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Modeling the Heterodyne Efficiency of Array Detector Systems in the Presence of Target Speckle

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Abstract: To solve the speckle effect problem in heterodyne detection, an array detector is proposed; heterodyne efficiency is modeled by accounting for speckle and using an additional parameter to simulate coherent summation processing. Heterodyne efficiency is derived by mixing a Monte Carlo simulated speckle field with a planar local oscillator field on a photodetector surface to evaluate the array detector performance at improving signal-to-noise ratio (SNR) of the coherent optical system. From the simulation results, the array detector is shown to effectively improve the SNR of the heterodyne system. For low SNR, the heterodyne efficiency of an $N \times N$ array detector increases linearly with N.

Index Terms: Laser radar, remote sensing, rough surfaces, spatial coherence, sensor arrays.

1. Introduction

The heterodyne technique is well recognized as a method of obtaining quantum-limited reception in spectral ranges where direct detection cannot be made quantum-limited. In general, the SNR of a heterodyne system reaches its maximum when both the amplitude and the phase distribution of the signal and the local oscillator fields are perfectly matched. Many applications relying on active detection encounter optically rough surfaces that produce speckle effects, which, consequently, degrade the system sensitivity or result in decoherent effects [1]-[8]. Hence, overcoming the speckle effect in coherent optical detection has become an important issue. Fink [9] published the first detailed theoretical analysis of the array detector method where a receiver constructed from an array of detectors can significantly improve the SNR of heterodyne systems if the detector elements are small enough and the phase shifts are correctly set. Chan et al. [10], [11] reported their experiments using a 2 \times 2 heterodyne detector array and derived an analytic expression for the ratio of the SNR of an $N \times N$ array detector system to the SNR of a single-detector system. These initial works started the investigation of the use of array detectors for heterodyne detection which has been ongoing for several decades [12]-[17]. At the same time, the related work of numerical models for simulating the dynamics of silicon optoelectronic devices promoted the development of array detector methods [18], [19]. However, to the best of our knowledge, no additional studies of the evaluation of the effectiveness of array detectors for improving the SNR of heterodyne systems in

the presence of target speckles have been reported. The recent spread of Monte Carlo methods offer a great opportunity to revisit the investigation of the array detector effectiveness.

Monte Carlo (MC) methods obtain numerical results based on probability and statistical theory. Consequently, MC methods found widespread application in the area of speckle field modeling [20]–[23], since the phase fluctuations of speckle fields can be regarded as a Gaussian random variable with a zero mean value. The statistical parameters of a speckle field can be described by its correlation length and its phase fluctuation root mean square value. A characteristic use of MC models in this paper is the generation of speckle fields on the surface of photodetectors.

A common characteristic used for evaluating the performance of coherent optical systems is the heterodyne efficiency. Heterodyne efficiency represents the matching state of the LO and the signal beams on the photodetector surface. Namely, Chambers [24] constructed a model for including aberrations in the determination of the heterodyne efficiency. Salem and Rolland [25] discussed the heterodyne efficiency as a function of the misalignment angle. Furthermore, Ren *et al.* [26] studied heterodyne efficiency in the presence of atmospheric turbulence. Yang *et al.* [27] investigated the heterodyne efficiency for coherent optical communication systems under the effects of polarization aberrations.

In this work, a method is presented for modeling the heterodyne efficiency of array detector systems in the presence of target speckle. We use the parameter k to simulate the coherent summation processing. The heterodyne efficiency is determined for an interfering MC speckle field and a planar LO field on an equidistantly divided photodetector surface. Consequently, the effectiveness of array detectors in improving the SNR of coherent optical systems in the presence of target speckle is determined. Based on the simulation results, it is proved that using an array detector method can effectively improve the SNR of heterodyne systems. Under low SNR conditions, the heterodyne efficiency of $N \times N$ array detector systems increases linearly with N.

2. Monte Carlo Model for Speckle Fields Interacting With Heterodyne Systems

In many engineering applications, optically rough surfaces—whose surface roughness is large compared to the laser wavelength—is inevitable. When a coherent laser beam is incident on an optically rough surface, the wave-front of the laser beam modulated by the surface will produce speckle and reduce the system sensitivity. In a heterodyne system, the intermediate frequency (IF) signals generated by different parts of the photodetector cancel each other out due to the random phase distribution of the speckle field, possibly resulting to decoherence effects [8].

The phase distribution of the speckle field can be expressed as a Gaussian random field with a zero mean value. The statistical parameters describing a speckle field are its correlation length and the RMS of its phase distribution. The phase distribution of the speckle field is defined by the function $\varphi(r)$:

$$\langle \phi(r) \rangle = 0, \tag{1}$$

$$\langle \phi(r)\phi(r')\rangle = \delta^2 e^{-|r-r'|^2/c^2},\tag{2}$$

where the angular brackets $\langle \cdot \rangle$ denote the statistical average, the parameter delta is the root mean square value of the phase distribution, and cl is the correlation length in the horizontal direction.

For modeling the speckle field on the photodetector surface, we set the laser wavelength to 1 μ m, the side length of the detector to 0.4 mm, and the size of the speckle field on the photodetector surface to 400 λ × 400 λ . The sampling rate was set to 3 points per wavelength so that the speckle field can be analytically expressed as a matrix of 1200 × 1200. Results of simulated speckle fields with various delta and cl obtained by the heterodyne system are shown in Fig. 1. The speckle fields in a single row were simulated using identical RMS values but different correlation lengths and share a common color bar. For identical RMS values, the smaller correlation length corresponds to more frequent phase changes. Speckle fields in a single column were simulated using identical



Fig. 1. Monte Carlo speckle models with various delta and cl, the parameter delta is the root mean square value of the phase distribution, and cl is the correlation length. The speckle models in the same row have identical RMS values but different correlation lengths. The speckle models in the same column have identical correlation lengths but different RMS values.

correlation lengths but different RMS values. Greater RMS values correspond to the greater phase distribution of the speckle fields.

3. Modeling Heterodyne Efficiency for the Array Detector System

Heterodyne efficiency is an important indicator of the performance of coherent optical systems. The heterodyne efficiency η_{μ} of coherent optical systems is described as [28]:

$$\eta_{\rm IF} = \frac{\left\{ \int_{A} |E_{S}| |E_{L}| \cos[\phi(r)] dA \right\}^{2} + \left\{ \int_{A} |E_{S}| |E_{L}| \sin[\phi(r)] dA \right\}^{2}}{\left(\int_{A} |E_{S}|^{2} dA \int_{A} |E_{L}|^{2} dA \right)},\tag{3}$$

where IF is the intermediate frequency, *A* is the area of the photodetector, E_S and E_L are the amplitudes of the signal and LO fields, respectively, $\varphi(r)$ is the phase mismatch between the two beams which is associated with the spatial position. Based on the assumption that the distributions of E_S and E_L are uniform, we have:

$$\eta_{\rm IF} = \frac{\left\{ \int_{A} \cos\left[\phi(r)\right] dA \right\}^{2} + \left\{ \int_{A} \sin[\phi(r)] dA \right\}^{2}}{A^{2}} \tag{4}$$

In array detector systems, a phase shift $\Psi(i, j)$ is attributed to the useful signal detected by the array element in the *i*th row and *j*th column. In addition, if the following condition is ensured:

$$\phi(r) + \psi(i, j) = \text{constant},\tag{5}$$

then, the heterodyne efficiency $\eta_{\rm F}$ can be rewritten as:

$$\eta_{\rm IF} = \frac{\left\{\int_A \cos[\phi(r) + \psi(i, j)]dA\right\}^2 + \left\{\int_A \sin[\phi(r) + \psi(i, j)]dA\right\}^2}{A^2} = 1$$
(6)

Consequently, the phase mismatch between the LO and signal beams are corrected and the useful IF signal of every array element can be considered in-phase. As given by Eq. 3, the heterodyne efficiency is related to the spatial phase difference between the signal and LO beams. Hence, the magnitude of this mismatch is related to the statistical parameters of the speckle field.

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Fig. 2. Example of a 3 \times 3 array detector system in the presence of a speckle field.

The speckle field model described in Section 2 can be expressed as:

$$\phi = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,1200} \\ \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,1200} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1200,1} & \phi_{1200,2} & \cdots & \phi_{1200,1200} \end{bmatrix}$$
(7)

An example of the model of a 3 × 3 array detector system in the presence of the target speckle is shown in Fig. 2. Fig. 2(a) shows the speckle field on the 3 × 3 array photodetector surface. The speckle field is divided into 9 uniform parts. The speckle field $\varphi(i, j)$ recorded by the array element in the *t*th row and *f*th column can be obtained as shown in Fig. 2(b).

In order to describe the $\varphi(i, j)$ as an analytic expression, we define *a*, *b*, and *c* as follows:

$$\begin{cases} a = (i - 1) \times INT(1200/N) + 1 \\ b = (j - 1) \times INT(1200/N) + 1 , \ i, j = 1, 2, \dots, N \\ c = INT(1200/N) - 1 \end{cases}$$
(8)

where $INT(\cdot)$ denotes the integer-valued function.

For an $N \times N$ array heterodyne system, the speckle field received by the array element in the *t*th row and *j*th column can be described as:

$$\phi(i,j) = \begin{bmatrix} \phi_{a,b} & \phi_{a,b+1} & \cdots & \phi_{a,b+c} \\ \phi_{a+1,b} & \phi_{a+1,b+1} & \cdots & \phi_{a+1,b+c} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{a+c,b} & \phi_{a+c,b+1} & \cdots & \phi_{a+c,b+c} \end{bmatrix}$$
(9)

Under the premise that the LO beam is a planar wave, the phase fluctuations of the speckle field in Eq. 9 can be used to describe the phase mismatch between the LO and signal beams. By substituting Eq. 9 into Eq. 4, the heterodyne efficiency of the array element in the t^{th} row and j^{th} column can be written as:

$$\eta_{IF}(i,j) = \frac{\left[\sum_{g=a}^{a+c} \sum_{h=b}^{b+c} \left[\cos\left(\phi_{g,h}(i,j)\right)\right]\right]^{2} + \left[\sum_{g=a}^{a+c} \sum_{h=b}^{b+c} \left[\sin\left(\phi_{g,h}(i,j)\right)\right]\right]^{2}}{(c+1)^{2}}$$
(10)

The heterodyne signal can be described as a superposition of cosine function. Hence, in the time domain, there are phase differences between the heterodyne signals recorded by the individual array elements due to the spatial phase mismatches. After summing the outputs from every element without any signal correction, array detectors exhibit the same performance as single detectors, i.e., no significant SNR improvement in heterodyne detection is achieved. The method of improving the SNR in heterodyne detection by adjusting the phase of the IF signal recorded by each individual



Fig. 3. Scheme of heterodyne efficiency sequence, where N = 3, i = 1, j = 3, k is the grouping index for speckle fields.

array element is referred to as coherent summation processing. In Ref. [16], a novel heterodyne system with two on-off controllers was designed. Consequently, the mismatch magnitude of the field distributions of the signal and LO beams could be calculated by relatively simple calculations. However, coherent summation processing cannot be applied to this novel system in our simulations. Accordingly, we introduce the parameter k to Eq. 10 to simulate the coherent summation processing. Consequently, we have:

$$\eta_{\rm IF}(i,j) = \frac{\max_{t=1,2,\dots,k} \left\{ \left[\sum_{g=a}^{a+c} \sum_{h=b}^{b+c} \left[\cos\left(\phi_{g,h}(i,j) + \frac{2\pi t}{k}\right) \right] \right]^2 + \left[\sum_{g=a}^{a+c} \sum_{h=b}^{b+c} \left[\sin\left(\phi_{g,h}(i,j) + \frac{2\pi t}{k}\right) \right] \right]^2 \right\}}{(c+1)^2}$$
(11)

where parameter *k* is the grouping index for speckle fields.

In Fig. 3, taking a 3 × 3 array detector system as an example, we get a group of speckle fields by using the parameter *k* to perform equal-interval sampling in $[0, 2\pi]$, similarly to an analog-to-digital converter (ADC) in the field of signal processing. The total number of the group of speckle fields is *k*. The phase difference between two adjacent speckle fields is $2\pi/k$. The heterodyne efficiency sequence $\{\eta_{IF}(1, 3, 1), \eta_{IF}(1, 3, 2), \eta_{IF}(1, 3, 3), \ldots, \eta_{IF}(1, 3, k)\}$ of the first row and the third column element in the 3 × 3 array detector is obtained by calculating the values of the heterodyne efficiency in the corresponding positions of the group of speckle fields. If *k* is large enough, the maximum value $\eta_{IF}(1, 3)$ in this heterodyne efficiency sequence can be considered as the maximum heterodyne efficiency of the array element in the first row and the third column in the 3 × 3 array detector.

By using Eq. 11, the maximum value of the heterodyne efficiency for each array element can be calculated. The accuracy of the heterodyne efficiency calculated by Eq. 11 is related to the value of the parameter k, while the maximum phase sampling error is equal to π/k . The determination of the parameter k will be discussed in Section 4.

By summing the maximum heterodyne efficiency values of every array element, the coherent summation processing is carried out. Meanwhile, the IF signal of each array element is an in-phase output in the time domain. The heterodyne efficiency of an $N \times N$ array heterodyne system can be expressed as:

$$\eta_{\rm IF} =$$

$$\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{\max_{t=1,2,\dots,k}\left\{\left[\sum_{g=a}^{a+c}\sum_{h=b}^{b+c}\left[\cos\left(\phi_{g,h}(i,j)+\frac{2\pi t}{k}\right)\right]\right]^{2}+\left[\sum_{g=a}^{a+c}\sum_{h=b}^{b+c}\left[\sin\left(\phi_{g,h}(i,j)+\frac{2\pi t}{k}\right)\right]\right]^{2}\right\}\right\}}{N^{2}(c+1)^{2}}.$$
(12)

After the processing described above, we can model the heterodyne efficiency of array detector systems in the presence of target speckle. Our method consists of three steps, as shown in Fig. 4.



Fig. 4. Process diagram of modeling the heterodyne efficiency for array detector systems.

The first step is utilizing a Monte Carlo method to generate speckle field models on the photodetector surface with various *delta* and *cl* values. The second step is to obtain the simulated speckle field at each array element by uniformly dividing the speckle field obtained in the first step according to Eqs. 7–9. Finally, the heterodyne efficiency of the array heterodyne system is approximately calculated by applying coherent summation processing according to Eqs. 11 and 12.

4. Numerical Results

In a previous study [11], the expression for the ratio of the SNR of the $N \times N$ array detector system to the SNR of a single detector system was written as:

$$\frac{\mathrm{SNR}_{N\times N}}{\mathrm{SNR}_{\mathrm{single}}} = N \left(\frac{D'_r}{D_r}\right)^2 \frac{1+\Omega^2}{1+\left(D'_r/D_r\right)^2 \Omega^2},\tag{13}$$

where D'_r is the diameter of the detector array element projected onto the receiver aperture plane and D_r is the receiver diameter. The parameter Ω is the coherence loss due to speckle. The quantities of the parameter Ω are related to the statistical parameters of the speckle field. Therefore, the ratio of the SNR of an $N \times N$ array heterodyne system to the SNR of a single detector heterodyne system is a linear function for certain parameters of the speckle field. Furthermore, the slope of the linear dependency is given by the statistical parameters of the speckle field.

During the calculations, the parameter *k* was set to effectively simulate the coherent summation processing. Consequently, the value of *k* is related to the accuracy of the simulation results. If *k* is large enough, the heterodyne efficiency of the array detector system calculated by Eq. 12 can be optimized. However, increasing the value of *k* results in longer computing time. Fig. 5 shows some typical results of the heterodyne efficiency of an $N \times N$ array system with various *k* values. The curves with the same symbol represent the calculation results for same statistical parameter value. The curves of the different colors represent the calculation results for various *k* values. The heterodyne-efficiency curves for k = 0 in Fig. 5 are the results calculated without applying coherent summation processing. It is evident that in this case the heterodyne efficiency is not significantly improved. Consistent with the previous analysis, the simulated results of the heterodyne efficiency values without applying coherent summation processing are incorrect. In addition, the values of the heterodyne efficiency shown in Fig. 5 coincide at k = 10 and 20 under the identical statistical parameters of speckle field.

Fig. 6 show the saturation of the slope increased with parameter k. The red curve in Fig. 6 is the nonlinear fitting result of discrete points. When k is greater than or equal to 10, changing the value of k has negligible influence on the heterodyne efficiency. According to the numerical results in Fig. 5 and the nonlinear fitting result in Fig. 6, it is appropriate to set the value of k to 10 for further analysis.

Fig. 7 shows the variation of the heterodyne efficiency η of $N \times N$ array systems as a function of *N*. The curves of different colors represent the results obtained from speckle fields on the photodetector surface with different correlation lengths values. The curves of different symbols represent the results obtained from speckle fields on the photodetector surface with different RMS values. The statistical parameters of the speckle fields have a significant impact on the heterodyne



Fig. 5. Schematic view of the effect of the value of k on the heterodyne efficiency curves. The curves with the same symbol represent the calculation results for same statistical parameter value. The curves of the different colors represent the calculation results for various k values.



Fig. 6. Variations of the slope of heterodyne efficiency curves with parameter *k*. Where delta = 3 rad, $cl = 20\lambda$.

efficiency: larger RMS values and smaller correlation lengths correspond to smaller heterodyne efficiencies.

As shown in Fig. 7, with N = 1 the heterodyne system behaves as a single-detector coherent system. When *delta* \geq 7 rad and *cl* \leq 40 λ (this condition is satisfied by almost every speckle field likely to be encountered in engineering applications), decoherence significantly affects the performance of single-detector coherent systems. The explanation for this observation lays in the phase fluctuation of the speckle field being a uniform distribution in $[-\pi,\pi]$. Consequently, the IF signals generated by different parts of the photodetector cancel each other out due to the random phase distribution of the speckle field. If the number of elementary contributions is large, the mean value of the heterodyne signal is approximately zero based on the central limit theorem.

In general, as shown in Fig. 7, the array detector method can effectively improve the SNR of heterodyne systems in the presence of target speckle. The number of independent scatterers obtained by the detector element is reduced with the decreasing area of the detector. Hence, the



Fig. 7. Variations of the heterodyne efficiency η of $N \times N$ array heterodyne systems as a function of N. The curves of different colors represent the results obtained from speckle fields on the photodetector surface with different correlation lengths values. The curves of different symbols represent the results obtained from speckle fields on the photodetector surface with different RMS values.

heterodyne efficiency increases with the increase in N. For low SNR values of the heterodyne system, the heterodyne efficiency increases linearly with the increase of the number of the detector elements N along a single side of the detector. In addition, the slope of the linear relationship between the number of detector elements and the heterodyne efficiency is related to the statistical parameters of the speckle field. By using this linearity relationship, one can estimate the trend of the heterodyne efficiency curve at low SNR values through two experimental points. This phenomenon could be a useful tool for designing array detector systems in practical applications, such as the device selection and the evaluation of the SNR of the array detector heterodyne system.

However, the linear relationship between the heterodyne efficiency and *N* is not maintained as the heterodyne efficiency approaches 1. For high SNR values of the heterodyne system, such as the curves which *delta* \leq 3 rad and *cl* \geq 40 λ in Fig. 7, the linear relationships can be maintained at the beginning of the heterodyne efficiency curves. With large enough *N* values the increase of the heterodyne efficiency slows down and the efficiency gradually approaches 1. Conversely, when the heterodyne detection efficiency approximately equals 1, the phase distribution over the plane of the detector element can be considered uniform and the decrease of the detector element size has little effect on the heterodyne efficiency.

Our method proved to be a valuable tool for analyzing the SNR of array detector systems which could be applied for designing coherent array detector systems in engineering applications. In addition, by changing the phase distribution of the signal field in Eq. 12, one can utilize the presented simulation framework to study the effectiveness of array detectors for improving the SNR of heterodyne systems in the presence of other factors, such as turbulence, misalignment angle, and aberrations.

5. Conclusion

Heterodyne efficiency represents the matching state of the LO and the signal beams on the photodetector surface. Consequently, the heterodyne efficiency can be used to evaluate the performance of coherent optical systems. A common method found in the literature for modeling the heterodyne efficiency of array detector systems in the presence of target speckle was expanded by the introduction of the parameter k to simulate the coherent summation processing. Based on the simulation results, it was proven that the array detector method can effectively improve the SNR of heterodyne systems in presence of target speckle. Under low SNR conditions, the heterodyne efficiency of $N \times N$ array detector systems increases linearly with N.

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