in (b) and (e) indicates the weaker amplification of light wave within the spectral band of WGM due to the loss of light wave at the position of WGM resonance. In order to quell mode competition and excite only one WGM for single-wavelength lasing (not single mode), we have to adjust the polarization state of waveguide mode and optimize the coupling of waveguide mode with certain WGM. Additionally, if we want to achieve single-mode lasing, a WGM resonance with linewidth 2κ less than FSR of the linear fiber cavity enclosed in the dashed rectangle is required. But for the whole resonance system shown in Fig. 1, only certain WGM can be excited with angular frequency ω_s satisfying resonance condition of standing wave:

$$\frac{2L_r\omega_s n_e}{c} + \theta = 2m\pi \tag{6}$$

where Lr is the distance of right mirror from the WGM resonator and θ is the sum of the phase of above derived r and the phase shift induced by the right mirror and m is an integer number. Therefore, only the mode with ω_s can be excited for lasing emission, and stable lasing operation without mode hopping is expected. In the following, we construct a fiber laser based on a linear fiber cavity coupled by a silica microsphere and stable operation is realized.

3. Experimental Demonstration of an EDF Laser with a Linear Fiber Cavity Coupled By a Silica Microsphere

The schematic diagram of the fiber laser is shown in Fig. 3, a ~976 nm single-mode LD with a single-mode fiber pigtail is used as the pump source and a 980/1550 nm WDM is for the coupling of pump power as a length of 3 m EDF (EDFL-980-HP, Nufern) functions as the gain material. A CFBG and a FM function as the left and right mirrors of the linear fiber cavity with the reflectivity of 90% and 99%, and a reflectivity bandwidth of ~18 nm and ~150 nm near 1.55 μm, respectively. We employ a silica microsphere as WGM resonator, which is fabricated by melting a silica fiber tip using a high power CO_2 laser and has a diameter of ~130 μm [21]. A fiber taper functions as a waveguide for optical coupling of the silica microsphere and the linear fiber cavity [23]. In our experiment, the taper has a waist diameter of ~2.0 μm and is in-contact coupled with the microsphere for the stable coupling. The resonance linewidth is measured and estimated from the transmission spectrum at 1.55 μm and is ~3 MHz for a chosen resonance and accordingly the loaded Q factor is ~6.5×10^7. The laser output is collected from the output port of CFBG and the optical and electric spectra are measured by an optical spectrum analyzer (OSA) and an electrical spectrum analyzer (ESA), respectively.

Firstly, we adjust the in-contact coupling position of the tapered fiber to the equator of the silica microsphere to excite only fundamental or fundamental-like WGMs [24], single-wavelength lasing operation without mode hopping is realized as the pump power P is tuned to 352 mW. A typical laser spectrum is shown in Fig. 4(a) as $P = 352$ mW, the laser emission has a signal to noise ratio (SNR)

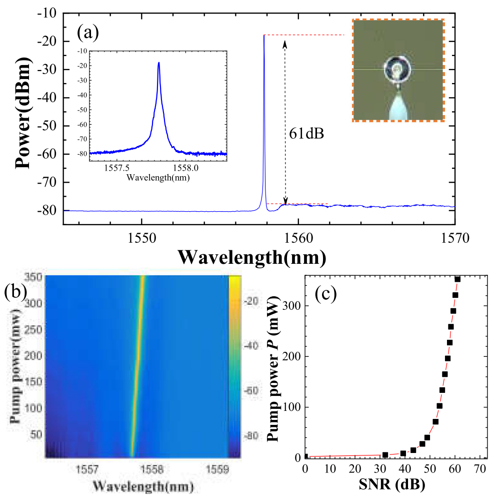

Fig. 4. (a) laser spectrum within a spectral band of 30 nm at pump power of 352 mW, the insets show the detailed lasing spectrum within a spectral band of 1 nm and a microscopic image of a taper coupled silica microsphere, (b) lasing spectra evolution, and (c) the SNR evolution as *P* increases from threshold to 352 mW.

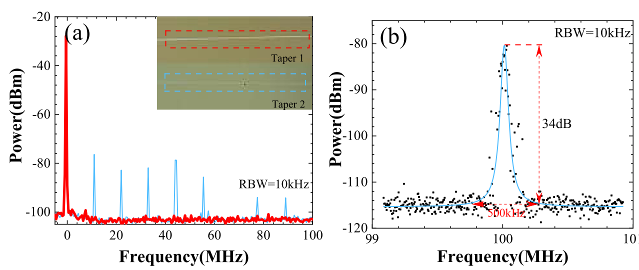

Fig. 5. (a) The beat signal of a single-wavelength lasing by using taper 1(red line) and taper 2 (cyan line), (b)The beat signal measured based on DSH method.

of ∼61 dB while the power of ASE noise in the reflection band of CFBG is reduced to ∼−80 dBm. The insets show the detailed lasing spectrum and the taper-coupled silica microsphere. Fig. 4(b) presents the lasing spectrum evolution versus as *P* increases from threshold of ∼4 mW to 352 mW, the stable operation without mode hopping is demonstrated and the slop efficiency of the laser is ∼0.62%. The evolution of the estimated SNR is shown in Fig 4(c), which increases from ∼33 dB to ∼61 dB. At the same time, we compared the performances of the fiber laser without the feedback of FM by simultaneously removing FM. The recorded optical spectra show that the lasing is very unstable due to mode hopping, and the corresponding output power fluctuates frequently. If we removed the WGM resonator, the laser cavity is a FP cavity and the multimode lasing with a band of ∼0.25 nm happens near the edge of refection band of CFBG. Here, the hybrid cavity allows the dual stabilizations of the lasing mode and enables mode hopping-free operation. Due to the thermal effect [25], the lasing wavelength shift of 0.175 nm (∼22 GHz) is observed as *P* increases from threshold to 352 mW. If the EDF is placed at the right hand side of the hybrid cavity, the lasing efficiency increases to ∼6.5% due to the higher amplification efficiency of light wave feedback from WGM resonator as the reflectivity of FM is greater than that of CFBG and additionally the SNR has no obvious improvement. In addition, if we replace the FM with a FBG with a lower reflectivity, a higher lasing efficiency can be obtained.

And then the electric spectra of single-wavelength lasing are measured for checking the lasing mode which may not be distinguished from the recorded optical spectrum with a resolution of 10 pm. Fig. 5(a) presents two spectra with a band of 100 MHz and resolution bandwidth (RBW) of 10 kHz by using two fiber tapers with different diameters at waist. Here, the pump power is 352 mW. The bolder line (red color), corresponding to the using of taper 1(labelled in inset) with diameter of

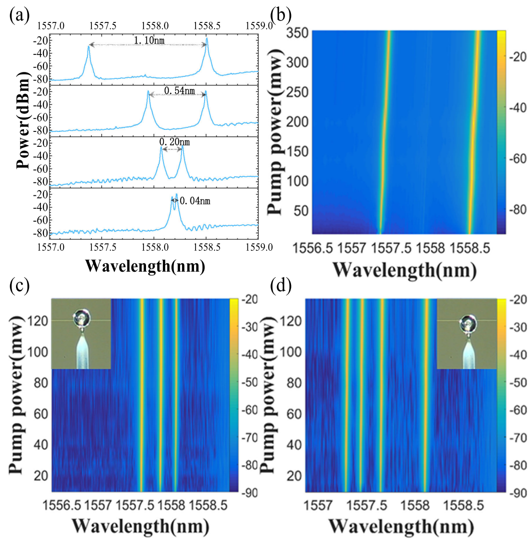

Fig. 6. Lasing spectra of (a) two-mode lasing with a gap of 0.04 nm, 0.20 nm, 0.54 nm and 1.1 nm, (b)–(d) laser spectra evolutions with two- to four-mode resonance as pump power increases. The insets in (c) and (d) show the taper positions with respect to the microsphere.

more than 2 μm(estimated with the reference of single-mode fiber with diameter of 125 μm from a micrograph), shows no side peak, indicating the lasing wave is a continuous and single-mode wave. The other spectrum (cyan line) shows several side peaks with an interval of ∼11 MHz, as a slenderer taper (labelled as taper 2 in inset) with more evanescent field leak or with a greater κ_e or κ is used for the coupling of WGM resonator. As predicted in Fig. 2(a) to (c), the lasing mode contains only one FP mode when the loaded linewidth of WGM 2κ is less than FSR of the linear fiber cavity (∼11 MHz for 9 meters long cavity). Thus, in order to achieve a single-mode fiber laser, either a WGM resonator with a 2κ less than FSR of the linear cavity or a short linear cavity with FSR greater than 2κ is required. In the following, the delay self-heterodyne (DSH) method with a 50km-long fiber (delay time ∼240 μs) and an acousto-optic modulator (100 MHz frequency shift) is used for measuring the linewidth of the single-mode laser. From the recorded electric spectrum shown in Fig. 5(b), the 3dB-linewidth of beat-signal is deduced to be ∼10 kHz from the estimated 500 kHz of 34 dB-linewidth and the actual linewidth of laser is ∼5 kHz, which is half of the self-beat signal linewidth [26].

4. Multi-Mode Laser with Stable Operation

We adjust the coupling position of the fiber taper slightly away from the equator (in-contact coupling is still kept), namely tuning the coupling match between the guided mode in taper and the WGMs of microsphere with different mode number (radial number or azimuthal number) as they share the field at the same position [24], two-mode lasings without mode hopping and with a gap ranging from 0.04 nm(∼5 GHz) to 1.1 nm (∼140 GHz) are realized. Due to that we can't determine which WGM can be excited, thus the separation of lasing wavelength and the lasing wavelength can't be pre-engineered. The lasing spectra are shown in Fig. 6(a). The two-mode lasing originated from the same cavity has high coherence and can be applied for the microwave generation with high quality [17]. Fig. 6(b) presents the evolution of a two-mode lasing spectrum and the lasing wavelength has a shift of ∼0.18 nm due to thermal effect as well. When the taper is further adjusted away from the equator of microsphere, as presented by Fig. 6(c) and (d) where the insets show the coupling position of taper with respect to the silica microsphere, stable operations with three- and four-mode lasing are realized as pump power is less than 123 mW. Thus, fine adjusting of the tapered fiber position with respect to the microsphere allows the controllable selection of the lasing modes and accordingly the single- to four-mode lasings.

5. Conclusion

In conclusion, we have proposed a WGM resonator-based hybrid cavity for the mode stabilization of a fiber laser and experimentally demonstrated a single- to four-mode fiber laser with stable operation by tuning the coupling between the linear fiber cavity and the WGM resonator. We wish the compact fiber laser with single- or multi-mode, narrow linewidth and high SNR can find applications in photonic microwave generation, high-resolution spectroscopy and nonlinear optics.

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