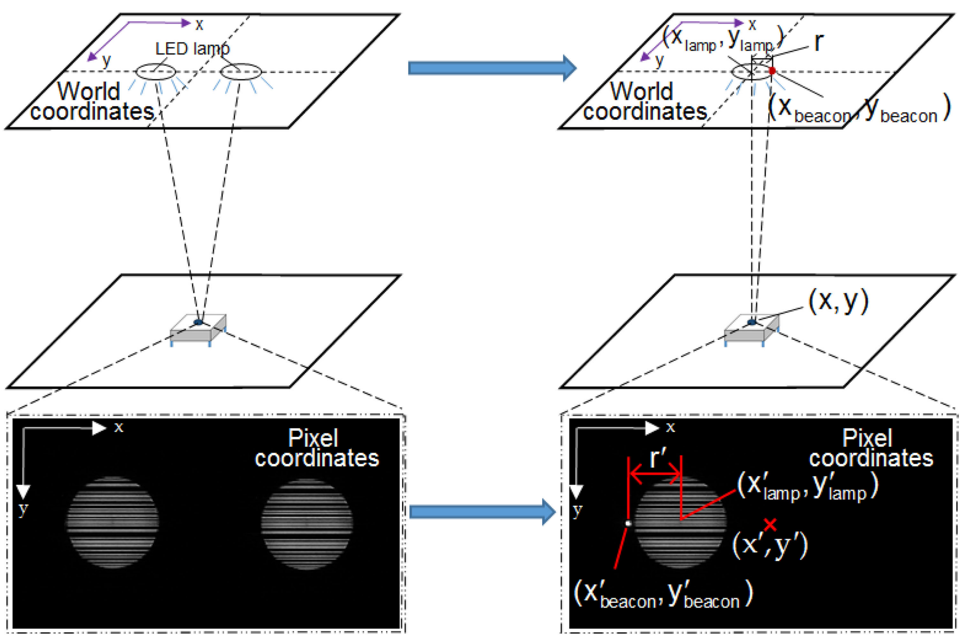
IEEE Photonics Journal
An IEEE Photonics Society Publication

Open Access

A Fast and High-Accuracy Real-Time Visible Light Positioning System Based on Single LED Lamp With a Beacon

Volume 12, Number 6, December 2020

Huaping Li
Hongbin Huang
Yongze Xu
Zhanhang Wei
Sichen Yuan
Puxi Lin
Hao Wu
Wen Lei
Junbin Fang
Zhe Chen



DOI: 10.1109/JPHOT.2020.3032448

IEEE photonics SOCIETY
IEEE

A Fast and High-Accuracy Real-Time Visible Light Positioning System Based on Single LED Lamp With a Beacon

Huaping Li,[1] **Hongbin Huang**,[1] **Yongze Xu**,[1] **Zhanhang Wei**,[1] **Sichen Yuan**,[1] **Puxi Lin**,[1] **Hao Wu**,[3] **Wen Lei**,[1] **Junbin Fang**,[1,2] **and Zhe Chen**[1]

[1]Guangzhou Key Laboratory for Visible Light Communication and Guangdong Engineering Technology Research Center for Visible Light Communication, Jinan University, Guangzhou 510632, China
[2]Cyberspace Security Research Center, Peng Cheng Laboratory, Shenzhen 518055, China
[3]Hunan Aerospace YuanWang Science & Technology Co. Ltd., Hunan University, Changsha 410205, China

DOI:10.1109/JPHOT.2020.3032448

Manuscript received October 13, 2020; accepted October 15, 2020. Date of publication October 20, 2020; date of current version November 6, 2020. This work was supported in part by the National Key Research and Development Program of China (2018YFB1801900), in part by the National Natural Science Foundation of China (61771222, 61872109), in part by the Science and Technology Project of Guangzhou (201803020023), in part by the Project of Guangzhou Industry Leading Talents (CXLJTD-201607), in part by the Science and Technology Project of Shenzhen (JCYJ20170815145900474), in part by the Peng Cheng Laboratory Project of Guangdong Province (PCL2018KP004), and in part by the Fundamental Research Funds for the Central Universities (21620439). Corresponding author: Junbin Fang (e-mail: tjunbinfang@jnu.edu.cn).

Abstract: With the advantages of high positioning accuracy and low cost, visible light positioning (VLP) is becoming a promising solution for practical indoor positioning system. However, most of the VLP systems require at least two VLP LED lamps for accurate position calculation. Therefore, the application of VLP in practical scenarios may be restricted due to this limitation. In this paper, we propose a fast and high-accuracy single-LED based VLP system. Firstly, an unbalanced single-LED VLP algorithm is proposed to increase the positioning accuracy and reduce the computational complexity. Secondly, a fast beacon searching algorithm is proposed to further reduce the processing time for each captured image. Finally, since the proposed algorithms have the advantages of high accuracy and low complexity, the proposed system can also be implemented on a low-end hardware platform. Experimental results show that the average positioning error of the proposed system is decreased to 2.26 cm at the height of 3 m, and the average positioning time is reduced to 6.3 ms on a laptop and 60ms on a low-end embedded platform.

Index Terms: Visible light positioning(VLP), single LED VLP, real-time, unbalanced single-LED VLP algorithm.

1. Introduction

With the widespread application of LED indoor lighting and the rapid development of visible light communication (VLC) technology, visible light positioning (VLP) technology is becoming a promising solution for indoor positioning systems. Compared with traditional indoor positioning technologies, such as WIFI [1], infrared (IR) [2], ultrasonic [3], bluetooth [4], and ultra-wideband (UWB) [5], VLP has the advantages of high positioning accuracy and low cost [6]–[8]. The positioning accuracy of image sensor (IS) based VLP system can reach centimeter-level, and the positioning time is about 23 ms on mobile terminals [9]–[11].

In principle, IS-based VLP system uses image sensor to capture the positioning signals from VLP LED lamps and utilizes the angle of arrival (AoA) information of VLP LED lamps for positioning calculation with triangulation functions [12], [13]. For most of the IS-based VLP systems, at least two VLP lamps are required to be captured simultaneously by the image sensor for solving the triangulation functions for accurate positioning. In practical, if the distance of VLP lamps is too long or some VLP lamps are shielded, there may be only one VLP lamp in the field of view (FoV) of the image sensor and the positioning accuracy will be significantly deteriorated. Therefore, the practicability of VLP system is limited. To solve this problem, several single-LED based VLP schemes have been proposed. Zhang et al. first proposed a single-LED based VLP system based on circle projection with a red marker which provides additional direction information [14]. However, the computational cost of circle projection is high while the positioning accuracy is only 17.52 cm, which does not meet the conditions for indoor mobile terminals. Hao et al. proposed to use the calibrated inertial sensor of smartphones to help single-LED position calculation with projective geometry and their system could achieve a positioning accuracy of 16 cm [15]. Xie et al. proposed to use mean shift algorithm and unscented Kalman filter to solve the LED light temporarily shielded problem [10], and the processing time per frame is 24.93 ms on a laptop with a 2.4 G Hz AMD A10-9600P CPU. In short, due to the complexity of the algorithm, the existing single-LED based VLP schemes have relatively low performance in terms of positioning accuracy and positioning speed, which also affects their practicability in existing devices on the market.

In this paper, we propose a fast and high-accuracy single-LED based VLP system to increase the positioning accuracy and positioning speed. Firstly, an unbalanced single-LED VLP algorithm is proposed to increase the positioning accuracy and reduce the computational complexity by designing a single-LED positioning lamp with a beacon and using imaging trigonometric relationship without complicated calculations. Secondly, a fast beacon searching algorithm is proposed to further reduce the processing time for each captured image. Taking advantages of the high-accuracy and low-complexity algorithms, the proposed system can be implemented on low-end hardware platform, which will further enhance the adaptability and the practicability of VLP technologies in practical scenarios. Experimental results show that the average positioning error of the proposed system is decreased to 2.26 cm at the height of 3 m, and the average positioning time is 6.3 ms on a laptop and 60 ms on a low-end embedded platform. Therefore, it is believed that the proposed system can provide fast and high-accuracy indoor positioning support to mobile robots, indoor parking and indoor pedestrian navigation.

The rest of the paper is organized as follows. The detailed design of the proposed system is introduced Section 2. The experimental setup and results are introduced and discussed in Section 3. Section 4 concludes this paper.

2. The Proposed Single-LED Based System

2.1 System Architecture

As shown in Fig. 1, the proposed system mainly has two hardware equipment, the single-LED VLP lamp as VLP signal sender and the positioning device as VLP signal receiver.

The single-LED VLP lamp is composed of an original VLP lamp which broadcasts VLP light signals and a small luminescent beacon which is mounted on the round edge of the original VLP lamp. And interleaved two five (ITF) code, which has a higher code efficiency than Manchester code [16], is used to encode VLP light data. Then the encoded VLP light date is modulated by on-off keying (OOK) [11], in which VLP lamp when bright is regarded as data “1” and VLP lamp when dark is regarded as data “0”. Since our modulation frequency is 18 kHz, which is far greater than 200 Hz, the flicker of the lamp is invisible to human eyes [17]. Besides, the luminescent beacon is much smaller than the VLP lamp. Therefore, the flickering of the VLP lamp and the luminescent beacon will not cause discomfort to the human eyes in common lighting.

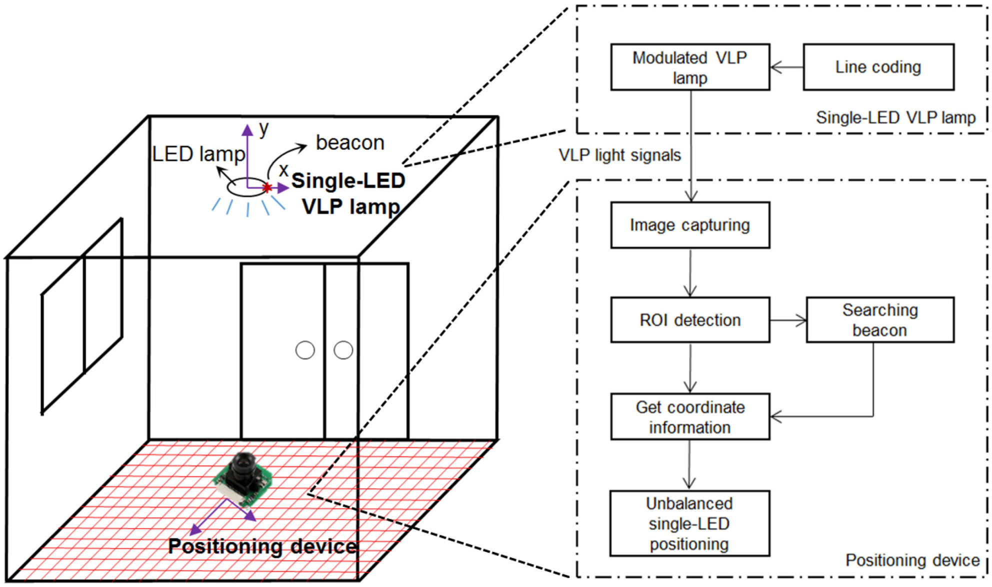


Fig. 1. System architecture.

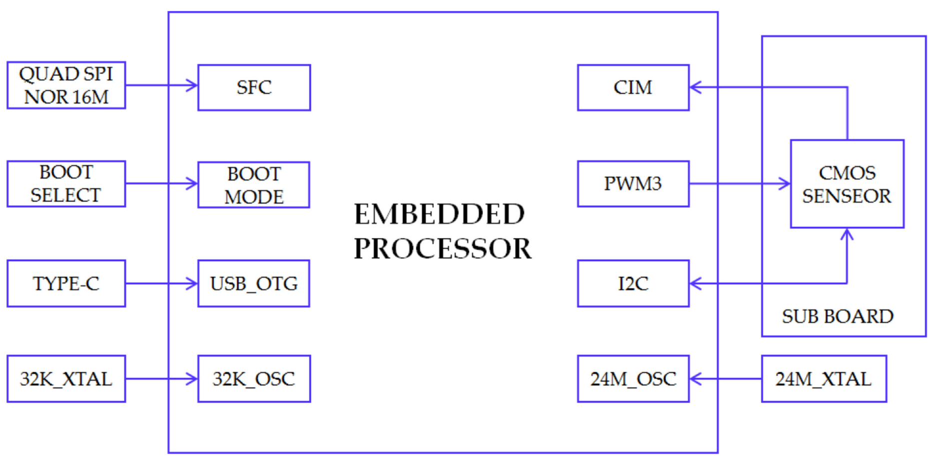


Fig. 2. The embedded processor architecture.

The positioning device is equipped with a complementary metal-oxide-semiconductor (CMOS) sensor and an embedded processor (as shown in Fig. 2). The working principle of CMOS sensor is called rolling shutter mechanism, that the pixel exposed and the data read out are performed row by row. Hence, when the CMOS sensor works in under-exposure mode, the bright/dark interlaced stripes containing the encoded data can be captured by the rolling shutter mechanism of the CMOS sensor. The principle of the bright/dark stripes formation is shown in Fig. 3. And the embedded processor is used to calculate the accurate position in a short time using the proposed unbalanced single-LED VLP algorithm and the fast beacon searching algorithm. When the stripe image is captured, the proposed VLP algorithm first uses region of interest (ROI) detection method to identify the area of the LED bright/dark stripes in the image and get the pixel coordinates of the circle center of the stripes area. Then, the fast beacon searching algorithm is used to locate the position of the beacon in the image and get its pixel coordinates. Based on the pixel coordinates and the world coordinates which are the physical coordinates of the VLP lamp and the relative coordinates of the

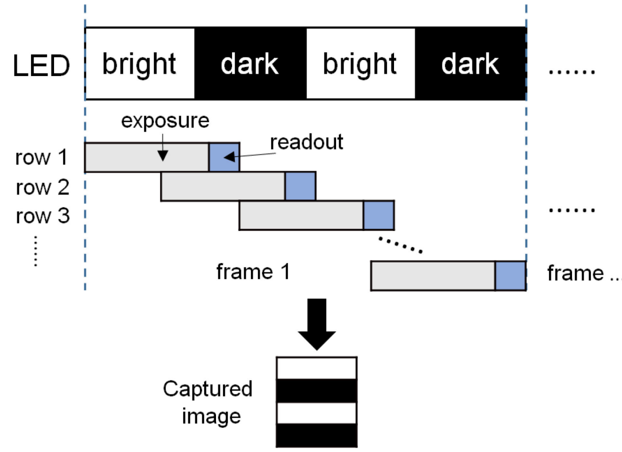


Fig. 3. The principle of the bright/dark stripes formation.

beacon, the accurate position of the positioning devices is calculated by solving the triangulation functions used in our previous VLP systems [11], [16]. The proposed unbalanced single-LED VLP algorithm and the fast beacon searching algorithm are detailed in follows.

2.2 The Unbalanced Single-LED VLP Algorithm

Since the CMOS image sensor may not be able to capture multiple VLP lamps simultaneously in practical scenarios due to the limitation of FOV or shielding, a single-LED VLP algorithm is desired to provide accurate positioning based on only one VLP lamp. In addition, in order to improve the practicability of the single-LED VLP system, the positioning speed is also the focus of our attention. Because the triangulation function based on imaging relationship can realize accurate positioning calculation and its computational complexity is low, if the method can be used in single-LED VLP, it will effectively improve the positioning accuracy and positioning speed of the single-LED VLP system. In this system, we propose an unbalanced single-LED VLP algorithm based on the dual-LED VLP algorithm in our previous VLP systems [11], [16].

Firstly, in hardware design, we shrink one of the VLP lamps in the dual-LED VLP system to a small luminescent beacon which can be mounted on the other one VLP lamp, as shown in Fig. 4. Then, the proposed algorithm takes the small beacon as an unbalanced VLP lamp instead of just a direction marker, compared with the previous single-LED VLP algorithm. Therefore, the proposed algorithm could fully utilize the coordinate information of the unbalanced VLP lamp (i.e., the small beacon) to achieve a high positioning accuracy as the original dual-LED VLP algorithm.

Secondly, the pixel coordinates and physical coordinates of the VLP lamp and beacon are obtained through image processing, respectively. The upper, lower, left and right boundaries as well as center pixel coordinates of the stripes could be found by comparing the number of pixels that meet the threshold in adjacent rows or columns sampled at large steps, as shown in Fig. 5. Note that if the number of pixels meeting the threshold in a row or column is zero, the current row or column is skipped. And the pixel coordinates of the beacon can be obtained through the beacon search algorithm described below. Hence, we get the pixel coordinates (x'_{lamp}, y'_{lamp}) and $(x'_{beacon}, y'_{beacon})$ of the VLP lamp and the beacon respectively. Next, a group of barcodes could be found according to the wider black interval between each group of barcodes, as shown in Fig. 6. Therefore, the physical coordinates (x_{lamp}, y_{lamp}) and (x_{beacon}, y_{beacon}) of VLP lamp and beacon can be obtained by scanning and decoding the pixels in the middle column of the ITF barcode [16]. Note that (x_{beacon}, y_{beacon}) are relative coordinates.

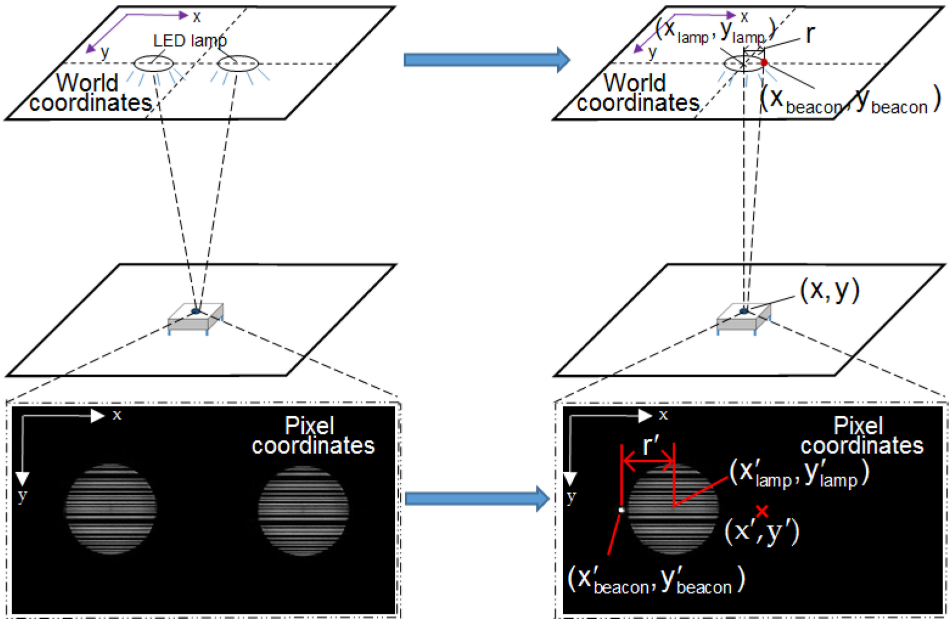


Fig. 4. The proposed unbalanced single-LED VLP algorithm.

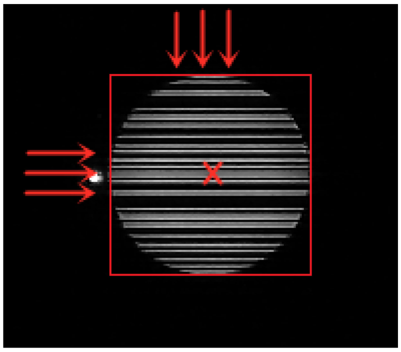


Fig. 5. The method of obtaining the area of the stripe.

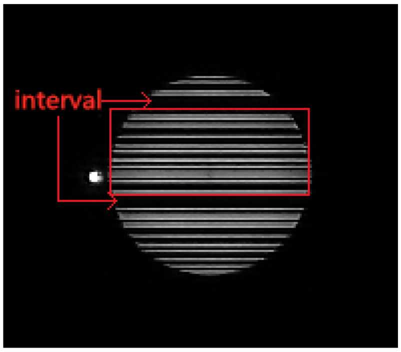


Fig. 6. The way a group of barcodes are found.

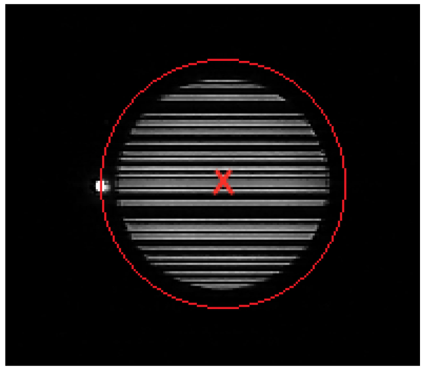


Fig. 7. The fast beacon searching algorithm.

Finally, as shown in Fig. 4, the position estimation of the positioning device is calculated using the physical coordinates and pixel coordinates obtained above. The formula is shown below.

$$\frac{r}{r'} = \frac{x_{lamp} + x_{beacon} - x}{x'_{beacon} - x'} = \frac{y_{lamp} + y_{beacon} - y}{y'_{beacon} - y'} \tag{1}$$

Where (x, y) and (x', y') are the physical coordinates and pixel coordinates of the positioning device, respectively, and (x', y') are also the center coordinates of the image. In addition, r and r' are the physical distance and pixel distance from the center of the LED to the beacon, respectively, and their expressions are shown below.

$$r' = \sqrt{\left(x'_{lamp} - x'_{beacon}\right)^2 + \left(y'_{lamp} - y'_{beacon}\right)^2} \tag{2}$$

$$r = \sqrt{x_{beacon}^2 + y_{beacon}^2} \tag{3}$$

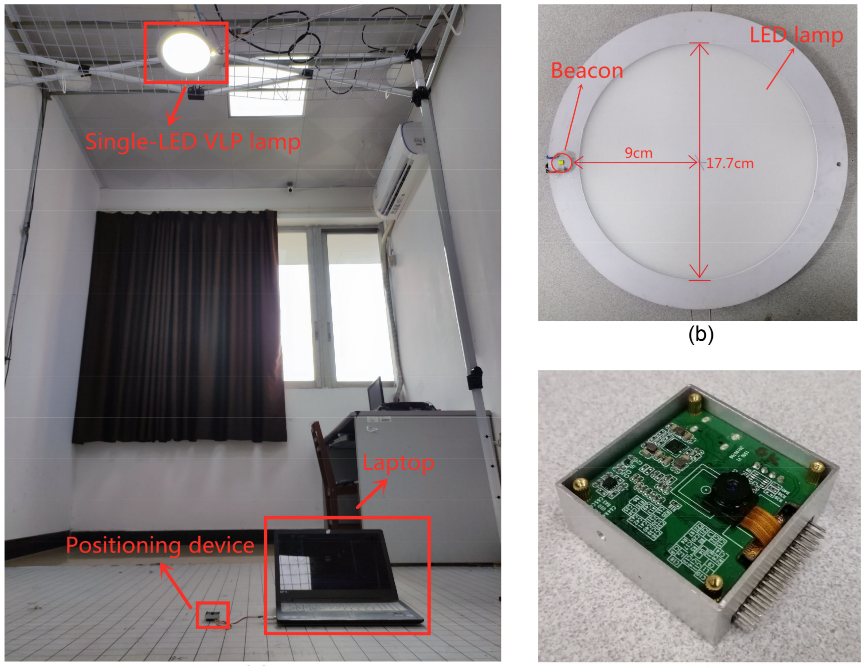
Take Equations (2), (3) into Equations (1), the physical coordinates of the positioning device are obtained by Equation (4), (5).

$$x = x_{lamp} + x_{beacon} + \left(x' - x'_{beacon}\right) \times \frac{\sqrt{x_{beacon}^2 + y_{beacon}^2}}{\sqrt{\left(x'_{lamp} - x'_{beacon}\right)^2 + \left(y'_{lamp} - y'_{beacon}\right)^2}} \tag{4}$$

$$y = y_{lamp} + y_{beacon} + \left(y' - y'_{beacon}\right) \times \frac{\sqrt{x_{beacon}^2 + y_{beacon}^2}}{\sqrt{\left(x'_{lamp} - x'_{beacon}\right)^2 + \left(y'_{lamp} - y'_{beacon}\right)^2}} \tag{5}$$

Furthermore, since the proposed single-LED VLP algorithm could calculate the accurate position based on the coordinate information of the unbalanced VLP lamp and the original VLP lamp, it can avoid the high-complexity computation such as circle projection and etc. which are used in the previous single-LED VLP algorithms. In fact, the computational cost of the image processing process is much greater than the mathematical calculation process. Compared with the dual-LED VLP algorithm, the image processing process of the proposed algorithm not only does not need to be binarized, but also only needs to compare the number of pixels in adjacent rows or columns, and only needs to process one group of LED bright/dark stripes in the image instead of two groups. In a word, the proposed algorithm not only has low computational complexity, but also can achieve similar positioning accuracy as the dual-LED VLP algorithm. It ensures that the proposed system can achieve fast and high-accuracy positioning even on low-end hardware platform.

Note that although our single-LED VLP lamp is similar to Reference [14], the positioning algorithm we use is different and the function of beacon is also different. Our beacon is regarded as an unbalanced VLP lamp and our image processing process has low-complexity, so we can achieve



(a) (b) (c)

Fig. 8. (a) The experimental environment. (b) The single-LED VLP lamp. (c) The positioning device.

fast and high-accuracy positioning by solving triangulation function. Therefore, since the proposed positioning algorithm has the advantages of high-accuracy and low-complexity, this increases the practicability of the single-LED VLP system in the field of indoor positioning and navigation.

2.3 A Fast Beacon Searching Algorithm

Since the small beacon is taken as one VLP lamp to provide coordinate information, the processing time for searching the beacon in the captured image also has impact on the real-time performance of the proposed VLP algorithm. Since the beacon is on the round edge of the LED lamp, its pixel coordinates are on a circle which has the same circle center as the LED stripes area, as shown in Fig. 7.

Therefore, the pixel coordinates (x'_{lamp}, y'_{lamp}) of the circle center identified in the previous step could be directly used for searching the beacon. As shown in Fig. 7, when the center of the LED stripes is identified, the brightness value of the pixels on the circle with a radius greater than r'_{stripes} (the radius of the area of the LED stripes) will be compared with a brightness threshold. Usually, the luminescent beacon has a higher brightness than the LED stripes and there are no other brighter points on the circle. So the pixel coordinates of the beacon can be searched through the following formula.

$$(x' - x'_{lamp})^2 + (y' - y'_{lamp})^2 > r'_{stripes} \tag{6}$$

Since the beacon searching algorithm only need to search several circles nearby to find the beacon point and the area of the LED stripes is a relatively small area in the captured image, the pixel coordinates of the luminescent beacon can be found by checking a small number of pixels. As a result, the proposed beacon searching algorithm can further reduce the image processing time for each captured image.

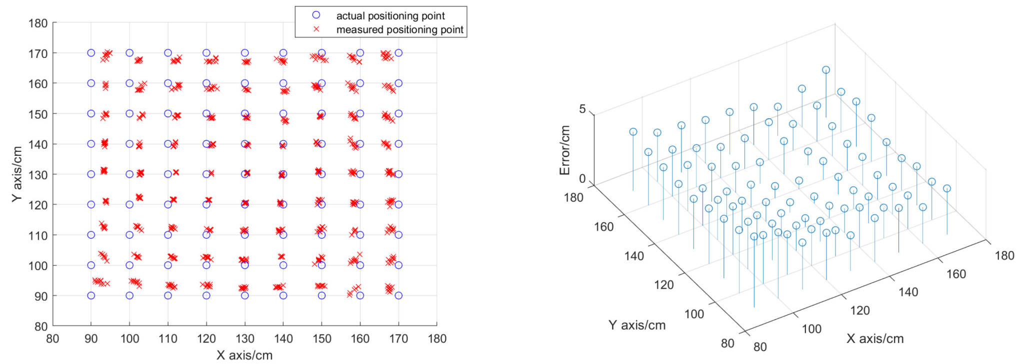
TABLE 1
Parameters of Experimental Facilities and Platforms

LED Lamp Specifications	
Model	DELIXISI RT-TD251
Working Voltage/V	85-265
Power/W	18
Diameter/cm	17.7
Positioning Device Specifications	
CPU	Ingenic X1500, Single Core, 1GHz
Driving Voltage/V	1.8
Driving Current/mA	300
Operation System	Linux
Chip Architecture	MIPS
Camera Specifications	
Model	OmniVision OV2640
Type of Shutter	Rolling Shutter
Size of Sensor/inch	1/4
Format of Output Image	YUV
Pixel	1600×1200
Frame Rate/FPS	15
Sensitivity	0.6V/Lux-sec
Signal to Noise Ratio (SNR)/dB	40
Dynamic Range/dB	50
Working Voltage/V	3.3
Field of View(FOV)/°	65
Lens Stop	F2.0
Lens Focus/mm	3.6
Working Temperature/°C	-30~70
Physical Coordinates Specifications	
LED (x, y)/cm	(130, 130)
Beacon (x, y)/cm	(10.8, 0)
Experimental Platform Specifications	
Size/cm³	180×180×300

3. Experimental Results and Discussion

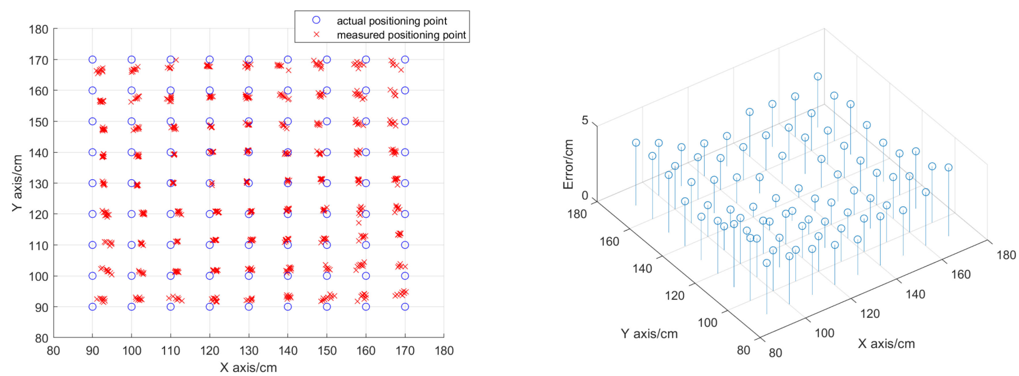
3.1 Experimental Setup

We conducted experiments to evaluate the positioning accuracy and positioning speed of the proposed system. The experimental environment is shown in Fig. 8(a), where a single-LED VLP lamp is installed on the ceiling with three different heights to ground and the positioning device is placed on ground. The size of our experimental platform is 180 cm $\times$ 180 cm $\times$ 300 cm. As shown in Fig. 8(b), the VLP lamp is a 17.7 cm diameter round LED lamp and the luminescent beacon is mounted with a distance of 9 cm to the center of the LED lamp. The positioning device shown in Fig. 8(c) is a custom embedded module equipped with an OmniVision OV2640 1600 $\times$ 1200 pixels CMOS image sensor and an Ingenic X1500 single core CPU with 1 G Hz operation frequency. A laptop with a 2.3 G Hz Intel i5-6200 CPU is also used to config the positioning module and read positioning results from the module. Detailed parameters of experimental facilities and platforms are shown in Table 1.



(a) (b)

Fig. 9. Positioning results with a height of 2m: (a) Distribution of the measured positioning points. (b) Distribution of the mean positioning error.



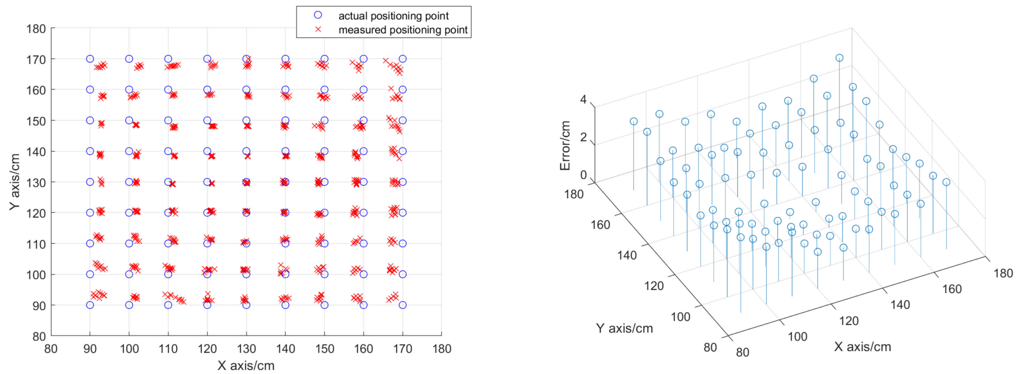
(a) (b)

Fig. 10. Positioning results with a height of 2.5m: (a) Distribution of the measured positioning points. (b) Distribution of the mean positioning error.

3.2 Positioning Accuracy

In general, the installation height of indoor LED lamp is between 2 m and 3 m. Therefore, in this experiment, we measured the positioning accuracy of the proposed VLP algorithm with three heights of 2 m, 2.5 m and 3 m. 81 points were tested in a 9×9 grid pattern with an interval of 10 cm and every grid point was measured 10 times and the measured results are shown in Fig. 9(a)-11(a). The blue circle represents the actual positioning point, and the red cross represents the measured positioning point. All the measured results are close to the actual points with slight deviation. The distribution of the mean positioning error is shown in Fig. 9(b)-11(b) and Table 2 for three different heights. When the height is 2 m, the mean positioning error is 2.55 cm with the minimum value of 0.36 cm and the maximum value of 5.76 cm. Note that the outer points in the experimental area have higher positioning error than inner points because the captured LED stripes for outer points are at the edge of the captured image and have slight distortion which increases the positioning error. The positioning errors at the height of 2.5 m and 3 m have similar distribution, while the mean positioning error at the height of 2.5 m is 2.44 cm and that at the height of 3 m is 2.26 cm. The higher the VLP lamps, the farther the captured stripes away from the image edge and the less the distortion. Therefore, the positioning error is relatively lower at the height of 3 m.

The Cumulative Distribution Function (CDF) of the positioning error is also shown in Fig. 12. When the height is 2 m, 2.5 m and 3 m, 90% of the positioning errors are within 3.81 cm, 3.77 cm and 3.36 respectively. The above experimental results show that the proposed system



(a) (b)

Fig. 11. Positioning results with a height of 3m: (a) Distribution of the measured positioning points. (b) Distribution of the mean positioning error.

TABLE 2
Positioning Errors at Three Different Heights

Height/m	Mean error/cm		
	x	y	x-y
2.0	1.64	1.58	2.55
2.5	1.44	1.63	2.44
3.0	1.39	1.53	2.26
Height/m	Maximum error/cm		
	x	y	x-y
2.0	4.94	5.10	5.76
2.5	4.93	5.17	5.18
3.0	4.33	4.73	4.79
Height/m	Minimum error/cm		
	x	y	x-y
2.0	0.00	0.00	0.36
2.5	0.00	0.00	0.07
3.0	0.00	0.00	0.14

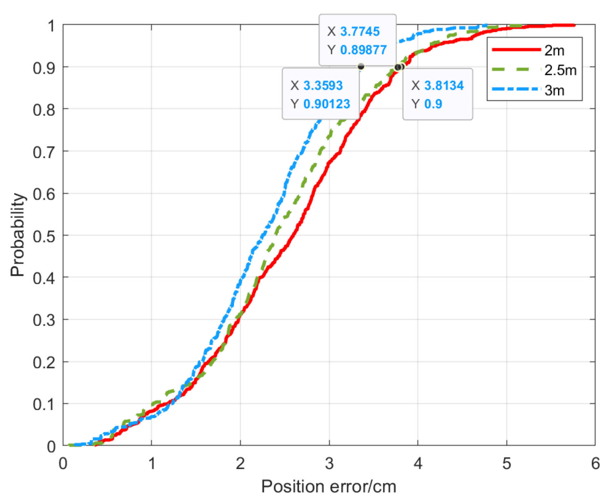


Fig. 12. CDF of the positioning error at three heights.

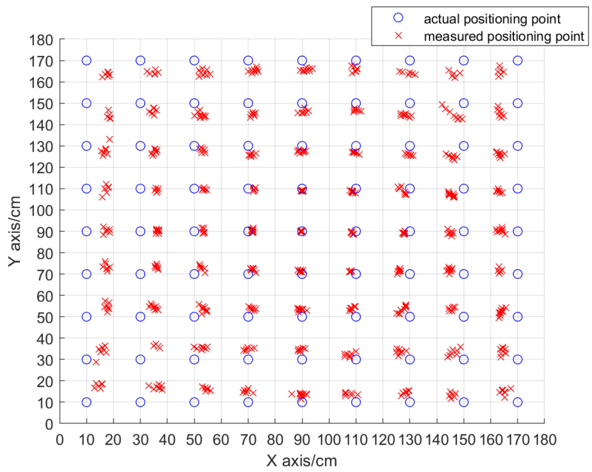
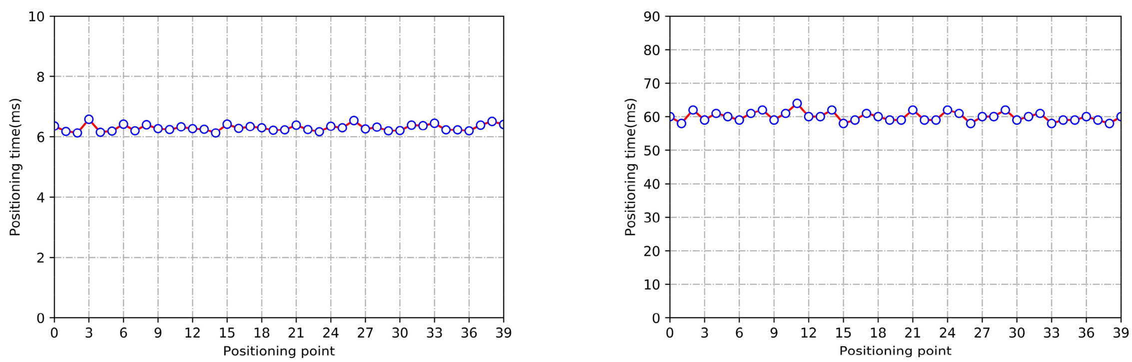


Fig. 13. Positioning results on the maximum area achieving positioning with a height of 3 m.



(a) (b)

Fig. 14. (a) The average positioning time of the proposed system on a laptop. (b) The average positioning time of the proposed system on low-end embedded hardware platform.

can achieve high-accuracy positioning, which meets the positioning accuracy requirements of most indoor terminals moving on the ground.

Furthermore, the maximum area that the proposed system can positioning is 80$\times$80 cm^2 when working at a height of 2 m and 160$\times$160 cm^2 when working at a height of 3 m, indicating that the higher the height is, the larger the area achieving positioning is. In fact, the maximum positioning area of the proposed system is also related to the FoV of the CMOS sensor, where the larger the FoV, the larger the positioning area. The mean error of 2 m is 2.55 cm and 3 m is 5.51 cm because positioning error of the outer points deteriorate dramatically with the height increases. Detailed positioning results are shown in Fig. 9(a) and Fig. 13, respectively.

3.3 Positioning Time

Positioning time has significant impact on the real-time performance for a practical positioning and navigation system. Higher positioning time means higher positioning latency which causes that the measured position cannot be updated in time and kept consistent with the actual position. In this experiment, we randomly measured the positioning time of 40 different positioning points in the experimental area, and each point was measured 5 times. First, we used the laptop to calculate the position using the proposed single-LED VLP algorithm. As shown in Fig. 14(a), the average positioning time of the proposed system using a laptop is about 6.3 ms, which is only 1/4 of the positioning time of Reference [10]. We also used a low-end embedded device (i.e., the positioning module) to run the proposed VLP algorithm and to calculate the position. Fig. 14(b) shows that the average positioning time of the proposed system is 60 ms on the low-end positioning module, which demonstrates that the proposed system can achieve high-accuracy real-time positioning on a low-end embedded platform.

In short, the proposed system can achieve fast and high-precision positioning. The proposed system can provide positioning and navigation services for devices with strong computing power at a fast-moving application scenario like cars running in tunnels and for devices with weak computing power at a relatively low speed application scenario like indoor parking or patrol robots.

4. Conclusion

In this paper, we propose a fast and high-accuracy single-LED based VLP system. Taking advantages of the proposed unbalanced single-LED VLP algorithm and the fast beacon searching algorithm, the proposed system can increase not only the positioning accuracy with one VLP lamp but also high positioning speed. Besides, the high-accuracy and low-complexity algorithms enables the proposed system to be implemented on a low-end embedded hardware platform with high-accuracy real-time positioning. Experimental results show that the average positioning error of the system is decreased to 2.26 cm, and the average positioning time is decreased to 6.3 ms on a laptop and 60 ms on a low-end hardware platform. Therefore, the proposed system can achieve fast and high-accuracy real-time positioning and can be applied to a lot of practical applications such as automated guided vehicle, unmanned factories and etc.

References

[1] C. Yang and H.-R. Shao, “WiFi-based indoor positioning,” *IEEE Commun. Mag.*, vol. 53, no. 3, pp. 150–157, Mar. 2015.

[2] K. Wang, A. Nirmalathas, C. Lim, K. Alameh, H. Li, and E. Skafidas, “Indoor infrared optical wireless localization system with background light power estimation capability,” *Opt. Express*, vol. 25, no. 19, pp. 22923–22931, Sep. 2017.

[3] S. Park and S. Hashimoto, “Autonomous mobile robot navigation using passive RFID in indoor environment,” *IEEE Trans. Ind. Electron.*, vol. 56, no. 7, pp. 2366–2373, Jul. 2009.

[4] J. Nieminen *et al.*, “Networking solutions for connecting bluetooth low energy enabled machines to the Internet of Things,” *IEEE Netw.*, vol. 28, no. 6, pp. 83–90, Nov./Dec. 2014.

[5] C. Zhang, M. J. Kuhn, B. C. Merkl, A. E. Fathy, and M. R. Mahfouz, “Realtime noncoherent UWB positioning radar with millimeter range accuracy: Theory and experiment,” *IEEE Trans. Microw. Theory Tech.*, vol. 58, no. 1, pp. 9–20, Jan. 2010.

[6] N. Pham, V. Rachim, and W. Chung, “High-accuracy vlc-based indoor positioning system using multi-level modulation,” *Opt. Express*, vol. 27, no. 5, pp. 7568–7584, Mar. 2019.

[7] Q. Wang *et al.*, “Light positioning: A high-accuracy visible light indoor positioning system based on attitude identification and propagation model,” *Int. J. Distrib. Sens. Netw.*, vol. 14, no. 2, Feb. 2018.

[8] E. W. Lam and T. D. C. Little, “Visible light positioning: Moving from 2D planes to no. 3D spaces [Invited],” *Chin. Opt. Lett.*, vol. 17, no. 3, 2019, Art. no. 030604.

[9] Y. Wu, W. Guan, X. Zhang, M. Huang, and J. Cao, “Visible light positioning system based on CMOS image sensor using particle filter tracking and detecting algorithm,” *Opt. Commun.*, vol. 444, pp. 9–20, Aug. 2019.

[10] Z. Xie, W. Guan, J. Zheng, X. Zhang, S. Chen, and B. Chen, “A high-precision, real-time, and robust indoor visible light positioning method based on mean shift algorithm and unscented Kalman filter,” *Sensors*, vol. 19, no. 5, Mar. 2019, Art. no. 1094.

[11] P. Lin *et al.*, “Real-time visible light positioning supporting fast moving speed,” *Opt. Express*, vol. 28, no. 10, pp. 14503–14510, May 2020.

[12] M. Afzalan and F. Jazizadeh, “Indoor positioning based on visible light communication: A performance-based survey of real-world prototypes,” *ACM Comput. Surv.*, vol. 52, no. 2, May 2019, Art. no. 35.

[13] Y. Zhuang *et al.*, “A survey of positioning systems using visible LED lights,” *IEEE Commun. Surveys Tuts.*, vol. 20, no. 3, pp. 1963–1988, 2018.

[14] R. Zhang, W. D. Zhong, K. Qian, and S. Zhang, “A single led positioning system based on circle projection,” *IEEE Photon. J.*, vol. 9, no. 4, Aug. 2017, Art. no. 7905209.

[15] J. Hao, J. Chen, and R. Wang, “Visible light positioning using a single led luminaire,” *IEEE Photon. J.*, vol. 11, no. 5, Oct. 2019, Art. no. 7905113.

[16] J. Fang *et al.*, “High speed indoor navigation system based on visible light and mobile phone,” *IEEE Photon. J*, vol. 9, no. 2, Apr. 2017, Art. no. 8200711.

[17] S. M. Berman, D. S. Greenhouse, I. L. Bailey, R. D. Clear, and T. W. Raasch, “Human electroretinogram responses to video displays, fluorescent lighting, and other high frequency sources,” *Optom. Vis. Sci.*, vol. 68, no. 8, pp. 645–662, Aug. 1991.