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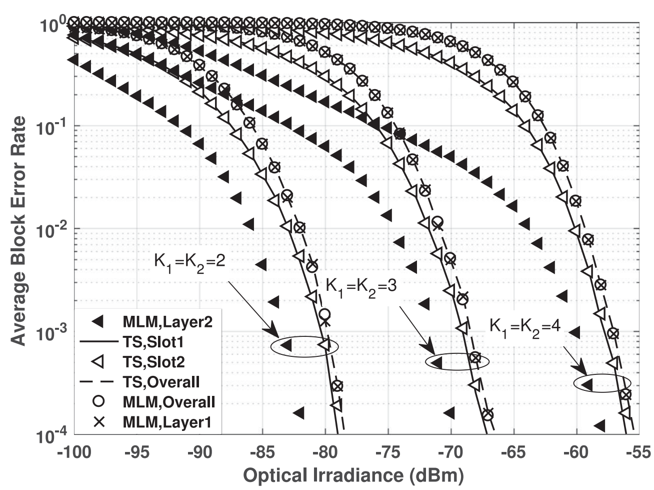

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Abstract: A multilayer modulation for peak-limited broadcast Poisson channel is designed and consists of three key elements: the transmitted symbol set, the users' data space, and the data-to-symbol mapping. For energy-efficiency, the transmitted symbols are modulated by the square pulse amplitude modulation. Meanwhile, a concept of uniquely decomposable constellation group is developed to resist the nonnegative multiuser interference in power domain. Combining this concept and the transmitted symbol structure, we design the users' data space and the data-to-symbol mapping such that, for this mapping, the transmitted symbol can be uniquely decomposed as elements of users' data space. The above described structure of the multilayer modulation enables not only efficient demodulation of the transmitted symbol from Poisson noisy received signal but also data decoding for each user from the estimation of the transmitted symbol. Extensive computer simulations indicate that the multilayer modulation not only has better overall error performance than the currently available time-sharing protocol, but also allows us to flexibly assign different priority. Additionally, a low-complexity receiver is established for the proposed multilayer modulation, whose error performance gradually approximates that of maximum likelihood receiver with the increment of optical power.

Index Terms: Optical wireless communications, discrete-time Poisson channels, broadcast channels, multi-layer modulation and uniquely decomposable constellation group.

1. Introduction

As a kind of standard model for optical communication, Poisson channel exists in various systems such as optical fiber systems [1], [2], free-space optical wireless communication systems [3], [4] and visible light communication systems [5], [6] etc. In particular, recently discrete-time Poisson (DTP) channel has drawn much attention due to the increasing researches on the intensity modulation direct detection (IM/DD) optical wireless communication (OWC) systems equipped with photomultiplier tubes (PMTs) or single photon avalanche diodes (SPADs) [7]–[9]. Like systems over Gaussian channel, these systems also meet the challenge for multiuser case [10], [11], for example, the download links from a light emitting diode (LED) to mobile stations equipped with PMT or SPAD naturally form a multiuser broadcast system. In light of the limited dynamic range of LEDs and SPAD's superior sensitivity over PMT, we consider a peak-limited broadcast SPAD OWC system in this paper.

Historically, the studies of multiuser DTP channels are limited [12]. On the one hand, those multi-access methods developed by information theoretical investigations for continuous-time Poisson (CTP) channels [13]–[15] can not be directly applied to multiuser DTP channels due to the capacity difference between these two kinds of Poisson channels [16], [17]. Meanwhile, so far the systematic design of multiuser DTP channels is a challenging task, since the channel capacity of DTP is still unsolved [18]. On the other hand, for information theoretical investigation, a main task is to manage the signal input distribution by coding such that the capacity is achieved [19]. While, for modern wireless communications, signal design and code design are equally important. However, for multiuser DTP channels, all of the existing researches concentrate on information theoretical investigation, partially because it is still an open problem to analyze the error performance of maximum likelihood (ML) detection in Poisson regime. The research status stimulates us to rethink the topic from a novel aspect. More recently, a square-root type transform was proposed to convert the DTP channel into a well-studied Gaussian channel [20], [21]. Based on a similar principle, the authors in [22]–[24] established Anscombe root (AR) receiver and further developed a Hellinger distance criterion for one-dimensional and multi-dimensional signal design. This inspires us to design a signal for multi-user case with the aid of the Hellinger distance criterion such that the multiuser interference is well managed.

For modern multiuser wireless communications, the treatment of multiuser interference has long been the central issue. To deal with this, as revealed in [25], the essential task for the signal design is to find a mapping scheme from the users' data space to finite-alphabet transmitted symbols such that users can recover their own data without mutual interference. At this point, the time-sharing protocol [26], as a common approach to communicating over a broadcast channel, can be considered to give such mapping. This method assigns exclusive time slots to users for their individual data transmission but implies time delay for awaiting the assigned time slot. Enlightened by this, we aim to design a multi-user signal for Poisson regime which enables data parallel transmission in time and frequency domain via allocating nonorthogonal resource to users in power domain. In [25], the authors devised a kind of multi-layer modulation over ideal AWGN VLC broadcast channels with the aid of a concept, additively uniquely decomposable constellation group (AUDCG), which confirms the unique identification of all users' data. However, due to the significant difference between AWGN channel and DTP channel, not only the concept of AUDCG is unsuitable for the considered channel, but also the multi-layer modulation designed in [25] is not energy-efficient any longer over this channel.

The aforementioned factors indeed motivate us to design an energy-efficient multi-layer modulation over multiuser broadcast DTP channels from an error performance perspective. Our main contributions can be summarized as follows: 1) We propose a concept of uniquely decomposable constellation group (UDCG), an more general extension of AUDCG, to serve as a guideline for managing multiuser interference over DTP channels; 2) With the aid of UDCG, we devise a group of users data set and its corresponding data-to-symbol mapping such that the transmitted symbol of the proposed multi-layer modulation can be uniquely decomposed to the data intended for all users; 3) By extending the scalar AR receiver, we provide a fast AR receiver to act as a demodulator and decoder for the proposed multi-layer constellation.

2. Channel Model and UDCG

Let us consider an OWC system with one transmitter LED and N users equipped with an SPAD respectively. At the transmitter, the data of N users are mapped to a symbol s by $s = f(x_1, \ldots, x_m, \ldots, x_N)$ where $s \in \mathcal{S}$, x_m is the data in mth layer intended for user n and belongs to a 2^K size set $\mathcal{X}_m$, f represents the mapping from users' data space to the transmitted signal space, i.e., $f : \mathcal{X}_1 \times \mathcal{X}_2 \cdots \times \mathcal{X}_N \mapsto \mathcal{S}$, where $\mathcal{X}_1 \times \mathcal{X}_2 \cdots \times \mathcal{X}_N = \{(x_1 \ldots, x_m \ldots, x_N) : x_m \in \mathcal{X}_m, 1 \leq m \leq N\}$ and $\mathcal{S} = \{s = f(x_1 \ldots, x_m \ldots, x_N) : x_m \in \mathcal{X}_m, 1 \leq m \leq N\}$. After transmitted by LED, s is received by user n and the received signal, y_n, is expressed as

$$y_n = \alpha s + \eta + p_n, \tag{1}$$

where α is the power gain caused by SPAD, η is the expected number of dark count in a symbol interval and p_n denotes the shot noise which makes signal y_n identically independently Poisson-distributed for $1 \le n \le N$ with mean $\lambda = \alpha s + \eta$, therefore, the probability density functions (PDFs) of y_n and p_n are respectively given by

$$\Pr(y_n = v) = \frac{\lambda^v}{v!} e^{-\lambda}, \Pr(p_n = \hat{v}) = \frac{\lambda^{(\hat{v}+\lambda)}}{(\hat{v}+\lambda)!} e^{-\lambda} \tag{2}$$

where v and $(\hat{v}+\lambda)$ are nonnegative integers. Based on these distributions, a maximum likelihood (ML) demodulation method for user n is given in what follows.

ML demodulation: The estimation of s, $\bar{s}$, is given by $\bar{s} = \arg\max_{s \in \mathcal{S}} \{v \log(\eta + \alpha s) - \alpha s\}$.

Though the transmitted symbol s can be estimated by this method, users can not decode their data properly unless there is one-to-one mapping between $\mathcal{X}_1 \times \mathcal{X}_2 \cdots \times \mathcal{X}_N$ and $\mathcal{S}$. For example, consider a 2 users system where $x_1, \hat{x}_1 \in \mathcal{X}_1, x_1 \ne \hat{x}_1$ and $x_2, \hat{x}_2 \in \mathcal{X}_2, x_2 \ne \hat{x}_2$. If $s = f(x_1, x_2) = f(\hat{x}_1, \hat{x}_2)$, users cannot uniquely identify whether (x_1, x_2) or $(\hat{x}_1, \hat{x}_2)$ even though they obtain exactly accurate estimation of s. Therefore, to avoid decoding error, symbol s should be uniquely divided as (x_1, x_2) and further we would like to give a more general definition to describe such character as follows.

Definition 1: $\mathcal{X}_1, \ldots, \mathcal{X}_m, \ldots, \mathcal{X}_N$ are the uniquely decomposable constellation group (UDCG) of $\mathcal{S}$ for given data-to-symbol mapping f, when $f(x_1 \ldots, x_m \ldots, x_N) = f(\hat{x}_1 \ldots, \hat{x}_m \ldots, \hat{x}_N)$ for any $x_m, \hat{x}_m \in \mathcal{X}_m$ holds if and only if $x_m = \hat{x}_m$ for $1 \le m \le N$.

The cardinality of the above mentioned sets respectively satisfy $|\mathcal{X}_m| = 2^{K_m}$ for $1 \le m \le N$ and $|S| = 2^{\sum_{m=1}^{N} K_m}$. Enlighten by the above analysis, we design the multi-layer modulation by meeting two demands that 1) the transmitted symbol is energy-efficient over our considered Poisson channel with nonnegative constraint of LED, saying, $s \in \mathcal{S}_e$ where $\mathcal{S}_e$ denotes the energy-efficient constellation designed for Poisson channel; 2) for given data-to-symbol mapping f, the users' data set group, $\mathcal{X}_1, \ldots, \mathcal{X}_m, \ldots, \mathcal{X}_N$, is the UDCG of $\mathcal{S}_e$.

3. Multi-Layer Modulation and Fast AR Receiver

Now, we formally state our main results in this paper.

3.1 Multi-Layer Modulation

Our first consideration is to make the proposed multi-layer modulation energy-efficient over Poisson channel by designing the constellation of transmitted symbol, which is equivalent to the one-dimensional constellation design problem over Poisson channel proposed in [22]. In [22], the energy-efficient constellation is constructed by maximizing the minimum Hellinger distance among all the distinct transmitted symbols with peak power limitation and turns out to be the square pulse amplitude modulation (SPAM) constellation. Specifically, $s \in \mathcal{S}_e = \{\frac{i^2}{(2^G-1)^2}\}_{i=0}^{i=2^G-1}$ with $G = \sum_{m=1}^{N} K_m$. Combining this conclusion with the concept of UDCG, we conclude the design problem of multi-layer modulation as follows.

Problem 1: Devise a group of users' data set, $\mathcal{X}_1, \ldots, \mathcal{X}_m, \ldots, \mathcal{X}_N$ with $|\mathcal{X}_m| = 2^{K_m}$ for $1 \le m \le N$, and corresponding data-to-symbol mapping $s = f(x_1 \ldots, x_m \ldots, x_N)$ such that $\{s : x_m \in \mathcal{X}_m, 1 \le m \le N\} = \mathcal{S}_e = \{\frac{i^2}{(2^G-1)^2}\}_{i=0}^{i=2^G-1}$ where $G = \sum_{m=1}^{N} K_m$. ■

To further reveal the necessity and the contribution for proposing UDCG, we would like to give the following observation related to AUDCG before providing a solution of this problem.

Lemma 1: The UDCG of $\mathcal{S}_e = \{\frac{i^2}{(2^G-1)^2}\}_{i=0}^{i=2^G-1}$ does not exist when data-to-symbol mapping is sum function, i.e., $s = f(x_1, \ldots, x_N) = \sum_{m=1}^{N} \frac{x_m}{(2^G-1)^2}$ for $x_m \in \mathcal{X}_m, 1 \le m \le N$.

Proof: Lemma 1 can be proved by contradiction. We assume that $\mathcal{X}_1, \ldots, \mathcal{X}_m, \ldots, \mathcal{X}_N$ consist of the UDCG of $\mathcal{S}_e$ for mapping $f(x_1, \ldots, x_m \ldots, x_N) = \sum_{m=1}^{N} \frac{x_m}{(2^G-1)^2}$, then, we have $s = \sum_{m=1}^{N} \frac{x_m}{(2^G-1)^2} \in \mathcal{S}_e$ for any $x_m \in \mathcal{X}_m, 1 \le m \le N$. Let $x_m, \hat{x}_m \in \mathcal{X}_m$ and $\hat{x}_m$ be the minimum number larger than x_m, thus, for $u_m \in \{x_m, \hat{x}_m\}$, we have

$$\sum_{m=1}^{N} \frac{x_m}{(2^G-1)^2} \leq \sum_{m=1}^{N} \frac{u_m}{(2^G-1)^2} \leq \sum_{m=1}^{N} \frac{\hat{x}_m}{(2^G-1)^2}. \quad (3)$$

For $s = \sum_{m=1}^{N} \frac{x_m}{(2^G-1)^2}$, let $s^{(j)}$ denote the result of the summation after replacing the jth addend, x_j, by $\hat{x}_j$, i.e., $s^{(j)} = \frac{x_1+\cdots+\hat{x}_j\cdots+x_m\cdots+x_N}{(2^G-1)^2}$. Likewise, for $\hat{s} = \sum_{m=1}^{N} \frac{\hat{x}_m}{(2^G-1)^2}$, we have $\hat{s}^{(j)} = \frac{\hat{x}_1+\cdots+x_j\cdots+\hat{x}_m\cdots+\hat{x}_N}{(2^G-1)^2}$. Combining this with (3), we have $s < s^{(j)} < \hat{s}$ and $s < \hat{s}^{(j)} < \hat{s}$. It can be verified by calculation that $(s - s^{(j)})^2 = (\hat{s} - \hat{s}^{(j)})^2 = [\frac{x_j}{(2^G-1)^2} - \frac{\hat{x}_j}{(2^G-1)^2}]^2$ for $1 \leq j \leq N$. Specifically, for $\jmath = \arg\min_{1\leq j\leq N} |x_j - \hat{x}_j|$, $s^{(\jmath)}$ is the minimum element in $\mathcal{S}_e$ lager than s, and $\hat{s}^{(\jmath)}$ is the maximum element in $\mathcal{S}_e$ smaller than $\hat{s}$. That is to say, if $s = \frac{l^2}{(2^G-1)^2}$ and $\hat{s} = \frac{h^2}{(2^G-1)^2}$ for $0 \leq l < h \leq 2^G - 1$, then $s^{(\jmath)} = (\frac{l+1}{2^G-1})^2$ and $\hat{s}^{(\jmath)} = (\frac{h-1}{2^G-1})^2$ further implying that $h - l > 2$. Finally we have $s^{(\jmath)} - s = \frac{2l+1}{(2^G-1)^2} < \frac{2h-1}{(2^G-1)^2} = \hat{s} - \hat{s}^{(\jmath)}$ which contradicts with the former conclusion we obtain, i.e., $s - s^{(\jmath)} = \hat{s} - \hat{s}^{(\jmath)}$. ■

This observation clarifies the limitation of the concept, AUDCG, over the considered Poisson channels. Therefore, in this paper, the data-to-symbol mapping and the UDCG of $\mathcal{S}_e$ are specially designed as follows:

Theorem 1: $\mathcal{X}_1 = \{i\}_{i=0}^{i=2^{K_1}-1}$ and $\mathcal{X}_m = \{i \times 2^{\sum_{j=1}^{m-1} K_j}\}_{i=0}^{i=2^{K_m}-1}$ for $2 \leq m \leq N$ are the UDCG of the constellation $\mathcal{S}_e = \{\frac{i^2}{(2^G-1)^2}\}_{i=0}^{i=2^G-1}$ for mapping $f(x_1, \ldots, x_N) = (\sum_{m=1}^{N} \frac{x_m}{2^G-1})^2$, saying, $\mathcal{S}_e = \{(\sum_{m=1}^{N} \frac{x_m}{2^G-1})^2 : x_m \in \mathcal{X}_m, 1 \leq m \leq N\} = \{\frac{i^2}{(2^G-1)^2}\}_{i=0}^{i=2^G-1}$.

Proof: To prove Theorem 1, let us first prove the operation $\sum_{m=1}^{N} x_m$ has $2^{\sum_{m=1}^{N} K_m}$ different results for $x_m \in \mathcal{X}_m, 1 \leq m \leq N$ by recurrence. 1) When $x_1 \in \mathcal{X}_1$ and $x_m = 0$ for $2 \leq m \leq N$, the operation $\sum_{m=1}^{N} x_m$ has 2^{K_1} different results and satisfies $0 \leq \sum_{m=1}^{N} x_m \leq 2^{K_1} - 1$; 2) Then, for $x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2$ and $x_m = 0$ with $3 \leq m \leq N$, we find $\sum_{m=1}^{N} x_m$ has $2^{K_1+K_2}$ different results and $0 \leq \sum_{m=1}^{N} x_m \leq 2^{K_1+K_2} - 1$; 3) Finally, by recurrence, for any $x_m \in \mathcal{X}_m$, we have $0 \leq \sum_{m=1}^{N} x_m \leq 2^{\sum_{m=1}^{N} K_m} - 1$, which proves that the operation $\sum_{m=1}^{N} x_m$ has $2^{\sum_{m=1}^{N} K_m}$ different results. Obviously, $(\sum_{m=1}^{N} \frac{x_m}{2^G-1})^2$ also has $2^{\sum_{m=1}^{N} K_m}$ different results for any $x_m \in \mathcal{X}_m, 1 \leq m \leq N$. Combine this with the fact that $(\sum_{m=1}^{N} \frac{x_m}{2^G-1})^2 \in \mathcal{S} = \{\frac{i^2}{(2^G-1)^2}\}_{i=0}^{i=2^G-1}$, we come to the desired conclusion and complete the proof of Theorem 1. ■

So far the design of multi-layer modulation is completed. It should be noticed that the proposed multi-layer modulation is constructed by mapping different data layers to sub-constellations with non-equal power. This feature enables us to allocate power flexibly according to different requirement. To further explain this, an example for $N = 2$ is provided in what follows.

Example 1:

1) *Equal rate with non-equal power:* When $N = 2$ and $K_1 = K_2 = 1$, the sub-constellations corresponding to layer1 and layer2 are $\mathcal{X}_1 = \{0, 1\}$ and $\mathcal{X}_2 = \{0, 2\}$ respectively, then, the constellation of multi-layer modulation is $\mathcal{S} = \{0, 1/9, 4/9, 1\}$.
2) *Low rate with more power:* For $N = 2$, the case, $K_1 = 2$ and $K_2 = 1$, means we allocate more power to the user with low data rate. For this strategy, the constellation is $\mathcal{S} = \{i^2/49\}_{i=0}^{i=7}$ with $\mathcal{X}_1 = \{0, 1, 2, 3\}$ and $\mathcal{X}_2 = \{0, 4\}$.
3) *High rate with more power:* For $N = 2$, the case, $K_1 = 1$ and $K_2 = 2$, means we allocate more power to the user with high data rate. For this strategy, the constellation is $\mathcal{S} = \{i^2/49\}_{i=0}^{i=7}$ with $\mathcal{X}_1 = \{0, 1\}$ and $\mathcal{X}_2 = \{0, 2, 4, 6\}$.

3.2 Low-Complexity Receiver

In this subsection, we devise a fast AR receiver to serve as a demodulator and decoder for the proposed multi-layer modulation based on the threshold-equally-spaced AR receiver proposed in [22]. For a better understanding, let us first recall the property of AR transform developed in [27].

Property 1: Let $y_n^{(AR)}$ denote the result obtained by applying AR transform on the signal received by user n, y_n, which is defined by (1). Then we have $y_n^{(AR)} = 2\sqrt{y_n + 3/8}$ and, when $\alpha s + \eta \gg 1/16$, $y_n^{(AR)}$ can be approximated by Gaussian variance, saying,

$$y_n^{(AR)} \approx \breve{s} + z_n \tag{4}$$

where $\breve{s} = 2\sqrt{\alpha s + \beta}$ with $\beta = \eta + 3/8$ and z_n is Gaussian noise with zero mean and variance being one.

Now let $\breve{\mathcal{S}}$ denote the constellation of $\breve{s}$ defined in (4) and P denote the peak optical power. As revealed by [22], when $\alpha s \gg \beta$ for $s \in \mathcal{S}_e = \{\frac{Pi^2}{(2^G-1)^2}\}_{i=0}^{i=2^G-1}$, the points in $\breve{\mathcal{S}}$ for $i \geq 1$ are approximately evenly distributed with the increment of P. Therefore, we further approximate $y_n^{(AR)}$ by

$$y_n^{(AR)} \approx \tilde{s} + z_n \tag{5}$$

where $\tilde{s} \in \tilde{\mathcal{S}} = \{\tilde{s}_i\}_{i=0}^{i=2^G-1}$ and $\tilde{\mathcal{S}}$ satisfies

$$\begin{cases} \tilde{s}_0 = 2\sqrt{\beta}, \tilde{s}_1 = 2\sqrt{\frac{P}{(2^G-1)^2} + \beta}, \tilde{s}_{2^G-1} = 2\sqrt{P + \beta} \\ \tilde{s}_i - \tilde{s}_{i-2} = 2\tilde{s}_{i-1}, \text{ for } 3 \leq i \leq 2^G - 1 \end{cases} \tag{6}$$

which makes $\tilde{\mathcal{S}}$ approximate $\breve{\mathcal{S}}$ in high power regime and meets the demand of peak power constrain. Solutions of (6) are $\tilde{s}_0 = 2\sqrt{\beta}$ and $\tilde{s}_i = \alpha'(i-1) + 2\sqrt{\frac{P}{(2^G-1)^2} + \beta}$ for $\alpha' = \frac{\sqrt{P+\beta} - \sqrt{P/(2^G-1)^2+\beta}}{2^{(G-1)}-1}$ respectively. Now, the transmitted signal can be estimated based on the Gaussian channel described by (5) and we propose the fast AR receiver as follows.

Fast AR receiver: User n can obtain the data in layer m from his received signal y_n by the following three steps:

1) AR transform: $y_n^{(AR)} = 2\sqrt{y_n + 3/8}$;
2) Demodulation: $\bar{s} = \begin{cases} 0, & y_n^{(AR)} \leq \frac{\tilde{s}_0+\tilde{s}_1}{2} \\ 1, & \frac{\tilde{s}_0+\tilde{s}_1}{2} < y_n^{(AR)} \leq \tilde{s}_1 \\ \lfloor \frac{y_n^{(AR)} + \alpha'/2}{\alpha'} \rfloor, & \tilde{s}_1 < y_n^{(AR)} \leq \tilde{s}_{2^G-1} \\ 2^G - 1, & y_n^{(AR)} > \tilde{s}_{2^G-1} \end{cases}$, where $\bar{s}$ is the estimation of the square root of transmitted signal;
3) Decode: let $\bar{x}_m$ for $1 \leq m \leq N$ denotes the estimation of the data in m layer, then $\bar{x}_1 = \bar{s} \bmod 2^{K_1}$ and $\bar{x}_m = 2^{\sum_{\ell=1}^{m-1} K_\ell}(((\bar{s} - \bar{s} \bmod 2^{\sum_{\ell=1}^{m-1} K_\ell})/2^{\sum_{\ell=1}^{m-1} K_\ell}) \bmod 2^{K_m})$ for $2 \leq m \leq N$.

4. Simulation Results

We carry out the computer simulations in the section with parameters $\alpha = 4.52 \times 10^{14}$ s/J and $\eta = 7.27$ [22].

4.1 Performance of Time-Sharing Protocol and UDCG

In this subsection, we carry out the error performance comparison between the time-sharing protocol and our proposed multi-layer modulation. The specific mathematic description of these two scheme are listed as follows:

1) **Time-sharing (TS) protocol**: The signal vector of this scheme is given by $\mathbf{s} = (s_1, \ldots, s_t, \ldots, s_N)$, where $s_t \in \{\frac{Pi^2}{(2^{NK_m}-1)^2}\}_{i=0}^{i=2^{NK_m}-1}$ with $1 \leq t \leq N$ and $1 \leq m \leq N$.
2) **Multi-layer modulation (MLM)**: The signal vector is given by $\mathbf{s} = (s_1, \ldots, s_t, \ldots, s_N)$, where $s_t \in \{\frac{Pi^2}{(2^G-1)^2}\}_{i=0}^{i=2^G-1}$ for $1 \leq t \leq N$ and $G = \sum_{m=1}^{N} K_m$.

We first consider the situation that the data intended for each user is transmitted at same bit rate. Fig. 1 displays the simulation results for two users system. As displayed by Fig. 1, for certain K, the overall block error rate of TS protocol and MLM are the same due to the coincidence of their geometrical structure. For the same reason the block error rate of slot1 is also identical to that

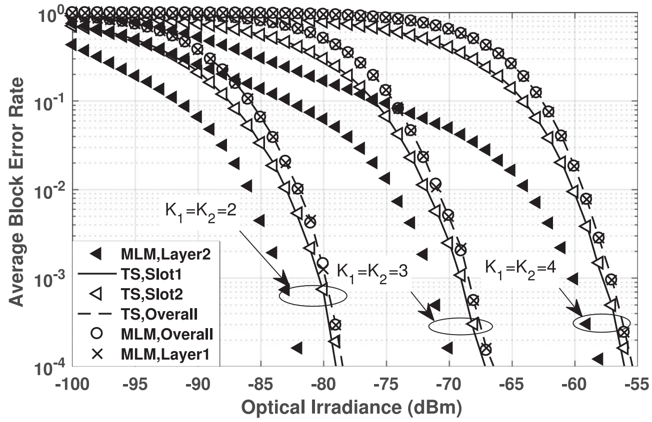

Fig. 1. Comparisons between MLM and TS protocol for $N = 2$ with same users' bit rate.

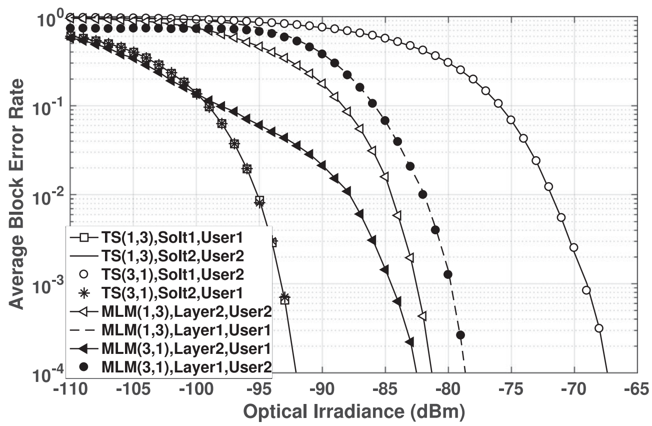

Fig. 2. Comparisons between MLM and TS protocol for $N = 2$ with different users' bit rate.

of slot2 but slightly lower than overall block error rate for TS protocol. Additionally, for MLM with certain K, since the transmitted constellation $\mathcal{S}_e$ and the subconstellation of layer1, $\mathcal{X}_1$, have the same minimum Hellinger distance, the overall block error rate curve coincides with that of layer1. Meanwhile, layer2 can perform better on data transmission than the other channels because more power is located to layer2. For example, when $K = 2, 3, 4$, layer2 offers 2, 3, 4 dB gains over layer1 respectively at the target error rate of 10^{-4}. This inspires us to arrange here the data with highest priority when users have same bit rate.

Then, we consider the situation that the data intended for each user is transmitted at different bit rate. For two users case, the data intended for user1 and user2 are transmitted at 1 bit/slot and 3 bits/slot respectively. In Fig. 2, the strategies that user1 and user2 are assigned layers and slots according to their identifier order are denoted by MLM(1,3) and TS(1,3) respectively, otherwise, they are denoted by MLM(3,1) and TS(3,1). For TS protocol, TS(1,3) and TS(3,1) provide same error performance for user1 and user2, since the users independently transmit data in their respective time slots and naturally the block error performance of slot1 and solt2 are only decided by the bit rate of their corresponding user. Such independence also causes a huge gap between the performance of user1 and user2. As depicted by Fig. 2, the power consumption of user1 suffers up to 20 dB increase compared with that of user2 at the target error rate of 10^{-4} indicating a shortcoming of TS protocol on overall error performance. To further reveal this, we compare the overall block error rate of TS and MLM for $N = 3$ with various bit rate. As illustrated by Fig. 3, the simulation results show that, when the resources (layers or slots) allocation strategies are (2,1,1), (2,2,1), (3,2,1) and (3,1,1) respectively, the corresponding power gains of UDCG over TS protocol are 11, 6, 17 and

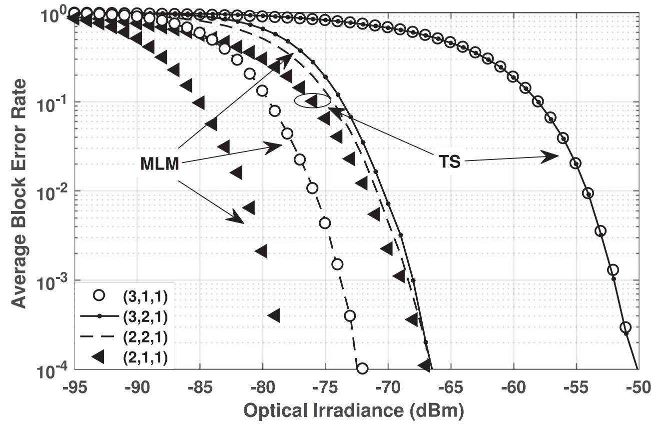

Fig. 3. Overall block error rate comparisons between MLM and TS protocol for $N = 3$ with different users' bit rate.

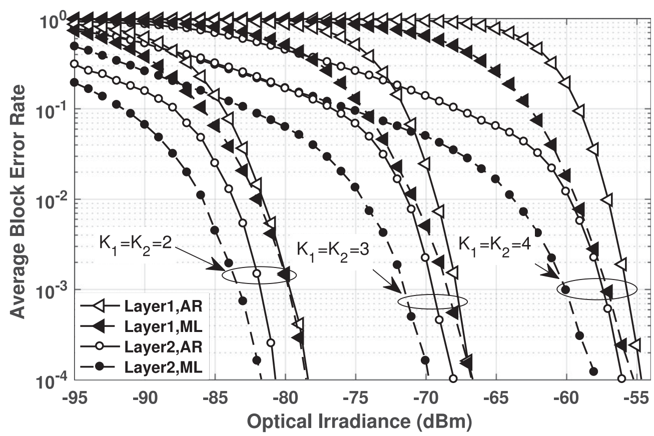

Fig. 4. Comparisons between fast AR receiver and ML receiver for $N = 2$ with same users' bit rate.

22 dB at the target rate of 10^{-4}. For the proposed MLM, Fig. 2 shows that, no matter which user occupies layer1, the average block error rate of this layer is unchanged. While, when transmitting data in layer2, user1, whose bit rate is lower than user2, obtains a better error performance than what user2 dose in the same way. This encourages us to allocate deeper layer to the user with lower bit rate when users' priority are the same.

4.2 Receivers Comparison

In this subsection, our simulation focuses on the performance comparison between ML receiver and AR receiver. We consider a two users system where the bit rate of these users are the same, specifically, $K = 2, 3, 4$. The average block error rate of these two receivers are illustrated by Fig. 4. It can be noticed that, with the increment of the optical power, the error performance curves of the two receivers gradually coincide, since the deviation caused by approximation (5) is become unnoticeable in high power regime. Specifically, compared with ML receiver, the respective power degradations caused by AR receiver for layer1 are 0.13, 0.08 and 0.53 dB for $K = 2, 3, 4$ at the target block error rate of 10^{-4}, while, for layer2, the power consumption gaps between the two receivers are 1, 1.75 and 1.9 dB respectively for $K = 2, 3, 4$ at the target block error rate of 10^{-4}. These results reveal the effectiveness of our proposed fast AR receiver.

5. Conclusions

In this paper, by developing and using the concept of UDCG, we designed a novel energy-efficient uniquely-decomposable multi-layer modulation constellation to manage the multiuser interference for broadcast system over Poisson channel. Firstly, energy-efficiency of the proposed multi-layer modulation is guaranteed by making the transmitted symbol square pulse amplitude modulated. Then, based on this structure, we proposed a mapping from the users' data space to the transmitted symbol set. For this mapping, the users data consist of a UDCG which is proved to be the unique decomposition of the multi-layer modulation. Finally, a fast AR receiver has been devised for users to obtain their data from the Poisson noisy multi-layer modulated signal. Comprehensive computer simulations have demonstrated that, compared with currently available time-sharing protocol, the multi-layer modulation can provide balanced error protection for all users, but this advantage comes at the cost that the different data layers are mapped to sub-constellations with non-equal power. Another important advantage is allowing us to flexibly assign different priority.

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