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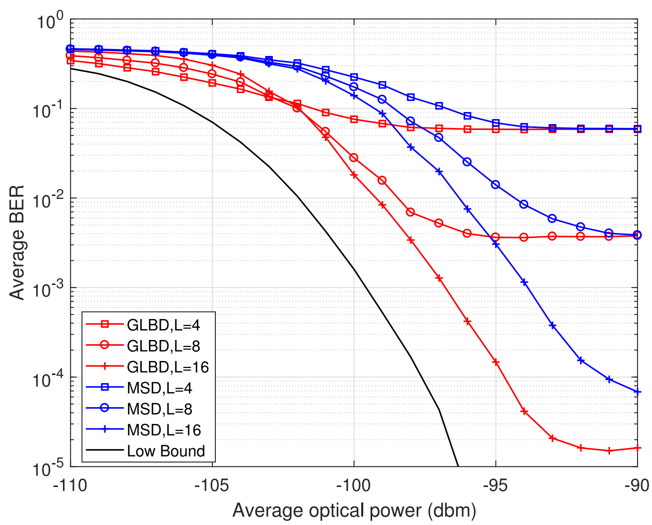

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Generalized Likelihood Block Detection for SPAD-Based Underwater VLC System

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Abstract: In the underwater visible light communication (VLC) system, the single-photon avalanche diode (SPAD) can expand communication distance. However, the output signals from practical SPADs under dead time limit are not Poisson distributed. In this paper, the generalized likelihood block detection (GLBD) receiver is developed for practical SPAD-based underwater VLC system with on–off keying modulation. The proposed receiver can detect the data sequence without any prior knowledge of the channel and the background radiation. Correspondingly, a fast search algorithm for GLBD receiver is also proposed. In addition, a block coding scheme is utilized to solve the error floor problem. Simulation results show that the bit error rate (BER) performance of the proposed receiver is closer to the BER low bound compared to the existing receiver. Moreover, the fast search algorithm reduces the computational complexity without any performance loss.

Index Terms: Underwater visible light communication, photon counting, blind detection.

1. Introduction

Now more than ever, so many researchers study the underwater wireless communication technology, which is a key enabler for ocean exploration [1]–[3]. Acoustic communication is the most widely used technology at present, whose performance is limited by low bandwidth, high latency and Doppler spread [3]. However, a high-speed communication technology with few Mbps data rates is required in various underwater applications such as high-throughput sensor networks, real-time video transmission, communication between underwater vehicles, etc. In case of this, underwater visible light communication (VLC) has attracted much attention due to the higher bandwidth, reliability, and flexibility [4]–[6].

However, the absorption and scattering of ocean lead to an exponential attenuation of optical power. The single photon avalanche diode (SPAD) is used as the photoelectric detector in underwater VLC system with green-blue LED to extend the communication distance, [7]. SPAD is the p-n diode operating in Geiger mode so that transconductance amplifiers are not required. Therefore, the signals from SPAD are photon counting pulses which are much different from analog signals. Generally, the photon counting detection is utilized in the SPAD-based system instead of direct detection [?], [8], [9]. In most previous studies, the receiver counting process is modelled as a Poisson process [10]. According to recent studies, the practical counting model of the SPAD is affected by the dead time and is not Poisson distributed [11], [12]. In [13], a complete analytical framework is presented for modelling the statistical behaviour of the SPAD under dead time limit. With perfect channel state information (CSI) at receivers, the one symbol maximum likelihood (ML) receiver is also proposed in [12], which obtains the decision threshold through the system parameters.

Oceanic turbulence is one of the main factors affecting the energy attenuation of light beams on the propagation path [14]. In [15], [16], the log-normal distribution is utilized to simulate the intensity fluctuations caused by weak oceanic turbulence. Training sequences are sent to improve system BER performances. To make full use of bandwidth and decrease the complexity in practical implementation, several blind detection methods for the photon-counting system in log-normal fading channel are investigated extensively [17], [18]. The multiple symbol detection (MSD) receiver for the SPAD-based system are proposed in [19]. However, the complexity of MSD receiver increases exponentially with the increase of the sequence length. In addition, the underwater background noise information is difficult to be implemented, which is needed in the MSD receiver.

In this paper, motivated by the challenges from turbulence and background noise, we proposed the generalized likelihood blind detection (GLBD) receiver for the practical SPAD-based underwater VLC system. The GLBD receiver no longer needs any prior knowledge about the channel and the background radiation via detecting data sequences by generalized likelihood ratio. Significantly, a fast search algorithm for GLBD receiver is proposed to reduce the computational complexity. Block coding schemes can solve the error floor problem, which are utilized to keep DC balance in the practical VLC system. Compared with the MSD receiver, our proposed receiver is more robust and efficient. Meanwhile, the BER performance of the ML receiver is presented as the low bound for the performance of receivers operating without CSI.

The paper is structured as follows. In Section 2, the system model is described. In Section 3, the low bound of the BER performance, the MSD receiver introduced in [19], and the GLBD receiver are illustrated separately. The fast search algorithm is also proposed in this section. In Section 4, the Monte-Carlo simulation results of error performance for different receivers are presented and discussed. Finally, some conclusions are given in Section 5.

2. System Model

We consider a SPAD-based underwater VLC system that employs NRZ-OOK and intensity modulation, operating over underwater turbulence induced fading channel.

2.1 Underwater Channel Model

In the underwater VLC system, the optical energy loss factor due to absorption and scattering is given as [10]

$$h_l = \exp(-cz) \tag{1}$$

where c is the extinction coefficient and z is the communication distance. In weak turbulence condition, the most widely accepted fading model is the log-normal distribution, in which the intensity variations is normally distributed [14]–[16]. The PDF of log-normal distribution is given by

$$p(h_t) = \frac{1}{2h_t\sqrt{2\pi\sigma_X^2}} \exp\left(-\frac{(\ln h_t - 2\mu_X)^2}{8\sigma_X^2}\right), \tag{2}$$

where σ_X and μ_X are respectively the variance and mean of the Gaussian distributed log-amplitude factor $X = (\ln h_t)/2$. To ensure that turbulence fading does not change the average power, we normalize the fading coefficients as $E(h_t) = 1$.

2.2 Signal Model

In SPAD-based underwater VLC system utilized intensity modulation, data bits are transmitted through changes of LED brightness. Different from Photo-Diode (PD) which directly detect light intensity, the SPAD-based receiver detects photons number arriving at the receiver.

During the kth symbol interval, the average number of the photons arrived at the receiver is denoted as $\lambda_s[k]$, which can be expressed as [12]

$$\lambda_s[k] = n_s h_t s[k] + n_b, \tag{3}$$

where $n_s = \eta P_r T/E_p$ is the mean count parameter due to the transmitted signal, $P_r = h_l P_s$ is the average signal optical power after absorption and scattering, P_s is the LED optical power, η denotes to the photon detection efficiency, T is the bit interval duration, $E_p = Cv/\lambda$ is the energy of one photon, C is the Planck constant, v is the speed of light, $s[k] \in \{0, 1\}$ is the transmitted NRZ-OOK information bit corresponding to the kth symbol interval, $n_b = \eta P_b T/E_p + N_{dcr} T$ is the mean count parameter due to the background radiation and the dark count, P_b is the power is background optical power, and N_{dcr} denotes the dark count ratio.

After a photon causes an avalanche in the SPAD, new photons cannot be detected for a period of time, which is known as the nonparalyzable dead time. The dead time can be regarded as a constant, which will not be prolonged. The photon counts of the SPAD receiver is affected by dead time and is not Poisson distributed. For this problem, the counting model considering the dead time is modelled as a double-random Poisson distribution [13].

The probability mass function (PMF) of photon counts of SPAD under the dead time t_d limit is

$$P(r[k]|\lambda_s[k]) = \begin{cases} \sum_{i=0}^{r[k]} \psi(i, \lambda_{r[k]}) - \sum_{i=0}^{r[k]-1} \psi(i, \lambda_{r[k]-1}) & r[k] < K_{\max}; \\ 1 - \sum_{i=0}^{r[k]-1} \psi(i, \lambda_{r[k]-1}) & r[k] = K_{\max}; \\ 0 & r[k] > K_{\max}, \end{cases} \tag{4}$$

where $\lambda_{r[k]} = \lambda_{s[k]}(1 - r[k]\delta)$ and $\psi(i, \lambda_{r[k]}) = \frac{\lambda_{r[k]}^i e^{-\lambda_{r[k]}}}{i!}$, $r[k]$ is the output counts in the kth symbol interval. $\delta = t_d/T$ denotes dead time ratio. It should be noticed that the photon counts in one symbol interval is limited by dead time, and the maximum photon counts can be expressed as $K_{\max} = \lceil 1/\delta \rceil$ [19]. $K_{\max} = \lceil 1/\delta \rceil$ denotes the maximum photon counts in one symbol interval

In a SPAD-based VLC system, P_r, h_t and n_b need to be known to detect the signal accurately. In order to make them easier to express, we define $n_r = h_t n_s$ in this paper as the the mean count parameter due to the received signal. Therefore, we can get $P(r[k]|\lambda_1[k]) = P(r[k]|n_r + n_b)$ and $P(r[k]|\lambda_0[k]) = P(r[k]|n_b)$.

3. Blind Detection Techniques

3.1 BER Low Bound

If all information, i.e., n_b and n_r, is available, the BER performance of the ML receiver serves as a low bound for the performance of receivers operating without CSI. In this case, the decision rule is given as

$$\frac{P(r[k]|n_r + n_b)}{P(r[k]|n_b)} \underset{\hat{s}[k]=0}{\overset{\hat{s}[k]=1}{\gtrless}} 1, \tag{5}$$

where $\hat{s}[k]$ denotes the decision on $s[k]$. However, it is difficult to obtain the analytical solution of (5). In [13], an approximate expression of the detection threshold is expressed as

$$\tau = \frac{n_r - n_r\delta}{n_r\delta + \ln \frac{n_r + n_b}{n_b}}. \tag{6}$$

Then, the decision rule can be approximated with τ as

$$r[k] \underset{\hat{s}[k]=0}{\overset{\hat{s}[k]=1}{\gtrless}} \tau. \tag{7}$$

Based on the decision rule, we can get the low bound of the BER performance as

$$BER_{low\ bound} = \frac{1 + \sum_{j=0}^{\tau} (P(j|1, n_r, n_b) - P(j|0, n_r, n_b))}{2}. \tag{8}$$

It should also be mentioned that the BER is related to n_r and n_b. Different from the other systems, the signal-to-noise ratio (SNR) value cannot determine the BER alone. For receivers using an inaccurate value of n_b, the BER performance is further degraded.

3.2 The MSD Receiver

The MSD receiver for the SPAD-based VLC system is introduced in [19]. It assumes that the CSI is unavailable and the value of n_b is constant and available at the receiver. At time k, the received signal sequence and transmitted data sequence are $\mathrm{r} = [r(k - L + 1), \ldots r(k)]$ and $\mathrm{s} = [s(k - L + 1), \ldots s(k)]$ respectively. L is the length of sequences.

The decision rule of the MSD scheme with channel distribution is given by

$$\hat{\mathrm{s}} = \arg\max_{\mathbf{s}} \left(\frac{Y_{on}}{(L_{on} - Y_{on}\delta)\, n_b} \right)^{Y_{on}} \exp(-Y_{on} + (L_{on} - Y_{on}\delta) n_b), \tag{9}$$

where $\hat{\mathbf{s}}$ is the estimated signal sequence, $L_{on} \in \{0, 1, 2, \ldots L\}$ is the number of ones in data sequence,

$$Y_{on} = \sum_{i=k-L+1}^{k} s(i) r(i) \tag{10}$$

is the sum of photon counts that correspond to the indexes if the ones in data sequence.

The MSD is a two step detection method which first estimates the h_t of the system and then substitute $\hat{h}_t$ into the decision rule. The longer the detection sequence length L, the closer the result obtained to the BER low bound. Clearly, one of the major drawbacks of the receiver is the computational complexity of its metrics. The complexity of a brute force search is $O(2^L/L)$ on a per symbol decision basis.

3.3 The GLBD Receiver

In this subsection, a new detection method for SPAD-based underwater VLC system is proposed. Since n_b and n_r are unavailable at the receiver side, the GLBD is a likelihood ratio detection which jointly estimates the system values and the transmitted data sequence directly.

At time kth symbol interval, the PMF of the received signal sequence is

$$P\{\mathbf{r}|\mathbf{s}, n_r, n_b\} = \prod_{i=1}^{L} P\{r[k]|s[k], n_r, n_b\}, \tag{11}$$

For a given $\mathbf{s}$, n_r and n_b can be estimated through the posterior information obtained by $\mathbf{r}$. We first consider the n_r, when bit one transmitted, the mean of photon counts can be expressed as

$$\bar{r} \approx \frac{\hat{n}_r + n_b}{1 + (\hat{n}_r + n_b)\,\delta} = \frac{Y_{on}}{L_{on}}. \tag{12}$$

The solution of (12) is

$$\hat{n}_r = \frac{Y_{on}}{L_{on} - Y_{on}\delta} - n_b. \tag{13}$$

Hence, by substituting for $n_r = \hat{n}_r$ from (11), the PMF of the received signal sequence $P\{\mathbf{r}|\mathbf{s}, n_r, n_b\}$ can be expressed as

$$P\{\mathbf{r}|\mathbf{s}, n_r, n_b\} = \left(\prod_{i=1}^{L} r(k)!\right)^{-1} \left(\frac{Y_{on}}{L_{on} - Y_{on}\delta}\right)^{Y_{on}} (n_b)^{Y_{off}} \exp\left(-Y_{on} - n_b\left(L_{off} - Y_{off}\delta\right)\right), \tag{14}$$

where L_{on} and N_{on} have been defined before respectively, $N_{off} \in \{0, 1, 2, \ldots L\}$ is the number of zeros in the data sequence,

$$Y_{off} = \sum_{i=k-L+1}^{k} (1 - s(i))r(i) \tag{15}$$

is the sum of photon counts that correspond to the indexes if the zeros in data sequence. When bit zero transmitted, the mean of photon counts can be expressed as

$$\bar{r} \approx \frac{\hat{n}_b}{1 + \hat{n}_b\delta} = \frac{Y_{off}}{L_{off}}. \tag{16}$$

Similar to n_r, n_b is calculated as the solution of (16), which is obtained as

$$\hat{n}_b = \frac{Y_{off}}{L_{off} - Y_{off}\delta}. \tag{17}$$

After substituting for $n_b = \hat{n}_b$ from (17), the PMF of the received signal sequence $P\{\mathbf{r}|\mathbf{s}, n_r, n_b\}$ is expressed as

$$P\{\mathbf{r}|\mathbf{s}, n_r, n_b\} = \left(\prod_{i=1}^{L} r(k)!\right)^{-1} \left(\frac{Y_{on}}{L_{on} - Y_{on}\delta}\right)^{Y_{on}} \left(\frac{Y_{off}}{L_{off} - Y_{off}\delta}\right)^{Y_{off}} \exp\left(-(Y_{off} + Y_{on})\right), \tag{18}$$

By eliminating irrelevant terms, we obtain the decision metric

$$\Lambda = \left(\frac{Y_{on}}{L_{on} - Y_{on}\delta}\right)^{Y_{on}} \left(\frac{Y_{off}}{L_{off} - Y_{off}\delta}\right)^{Y_{off}}. \tag{19}$$

However, in simulation, we observed that the values of some parts in the (19) would become too large and cause a memory overflow problem on the computer. Hence, by taking the logarithm of the right side of (19), we obtain our GLBD sequence receivers decision metric

$$\Lambda = Y_{on} \ln\left(\frac{Y_{on}}{L_{on} - Y_{on}\delta}\right) + Y_{off} \ln\left(\frac{Y_{off}}{L_{off} - Y_{off}\delta}\right). \tag{20}$$

Finally, similar to (9), the decision rule of the GLBD scheme is given by

$$\hat{\mathbf{s}} = \arg\max_{\mathbf{s}} \Lambda. \tag{21}$$

The GLBD receiver cannot distinguish the data sequences constituted by all zeros and all ones. In the practical system, GLBD receiver requires at least one symbol in the transmitted data sequence. Therefore, considering the same possibility of all transmission sequences, the BER performance of the proposed receiver is limited by the error floor, similarly to the MSD receiver. The BER floor could be calculated by $BER_{floor} = 1/2^L$.

It is noticed that information codes are not directly used to the electric light transformation in VLC systems. Block coding schemes are utilized to keep DC balance which reduce the maximum number of the successive same symbols. So we find that the error floor problem can be solved by block coding schemes such as mBnB code in practical VLC systems.

3.4 Fast Search Algorithm

The computation complexity of exponential growth limits the efficiency of receivers. To solve this problem, we propose a fast search algorithm to reduce the computation complexity in this subsection. We first determine the monotonicity of the objective function in order to simplify the derivation process (20). When the data block is not all zeros or ones, we can get the derivative of Λ with respect to Y_{on} as

$$\begin{aligned}\Lambda'(Y_{on}) &= \ln\left(\frac{Y_{on}}{L_{on} - Y_{on}\delta}\right) - \ln\left(\frac{Y - Y_{on}}{(L - L_{on}) - (Y - Y_{on})\delta}\right) + \frac{L_{on}}{L_{on} - Y_{on}\delta} - \frac{L - L_{on}}{(L - L_{on}) - (Y - Y_{on})\delta} \\ &= \ln\left(\frac{(L - L_{on})/(Y - Y_{on}) - \delta}{L_{on}/Y_{on} - \delta}\right) + \frac{(Y_{on}/L_{on} - (Y - Y_{on})/(L - L_{on}))\delta}{(1 - Y_{on}\delta/L_{on})(1 - (Y - Y_{on})\delta/(L - L_{on}))},\end{aligned} \tag{22}$$

where $Y = Y_{on} + Y_{off}$. It is clearly that in the high SNR region the range of Y_{on} is

$$\frac{YL_{on}}{L} < Y_{on} \leq m\{\mathbf{r}, Y_{on}\}, \tag{23}$$

where $m\{\mathbf{r}, L_{on}\}$ is defined as the sum of the L_{on} largest numbers in the $\mathbf{r}$. Hence, $\Lambda' > 0$ is always admitted and Λ is a monotonous increasing function. Therefore, the maximum value of Λ can be obtained at the maximum of Y_{on} with every L_{on}, which can be defined as $\Lambda_{\max}(L_{on})$. For the $\Lambda_{\max}(L_{on})$, the corresponding signal block can be defined as $\mathbf{s}_{\max}(L_{on})$. Our goal becomes to get the maximum value of $m\{\mathbf{r}, L_{on}\}$, which is transformed into a sequencing problem. Based on this, we can reduce the search scope from 2^L to L.

All the above discussions can be summarized as the following algorithm.

Algorithm 1: Fast GLBD Algorithm.

1) Set $L_{on} = L$, which means that $\mathbf{s}_{\max}(L)$ is all ones, and calculate $\Lambda_{\max}(L)$.
2) Search the minimum value in the $\mathbf{r}$ corresponding the ones in $\mathbf{s}_{\max}(L_{on})$, update $\Lambda_{\max}(L_{on})$ and $\mathbf{s}_{\max}(L_{on})$ with $L_{on} = L_{on} - 1$ until $L_{on} = 0$.
3) The number of ones in data sequence can be estimated as $\hat{L}_{on} = \arg\max_{L_{on}} \Lambda_{\max}(L_{on})$ and $\mathbf{s}$ can be estimated as $\hat{\mathbf{s}} = \mathbf{s}_{\max}(\hat{L}_{on})$.

It is obvious that the main complexity of the algorithm comes from sorting the received signal. Hence, the overall complexity of the proposed algorithm is $O(L \log L)$, which is much lower than the traversal search algorithm.

4. Simulation Results

In this section, Monte-Carlo simulation results for the underwater VLC system BER performance of the proposed receiver are presented under various turbulence conditions. Furthermore, the low bound given by the receiver with perfect CSI is also included for reference. In order to simplify the discussion, P_r is utilized to instead of P_s in simulation as [?]. The transmission rate is assumed to be 1 Mbps and P_b is calculated according to [4] in 40 meters deep ocean. Other important parameters of the simulation are shown in Table 1 including the transmitter and receiver specifications.

In Fig. 1, the BER performances of the proposed GLBD receiver with the MSD receiver are compared with various detection sequence lengths over log-normal turbulence channels with standard deviation $\sigma_X = 0.1$. As clearly described in the figure, simulation results show that GLBD obviously outperforms MSD with the same detection sequence length. However, the BER performances of both receivers are limited by the error floor. Increase the average optical power does not cause the BER continues to decrease and an error floor appears. Due to the transmitted data sequence whose entries are all zeros, the error floor can not vanish even though the P_b approaches infinity. When the detection sequence length becomes longer, the gap between receivers performances

TABLE 1
Parameters for the Simulation System

Coefficient	Value
Wavelength of light	532 nm
Dead time, t_d	20 ns
Dark count, N_{dcr}	50 Counts/s
The light speed in water v	2.25×10^8 m/s
Photon detection efficiency, η	0.2

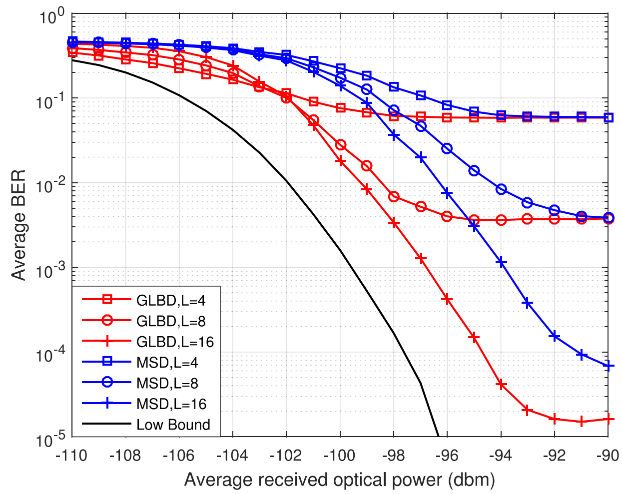

Fig. 1. BER performance comparisons with GLBD receiver and MSD receiver in log-normal channel of $\sigma_X = 0.1$.

decreases, and the error floor becomes lower since we can estimate the system parameters more accurately. However, the detection efficiency decreases with the sequence length, because the computational complexity of the algorithm increases exponentially.

Fig. 2 shows the corresponding results assuming $\sigma_X = 0.3$. The underwater turbulence reduces the system BER performance. Meanwhile, when the channel turbulence scintillation index increases, the BER gaps between the GLBD and MSD increase. That shows the GLBD receiver is more robust.

Block coding scheme is utilized to eliminate the error floor in this paper by eliminating the sequences of successive same symbols. Fig. 3 shows the BER performance of the GLBD receiver with and without mBnB coding with standard deviation $\sigma_X = 0.1$. It is obvious that, with the block coding scheme, the GLBD receiver completely avoids the error floor and also performs much better than uncoded scheme. When $L = 32$, the receiver BER performance approaches the low bound as the optical power increases.

In Fig. 4, The accuracy of the fast search algorithm is verified over log-normal turbulence channels with standard deviation $\sigma_X = 0.1$. The proposed algorithm does not reduce the receiver performance in the case of reducing search range. The computational complexity per symbol is reduced from $\mathbf{O}(2^L/L)$ to $\mathbf{O}(\log L)$.

In Fig. 5, the BER performance of the GLBD receiver and the Low Bound are compared under different symbol rates with standard deviation $\sigma_X = 0.1$ and sequence length $L = 8$. Influenced by dead time, the BER performance of the system becomes better with the decrease of symbol rate. Meanwhile, the gap between the GLBD receiver performance and the Low Bound reduced gradually.

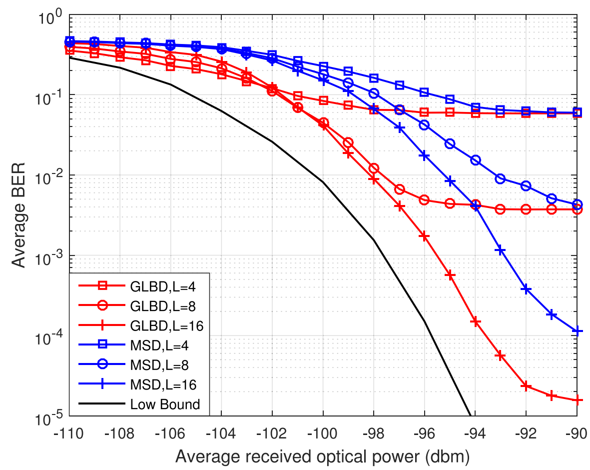

Fig. 2. BER performance comparisons with GLBD receiver and MSD receiver in log-normal channel of $\sigma_X = 0.3$.

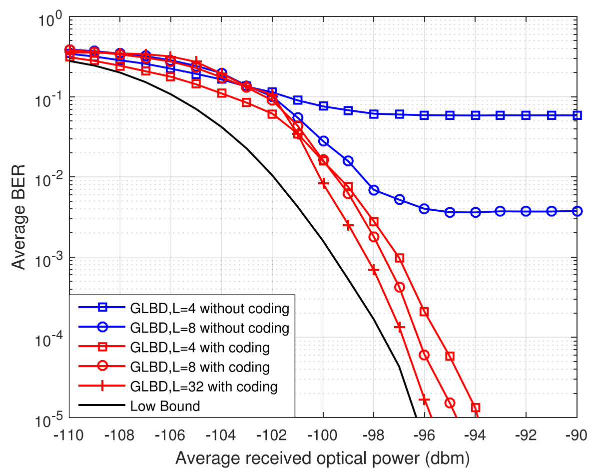

Fig. 3. Performance comparisons with different algorithms in log-normal channel of $\sigma_X = 0.1$.

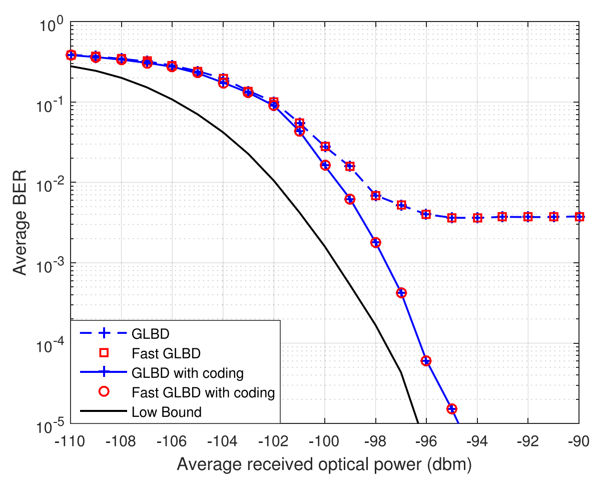

Fig. 4. Performance comparisons with different search algorithms in log-normal channel of $\sigma_X = 0.1$.

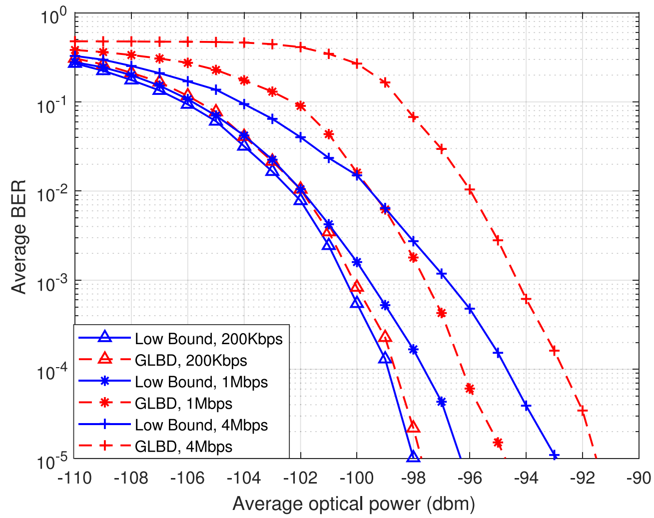

Fig. 5. BER performance comparisons with different symbol rates.

5. Conclusion

In this paper, we proposed the GLBD receiver for SPAD-based underwater VLC systems in the presence of turbulence-induced fading. Correspondingly, we proposed a fast search algorithm for the GLBD receiver without any BER performance loss. The proposed detection shame is based on the double-random Poisson photon counting model considering dead time. As compared with existing MSD receiver, the GLBD receiver has better robustness and efficiency. Simulation results indicate that for a sufficiently lone detection sequence length, the proposed receiver can attain performance comparable to low bound. In the last, the detection method for the MIMO SPAD-based underwater VLC system will be studied in the future work.

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