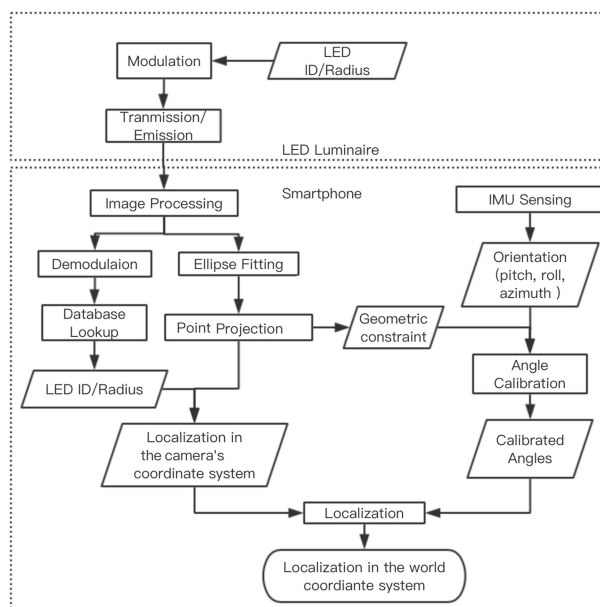
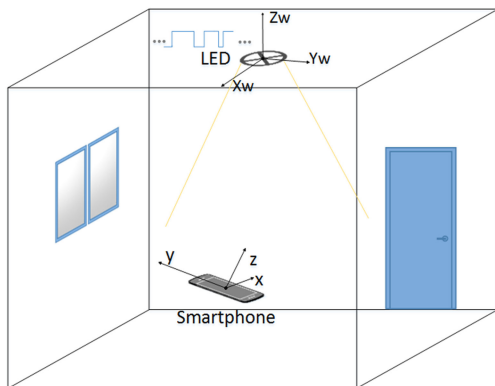


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Volume 11, Number 5, October 2019

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DOI: 10.1109/JPHOT.2019.2930209

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DOI:10.1109/JPHOT.2019.2930209

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Manuscript received June 1, 2019; revised July 13, 2019; accepted July 17, 2019. Date of publication July 22, 2019; date of current version August 8, 2019. This work was supported in part by the National Natural Science Foundation of China under Grants 61602242 and 61572253, in part by the Natural Science Foundation of the Jiangsu Province under Grants BK20160807 and BK20160812, in part by the China Postdoctoral Science Foundation under Grants 2017M611805 and 2017M611806, in part by Jiangsu Planned Projects for Postdoctoral Research Funds under Projects 1701138B and 1701137A, in part by the China Postdoctoral Science Foundation No. 11 Special Fund under Grant 2018T110498, and in part by the Aeronautical Science Foundation of China under Grant 2016ZC52030. Corresponding author: Jie Hao (e-mail: haojie@nuaa.edu.cn).

**Abstract:** Visible light positioning (VLP) has developed rapidly with the ubiquitous lighting infrastructure in recent years. Based on classic trilateration or triangulation, VLP usually requires at least three light sources which makes it infeasible in some scenarios where sparse LED luminaires are mounted. In this paper, we focus on VLP in such scenarios and propose a VLP method based on a single LED luminaire as the light transmitter and commercial off-the-shelf smartphone as the receiver. A critical problem that needs to be addressed is to derive the accurate orientation of the smartphone. Although the inertial measurement unit of the smartphone can be used to obtain the orientation, the measurement error severely degrades the positioning accuracy. To address the measurement error, this paper innovates in exploiting projective geometry to calibrate the orientation measurements. We build an experimental platform to evaluate the proposed VLP method. The experimental results show that the proposed VLP method can achieve a submeter accuracy and the indoor positioning errors are within 0.16 m by 90%.

**Index Terms:** Visible light positioning, computer vision, inertial measurement unit, image sensor.

## 1. Introduction

Indoor localization has been a hot topic for decades for its massive applications, such as indoor navigation, location based advertisement distribution in supermarkets and shopping malls. For decades, there have been extensive studies on indoor localization mainly using WiFi, RFID, UWB, iBeacon, ultrasound technique and so on. Recently, with the popularity of LED luminaires, visible light positioning (VLP) has shown its promise for its reuse of existing lighting infrastructure and low deployment and energy cost. VLP can mainly be catalogued into two classes according to the receiver type, photodiode (PD) based and camera based VLP. The former utilizes Received Signal Strength (RSS) to infer distance information according to the light channel model and then uses trilateration for positioning. It generally has the limitations such as unpredictability of transmit power, customized hardware, and additional smartphone attachments [1], which make the VLP



Fig. 1. Only a single LED luminaire can be captured by the camera.

system impractical to be deployed. Camera based VLP firstly identifies the LED luminaires in the captured image and then perform angle-of-arrival (AoA) localization algorithm to determine the smartphone's absolute position and orientation. Generally, the working principle of VLP is not different from classic trilateration or triangulation. Hence, most VLP methods require at least three LED luminaires as light transmitters.

However, in reality there exist scenarios in which only one LED luminaire can be sensed or decoded by the receiver. In a sparse scenario, such as in long corridor or tunnel, a serial of LED luminaires are usually deployed with a long distance interval. In such cases, only one or two LED luminaires can be sensed or decoded by the receiver (a light sensor or a smartphone camera) and typical trilateration or triangulation based localization algorithm cannot work any longer. Fig. 1 shows some example scenarios where only one LED luminaire is captured. Therefore, recently work has focused on VLP using a single luminaire. However, the existing work mainly depends on dedicated hardware [2]–[5] or the user's hand gestures [2], [6]. These limitations bring in big difficulties to deploy practical VLP systems in the indoor environments in which the users are more willing to use their own smartphones for localization. To address this problem, we present a VLP method based on only one commercial off-the-shelf (COTS) LED luminaire as the transmitter and a COTS smartphone as the receiver. The positioning principle is to integrate the inertial measurements to obtain the orientation and exploit projection geometric features to address the inertial measurement error.

## 2. Related Work

In this section, we review the literature on VLP using three or more LED luminaires and VLP using only one LED luminaire. The literature on VLP is summarized in Table 1.

### 2.1 VLP Using Three or More LED Luminaires

In the very beginning, VLP relies on at least three LED luminaires for localization and trilateration or triangulation is applied for positioning. Receiver Signal Strength (RSS)-based localization [1] utilizes the optical channel model and RSS measurements. Firstly, incidence angle and irradiation angle are derived, localization with triangulation is then carried out by combining the optical channel models corresponding to more than three LED luminaires. By taking a camera as an angle-of-arrival (AoA) sensor, Luxapose system [7] captures at least three LED luminaires in a single image frame and

TABLE 1  
Literature Review

Literature	Method	Accuracy (cm)	Limitations
LIPS [2]	Multi-sensor	< 100	Customized Receiver
Epsilon [1]	Trilateration	< 100	Dense luminaires
LiPro [6]	Multilateration	< 100	Customized receiver
Hou et al. [9]	RSS/AoA	< 26	Vulnerable to orientation measurement error
AoA-ABR [5]	AoA	< 20	Customized Receiver
Red Marker based VLP [4]	Geometric features	< 27.25	Extra mark on LED luminaire
Luxapose [7]	AoA	< 100	Dense Luminaires
SmartLight [3]	LED Array	< 100	Dense luminaires

then perform AoA based triangulation. To solve the unknown tilting angle of camera, Zhu *et al.* [8] propose a VLP method based on angle difference of arrival (ADOA). ADOA does not need the receiver (camera) to know its tilting angle, and the position estimation is modeled as a three-variate optimization problem in a finite search region.

## 2.2 VLP Using Less Than Three LED Luminaires

Recent work has focused on VLP using a single luminaire and the key working principle is to create transmitter/receiver diversity to achieve trilateration or triangulation. Epsilon system [1] explores how to perform RSS based localization with the user involvement by using one single LED luminaire. With the help of the guided hand motion for the user and recorded inertial measurements, more than three RSS measurements can be collected by moving the light sensor and enable the usage of classic trilateration. The same design thought is adopted by LIPS system [2] and LiPro system [6]. Obviously, this method suffers from the high inertial measurement error and requires dedicated calibration.

Besides changing the location or orientation of the receiver to create diversity, directly designing a sophisticated light receiver and transmitter can avoid the rotation calibration and achieve more accurate positioning. In LiPro system [2], a dedicated light receiver deploys three (or more) properly oriented sensors to collect signal strengths as well as the sensors' orientation measurements to enable trilateration. In [5], the receiver lays out multiple PDs combined with apertures in order to have angular diversity, implying it can detect the direction from which light is coming by simply comparing the relative differences in received signal strength values in the different PDs. Hou *et al.* [9] use a PD and an image sensor as two light receivers to measure RSS, the image sensor to measure the incident light azimuth angle, and a magnetic field sensor to measure the receiver orientation. All data are utilized to perform AoA based localization. In contrast, [3] and [4] transfer the design complexity to the LED transmitter. In SmartLight system [3], the LED transmitter is composed of an LED array and a convex lens. Because of the light splitting properties of the convex lens, the light receiver will capture different light patterns at different locations which makes localization feasible. Zhang *et al.* [4] design a circular LED transmitter with a red point marker and then utilize the projective geometry to compute the receiver's orientation and location. All these VLP methods require to re-design the LED transmitter or receiver.

In order to present a VLP using only one COTS LED luminaire as the transmitter and COTS smartphone as the receiver, we present a VLP method that integrates the inertial measurements and exploit geometric features to address the inertial measurement error.

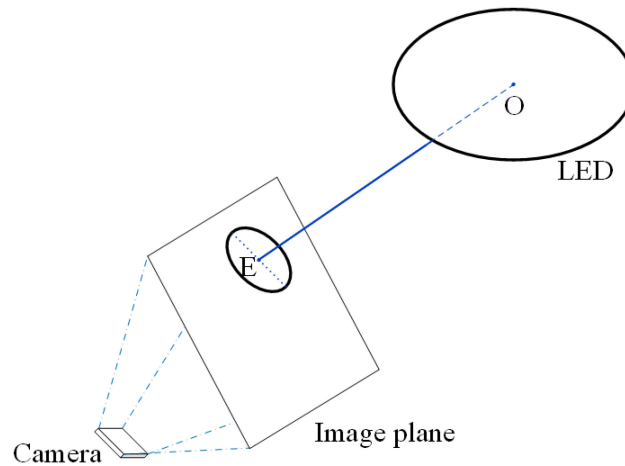


Fig. 2. Shooting a circular object will yield an ellipse projection on the image.

### 3. Visible Light Positioning

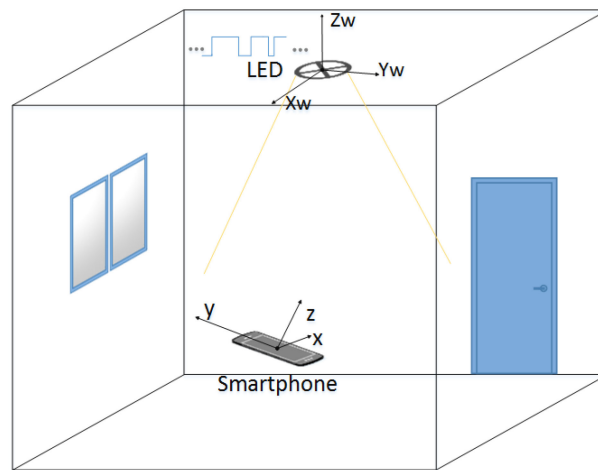
In an indoor environment, sparse LED luminaires are mounted on the ceiling. For example, in a long lane, the LED luminaires are deployed in a line with a long distance between neighboring LED luminaires. In such cases, only one LED luminaire can be captured by the camera held by the user or only the data transmitted from one LED luminaire can be successfully decoded by the camera.

The basic idea is to achieve single LED luminaire based VLP by combining the projective geometry and internal sensor measurements. While the inertial sensor measurements can be used to provide orientation information for positioning, they are not accurate enough and need to be calibrated. As well known, the projective geometry implies rich orientation information. For example, when shooting a circular LED luminaire, the higher orientation angle of the camera along the horizontal axis, the flatter the ellipse we can see in the captured image. Although we cannot derive the orientation directly from the projective geometry, we can exploit the projective geometry to calibrate the inertial readings.

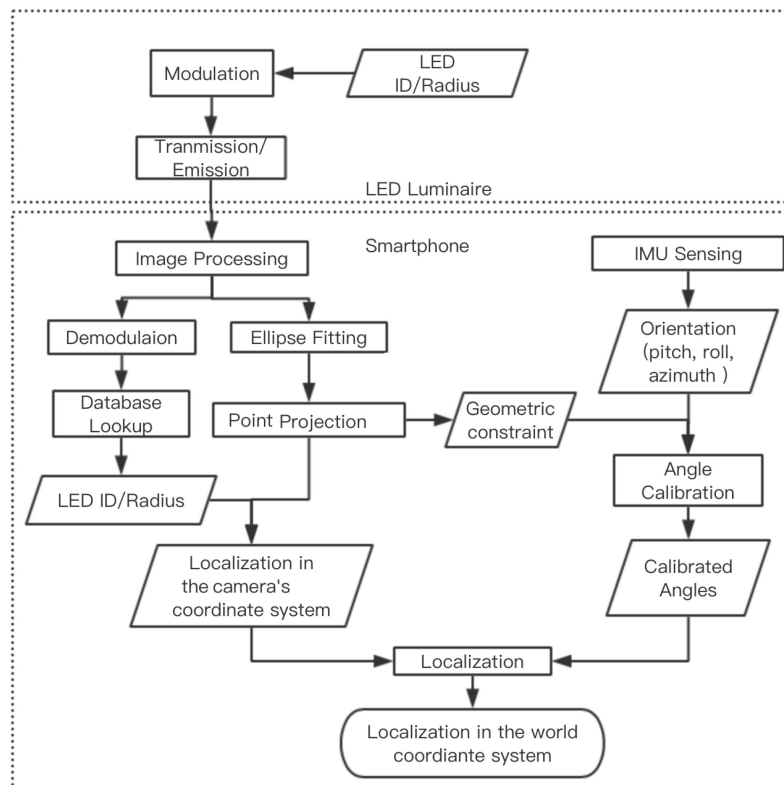
In the following, we will firstly outline the system overview, then describe how we process the inertial measurements and the captured image, and finally detail the positioning method followed by a discussion.

#### 3.1 System Overview

Fig. 3 shows an overview of the proposed indoor VLP system using a single LED luminaire. The circular LED luminaires are mounted on the ceiling of the target indoor environment. Each LED luminaire is assigned with a unique ID number which is stored in an ID-location database and broadcasts its ID number repeatedly by modulated light intensity (the readers may refer to [10] for a reliable LED-camera communication). The dimensions of the LED luminaire (i.e., the radius) can either be stored in the database or transmitted by the LED luminaire. A COTS smartphone works as the receiver. When a user holding a smartphone wants to know his/her location, the smartphone will capture the LED luminaire and record the inertial readings at the same time. The smartphone will firstly demodulate the ID information, infers the location of the LED luminaire in the world coordinate system. Meanwhile, it extracts the ellipse contour of the LED luminaire in the image and the geometric features. Thirdly, the smartphone records the IMU sensing readings and calculates the orientation angles at the same time. By utilizing the point projection of the center of luminaire, the location of the LED luminaire in the camera's coordinate system is derived. Utilizing the point projection of the rightmost point, a geometric constraint is imposed on the orientation angles and angle calibration is carried out to obtain the refined orientation angles. Finally, based



(a)



(b)

Fig. 3. System overview.

on the location of the LED luminaire in the camera's coordinate system and the refined orientation angles, the location of the LED luminaire in the world coordinate system is finally obtained. In this paper, we assume the LED-camera communication can always succeed and the communication failure is not considered in this paper.

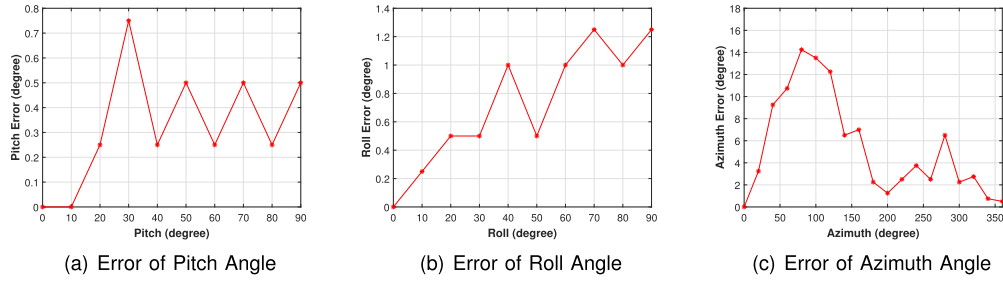


Fig. 4. Errors of Pitch, Roll and Azimuth angles.

### 3.2 Inertial Measurement

We use the inertial measurement unit (IMU) of the smartphone to obtain its orientation, i.e., pitch, roll, and azimuth angle which are denoted as  $\alpha$ ,  $\beta$ ,  $\gamma$ , respectively. Fig. 4 shows the measurement errors of the three angles. Pitch and roll angle can be measured with accuracy of  $1.5^\circ$  while azimuth angle has the error of up to  $15^\circ$ . The reason of the high error in the azimuth angle is the high noise in compass sensor. This result conforms with several research work [11]. Therefore, we assume that the pitch and roll angle readings can be directly used for localization while the azimuth angle needs to be calibrated during localization. The calibration will be described in the following section.

### 3.3 Image Processing

We utilize the rolling shutter effect of CMOS camera to enable visible light communication (VLC). For VLC, we need to set the camera at a sufficiently low exposure time and consequently obtain a banded image where the bands imply the encoded data. The encoding and decoding procedure is omitted here. The readers may refer to [10] for detailed description.

As shown in Fig. 2, a common circular LED luminaire is projected into an ellipse in the image plane. To extract the ellipse, we firstly use Gaussian blur and extract the contour with Sobel operator. With the obtained scattered points, we use the least squares method to obtain the coefficients of the elliptic equation as

$$F(x, y) = s_1x^2 + s_2xy + s_3y^2 + s_4x + s_5y + s_6 = 0, \quad (1)$$

where  $S = [s_1, s_2, s_3, s_4, s_5, s_6]^T$  is the coefficient of the elliptic equation. Using the six coefficients, we can calculate the geometric parameters of interest including the rotation angle  $w$ , the major axis length  $2a$ , the minor axis length  $2b$  and the center  $[u_0, v_0]^T$ . The detailed expressions are shown in the Appendix.

### 3.4 Localization

We assume the LED luminaire lies on the plane  $z = 0$  and its world coordinate (the coordinate of the centre) is  $O = (0, 0, 0)$ . We need to derive the localization of the camera  $P_c = [P_x, P_y, P_z]^T$ . Each point on the LED luminaire  $P$  projects to  $p$  by

$$p = KR(P - P_c), \quad (2)$$

where  $K = \text{diag}\{f, f, 1\}$  is the camera calibration matrix which is calibrated in advance and  $f$  is the focal length,  $p = [u, v, 1]$  is the image coordinate, and  $R$  is the rotation matrix as shown in (3),

$$R = R_z(\gamma)R_y(\beta)R_x(\alpha) = \begin{bmatrix} c_\gamma c_\beta & c_\gamma s_\beta s_\alpha - s_\gamma c_\alpha & c_\gamma s_\beta c_\alpha + s_\gamma s_\alpha \\ s_\gamma c_\beta & s_\gamma s_\beta s_\alpha + c_\gamma c_\alpha & s_\gamma s_\beta c_\alpha - c_\gamma s_\alpha \\ -s_\beta & c_\beta s_\alpha & c_\beta c_\alpha \end{bmatrix} \quad (3)$$

where  $s$  and  $c$  represent sine and cosine (e.g.,  $s_\alpha$  represents  $\sin \alpha$ ).

Let  $[X, Y, Z]^T = R(P - P_c)$ , the projection can be re-written as

$$u = fX/Z \quad (4)$$

$$v = fY/Z. \quad (5)$$

If the dimension of LED luminaire is small enough compared with the distance between the camera and LED luminaire, we can approximate the perspective projection by weak perspective projection [12]. That is we can assume that  $Z$  for every point  $P$  on the LED luminaire is the same. Also as shown in Fig. 2 under weak perspective projection, the major axis of the ellipse is always perpendicular to OE [13] and there always exists a diameter which is perpendicular to OE in the circle. Hence, we can approximate the projection by using a scale factor  $a/r$ , where  $a$  is the major axis of the ellipse,  $r$  is the diameter of the circular LED luminaire. Specifically, we have approximately

$$u = \frac{a}{r}X = fX/\bar{Z} \quad (6)$$

$$v = \frac{a}{r}Y = fY/\bar{Z}, \quad (7)$$

where  $\bar{Z}$  is the average  $Z$  for all points on the LED luminaire.

When we substitute  $O$  into (6)(7), we denote  $[X_0, Y_0, Z_0]^T = R(O - P_c) = -RP_c$  and the projected  $p_0 = [u_0, v_0]$  (i.e., the ellipse center). We can easily obtain  $\bar{Z}$  as  $Z_0$  and  $X_0, Y_0$ . Thus, we obtain the location of the LED luminaire in the camera's coordinate system, i.e.,  $[X_0, Y_0, Z_0]^T$ . Sequentially, with the obtained  $[X_0, Y_0, Z_0]^T$ , we can derive  $P_c$  easily by  $P_c = -R^{-1}[X_0, Y_0, Z_0]^T$ .

However, the IMU measurements are not accurate enough as mentioned above, particularly, the calibration of azimuth angle is in urgent need. In the next subsection, we'll describe the calibration procedure.

### 3.5 Angle Calibration

Most existing work requires the user to manually calibrate the IMU measurements [6] or customize a dedicated hardware with multiple sensors [11]. In our paper, we exploit the projective geometry for angle calibration.

Recall the circular LED luminaire is projected to an ellipse on the image plane. Each point on the ellipse can be denoted as

$$\begin{bmatrix} u - u_0 \\ v - v_0 \end{bmatrix} = \begin{bmatrix} \cos w & -\sin w \\ \sin w & \cos w \end{bmatrix} \times \begin{bmatrix} a \cos \theta \\ b \sin \theta \end{bmatrix} \quad (8)$$

$$= \begin{bmatrix} a \cos \theta \cos w - b \sin \theta \sin w \\ a \cos \theta \sin w + b \sin \theta \cos w \end{bmatrix} \quad (9)$$

where  $\theta \in [0, 2\pi]$ ,  $w$  is the rotation angle counterclockwise,  $a, b$  are the semi-major and semi-minor axis respectively and  $a \geq b$  which are obtained as shown in Appendix.

The rightmost point of the ellipse is the point such that  $a \cos \theta \cos w - b \sin \theta \sin w$  achieves its maximum, i.e.,  $u_0 + \sqrt{a^2 \cos^2 w + b^2 \sin^2 w}$ . In other words, the abscissa of the rightmost point is  $u_0 + \sqrt{a^2 \cos^2 w + b^2 \sin^2 w}$ . Also, from the projection we have

$$u - u_0 = \frac{a}{r}(X - X_0), \quad (10)$$

$$v - v_0 = \frac{a}{r}(Y - Y_0). \quad (11)$$

where  $[X - X_0, Y - Y_0, Z - Z_0]^T = R(P - O) = RP$ . With a point  $P = [x, y, 0]$  on the circular contour of the LED luminaire, we have

$$u - u_0 = \frac{a}{r}[(\cos \gamma \cos \beta)x + (\cos \gamma \sin \beta \sin \alpha - \sin \gamma \cos \alpha)y] \quad (12)$$



TABLE 2  
Error Sources

Error source	Average Accuracy
Measurement error of $\alpha$	$1^\circ$
Measurement error of $\beta$	$1^\circ$
Calibration error of $\gamma$	$1.96^\circ$
Assumption of weak perspective projection	—
Image processing	Dependent on the contour fitting method

From (12), the abscissa of the rightmost point is

$$u_0 + a\sqrt{\cos^2 \gamma \cos^2 \beta + (\cos \gamma \sin \beta \sin \alpha - \sin \gamma \cos \alpha)^2} \quad (13)$$

Therefore, we have the constraint imposed by the projective geometry as follows,

$$\frac{a^2 \cos^2 w + b^2 \sin^2 w}{a^2} = \cos^2 \gamma \cos^2 \beta + (\cos \gamma \sin \beta \sin \alpha - \sin \gamma \cos \alpha)^2 \quad (14)$$

In this way, we can derive  $\gamma$  from the geometric constraint (14).

However, it's rather difficult to obtain an explicit solution from equation (14) and there might exist multiple solutions. Fortunately, we already have a measured  $\gamma$  from IMU sensors. Although, it's not accurate enough, it is a good bound to search for a proper solution. A proper solution can be derived by using classic optimization methods such as Newton iteration that minimizes  $(A - B)^2$ , where  $A, B$  are the left part and right part of equation (14). In this paper, as the search radius is limited (the measurement error is within  $15^\circ$  with a high probability as shown in Fig. 4(c), we simply use brute force search to seek a solution with search step  $1^\circ$ .

Moreover, considering practical constraints, we need to check if the constraints are satisfied during the brute force search. The user usually holds the smartphone with the phone screen facing himself/herself and the camera should generally face up for shooting the LED luminaire on the ceiling. These impose two constraints that the normal vector of the front camera after the rotation should have positive y-component and z-component. Mathematically, let  $[0, 0, 1]^T$  be the original normal vector of the front camera,  $[q_1, q_2, q_3]^T = R[0, 0, 1]^T$  be the normal vector after the rotation, we have two constraints  $q_2 > 0$  and  $q_3 > 0$ .

Finally, we obtain the calibrated angle  $\gamma^*$  that matches the above constraint. With the obtained angles, we localize the camera (the user) as  $P_c = -R^{-1}[X_0, Y_0, Z_0]^T$ .

### 3.6 Discussion

In this subsection, we will firstly discuss the limitations of the proposed VLP method, including the error sources, limited working range, and the assumption of circular LED luminaire and then we discuss the potential improvements.

**3.6.1 Limitations:** Like any practical system, the proposed VLP method has inevitable error. We deem the localization error mainly comes from the image processing (including the ellipse fitting and geometric features extraction), the assumption of weak perspective projection and the inertial measurements of  $\alpha, \beta$  and also the calibration error of  $\gamma$ . The error analysis is summarized in Table 2, in which the calibration error of  $\gamma$  will be evaluated in Section 4.1.

The working distance is mainly limited by the communication range. As the proposed VLP relies on the LED-camera communication to convey the ID information of the LED luminaire, the proposed VLP method can only work within the distance that the camera can successfully decode the ID information. Thus, the working distance depends on the dimension of LED luminaire, the modulation scheme, the rolling shutter frequency and so on [10]. Yet, the above working distance constraint is mandatory for all camera based VLP methods since they all rely on LED-camera communication. Therefore, if the same LED-camera communication method is adopted, the proposed VLP method has the same working range with existing VLP methods based on a single luminaire and much

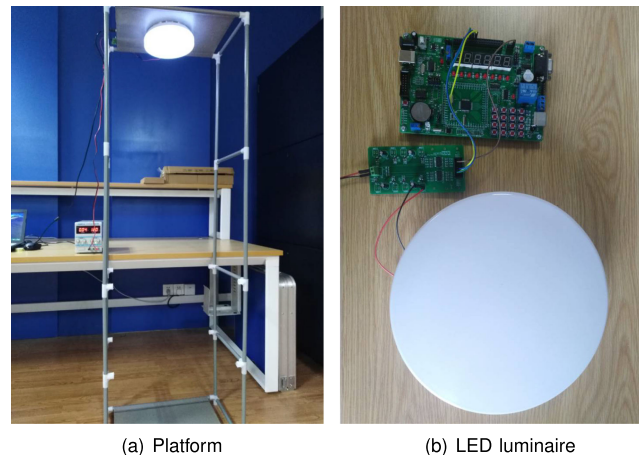


Fig. 5. System experimental platform.

larger working distance than the VLP method based on more than three luminaires. Thus, we can conclude that the working range of the proposed VLP method is no less than the state-of-the-art camera based VLP methods.

In this paper, we assume that the LED luminaire is circular. However, the thought that exploits projective geometry for angle calibration can be applied for VLP with the LED luminaires of the other shapes. For example, if the LED luminaire is square, which is also common in an indoor environment, we can use the projections of vertexes for calibration. The key issue is to determine the exact projection as there exist multiple vertex projections. Similar with the calibration based on circular contour, we can also utilize the rotation angles measured by IMU and assume they are close to the ground truth. Based on this assumption, among all calibrated angles (i.e.,  $\gamma$ ) associated with all possible projections, we choose the one closest to the measured one. Finally, we can derive the localization in the same way with the proposed VLP method using circular luminaire.

**3.6.2 Potential Improvement:** In this paper, we utilize geometry to calibrate the rotation angles. However, a standard manual calibration can also be applied before localization. In a manual calibration [14], [15], a 3D rotation of several 2D rotations are performed to collect magnetic field data and use the least squares fitting method to recover the parameters of the ellipsoid and further correct the raw data.

In this paper we only utilize a single image to calibrate the IMU readings. Using a video, i.e., a sequence of images can further improve the calibration performance and more sophisticated methods such as Kalman filtering (e.g., in [16]) can be applied.

## 4. Evaluation

In order to evaluate the VLP method presented in this paper, we build an experimental platform in a laboratory. Fig. 5 shows the system experimental platform in which a circular LED luminaire with a diameter of 21 cm is mounted on the ceiling of a metal frame. The LED luminaire is supplied with a rated voltage of 12 V. It transmits its radius information and position coordinate at a frequency of 3 KHz repeatedly. As for the receiving end, the back faced camera of HUAWEI TRT-AL00A smartphone is used to decode the LED's ID and radius information. In this paper, we do not consider the impact of the LED-camera communication failure.

### 4.1 Calibration Performance

Before the evaluation of localization performance, we firstly demonstrate the effectiveness of the proposed geometric projection based calibration approach. In Fig. 6, we shoot the LED luminaire

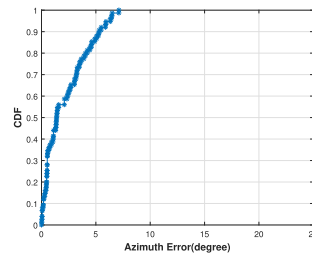


Fig. 6. Cumulative Distribution Function (CDF) of the calibration error.

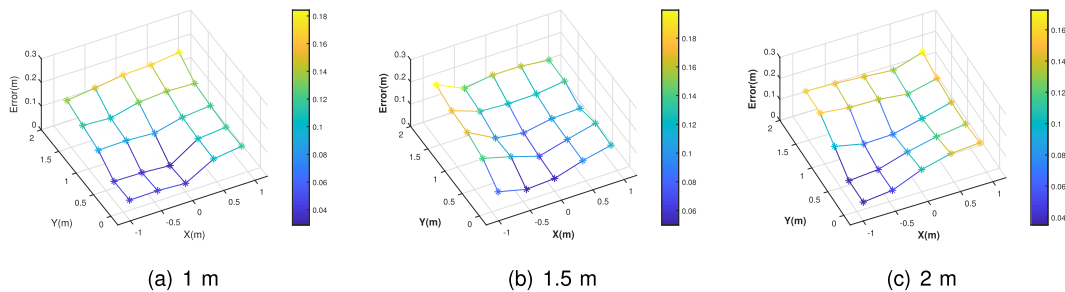


Fig. 7. Localization errors in each grid at the heights of 1 m, 1.5 m, and 2 m.

with the same  $\alpha$  and  $\beta$  but from different directions, i.e., with different  $\gamma$ . We fix  $\alpha = \pm 30^\circ$  and  $\beta = 0^\circ$  as it's convenient for the user to shoot the LED luminaire on the ceiling. It is demonstrated in Fig. 6 that the calibrated angle error is below  $5^\circ$  by roughly 90%. The angle error after calibration is reduced to  $1.96^\circ$  on average. Compared with the angle calibration with extensive user involvement, such as rotating the smartphone  $360^\circ$  [14], [15], the proposed geometric projection constrained calibration approach can achieve high accuracy in a rather lightweight way.

#### 4.2 Localization Performance

To evaluate the localization accuracy, we conduct experiments at three different heights between the smartphone and the ceiling (the plane where the LED luminaire is located). As the typical height of a building is about 2.8 m to 3 m and the camera held by a user is usually at the height of more than 1 m, we choose three heights in this experiment, i.e., 1 m, 1.5 m, and 2 m. In each height, we divide the target horizontal plane of size  $1.8\text{ m} \times 1.8\text{ m}$  into grids of size  $45\text{ cm} \times 45\text{ cm}$  and perform localization in each grid ten times with different orientation angles. The positioning results in Fig. 7 are the average errors of the ten localizations. At the same time, we implement the localization method by Zhang *et al.* [4] for comparison and plot the localization results in Fig. 8. Please recall that [4] adds a red mark on the LED lamp for calculating the smartphone's orientation angles.

In Fig. 7, it is shown that the proposed positioning method in this paper achieves sub-meter positioning accuracy. All the positioning errors are below 20 cm and can be down to about 2.9 cm. Meanwhile, we also find that generally the higher the horizontal distance (thus the higher Euclidean distance) between the smartphone and the LED luminaire, the higher the positioning error. When the smartphone is just below the LED luminaire, we obtain the lowest positioning error down to 3-4 cm. This result conforms with our intuitive understanding.

When we compare our method with red marker based VLP [4], Fig. 8 shows that adding a red marker on the LED luminaire can help achieve sub-meter localization accuracy too. Fig. 9 shows us more clearly how the two methods perform in terms of Cumulative Distribution Function (CDF) of the localization errors. The errors achieved by ours are below 16 cm by the percentage of 90%

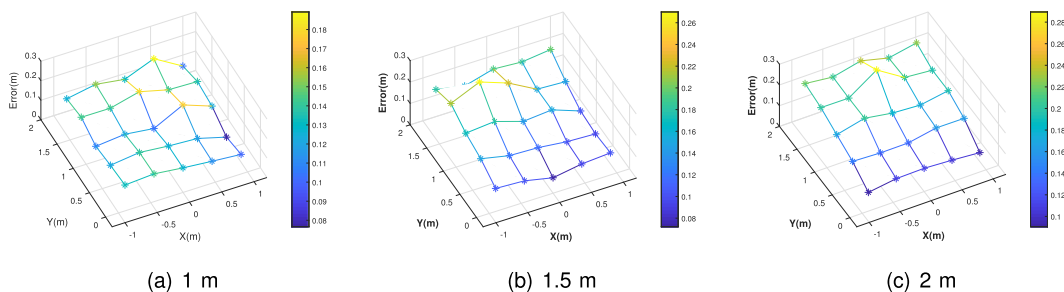


Fig. 8. Localization errors of red marker based VLP [4] in each grid.

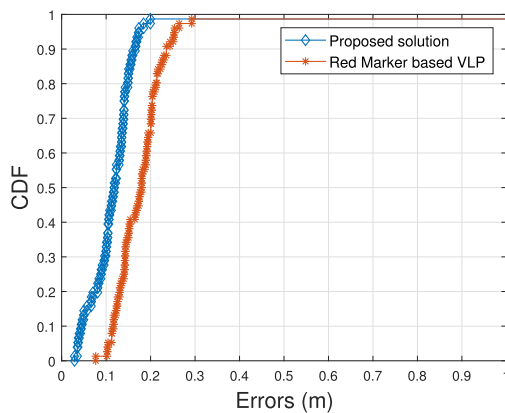


Fig. 9. CDF of the localization errors.

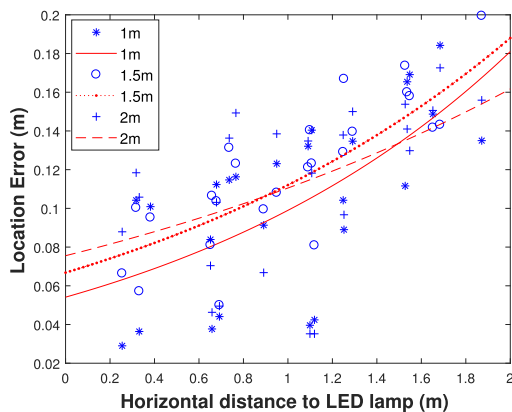


Fig. 10. Positioning errors with varying horizontal distance. Scattered points denote the measured results and the lines denote the fitting curves.

and the average error is down to roughly 11.2 cm. The two methods have comparable positioning performance and ours need not change the appearance of LED luminaires.

Overall, our localization method that exploits the projective geometry to calibrate the rotation angles can achieve high localization accuracy.

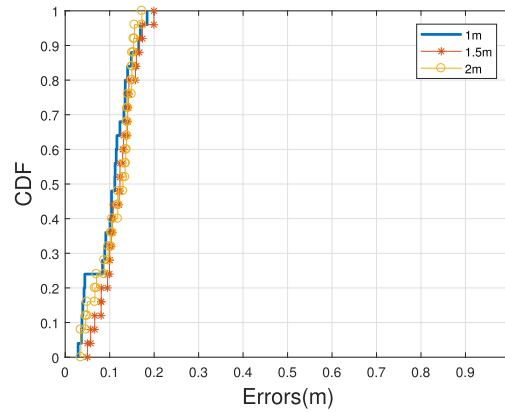


Fig. 11. CDF of the localization errors at different heights.

#### 4.3 Impact of Distance

We also evaluate our VLP method when performing localization at different distance as shown in Figs. 10 and 11. In Fig. 10, the scatter points are the average errors in each location and the curve is the fitting curve by using exponential function which is plotted to show the change trend of the positioning errors more clearly. As the horizontal distance between the smartphone with the LED luminaire increases, the positioning error increases from about 5 cm to about 18 cm and remains sub-meter. This conclusion conforms with that in Fig. 7. Fig. 11 gives the same conclusion that the CDF of localization error at the height of 1 m is lower than that at the height of 1.5 m and 2 m. However, the difference is so slight that we can conclude the proposed localization method can achieve a high accuracy at varying distance.

### 5. Conclusion

In this paper, we focus on the VLP problem using only one LED luminaire in the indoor environments with sparse LED luminaires deployed. To address the missing distance and orientation information for positioning, we utilize the geometric features of the LED luminaire. Specifically, we use the dimension of the LED luminaire to obtain the distance information and IMU measurements for orientation. Moreover, to cope with the high IMU measurement errors, projective geometry is exploited for measurements calibration. The experimental results demonstrate that the proposed VLP method is able to achieve state-of-the-art positioning accuracy. In particular, the positioning errors are below 0.16 m by the percentage of 90%.

### Appendix

The parameters of interest can be calculates as follows.

$$w = \frac{1}{2} \arctan \frac{s_2}{s_1 - s_3}, \quad (15)$$

where  $w$  is the rotational angle of the ellipse. The center of ellipse is shown as

$$[u_0, v_0]^T = \left[ \frac{-n_1}{2m_1}, \frac{n_2}{2m_2} \right]^T \quad (16)$$

where  $m_1 = s_1 \cos^2 w + s_2 \sin w \cos w + s_3 \sin^2 w$ ,  
 $m_2 = s_1 \sin^2 w - s_2 \sin w \cos w + s_3 \cos^2 w$ ,  
 $n_1 = s_4 \cos w + s_5 \sin w$ ,  $n_2 = -s_4 \sin w + s_5 \cos w$ .

The major axis length  $2a$  and the minor axis length  $2b$  are shown as

$$2a = \sqrt{\frac{m_2 n_1^2 + m_1 n_2^2 - 4m_1 m_2 s_6}{4m_1^2 m_2}}, \quad (17)$$

$$2b = \sqrt{\frac{m_2 n_1^2 + m_1 n_2^2 - 4m_1 m_2 s_6}{4m_1 m_2^2}}. \quad (18)$$

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