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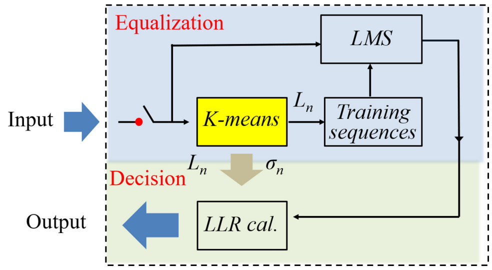

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Enabling Equalization and Soft Decision by k-Means for VCSEL-Based PAM-4 Optical Interconnection

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Abstract: As for vertical-cavity surface-emitting lasers (VCSEL)-based optical pulse amplitude modulation (PAM) interconnection system, apart from the universal signal distortions like Gaussian noise and fiber dispersion, direct modulation of VCSEL specially induces level nonlinearity and level-dependent noise. To mitigate these two specific distortions in VCSEL-based link, a k-means assisted PAM-4 signal receiver including equalization and soft decision (SD) is proposed. Based on learned mean values and standard deviations for individual levels through k-means approach, probability density function of nonlinearly distorted PAM-4 symbols can thus be approximated more precisely than conventional SD, leading to improved decision performance. In addition, depending on k-means result, least means square (LMS) equalization with adapted levels can converge to a lower error value than conventional LMS, thanks to level nonlinearity mitigation. Consequently, by using proposed k-means equalization and SD, improved signaling performance can be obtained by mitigating the level nonlinearity and level-dependent noise. Both simulation and experiment are performed for evaluating the proposed method in a 100-Gb/s VCSEL-MMF PAM-4 optical interconnection system, by comparing with conventional means.

Index Terms: Optical interconnects, fiber optics systems.

1. Introduction

Recently, emerging applications like cloud computing/storage, 4G/5G wireless communication, artificial intelligence and so on have been boosting the growth of data amount. High-speed data communication technologies especially in the field of short-reach data interconnection are thus urgently demanded. Next-generation short-reach interconnection aims to find the optimal trade-off among capacity, power consumption as well as density. Currently, intensity modulation and direct detection (IM-DD) solution is treated as one of the most balanced approach. For IM-DD, novel optoelectronic devices [1]–[3], modulation formats [4]–[9] and digital signal processing (DSP) methods [10]–[13] are continuously emerged in attempts to improve signaling performance. Among these solutions, 4-level pulse amplitude modulation (PAM-4) signaling based on vertical-cavity

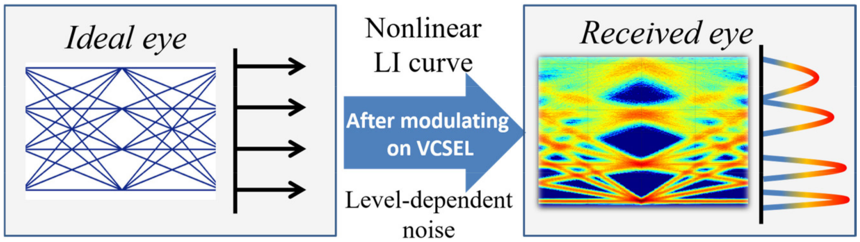

Fig. 1. Level nonlinearity and level-dependent noise induced by VCSEL modulation.

surface-emitting lasers (VCSEL) and multimode fiber (MMF) has been broadly adopted for practical industrial implementation by standards like IEEE 802.3bs Task Force for the 400-Gbps interconnects [14]. That is because VCSEL exhibits attractive features of low driving power, small size and massive fabrication capability. Moreover, VCSEL combined with PAM modulation can provide a quite balanced performance of implementation complexity and signaling capacity. As a result, cost-effective DSP approaches to enhance the capacity of the VCSEL-MMF based optical PAM-4 system are highly desired.

There have been extensive DSP methods proposed and most of them focused on the mitigation of inter-symbol interference (ISI), by the means of pre-filtering [15], [16], post equalization [17]–[19] and coding [20]. However, as for the VCSEL-MMF link, the challenge becomes even more complicated due to the presence of ISI together with level nonlinearity [21] and level-dependent noise [11]–[22], as schematically plotted in Fig. 1. In detail, level nonlinearity is mainly caused by the thermal characteristic of VCSEL. Besides, skewed eyes of PAM-4 have been observed after VCSEL modulation [23], [24], which further degrade amplitude linearity. Apart from level nonlinearity, shot noise and relative intensity noise will result in level-dependent noise of PAM-4, namely different noise powers over different levels. Furthermore, multimode coupling and transmission will aggravate intensity noise due to mode partition noise of VCSEL. Until now, it is still challenging to efficiently mitigate such level-dependent noises, particularly with the consideration of level nonlinearity. Previously, a machine-learning detector based on support vector machine (SVM) has been proposed to address the modulation nonlinearity of VCSEL [25] and micro-ring modulator [12]. However, SVM classifier requires to train the decision boundaries based on prior-known sequences, thus it is difficult to get utilized for real-time implementation.

To mitigate the above distortions of level nonlinearity and level-dependent noise, a k-means assisted signal detection method including both equalization and soft decision is proposed for VCSEL-MMF based optical PAM-4 interconnection systems. Firstly, reference sequences' amplitudes of LMS process can be adapted according to the learned levels by k-means, interacted error can thus converge to a lower value than conventional LMS, due to extra consideration of level nonlinearity. Secondly, standard deviations of individual levels can be attained as well by k-means with no use of prior-know sequences. Thus, more precise estimation of symbol's probability density function (PDF) can be realized based on learned levels and deviations. Benefitting from which, the precision of log-likelihood ratio (LLR) estimation can be correspondingly enhanced, which leads to improved performance of soft decision (SD). Unlike other machine-learning detectors for obtaining adaptive decision boundaries which are more likely as hard decision [12]–[25], this work realized an intelligent SD based on PDF estimation, which can extract and mitigate level nonlinearity and level-dependent noise simultaneously [26]. Even though acquisition of LMS filter coefficients require training process, it no longer needs further training after coefficients initialization [27], [28]. Moreover, proposed SD can also get rid of training based on prior-known sequence. As a consequence, the proposed equalization and SD exhibit lowered complexity, compared to other approaches [12]–[25].

In fact, k-means is no longer a new technique for DSP. In [29], k-means was utilized to obtain the hard-decision thresholds of the rotated constellations. Similar application in phase recovery has also been demonstrated on 8-PSK system [30]. They performed k-means by processing 2-D

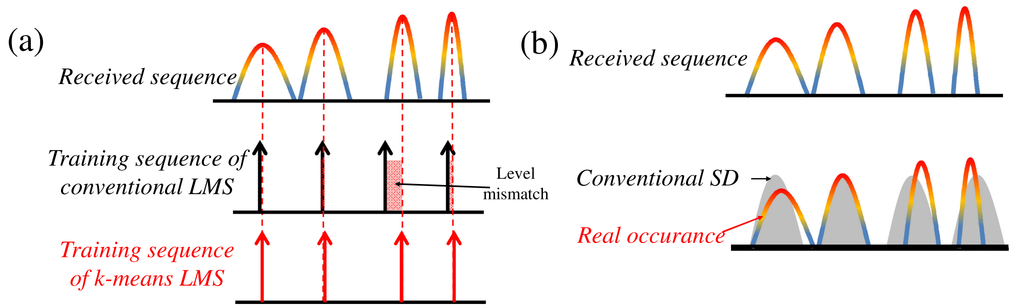

Fig. 2. Proposed k-means assisted LMS: (a) equalization and (b) soft decision, compared with conventional methods.

constellations, for obtaining an optimal sampling. Besides, a k-means assisted decision has been proposed in [31] on a 75-Gbps QAM-64 system, it is more likely as a hard decision because the decision is performed based on Euclidean distances. Other possible applications of k-means include modulation formats identification [32]. The intrinsic principle of these works basically depends on 2-D symbols classification. Although k-means is not a new method itself, its application in LMS equalization and LLR-based SD is at the first-time proposed in this paper for mitigating level nonlinearity and level-dependent noise in VCSEL-MMF link.

As an extended study of [26], k-means equalization combined with SD is evaluated through both simulation and experiments in this paper. In detail, simulations have been conducted based on a 100-Gbps VCSEL-based optical PAM-4 system on *VPItransmissionMakers*. Significant BER reduction (over 2-order BER reduction) is achieved by using the proposed k-means approach, with severe level nonlinearity and level-dependent noise. Meanwhile, 100-Gbps and 100-m transmission has been experimentally realized using a commercial-production-level VCSEL chip and OM3 multimode fiber (MMF). 0.54-dB reduced optical power requirement is obtained for achieving error free under the assumption of 25% SD-FEC assistance, based on generalized mutual information (GMI) analysis.

2. Principle and Simulations

Generally, a normal PAM-4 eye has equally spaced levels, and consistent noise powers over individual levels. Under this circumstance, equalization and decision at the receiver is simple without extra requirement of level linearity operation. Most widely utilized equalization methods are based on finite impulse response (FIR) filter, whose tap coefficients adapted through LMS process. Relative error between filtered sequence and reference sequence converges to its minimum value along with the interactions. But for VCSEL-MMF link, PAM-4 suffers extra level nonlinearity after VCSEL modulation, which degrades the convergence of LMS process due to the mismatched levels between received samples and reference sequence, as schematically described in Fig. 2(a). On the other hand, SD based on LLR estimation exhibits enhanced performance than hard decision, because it decides symbols according to PDF through multiple bits quantization [33]. For conventional SD of PAM-4 symbols, two independent LLR tributaries are estimated based on known levels and standard deviations of noise. Specifically, two tributaries of LLR for Gray-coded PAM-4 are given as Eq. 1 and Eq. 2, in which R is the sampled sequence, Ln ($n = 1, 2, 3, 4$) stands for the mean value of level, σ is the standard deviation. For conventional SD without considering level nonlinearity and level-dependent noise, Ln is equally spaced and σ is consistent over different levels. The resulted PDF is plotted in gray color in Fig. 2(b).

$$LLR_1 = \log_2\left(\frac{e^{-(R-L_3)^2/2\sigma^2} + e^{-(R-L_4)^2/2\sigma^2}}{e^{-(R-L_1)^2/2\sigma^2} + e^{-(R-L_2)^2/2\sigma^2}}\right) \tag{1}$$

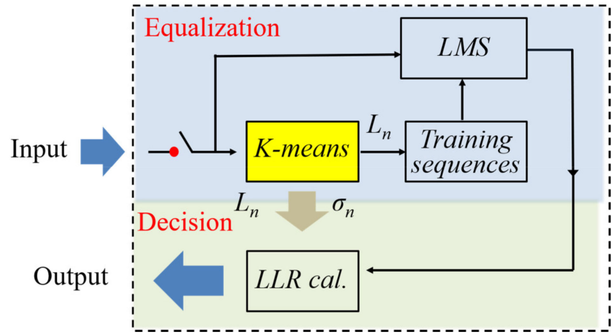

Fig. 3. Procedure of proposed k-means equalization and SD.

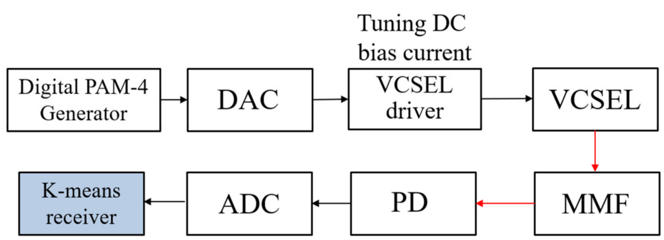

Fig. 4. VCSEL-MMF based optical PAM-4 system for simulation of the proposed k-means signal processor.

$$LLR_2 = \log_2\left(\frac{e^{-(R-L_2)^2/2\sigma^2} + e^{-(R-L_3)^2/2\sigma^2}}{e^{-(R-L_1)^2/2\sigma^2} + e^{-(R-L_4)^2/2\sigma^2}}\right) \tag{2}$$

Nevertheless, PAM-4 symbols no longer have equal-spaced levels and consistent noise powers over individual levels after VCSEL modulation, due to the presence of level nonlinearity and level-dependent noise. As a consequence, conventional LMS equalization as well as SD cannot work well without taking these two factors into consideration. To deal with it, we propose k-means assisted equalization and SD which can learn and thus mitigate these two distortions simultaneously.

The procedure of k-means assisted equalization and SD is plotted in Fig. 3. K-means clustering is performed after resampling, to adaptively learn the mean values and noise variances for individual levels, denoted as Ln and σ_n. The upper blue box is the equalization part. Tap coefficients are trained through LMS process. Different from conventional LMS, levels of training sequence are altered adaptively according to the learned level means. The lower box is the decision part. Based on learned level Ln ($n = 1, 2, 3, 4$) and noise variances σ_n, the corresponding LLRs can be calculated. Because it takes level nonlinearity (affecting Ln) and level-dependent noise (affecting σ_n) into consideration, the proposed SD is expected with improved decision precision, as shown in Fig. 2(b). To evaluate the k-means assisted signal processor, simulation is carried out on a 100-Gbps VCSEL-MMF PAM-4 system using *VPItransmissionMakers*. The system configuration is shown in Fig. 4. Transmitter includes a digital PAM-4 signal generator, a digital-to-analogue converter (DAC), a VCSEL driver and a VCSEL working at 850nm. At the receiver side, an ADC is used to record the sampled signals, which following a photo detector (PD). The detailed simulation parameters are given in Table 1. Nonlinear LI and stimulated emission characteristics of VCSEL are simulated using the thermal-dependent VCSEL model embedded in VPI, which was proposed in [21]. At last, recorded signals are processed including the proposed equalization, soft decision and BER counting.

TABLE 1
Simulation Settings

Device	Setting
PAM-4 generator	PRBS-11, Gray coding
DAC	8-bit resolution, 40-GHz bandwidth
VCSEL	Thermal model in [21]
Fiber	Multimode fiber
PD	0.4-A/W sensitivity, 40-GHz bandwidth
ADC	8-bit resolution, 40-GHz bandwidth
K-means clustering	100 replications on 1k symbols, optimizing Euclidean distance

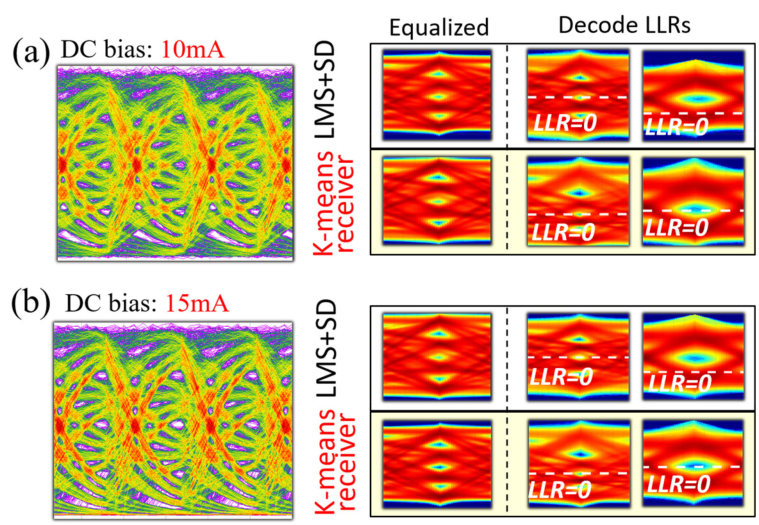

Fig. 5. Equalized eyes and LLRs using conventional LMS and proposed K-means receiver, with DC bias current of (a) 10 mA and (b) 15 mA.

As for the direct modulation of VCSEL, severer level nonlinearity is observed when bias is either too large or too small, exceeding the limited linear operation range. The captured eyes at 10-mA and 15-mA bias currents are plotted in Fig. 5(a) and (b), respectively. Eye suffers severe level nonlinearity and level dependent noise, which results in different eye heights. In detail, level-dependent noise is observably larger at the bias current of 10 mA than that of 15 mA, which matches the fact that VCSEL's RIN is larger when it is biased at lower current. However, level nonlinearity distortion is not so distinguished as level-dependent noise, because signal is too noisy. In addition, eye becomes skewed when bias current is 15 mA, which leads to increased level nonlinearity after sampling. For LMS process, the tap number of FIR filter is 50, with 50000 samples for interacting. K-means clustering is performed over 1000 samples, with optimizing Euclidean distance. The results of conventional LMS equalization and LLR estimation are depicted in the upper box, while the ones of proposed k-means receiver in the lower box. In the same circumstance, equalized eye becomes clearer by using k-means assisted LMS. For conventional SD, although the decoded LLR eyes have similar shape to the ones by k-means SD, the decision line (LLR $=0$) is not placed in the position with the largest eye width. In this case, decided binary sequences based on threshold of 'LLR $=0$'

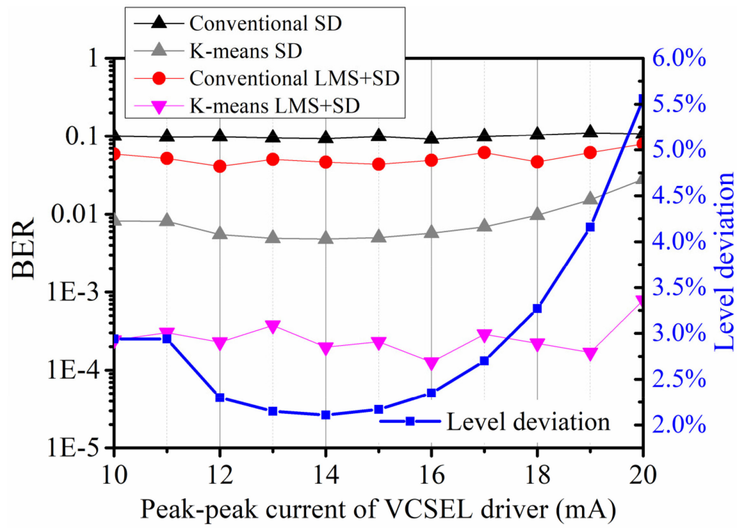

Fig. 6. BERs and LD values under varying driving current of VCSEL.

will exhibit large amount of error. In contrast, the proposed k-means SD exhibits its decision line in the optimal position (largest eye width) of LLR eyes.

For further investigation, level deviation (LD) is chosen as the parameter for numerically evaluating the level nonlinearity under varying bias current, as Eq. 3 [34] where Vpp is the peak-to-peak voltage and D denotes the eye-height of three eyes. According to the LD values, the optimal DC bias of this VCSEL model is near 14 mA. Too high or too low bias currents lead to the increase of LD values. BERs at varying bias currents are counted using conventional SD and k-means SD, as plotted in Fig. 6. For conventional LMS and SD, the increasing LD values do not influence BERs too much, because BERs are so high (nearly 0.1). By using k-means SD only, BERs are significantly reduced under 0.01. When using 50-tap FIR whose coefficients are adapted through k-means LMS, the lowest BER gets to $1 \times 10 - 4$. Consequently, over 2-order BER reduction is achieved by the mean of k-means receiver, comparing to conventional LMS and SD.

$$LD = \left(\frac{|D32 - Vpp/3|}{Vpp} + \frac{|D21 - Vpp/3|}{Vpp} + \frac{|D10 - Vpp/3|}{Vpp} \right) \times 100\% \tag{3}$$

3. Experimental Results and Discussions

Experiments have been carried out for PAM-4 signaling over a VCSEL-MMF optical interconnection link, to verify the performance of the proposed k-means receiver. Corresponding setup is illustrated in Fig. 7(a). A 25-GHz arbitrary waveform generator (AWG) is used for electrical signal generation. A multi-dimensional alignment platform is built up for the modulation of VCSEL, including the optically coupling between a commercial-product-level VCSEL and MMF, and the electrically coupling of the high-speed signals to VCSEL through a high-speed probe. The VCSEL chip operates at around 850nm, and MMF used in this work is OM3 fiber. A photo-detector (PD) with 22-GHz bandwidth working at 850nm is used to detect the optical signals. At the end of the channel, a digital storage oscilloscope (DSO) is used for recording and offline processing of signals with a sampling rate of 160 GSa/s. Offline DSP is performed on Matlab. In the transmitter, digital PAM-4 sequences are generated by binary-to-integer mapping of PRBS11 sequence. Before digital-to-analogue conversion, a pre-distortion on frequency domain has been implemented in AWG [35]. For the receiver part, the resampling process includes synchronization and down-sampling, to obtain 1 sample/symbol sequences. Then conventional LMS algorithm and SD are performed as the based-line comparison

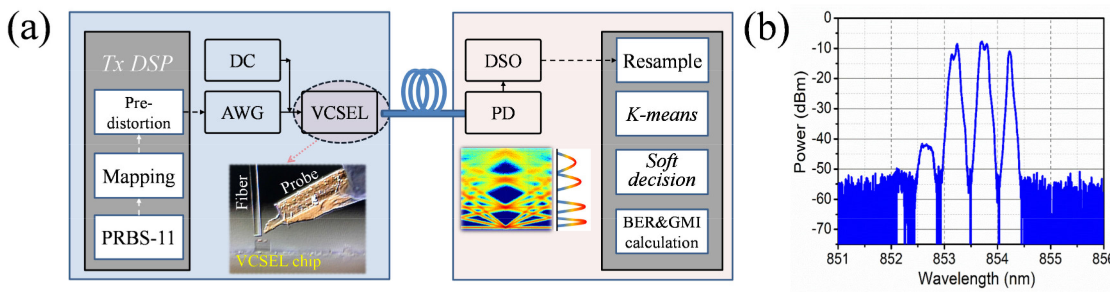

Fig. 7. (a) Experimental setup. (b) Optical spectrum of VCSEL with bias current of 15 mA.

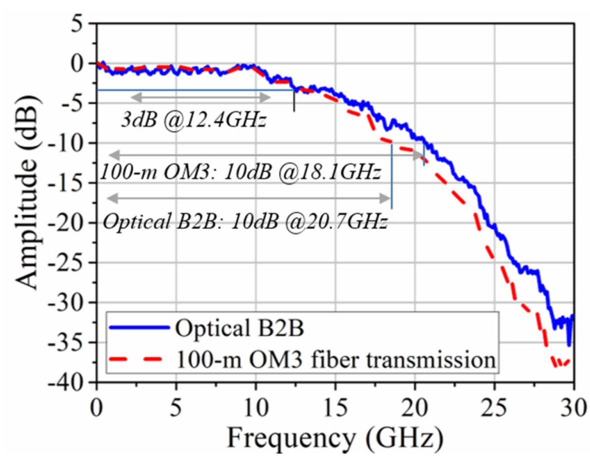

Fig. 8. Frequency responses in the case of optical B2B and after 100-m OM3 transmission.

of proposed k-means receiver. At last, BERs (namely pre-FEC BER [33]) are counted for evaluating the precision of conventional SD and proposed k-means SD. LLR calculation estimates bits by probability approach, thus BER of individual bit can be obtained by deciding corresponding LLR values with the decision threshold of '$\text{LLR} = 0$'. Worth to mention that, generalized mutual information (GMI) has been proved as an effective parameter for predicting post-FEC BER. Consequently, GMI values under varying received optical power are also calculated for investigating the reduction of optical power requirement to achieve error free signaling, assuming 25% SD-FEC assistance. VCSEL in this test is a commercial bare chip with bandwidth of 20 GHz. The measured spectrum of VCSEL is plotted in Fig. 7(b), with a bias current of 15 mA. There are four transverse modes emitted simultaneously, which may lead to severe modal dispersion during multimode transmission. Frequency responses of end-to-end link are measured by sending multi-tone signal into system. Through decoding the received signal at the receiver, SNR value for every subcarrier can be obtained. Normalized frequency responses of the optical back-to-back (B2B) link and the 100-m long OM3 fiber transmission link are plotted in Fig. 8. Because of constrained bandwidth of the optoelectronic devices, the SNR response rolls downward severely. After 100-m OM3 fiber transmission, the accumulated dispersion further reduces the channel bandwidth.

To realize 90-Gbps and 100-Gbps PAM-4 signaling, the symbol rate of AWG is set at 45 Gbaud and 50 Gbaud, respectively. The DC bias current is fixed at 15 mA in the experiment for obtaining optimal performance. And peak-to-peak voltage of electrical signal is 500mV. To process the PAM-4 sequences, the sampled signals by DSO have to be re-sampled to 1 sample/symbol. Then the samples are sent into LMS for equalization, errors between training sequence and filtered one are recorded for every interaction. Error convergence curves for 90-Gbps and 100-Gbps PAM-4 in

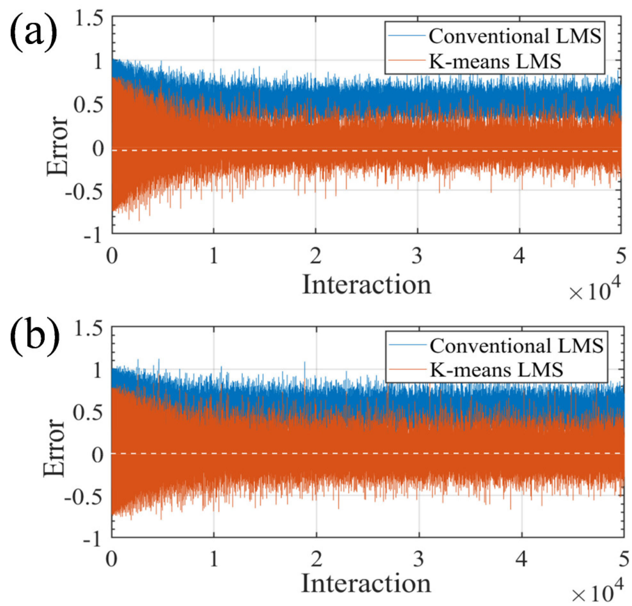

Fig. 9. Errors between training sequence and filtered one during LMS interaction for optical B2B case: (a) 90-Gbps PAM-4, (b) 100-Gbps PAM-4.

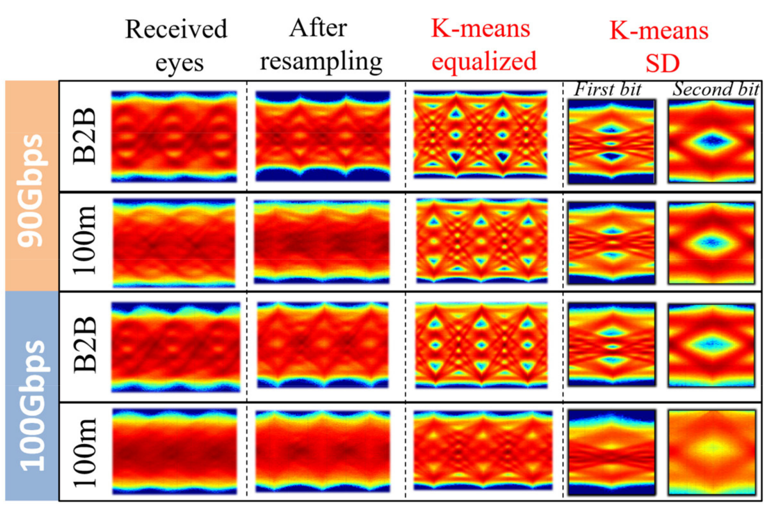

Fig. 10. Results of k-means equalization and SD for 90-Gbps and 100-Gbps PAM-4.

optical B2B case are shown in Fig. 9 (a) and (b) respectively, with received optical power of 3 dBm. The residual errors are mainly induced by random noise and residual ISI. After 100-m transmission, residual errors are higher than the case of optical B2B. It can also be seen that, for k-means LMS (red line), converged error is lower than conventional LMS (blue line), with respect to zero. It is because that for conventional LMS, the levels are mismatched between training sequences and received samples, which deteriorates the convergence performance. This result indicates that, lower residual error after filtering can be achieved by k-means LMS.

The eye-diagrams of original sequences, equalized ones as well as LLRs are depicted in Fig. 10. 90-Gbps PAM-4 in optical B2B case shows clearer eyes than others, that is because less ISI occurs in this circumstance. While for 100-Gbps signaling after 100-m OM3 fiber transmission, the eye

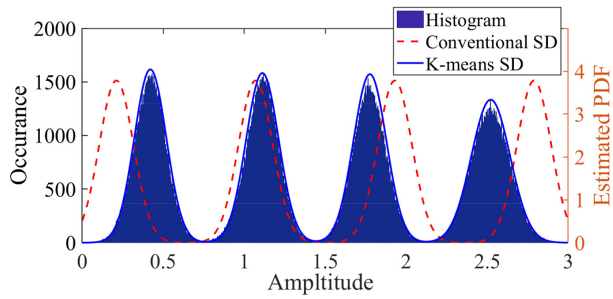

Fig. 11. Estimated PDFs based on conventional SD and k-means SD.

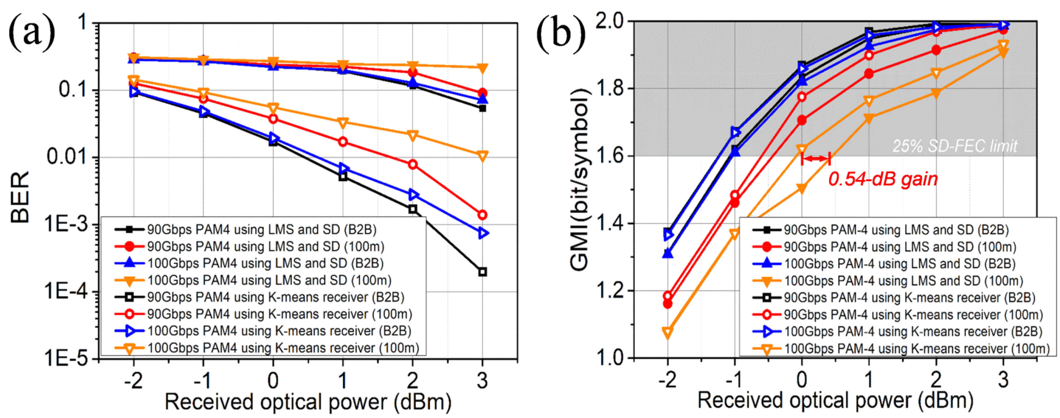

Fig. 12. Results of conventional SD and k-means SD, for 90-Gbps and 100-Gbps PAM-4 in optical B2B case and 100-m transmission. (a) BER values. (b) GMI values.

severely closes with undistinguished four levels. As indicated by 90-Gbps data in optical B2B case, eye-skewing as well as level nonlinearity have been observed with differing eye closures for three sub-eyes. Then, resampling is performed to obtain 1 sample/symbol sequences, whose eyes are depicted in the second column in Fig. 10. The phenomenon has been found that the middle eye pattern of equalized signal exhibits severer eye closure than the other two. That is because that obtained level mean values through k-means process are not equally spaced. By using k-means equalization, the corresponding eyes are opened obviously with observable four levels, as shown in the third column. While in the case of 100-Gbps and 100-m transmission, the corresponding eye is still noisy, that is because the residual ISI cannot be effectively eliminated. At the fourth column, the two LLR tributaries are calculated through Eq. 1 and 2 according to learned level means and deviations as demonstrated by the mean of eye diagram.

As described in Fig. 2, LLR calculation is conducted based on PDF extraction. Conventional SD assumes equally-spaced levels and consistent Gaussian distribution over individual levels, thus it cannot guarantee SD precision when signal is nonlinearly distorted. For investigating that, the symbol histogram of equalized 100-Gbps PAM-4 is plotted in Fig. 11. It can be seen that mean values as well as standard deviations of four levels are different. To realize conventional SD, PDF of individual symbol needs to be estimated. For conventional SD, levels are equally spaced. Thus, the estimated PDF by conventional SD is plotted as dashed red line, which mismatches the real occurance. While by using k-means approach, estimated PDF is more precise.

Based on the calculated LLRs, corresponding binary sequences are obtained by deciding bit '1' or '0' with decision line of 'LLR = 0'. Then BERs are counted for PAM-4 signals with different received optical power. Fig. 12(a) shows the BER results by using k-means assisted equalization and SD, compared to conventional ones. With increasing of either data rate or transmission distance, BER is increased. By using conventional approaches, BERs are too high (nearly 0.1). Fortunately, BER reduction is obvious assisted by k-means. Nearly 2-order BER reduction is achieved when optical power is over 2 dBm. In detail, BER can be reduced to $1 \times 10 - 2$ for 100-Gbps PAM-4 signaling after 100-m transmission, with optical power of 3 dBm. This result is contributed to the enhanced precision of LLR estimation, which can be proved by Fig. 5 where k-means SD has its 'LLR = 0' in the center position with largest eye width. At the same time, GMI values are also calculated. According to the results in Fig. 12(b), the GMI improvement by k-means is also obvious. Assuming 25% SD-FEC assistance whose code rate is 0.8, the corresponding GMI limit of error free is 1.6 bit/symbol, according to [36]. Consequently, 0.54-dB optical power reduction can be obtained to achieve error free, for realized 100-Gbps and 100-m signaling.

4. Conclusions

In this paper, an adaptive receiver to level nonlinearity and level-dependent noise based on k-means clustering is proposed in VCSEL-based PAM-4 system. Proof-of-concept investigations have been carried out via both simulations and experiments. Nearly 2-order BER reduction is theoretically obtained with severe distortions of level nonlinearity and level-dependent noise. In addition, optical signaling was realized experimentally using a commercial VCSEL at a 100 Gbps data rate. Optical sensitivity gain of 0.54 dB is achieved for 100-Gbps and 100-m transmission, thanks to the proposed k-means receiver. Moreover, since there is no requirement of training with prior-known sequences in decision part, k-means approach is expected to be more applicable for real-time implementation. This method can also be utilized for other systems, including not only IM-DD but also coherent optical communications.

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