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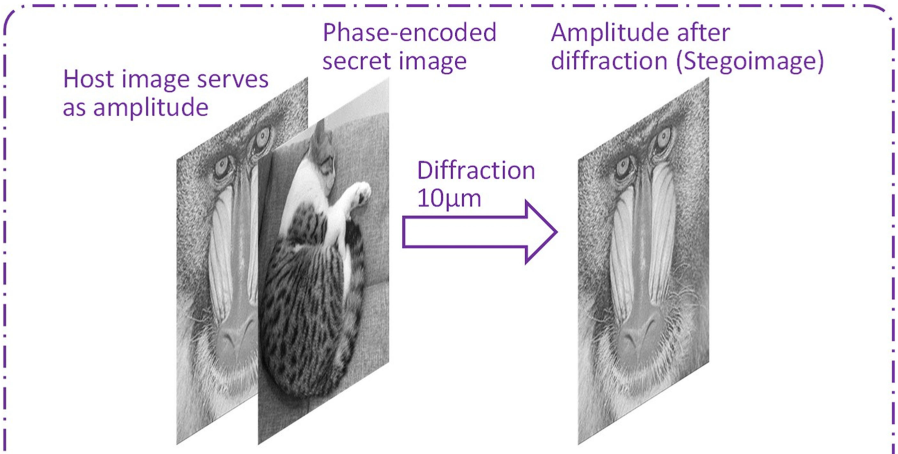
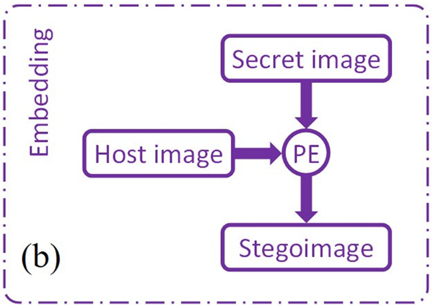


(a) The diagram of the proposed steganography method.



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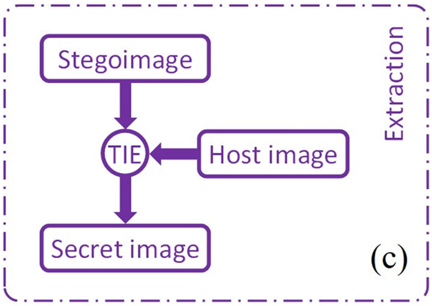

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Abstract: A high capacity steganography based on optical phase-encoding and transport of intensity equation (TIE) is proposed. The secret image is phase-encoded and then attached a real-valued host image to form a complex light field, which diffracts a short distance and generate a slightly different complex amplitude. Its intensity is recorded and the corresponding amplitude is calculated as the stegoimage. For extraction, the secret image could be reconstructed near perfectly by using the TIE method. It is well known that the capacity and the imperceptibility are two fundamental but contrary attributes in any a steganography scheme. However, in this work, the high capacity could be attained while the imperceptibility is well preserved based on the phase-encoding and TIE method. As the steganography is applied mainly on the digital media which are transmitted through the Internet, the optical experiments are not essential in this situation, and we only provide the simulation results to verify the validity and feasibility of our method.

Index Terms: Information security, steganography, optical phase-encoding, transport of intensity equation, TIE.

1. Introduction

Steganography is the technique of covert communication. It works by hiding message in inconspicuous objects in such a way that it is not possible even to detect the existence of the secret message [1], [2]. Therefore, its most important feature is the imperceptibility. Researchers proposed various digital steganography methods, which embed messages into spatial domain, transform domains or feature domains of the host object. Compared with traditional digital steganography methods, optical information technique provides multiple parameters such as amplitude, phase and polarization for encoding. Therefore, various optical methods were used in the steganography, such as holography [3], temporal phase modulation [4], amplified spontaneous emission embedding [5]–[14], discrete prolate spheroidal sequences encoding [15], polarization modulator based code-shift-keying [16] and so on [17]–[20].

Steganography and watermarking are two important subdisciplines of information hiding. Different from watermarking which has the additional requirement of robustness against possible attacks,

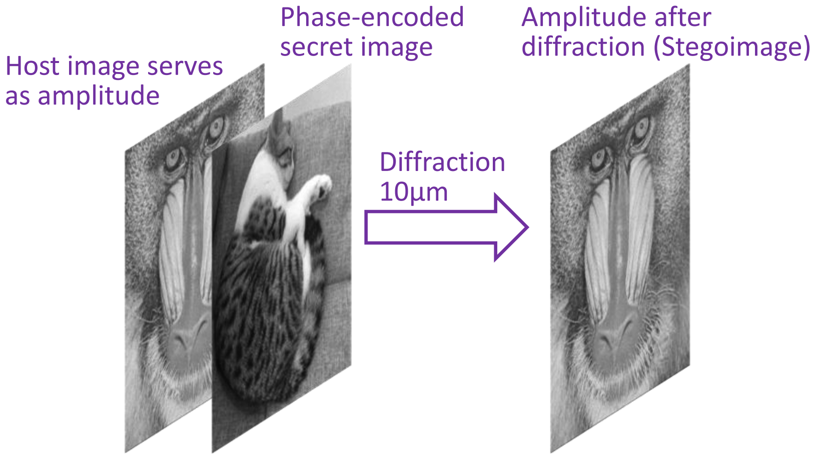

Fig. 1. The diagram of the proposed steganography method.

the steganography, as a technique of covert communication, only put emphasis on imperceptibility and communication capacity. Nevertheless, the communication capacity and the imperceptibility are two contradictory attributes. Simply put, more information embedding will give rise to more change of the host object, which will cause security risks of exposure. For remedying this issue, mathematical methods such as compressive sensing have been introduced to improve the capacity [21], [22]. Most of the previous works embedded the information directly into the spatial or transform domains, and as a result, the high capacity would do harm to the imperceptibility. We choose the optical phase-encoding method, which is an indirect way to embed the secret message into the host image, and the imperceptibility could be well preserved while the capacity could achieve a high level in such a manner. In this paper, we propose a high capacity optical steganography technique based on the phase-encoding and transport of intensity equation (TIE) [23], [24]. A phase-encoded secret image bonds with a real-valued host image to form a complex light field, which diffracts a short distance and generates a slightly different complex amplitude. The corresponding intensity is recorded and its amplitude is calculated as the stegoimage. As the diffraction distance is very short, the generated amplitude changes slightly and therefore, the imperceptibility is well preserved. On the other hand, the phase-encoding technique provides a high capacity while the TIE guarantees a high-quality extraction of the secret message. The issue of capacity and imperceptibility could be well addressed by leveraging our phase-encoding and TIE scheme. The rest of this paper is organized as follows. In Section 2, we give a theoretical description of the proposed steganography method. Section 3 provides simulation results. At last, a conclusion is made in Section 4.

2. Theoretical Description

As shown in the Fig. 1, the embedding process of the proposed steganography method is based on the optical phase encoding and Fresnel diffraction. A secret image is encoded into the phase while the host image is served as the amplitude, and the resultant complex amplitude diffracts a short distance to generate a slightly different complex amplitude, which could be recorded by a camera and its corresponding amplitude is calculated as the stegoimage. By leveraging this optical phase-encoding technique, an $N \times N$ pixels secret image could be embedded into an $N \times N$ pixels real-valued host image, which is a fairly high capacity level compared with the current methods. On the other side, its imperceptibility could be well preserved on the condition that the diffraction distance is short enough. For extraction, the transport of intensity equation (TIE) method is adopted and the secret image could be extracted near perfectly from the stegoimage.

2.1 Embedding

In the embedding process, as shown in the Fig. 1, the secret image ‘Sleeping cat’ is phase-encoded as $\exp[i\pi f(x,y)]$ in a range of $[0, \pi]$, which attaches a real-valued host image, to form a complex light field:

$$u_0(x,y) = |u_0(x,y)| \exp[i\pi f(x,y)] \tag{1}$$

Where $f(x, y)$ is the two-dimensional (2D) distribution of the secret image ‘Sleeping cat’, and $|u_0(x, y)|$ is the 2D distribution of the real-valued host image ‘Baboon’. The corresponding complex amplitude transmits through the free space, which could be formulated by the angular spectrum method:

$$u_z(x,y) = \mathrm{IFT}\left\{\mathrm{FT}\left[u_0(x,y)\right] H(q_x, q_y)\right\} \tag{2}$$

Where FT and IFT denote the Fourier transform (FT) and inverse FT, (q_x, q_y) is the Fourier conjugate variable and $H(q_x, q_y)$ is the transfer function:

$$H(q_x, q_y) = \exp\left[ikdz\sqrt{1-(\lambda q_x)^2-(\lambda q_y)^2}\right] \tag{3}$$

Where $dz = 10\ \mu\mathrm{m}$, which is a short diffraction distance, $k = 2\pi/\lambda$ and λ is the wavelength. And the parameter value of the diffraction distance ($dz = 10\ \mu\mathrm{m}$) is feasible and reasonable because it has been verified by optical experiments and computer simulations in the references [25], [26]. A camera could record the intensity of the complex amplitude after diffraction:

$$I_z(x,y) = |u_z(x,y)|^2 \tag{4}$$

And the corresponding amplitude $|u_z(x, y)|$ could be calculated and saved as the stegoimage:

$$|u_z(x,y)| = \sqrt{I_z(x,y)} \tag{5}$$

2.2 Extraction

As the receivers obtain the stegoimage, the secret image $f(x, y)$ could be calculated from the stegoimage and the host image by leveraging the TIE. Generally speaking, it is difficult to retrieve the true phase from two amplitude constraints using an iterative phase retrieval algorithms. However, if the two amplitudes are in close proximity, its phase distribution could be recovered from the finite difference of the two amplitudes’ intensities using the TIE method [23]–[27]. Here we give a brief derivation of the TIE formulation. A complex amplitude satisfies approximately the parabolic [23]:

$$\left(i\frac{\partial}{\partial z} + \frac{\nabla^2}{2k} + k\right) u_0(x,y) = 0 \tag{6}$$

Where $k = 2\pi/\lambda,\ \nabla = \vec{x}\frac{\partial}{\partial x} + \vec{y}\frac{\partial}{\partial y},\ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and $u_0(x, y)$ is such that,

$$I_0(x,y) = |u_0(x,y)|^2 \tag{7}$$

Where $u_0(x, y)$ is defined in the (1). So that the complex amplitude could be expressed in terms of the intensity and the phase:

$$u_0(x,y) = [I_0(x,y)]^{1/2} \exp[i\varphi(x,y)] \tag{8}$$

Let (6) be multiplied by $u_0{}^*(x, y)$ on the left-hand side and the complex conjugate of (6) be multiplied by $u_0(x, y)$ on the left-hand:

$$\left(i\frac{\partial}{\partial z} + \frac{\nabla^2}{2k} + k\right) u_0(x,y) u_0{}^*(x,y) = 0$$

$$\left[\left(i\frac{\partial}{\partial z} + \frac{\nabla^2}{2k} + k\right) u_0(x,y)\right]^* u_0(x,y) = 0 \tag{9}$$

Which could be expanded as:

$$\begin{aligned} &i{u_0}^*(x,y)\frac{\partial}{\partial z}u_0(x,y)+{u_0}^*(x,y)\frac{\nabla^2}{2k}u_0(x,y)+{u_0}^*(x,y)ku_0(x,y)=0\\ &-iu_0(x,y)\frac{\partial}{\partial z}{u_0}^*(x,y)+u_0(x,y)\frac{\nabla^2}{2k}{u_0}^*(x,y)+u_0(x,y)k{u_0}^*(x,y)=0 \end{aligned} \tag{10}$$

To subtract the two resulting equations one gets:

$$\begin{aligned} &i\left[{u_0}^*(x,y)\frac{\partial}{\partial z}u_0(x,y)+u_0(x,y)\frac{\partial}{\partial z}{u_0}^*(x,y)\right]\\ &+\frac{1}{2k}\left\{\nabla\left[{u_0}^*(x,y)\nabla u_0(x,y)-u_0(x,y)\nabla{u_0}^*(x,y)\right]\right\}=0 \end{aligned} \tag{11}$$

According to the (8), it could be formulated as:

$$i\frac{\partial I_0(x,y)}{\partial z}+\frac{1}{2k}\nabla\cdot[2i\cdot I_0(x,y)\nabla\varphi(x,y)]=0 \tag{12}$$

And one gets:

$$k\frac{\partial I_0(x,y)}{\partial z}=-\nabla\cdot[I_0(x,y)\nabla\varphi(x,y)] \tag{13}$$

Now, we need further introduce an auxiliary function $\psi(x,y)$, which is defined as:

$$\nabla\psi(x,y)=I_0(x,y)\nabla\varphi(x,y) \tag{14}$$

Then the (13) could be rewritten as:

$$\nabla^2\psi(x,y)=-k\frac{\partial I_0(x,y)}{\partial z} \tag{15}$$

According to the Fourier derivative theorem:

$$\mathrm{FT}\left[\partial_x^{(n)}w(x,y)\right]=i^n q_x^n\mathrm{FT}[w(x,y)] \tag{16}$$

So we get:

$$\psi(x,y)=\mathrm{IFT}\left[\left(q_x^2+q_y^2\right)^{-1}\mathrm{FT}\left(k\frac{\partial I_0(x,y)}{\partial z}\right)\right] \tag{17}$$

The longitude intensity derivative $\partial I_0(x,y)/\partial z$ in (17) along the Fresnel diffraction could not be directly measured [28]. Conventionally, it has to make an approximation by a finite difference between two adjacent intensities. In the embedding process, as shown in the Fig. 1, we set a very short diffraction distance, which ensures that the derivative $\partial I_0(x,y)/\partial z$ could be approximated by its finite difference. In the extraction process, the derivative is approximated as follows:

$$\frac{\partial I_0(x,y)}{\partial z}\approx\frac{I_z(x,y)-I_0(x,y)}{dz} \tag{18}$$

Where $I_z(x,y)$ and $I_0(x,y)$ have been defined by the (4) and (7), dz is the diffraction distance. Substitute (17) into (14) and we could get:

$$\varphi(x,y)=-\mathrm{IFT}\{\left(q_x^2+q_y^2\right)^{-1}\mathrm{FT}\left\{\nabla\cdot\left[\nabla\psi(x,y)/I_0(x,y)\right]\right\}\} \tag{19}$$

The secret image could be calculated straightforward:

$$f(x,y)=\varphi(x,y)/\pi \tag{20}$$

A flow chart of the above-mentioned embedding and extraction process is illustrated in the Fig. 2. In the embedding process, the high capacity is achieved while the imperceptibility is well preserved through the phase-encoding technique. On the other side, the secret image could be reconstructed near perfectly in the extraction process by leverage the TIE method.

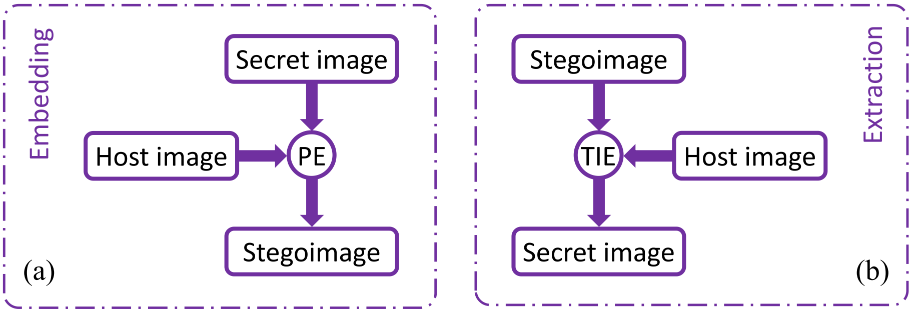

Fig. 2. The flowchart of the steganography. (a) Embedding process, (b) extraction process. PE: phase-encoding, TIE: transport of intensity equation.

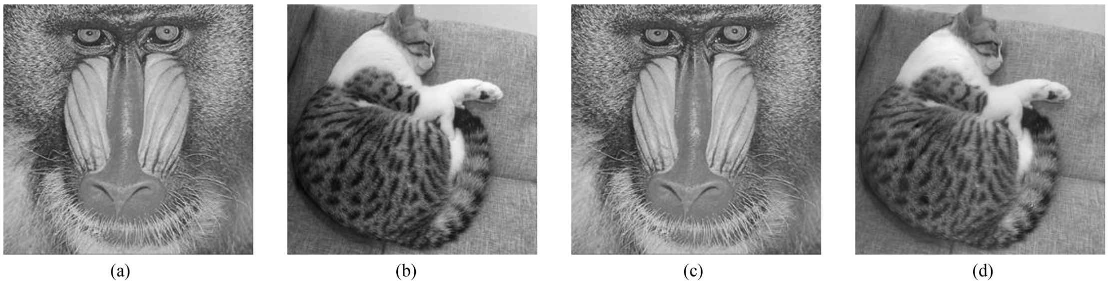
(a) (b) (c) (d)

Fig. 3. Simulation results. (a) Host image 'Baboon', 256 × 256 pixels. (b) Secret image 'Sleeping cat', 256 × 256 pixels. (c) Stegoimage. The CC value between the stegoimage (c) and the host image (a) is 0.9998. (d) Extracted image from the stegoimage. The CC value between the extracted image (d) and the secret image (b) is 0.9883.

3. Simulation Results and Discussion

To verify our method, we perform computer simulations under the environment of MATLAB R2013b. The wavelength λ=632.8nm, the diffraction distance $dz = 10\ \mu$m and the pixel size is set as 8 μm. These parameters have been verified by computer simulations and optical experiments and thus are feasible and reasonable [25], [26]. And there is no doubt that this method will still work if the parameters choose other reasonable values. A Gray-scale image 'Baboon' (256 × 256 pixels) with a gray value in [0, 1] is rescaled into [0.16, 1] for avoiding the division by zero error as formulated in the (19), and it is adopted as the host image shown in the Fig. 3(a). The secret image 'Sleeping cat' (256 × 256 pixels, Fig. 3(b)) is phase-encoded in a range of [0, π] and embedded into the host image using the proposed phase-encoding steganography method. The generated stegoimage is quantified into 8-bit unsigned integers and saved as an image (Fig. 3(c)), which has a correlation coefficient (CC) value 0.9998 with the host image. By leveraging the TIE method, the secret image could be reconstructed after extraction, which is shown in the Fig. 3(d), and its CC value with the original secret image is 0.9883. As shown in the simulation, the imperceptibility could be well preserved while the capacity could achieve a high level by using the phase-encoding method. And the CC is defined as:

$$CC = \frac{\sum_m \sum_n (A_{mn} - \overline{A})(B_{mn} - \overline{B})}{\sqrt{\left(\sum_m \sum_n (A_{mn} - \overline{A})^2\right)\left(\sum_m \sum_n (B_{mn} - \overline{B})^2\right)}} \tag{21}$$

Where A and B denote two different two-dimensional distributions, m and n are the indexes of the rows and columns, respectively.

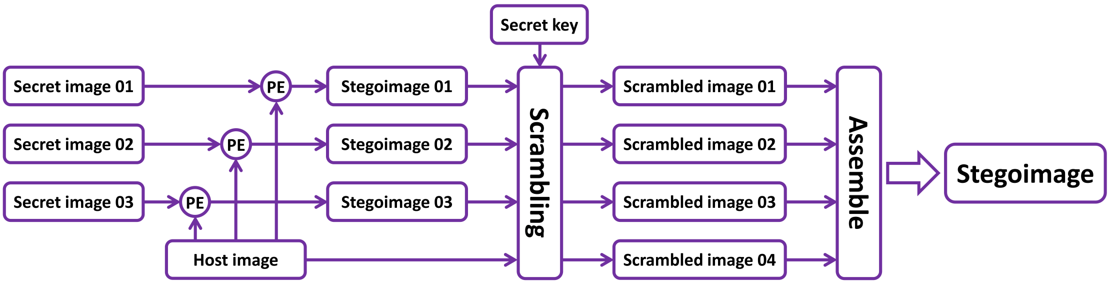

Fig. 4. The embedding process of the SKS. PE: phase-encoding.

As shown in the Fig. 2, the receivers could reconstruct the secret image only when they possess both the stegoimage and the host image. With the increasing use of digital media as well as the rapid expansion of the Internet, the proposed steganography system could work based on the web-based photo gallery. However, it is not convenient under some certain circumstances. Moreover, its security is a potential problem because that the embedding process is not encrypted by a key. For remedying these issues, we design a secret key based scheme (SKS), which is illustrated in the Fig. 4.

As shown in the Fig. 4 and Fig. 5, the embedding process of the SKS mainly consists of three steps:

Step 1: Three different secret image E1-E3 are embedded into the same host image H, which generate three slightly different stegoimage S1-S3.

Step 2: The stegoimage S1-S3 and the host image H are encrypted using a pixel scrambling method according to a Random Index Map K(m, n) (RIM, which is served as the secret key) and m, n are the indexes of the rows and columns of the image. For instance, we suppose that the K(3, 7) = {3, 0, 2, 1}, which means that:

$$\begin{array}{c} C1(3,7) = S3(3,7); \quad C2(3,7) = H(3,7) \\ C3(3,7) = S2(3,7); \quad C4(3,7) = S1(3,7) \end{array} \tag{22}$$

Step 3: The four scrambled images 01-04 are assembled as one stegoimage T:

$$\begin{array}{c} T(1:2:512, 1:2:512) = C1 \\ T(2:2:512, 1:2:512) = C2 \\ T(1:2:512, 2:2:512) = C3 \\ T(2:2:512, 2:2:512) = C4 \end{array} \tag{23}$$

As the Fig. 5 shows, three secret images ‘Treasure map’, ‘Couple’ and ‘Sleeping cat’ are embedding into the host image ‘Baboon’ independently and generating corresponding intermediate stegoimages (S1-S3 in the Fig. 5). And the CC values are 0.9994, 0.9999 and 0.9998 between S1-S3 and H, respectively. After encryption process according to the step 2, the corresponding pixels are scrambled and then generated C1-C4, which have the same CC value of 0.9998 with the host image H. After that, the C1-C4 are assembled as the final stegoimage according to the step 3, which is shown in the Fig. 5T. And the extraction process is the inverse of the Step 1–3. By leveraging the TIE method with the help of the secret key, the secret image R1-R3 could be extracted and the CC values with the E1-E3 are 0.9930, 0.9854 and 0.9883, respectively. As shown in the Fig. 5, the imperceptibility is well preserved by using the phase-encoding method through setting a short diffraction distance. On the other hand, the secret images could be extracted near perfectly from the stegoimage by leveraging the TIE method. Meanwhile, the capacity could achieve a high level by using the proposed phase-encoding method. In terms of security, eavesdroppers or monitors would try to extract the secret image from the final stegoimage. However, only meaningless patterns could be extracted without using the secret key, which is shown in the Fig. 5 W1-W3.

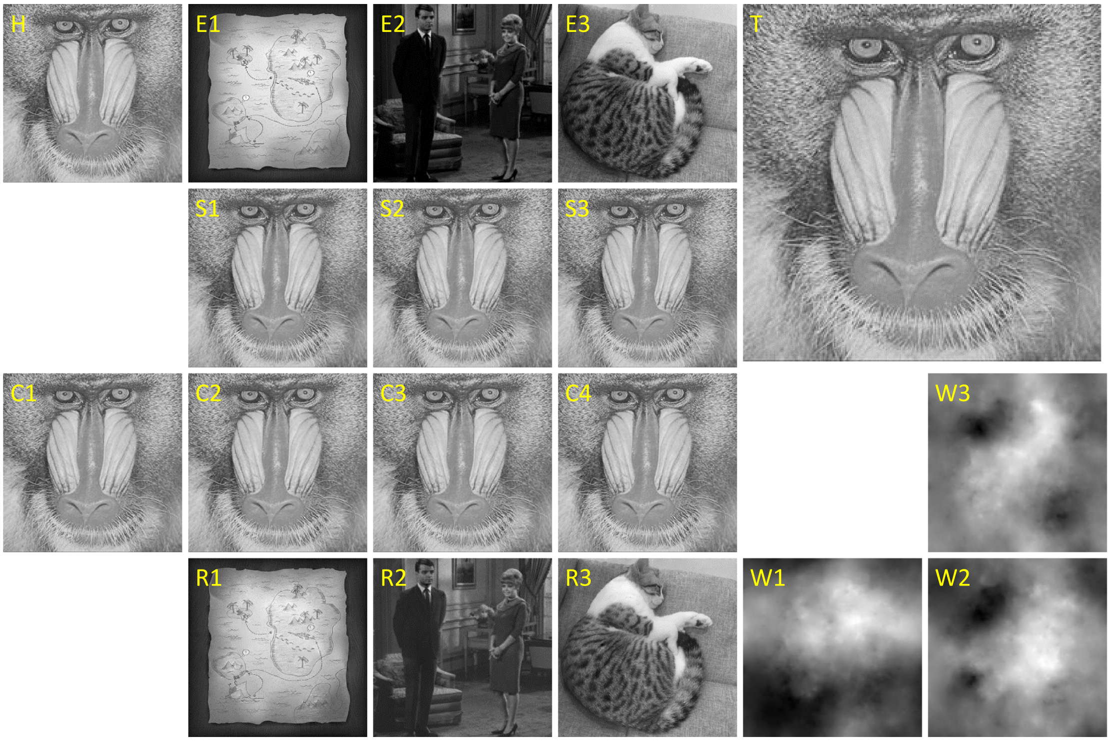

Fig. 5. Simulation results of the SKS. (H) Host image 'Baboon', 256 × 256 pixels. (E1) Secret image 'Treasure map', 256 × 256 pixels. (E2) Secret image 'Couple', 256 × 256 pixels. (E3) Secret image 'Sleeping cat', 256 × 256 pixels. (S1) Stegoimage corresponding to E1. The CC value between the stegoimage (S1) and the host image (H) is 0.9994. (S2) Stegoimage corresponding to E2. The CC value between S2 and H is 0.9999. (S3) Stegoimage corresponding to E3. The CC value between S3 and H is 0.9998. (C1-C4) The four scrambled images. (T) The final stegoimage. (R1-R3) Extracted images from the final stegoimage using the secret key. The CC values between the extracted image (R1-R3) and the secret image (E1-E3) is 0.9930, 0.9854 and 0.9883. (W1-W3) Extracted images from the final stegoimage without using the secret key.

The capacity and imperceptibility are two contradictory attributes. However, the optical phase-encoding method provides a high capacity method for embedding, and meanwhile, the imperceptibility could be well preserved by setting a short diffract distance. Therefore, the issue of capacity and imperceptibility in traditional steganography could be well addressed by leveraging the concepts and theories of optics. And we have to emphasize that the steganography is used mainly on the digital media which are transmitted through the Internet. Therefore, the optical experiments are not essential and the noise issue is also not considered in this paper.

4. Conclusion

In summary, we have proposed a high capacity optical steganography technique based on the phase-encoding and transport of intensity equation (TIE) method. In the current steganography, the communication capacity and the imperceptibility are two contradictory attributes. For remedying this issue, a phase-encoding method is adopted to achieve a high capacity and meanwhile, the imperceptibility could be well preserved by setting a short diffraction distance. On the other hand, the secret image could be extracted near perfectly by leveraging the TIE method. In terms of security, a secret key based scheme is also proposed and the eavesdroppers or monitors could only crack meaningless patterns without using the secret key. A series of simulation results are provided to verify the validity and feasibility of the proposed method.

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References

[1] S. Katzenbeisser and F. A. P. Petitcolas, *Information Hiding Techniques For Steganography and Digital Watermarking*. Norwood, NJ, USA: Artech House Press, 2000.

[2] J. Fridrich, *Steganography in Digital Media: Principles, Algorithms, and Applications*. Cambridge, U.K.: Cambridge Univ. Press, 2009.

[3] H. Hamam, “Digital holography-based steganography,” *Opt. Lett.*, vol. 35, no. 24, pp. 4175–4177, 2010.

[4] Z. Wang, M. P. Fok, L. Xu, J. Chang, and P. R. Prucnal, “Improving the privacy of optical steganography with temporal phase masks,” *Opt. Exp.*, vol. 18, no. 6, pp. 6079–6088, 2010.

[5] B. Wu *et al.*, “Optical steganography based on amplified spontaneous emission noise,” *Opt. Exp.*, vol. 21, no. 2, pp. 2065–2071, 2013.

[6] B. Wu, A. N. Tait, M. P. Chang, and P. R. Prucnal, “WDM optical steganography based on amplified spontaneous emission noise,” *Opt. Lett.*, vol. 39, no. 20, pp. 5925–5928, 2014.

[7] K. Xu, “Integrated silicon directly modulated light source using p-Well in standard CMOS technology,” *IEEE Sens. J.*, vol. 16, no. 16, pp. 6184–6191, Aug. 2016.

[8] B. Wu, Z. Wang, B. J. Shastri, M. P. Chang, N. A. Frost, and P. R. Prucnal, “Temporal phase mask encrypted optical steganography carried by amplified spontaneous emission noise,” *Opt. Exp.*, vol. 22, no. 1, pp. 954–961, 2014.

[9] B. Wu, B. J. Shastri, and P. R. Prucnal, “System performance measurement and analysis of optical steganography based on noise,” *IEEE Photonic. Tech. Lett.*, vol. 26, no. 19, pp. 1920–1923, Oct. 2014.

[10] B. Wu, M. P. Chang, N. R. Caldwell, M. E. Caldwell, and P. R. Prucnal, “Amplifier noise based optical steganography with coherent detection,” *Coherent Opt. Phenom.*, vol. 2, pp. 13–18, 2014.

[11] B. Wu, M. P. Chang, B. J. Shastri, P. Y. Ma, and P. R. Prucnal, “Dispersion deployment and compensation for optical steganography based on noise,” *IEEE Photonic. Tech. Lett.*, vol. 28, no. 4, pp. 421–424, Feb. 2016.

[12] C. Wang, H. Wang, and Y. Ji, “Multi-bit wavelength coding phase-shift-keying optical steganography based on amplified spontaneous emission noise,” *Opt. Commun.*, vol. 407, pp. 1–8, 2018.

[13] E. Wohlgemuth, Y. Yoffe, T. Yeminy, Z. Zalevsky, and D. Sadot, “Photonic-layer encryption and steganography over IM/DD communication system,” *Opt. Exp.*, vol. 26, no. 25, pp. 32691–32703, 2018.

[14] W. Lin *et al.*, “10 m/500 Mbps WDM visible light communication systems,” *Opt. Exp.*, vol. 20, no. 9, pp. 9919–9924, 2012.

[15] I. B. Djordjevic, A. H. Saleh, and F. Küppers, “Design of DPSS based fiber bragg gratings and their application in all-optical encryption, OCDMA, optical steganography, and orthogonal-division multiplexing,” *Opt. Exp.*, vol. 22, no. 9, pp. 10882–10897, 2014.

[16] H. Zhu *et al.*, “Optical steganography of code-shift-keying OCDMA signal based on incoherent light source,” *IEEE Photon. J.*, vol. 7, no. 3, Jun. 2015, Art. no. 6801607.

[17] H. Zhu *et al.*, “Experimental demonstration of optical stealth transmission over wavelength-division multiplexing network,” *App. Opt.*, vol. 55, no. 23, pp. 6394–6398, 2016.

[18] W. Loh, S. Yegnanarayanan, K. E. Kolodziej, and P. W. Juodawlkis, “Optical unmasking of spectrally overlapping RF signals,” *Opt. Exp.*, vol. 25, no. 22, pp. 26581–26590, 2017.

[19] P. Y. Ma, B. Wu, B. J. Shastri, A. N. Tait, P. Mittal, and P. R. Prucnal, “Steganographic communication via spread optical noise: A link-level eavesdropping resilient system,” *J. Lightw. Technol.*, vol. 36, no. 23, pp. 5344–5357, Dec. 2018.

[20] M. Song, Z. A. Kudyshev, H. Yu, A. Boltasseva, V. M. Shalaev, and A. V. Kildishev, “Achieving full-color generation with polarization-tunable perfect light absorption,” *Opt. Mater. Exp.*, vol. 9, no. 2, pp. 779–787, 2019.

[21] J. S. Pan, W. Li, C. S. Yang, and L. J. Yan, “Image steganography based on subsampling and compressive sensing,” *Multimed. Tools Appl.*, vol. 74, no. 21, pp. 9191–9205, 2015.

[22] C. Zhang *et al.*, “Compressive optical steganography via single-pixel imaging,” *Opt. Exp.*, vol. 27, no. 9, pp. 13469–13478, 2019.

[23] M. R. Teague, “Deterministic phase retrieval: A green’s function solution,” *J. Opt. Soc. Am.*, vol. 73, no. 11, pp. 1434–1441, 1983.

[24] L. J. Allen and M. P. Oxley, “Phase retrieval from series of images obtained by defocus variation,” *Opt. Commun.*, vol. 199, no. 1-4, pp. 65–75, 2001.

[25] J. Wu, X. Lin, Y. Liu, J. Suo, and Q. Dai, “Coded aperture pair for quantitative phase imaging,” *Opt. Lett.*, vol. 39, no. 19, pp. 5776–5779, 2014.

[26] C. Zuo, Q. Chen, and A. Asundi, “Boundary-artifact-free phase retrieval with the transport of intensity equation: Fast solution with use of discrete cosine transform,” *Opt. Exp.*, vol. 22, no. 8, pp. 9220–9244, 2014.

[27] V. V. Volkov, Y. Zhu, and M. Graef, “A new symmetrized solution for phase retrieval using the transport of intensity equation,” *Micron*, vol. 33, pp. 411–416, 2002.

[28] K. Xu, “Monolithically integrated si gate-controlled light-emitting device: Science and properties,” *J. Opt.-U.K.*, vol. 20, no. 2, 2017, Art. no. 024014.