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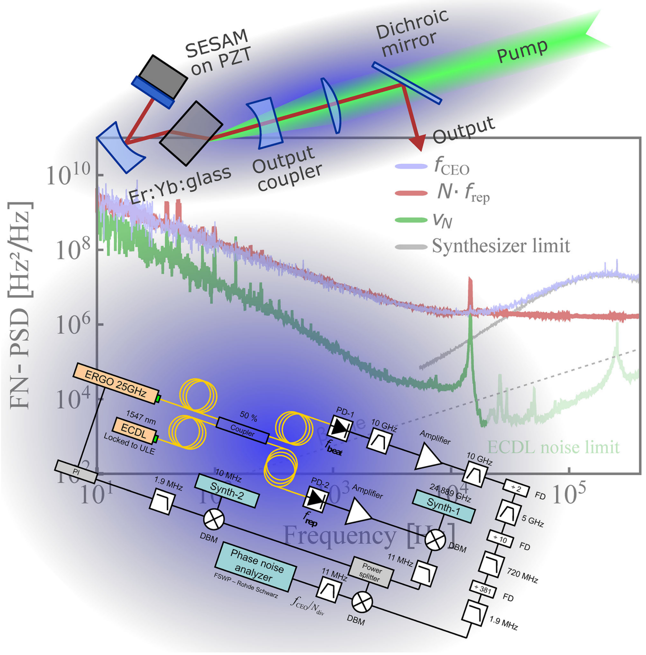

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Frequency Noise Characterization of a 25-GHz Diode-Pumped Mode-Locked Laser With Indirect Carrier-Envelope Offset Noise Assessment

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Abstract: We present a detailed frequency noise characterization of an ultrafast diode-pumped solid-state laser operating at 25-GHz repetition rate. The laser is based on the gain material Er:Yb:glass and operates at a wavelength of 1.55 μm. Using a beating measurement with an ultralow-noise continuous-wave laser in combination with a dedicated electrical scheme, we measured the frequency noise properties of an optical mode of the 25-GHz laser, of its repetition rate and indirectly of its carrier-envelope offset (CEO) signal without detecting the CEO frequency by the standard approach of nonlinear interferometry. We observed a strong anticorrelation between the frequency noise of the indirect CEO signal and of the repetition rate in our laser, leading to optical modes with a linewidth below 300 kHz in the free-running laser (at 100-ms integration time), much narrower than the individual contributions of the carrier envelope offset and repetition rate. We explain this behavior by the presence of a fixed point located close to the optical carrier in the laser spectrum for the dominant noise source.

Index Terms: Laser stability, mode-locked lasers, phase noise

1. Introduction

Mode-locked lasers with a high repetition rate are attractive for applications in optical telecommunications, such as ultra-high speed transmission systems up to 30 Tbits/s [1], optical clocking or multi-wavelength sources [2]. With a mode spacing of 12.5 GHz, 25 GHz, 50 GHz or 100 GHz in the C or L spectral band, a mode-locked laser can act as a single light source simultaneously covering a large number of channels in dense wavelength division multiplexing (DWDM) optical telecommunications. This can advantageously replace a large number of continuous-wave single-mode distributed feedback (DFB) lasers with their associated own drive electronics and temperature control, which each acts as an optical source for a single wavelength channel only. In addition, the high frequency spacing between the modes of an optical frequency comb with a high repetition rate can be beneficial to other applications such as ultralow-noise microwave generation [3], [4] or astronomical spectrographs calibration [5], [6]. However, a frequency comb requires the second degree of freedom of the mode-locked laser spectrum, i.e., the carrier-envelope offset (CEO) frequency f_{CEO}, to be stabilized or at least detected, which constitutes the fundamental difference between any mode-locked laser and a frequency comb [7].

Diode-pumped solid-state lasers (DPSSLs) constitute a proven technology for high repetition rate mode-locked lasers, with repetition rates up to 100 GHz achieved in fundamentally mode-locked operation [8]. They are also particularly suitable for optical frequency combs with a large mode spacing. The standard approach to detect the CEO beat based on supercontinuum spectrum generation in a non-linear fiber and f-to-$2f$ interferometry [9] is particularly challenging with high repetition rate lasers due to their low peak power and usually long pulse duration. For this reason, the highest repetition rates for a frequency comb produced from a mode-locked laser are 10 GHz demonstrated for a self-referenced Ti:Sapphire mode-locked laser [10] and 15 GHz reported more recently for an Yb:Y_2O_3 ceramic DPSSL locked to an ultra-stable laser [11]. However, this comb was not self-referenced and suffered from the frequency drift of the reference laser.

Even if not CEO-stabilized, a high repetition rate mode-locked laser can act as a regular grid of equispaced optical frequencies when its repetition rate is phase-locked to a stable radio-frequency (RF) reference source, which can be straightforwardly realized by controlling the cavity length using a piezo-electric transducer (PZT). Such stabilized laser could serve as a multi-wavelength light source for DWDM optical networks using multi-level quadrature-amplitude modulation (mQAM) with digital coherent receivers to address the needs for increased network capacity [12]. A critical constraint for such coherent detection systems is the phase noise of the laser source and the corresponding linewidth of the laser modes, which typically needs to be significantly smaller than 100 kHz in coherent transmission systems [13]. A naïve expectation from the stabilization of the repetition rate is that the frequency stability of the individual laser modes would be improved at the same time (i.e., their low-frequency noise would be reduced). Here, we present a study of the noise properties of a mode-locked DPSSL with 25-GHz repetition rate, notably obtained by beating the laser with a narrow-linewidth continuous-wave reference laser. We locked the repetition rate of the mode-locked laser to a stable radio-frequency reference source and show that it leads to a degradation of the phase noise of the analyzed optical line and to a corresponding increase of its linewidth. To understand this behavior, we separately characterized the phase noise of the optical line and of the repetition rate frequency of the laser, which is the controlled parameter in the considered stabilization scheme of our laser. As the optical spectrum of a mode-locked laser is fully determined by only two parameters, the mode spacing f_{rep} and the global frequency offset f_{CEO}, we furthermore implemented a dedicated scheme to characterize the noise properties of the second degree of freedom f_{CEO}, to have a complete picture. Hence, the two characteristic frequencies defining the optical modes of the DPSSL spectrum have been separately analyzed. The measurement of the phase noise of the CEO frequency involves an electrical scheme that circumvents the standard f-to-$2f$ interferometry method that cannot be implemented here due to the too low output power and too long pulses emitted by the laser for coherent octave-spanning supercontinuum spectrum generation. Nevertheless, we have been able to indirectly measure the frequency noise spectrum of the CEO signal without detecting f_{CEO}. This result constitutes to the best of our knowledge the highest repetition rate CEO noise measurement realized for a mode-locked laser. We observed a very similar and anti-correlated phase noise between f_{rep} and f_{CEO}, leading to a much lower phase noise of the optical mode. We explain this behavior using the elastic tape model of a frequency comb [14], [15] and the presence of a fixed point located close to the carrier frequency in the laser spectrum.

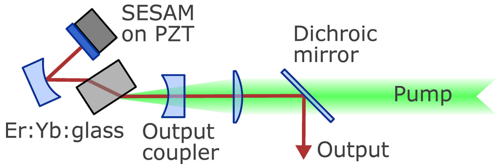


Fig. 1. Layout of the 25-GHz ERGO laser. The gain medium is an Er:Yb-doped glass plate and a quantum well SESAM is placed at the end of the cavity, mounted on a PZT for cavity length control.

2. Experimental Setup

We have used a picosecond Er:Yb:glass laser oscillator (ERGO), which is schematized Fig. 1. It is made of a V-shaped cavity, consisting of an output coupler with 0.5% transmission, a 1-mm thick Er:Yb-doped glass plate (QX/Er from Kigre) as gain medium, a folding high-reflectivity mirror, and a quantum-well-based semiconductor saturable absorber mirror (SESAM). The laser is based on the same design as previously described for a similar 10-GHz ERGO laser [16] and also used in a 12.5-GHz ERGO laser employed for some optical communications proof-of-principle demonstrations [1], [13]. The same components such as gain medium and SESAM are used, only the folding mirror and output coupler have a different radius of curvature that is shorter than 2 mm resulting into a compact standing-wave cavity with an optical length of only 6 mm between its end mirrors. All optics are glued on stainless steel parts. The laser is fundamentally mode-locked, meaning that a single pulse circulates within the cavity with 25-GHz pulse repetition rate of the output beam. The SESAM is mounted on a PZT for fine cavity length adjustment and stabilization. The laser is pumped by a single-mode fiber-pigtailed diode, emitting up to 600 mW at 976 nm (model 300076 from EM4), but only a fraction of up to ∼300 mW of pump power was used here. This is the same type of pump diode used at a similar power as in the 10-GHz ERGO laser reported in [16].The pump beam is coupled into the laser cavity through a dichroic mirror placed in the output beam, after the output coupler (see Fig. 1). The laser generates pulses with a duration of ∼3 ps at a center wavelength of 1547 nm. Passive fundamental SESAM-mode-locking enables ultralow pulse timing jitter and optical pulse-to-pulse phase coherence, which is essential for high data rate communication systems with coherent modulation formats [1]. As the 25-GHz ERGO laser is very similar to the previously reported versions with 10-GHz and 12.5-GHz repetition rates [1], [13], [16], which are built on the same mechanical platform and pumped by the same type of laser diode, similar noise properties are expected, just scaled up by the higher repetition rate. However, these noise properties have not been studied in detail before. In a basic heterodyne beat measurement made with a narrow-linewidth continuous-wave (CW) laser, a linewidth of an optical mode of the 12.5-GHz ERGO laser was claimed to be at the kHz level for an observation time of a few ms only [13]. However, this result was obtained by fitting the measured beat signal by a Lorentzian lineshape in a logarithmic scale, so that the 3-dB linewidth was not properly assessed and strongly underestimated in our opinion. Here, we present a detailed analysis of the noise properties of the 25-GHz mode-locked laser obtained by measuring the complete frequency noise power spectral density (FN-PSD) of an optical line, as well as of the individual contributions of the repetition rate and CEO frequency down to a Fourier frequency of 1 Hz. The FN-PSD is the most complete quantity to characterize the noise properties of a signal, and contains much more information than the power spectrum [17].

The experimental setup shown in Fig. 2 was implemented to beat one mode of the laser with an ultralow-noise continuous-wave reference laser to measure the frequency noise of one of its individual modes. Additionally, this setup enabled us to indirectly assess the frequency noise of the CEO signal of the ERGO laser without directly detecting the CEO beat by the traditional method of f-to-$2f$ interferometry. This was not possible with this laser as the required octave-spanning supercontinuum spectrum cannot be achieved with the present laser power and pulse duration. The general principle of the implemented approach was previously presented and validated using a self-referenced Er:fiber frequency comb [18]. We recently applied it to perform the first detailed investigation of the noise properties of the CEO frequency in a mode-locked semiconductor laser

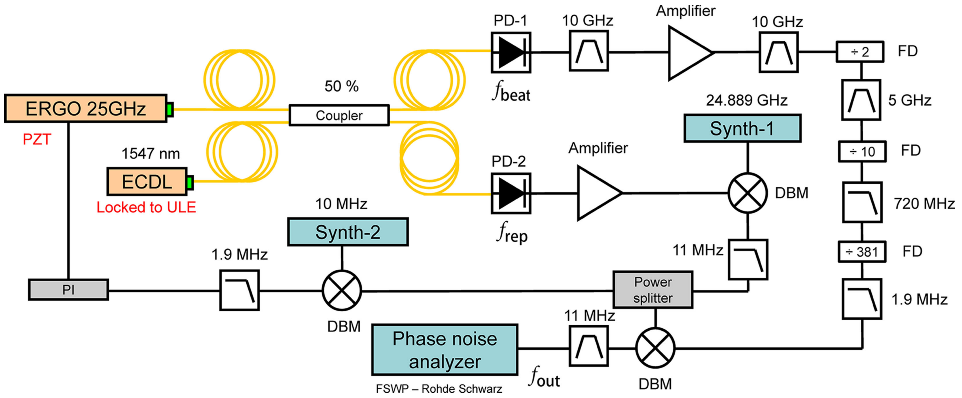
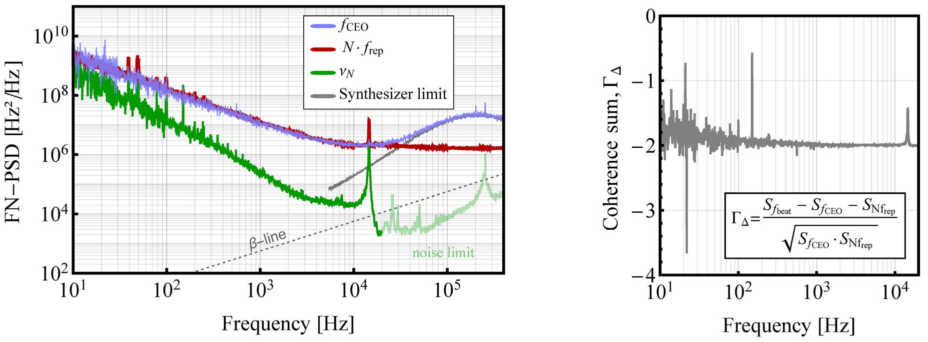


Fig. 2. Overall scheme of the experimental setup implemented to measure the frequency noise of the unknown free-running CEO signal of the 25-GHz ERGO laser and to lock its repetition rate to an external frequency reference. The CEO frequency noise is indirectly measured by separately detecting the heterodyne beat between one optical line of the ERGO laser and a narrow-linewidth external cavity diode laser (ECDL) at 1547 nm, filtered and frequency-divided (FD) by a large number $N_{div} = 7,620$ (upper branch of the scheme) and the repetition rate f_{rep}, down-converted to a low frequency of 10 MHz (lower branch of the scheme). The two signals are then combined (mixed) to remove the noise contribution of f_{rep}, such that the resulting signal f_{out} is representative of the noise of f_{CEO}. The down-converted repetition rate signal is also used for phase stabilization using a proportional-integral (PI) servo-controller and a feedback signal applied to a PZT in the laser cavity (bottom part of the scheme).

[19]. Here we have applied it in a similar way to the 25-GHz ERGO laser. In brief, the method consists of suppressing the contribution of the repetition rate in the frequency noise of an optical line $\nu_N = (N \cdot f_{rep} + f_{CEO})$ of the mode-locked laser, measured from the heterodyne beat with a narrow-linewidth laser. The repetition rate suppression was achieved using an electrical scheme (see Fig. 2) and not optically as with f-to-$2f$ interferometry. As a result, the frequency noise originating from f_{CEO} is the dominant noise contribution in the analyzed signal, which could be measured even if the CEO frequency itself remained unknown.

More specifically, we first detected the heterodyne beat signal $f_{beat} = \nu_{laser} - N \cdot f_{rep} - f_{CEO}$ between a CW reference laser of frequency ν_{laser} and one mode ν_N of the 25-GHz ERGO laser at $\sim$1547 nm, corresponding to a mode number $N \approx 7{,}760$. The reference laser was an external cavity diode laser (ECDL) stabilized to an ultralow thermal expansion (ULE) optical cavity using the Pound-Drever-Hall locking technique [20]. It has a negligible frequency noise (with a corresponding linewidth in a range of a few Hz) compared to the CEO signal to be analyzed. The 10-GHz heterodyne beat signal f_{beat} was detected using a fast photodiode (Discovery Semiconductors DSC40S, PD-1 in Fig. 2). It was subsequently band-pass filtered, amplified and frequency-divided by a large integer number $N_{div} = 7,620$ using three successive off-the-shelf frequency dividers with a division factor of 2, 10 and 381, respectively. The resulting frequency-divided signal $f_A = f_{beat}/N_{div} \approx 1.3\,\text{MHz}$ contained the frequency fluctuations Δf_{CEO} of the unknown CEO signal divided by the large number N_{div}, plus the frequency fluctuations Δf_{rep} of the repetition rate scaled by the factor $(N/N_{div}) \approx 1$, i.e., $\Delta f_A = (N/N_{div}) \cdot \Delta f_{rep} + \Delta f_{CEO}/N_{div}$. In this expression, the frequency fluctuations $\Delta\nu_{laser}$ of the reference laser have been omitted as they are negligible compared to the noise of the CEO signal. The noise of the repetition rate was removed by mixing the frequency-divided beat signal f_A with the repetition rate f_{rep} separately detected using another fast photodiode (New-Focus 1014-IR, PD-2 in Fig. 2). A perfect cancellation of the noise of f_{rep} was not achieved as the required condition $N_{div} = N$ was not exactly fulfilled in the setup. However, the residual noise contribution of f_{rep} was negligible as $N/N_{div} \approx 1$. Due to the large frequency difference occurring between the two signals $f_A \approx 1.3\,\text{MHz}$ and $f_{rep} = 24.899\,\text{GHz}$, the repetition rate was first frequency down-converted using a frequency synthesizer (Agilent E8257D, Synth-1 in Fig. 2) tuned at $f_{synth} \approx 24.889\,\text{GHz}$, prior to be mixed with the frequency-divided beat signal f_A in order to make possible the filtering of the proper signal at the mixer output. Alter-



(a) (b)

Fig. 3. (a) Frequency noise power spectral density (FN-PSD) measured for an optical mode ν_N (green), in comparison to the laser repetition rate scaled to the optical frequency ($N \cdot f_{\mathrm{rep}}$, red) and indirectly assessed for the CEO signal (light blue). At high Fourier frequency, the noise spectrum of f_{CEO} is limited by the 25-GHz synthesizer used for frequency down-conversion (grey curve), whereas the noise spectrum of the comb line is limited by the reference CW laser and some RF components used in the setup (displayed by the light green part of the curve). The noise peak at ~14 kHz visible in the spectra of $N \cdot f_{\mathrm{rep}}$ and ν_N arises from a spurious noise peak present in the output signal of the PID controller that was connected to the laser cavity PZT. (b) Frequency dependence of the sum of the complex coherences Γ_Δ between the frequency variations of f_{CEO} and f_{rep} in the free-running ERGO laser.

nately, a single-sideband mixer could be used to circumvent the use of Synth-1. The resulting signal $f_{\mathrm{out}} = f_A + (f_{\mathrm{rep}} - f_{\mathrm{synth}}) \approx (\nu_{\mathrm{laser}} - f_{\mathrm{CEO}})/N_{\mathrm{div}} - f_{\mathrm{synth}} \approx 11.3\,\mathrm{MHz}$ comprised the frequency noise of the free-running CEO signal, scaled down by the large number N_{div}, i.e., $\Delta f_{\mathrm{out}} \approx \Delta f_{\mathrm{CEO}}/N_{\mathrm{div}}$. This signal was analyzed using a phase noise analyzer (Rohde & Schwarz FSWP26).

To stabilize the repetition rate at ~25 GHz, the down-converted repetition rate signal ($f_{\mathrm{rep}} - f_{\mathrm{synth}}$) was phase-locked to a 10-MHz signal (Agilent 33250A, Synth-2 in Fig. 2) by applying feedback to the PZT holding the SESAM in the laser cavity. For this stabilization, both synthesizers Synth-1 and Synth-2 (at 24.899 GHz and 10 MHz, respectively) were referenced to an H-maser to ensure their long-term frequency stability.

3. Experimental Results

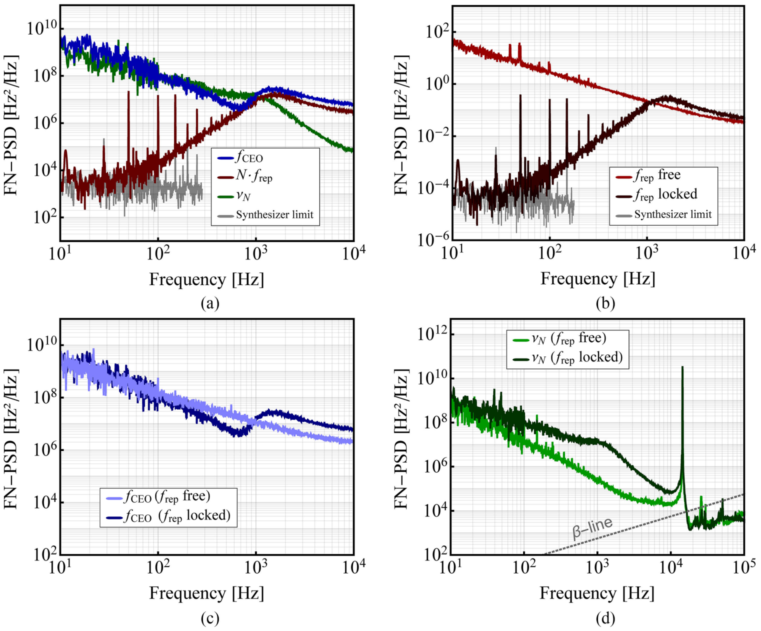
3.1. ERGO Laser Frequency Noise Characterization

The FN-PSD measured for an optical mode ν_N of the free-running ERGO laser (from the beat signal with the narrow-linewidth CW laser) is displayed in Fig. 3. For comparison, the frequency noise spectra separately measured for the repetition rate f_{rep} and indirectly assessed for the unknown CEO signal using our experimental scheme are also plotted on the same graph. A very similar noise spectrum is observed for the indirect CEO signal and for the repetition rate up-scaled to the optical frequency (i.e., $N \cdot f_{\mathrm{rep}}$), whereas the noise of the optical line is significantly lower. This indicates that the fluctuations of f_{rep} and f_{CEO} are anti-correlated and partially compensate each other in the optical line. This behavior indicates the existence of a fixed point [21], [22] in the vicinity of the optical carrier in the spectrum of our laser according to the elastic tape model of the frequency comb [14], [15]. This fixed point results from the dominant noise source in our laser, which is believed to be the amplitude noise of the pump laser. It has been previously shown that pump noise leads to a fixed point located at the carrier frequency in diode-pumped frequency combs [21] and we recently observed that the pump noise was also the main contribution in the frequency noise of the CEO beat in a 1-GHz DPSSL frequency comb [23]. However, the noise of an optical line of the comb was dominated in this case by mechanical noise in the laser cavity, which strongly affected the repetition rate.

At Fourier frequencies higher than ~30 kHz, the measured CEO FN-PSD is limited by the experimental noise floor arising from the synthesizer used to frequency down-convert the repetition rate signal (Synth-1 in the setup of Fig. 2). However, from the very similar noise observed at lower frequencies for $N \cdot f_{\mathrm{rep}}$ and f_{CEO} and its interpretation in terms of the laser fixed point, one can also expect a comparable correspondence at high frequency. Therefore, one can assume the presence of the same white frequency noise plateau in the FN-PSD of f_{CEO} as for $N \cdot f_{\mathrm{rep}}$, at a level of $\sim 2 \cdot 10^6\ \mathrm{Hz}^2/\mathrm{Hz}$. This assumption is justified by the flattening of the indirect CEO signal FN-PSD observed in the range of 10 to 30 kHz compared to its regular $1/f$ behavior occurring at lower frequencies. Furthermore, the FN-PSD of the optical beat is much lower than the measured noise of $N \cdot f_{\mathrm{rep}}$ in this frequency range, which shows that here also f_{CEO} and $N \cdot f_{\mathrm{rep}}$ have a similar noise level and are anti-correlated.

By considering the same white frequency noise as for $N \cdot f_{\mathrm{rep}}$, a CEO linewidth of $\sim$5 MHz is obtained at an integration time of 100 ms using the approximation of the β-separation line [24]. This value results to a large extent from the high white FN-PSD of f_{CEO} at Fourier frequencies above $\sim$30 kHz. This high white frequency noise is believed to result from the low optical power of the laser ($\sim$4 mW output power), leading to high quantum noise. Using typical parameters for our laser (0.5% output coupler, $\sim$0.5% parasitic losses leading to $\sim$1% gain per round trip, $\sim$3-ps pulse duration), the quantum noise level estimated from the theoretical formula introduced by Schlatter *et al.* [25] is on the same order of magnitude as the observed white noise level. However, this estimation is fairly imprecise due to the poor knowledge of some of the relevant laser parameters for this calculation. In contrast, the resulting linewidth estimated for the optical mode $\nu_{\mathrm{N}} = (N \cdot f_{\mathrm{rep}} + f_{\mathrm{CEO}})$ is only $\sim$270 kHz at 100-ms integration time, which is indeed significantly narrower than the estimated CEO linewidth of $\sim$5 MHz. This is also much smaller than the value of $\sim$10 MHz that would result from a completely uncorrelated noise of f_{CEO} and f_{rep}. The noise observed at Fourier frequencies higher than 20 kHz in the FN-PSD of the optical beat note has not been considered in the linewidth calculation, as it mainly arises from the CW laser locked to the cavity (especially its servo bump at $\sim$250 kHz) and from other RF components used in the measurement scheme, therefore it is not representative of the actual laser noise. The noise peak occurring at $\sim$14 kHz in the FN-PSD of f_{rep}, but not of f_{CEO}, does not originate directly from the mode-locked laser. We discovered afterwards that it arose from a parasitic noise peak occurring in the servo-controller used in the repetition rate stabilization loop. This technical noise peak was present even when f_{rep} was not stabilized, but with the PZT connected to the servo-controller. This technical noise peak was amplified in the optical beat as there is no compensation by the CEO signal. However, its contribution to the calculated linewidth of the optical line is small (it contributes only for 10 kHz in the linewidth calculated at 100-ms integration time).

The degree of correlation between the frequency noise of $N \cdot f_{\mathrm{rep}}$ and f_{CEO} was assessed from our measurements to further confirm our previous statement about the laser fixed point. It was obtained by calculating the sum of the complex coherence Γ_{Δ} between the frequency variations of the indirect CEO signal and repetition rate in the free-running ERGO laser from the measured FN-PSD of ν_{N}, f_{rep} and f_{CEO} following the approach presented by Dolgovskiy *et al.* [26]. The calculated value of Γ_{Δ} shown in Fig. 3(b) is close to –2 in the entire spectral range of 10 Hz to $>$10 kHz, which demonstrates a strong anti-correlation of the frequency noise of $N \cdot f_{\mathrm{rep}}$ and f_{CEO}. This outcome, combined with the similar amplitude of the FN-PSD separately measured for f_{rep} and indirectly for f_{CEO}, confirms the existence of a fixed point in the spectrum of our laser. This explains the lower observed frequency noise of the optical mode of the ERGO laser compared to the individual noise contributions of $N \cdot f_{\mathrm{rep}}$ and f_{CEO}. The Γ_{Δ} value matches -2 in the frequency range of 1 kHz to 10 kHz, whereas it is somehow smaller (in absolute value) at lower Fourier frequencies. This indicates that there is in fact not a true fixed point in the spectrum of our laser as generally considered by the simplified comb elastic tape model [14], [15]. Instead, the fixed point varies with the considered noise frequency as previously observed in an Er:fiber comb and described in details in [26]. This is the reason why the FN-PSD of the optical mode is much lower than the individual noise of f_{rep} or f_{CEO} in the frequency range of 1 kHz to 10 kHz (by two orders of magnitude), whereas the difference is reduced at lower frequency (only one order of magnitude at $f = 100$ Hz) despite the similar amplitude of the frequency noise of f_{rep} and f_{CEO}. This may indicate that another noise source affects the laser at low Fourier



(a) (b) (c) (d)

Fig. 4. (a) Comparison of the FN-PSD of the repetition rate up-scaled to the optical frequency ($N \cdot f_{\text{rep}}$, red), of the indirect CEO signal (blue) and of the optical line ν_{N} (green) of the ERGO laser measured when the repetition rate is locked. The low-frequency noise floor resulting from the ~25-GHz synthesizer used as a reference in the stabilization of f_{rep} is also displayed (gray), showing that it limits the FN-PSD of the repetition rate at frequencies below ~30 Hz. (b)–(d) FN-PSD of the repetition rate (b), of the indirect CEO signal (c) and of the optical line (d) of the ERGO laser in locked (darker colors) and free-running (lighter colors) conditions. The strong noise peak at ~14 kHz visible in the spectra of the optical line arises from a spurious noise peak present in the output signal of the PID controller and is not inherent to the laser itself.

frequencies, and has a different impact on f_{CEO} and f_{rep} (in other words it corresponds to a different fixed point).

3.2 Repetition Rate Stabilization and Effect on the ERGO Noise

We stabilized the repetition rate of our laser to a reference signal from a synthesizer as schematized in Fig. 2 and we measured the resulting frequency noise spectra of f_{rep}, f_{CEO} and of the optical line ν_{N} in a similar way as for the free-running laser. Results obtained with the locked repetition rate are displayed in Fig. 4(a) and a comparison between the free-running and stabilized cases is separately shown in Fig. 4(b)–(d) for f_{rep}, the indirect CEO signal and the optical line ν_{N}. One observes that the repetition rate is properly stabilized with a feedback bandwidth of around 1.5 kHz, assessed from the servo bump in its FN-PSD [see Fig. 4(b)]. The achieved FN-PSD coincides with the noise of the reference synthesizer at Fourier frequencies below ~50 Hz. However, the frequency noise of the optical line ν_{N} is significantly degraded by the repetition rate stabilization, while the frequency noise of f_{CEO} is not reduced by the feedback loop, despite its strong negative correlation with the noise of f_{rep}. The reason is that the effect of the PZT on the laser operation corresponds to a different fixed point than the principal noise source that affects the free-running laser, which is believed to be the amplitude noise of the pump diode.

To further investigate the noise behavior observed in Fig. 4 for the stabilized repetition rate, we measured the transfer function of ν_{N}, f_{rep} and f_{CEO} for a modulation of the cavity length. This was realized by modulating the PZT voltage with a sine waveform and by demodulating the corresponding signals (optical beat, repetition rate or indirect CEO signal) using a frequency discriminator [27] and a lock-in amplifier. The indirect CEO signal was obtained using the scheme of Fig. 2. The different transfer functions, displayed in Fig. 5 present the same behavior, both in amplitude and in phase,

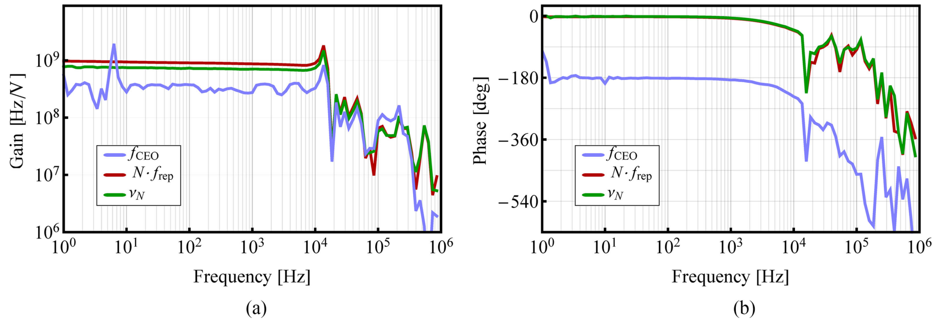


Fig. 5. Transfer functions in amplitude (a) and phase (b) of $N \cdot f_{\mathrm{rep}}$ (red), f_{CEO} (light blue) and ν_{N} (optical line, green) for a modulation of the PZT voltage.

with a constant amplitude response up to ~10 kHz. A phase shift of 180° is observed between the transfer functions of f_{rep} and f_{CEO}, which indicates that a change of the cavity length produces an opposite effect on f_{rep} and f_{CEO}. The PZT produces a significant change of f_{CEO}, which is a factor ~3,000 larger than the corresponding tuning coefficient of f_{rep}. This corresponds to a fixed point $N_{\mathrm{fix}} \approx 3,000$ or to a fixed frequency $N_{\mathrm{fix}} \cdot f_{\mathrm{rep}} \approx 75\,\mathrm{THz}$ for the cavity length change, which is substantially lower than the fixed frequency resulting from the noise in the free-running laser, located close to the optical carrier of ~194 THz. The fixed frequency for a change of the cavity length is two orders of magnitude higher than values reported in a 250-MHz Er:fiber frequency comb [26] and in another Er:Yb:glass DPSSL comb with 75-MHz repetition rate [28], both in the same wavelength range of 1.56 μm. This factor is of the same order of magnitude as the ratio of the considered repetition rates.

The repetition rate has a slightly more important contribution (by a factor ~2.7) than f_{CEO} to the modulation of the optical line ν_{N} resulting from a change of the cavity length, as a result of its scaling by the mode number $N \approx 7{,}760$. From the theoretical model of coupled stabilization loops in a frequency comb presented by Dolgovskiy *et al.* [26], the impact of the repetition rate stabilization loop on the CEO signal is given by the following expression if a fully anti-correlated noise of similar amplitude is considered between f_{CEO} and $N \cdot f_{\mathrm{rep}}$ (i.e., $\Delta f_{\mathrm{CEO}} \approx -N \cdot \Delta f_{\mathrm{rep}}$):

$$\delta f_{\mathrm{CEO}} = \Delta f_{\mathrm{CEO}} \frac{1+\left[1-\left(C_{f_{\mathrm{CEO}}}/C_{f_{\mathrm{rep}}}\right)/N\right]H}{1\,+\,H}, \tag{1}$$

where Δf_{CEO} and δf_{CEO} represent the frequency fluctuations of f_{CEO} with the repetition rate stabilization loop open and closed, respectively, $H(f)$ is the total open loop transfer function of the repetition rate stabilization, while $C_{f_{\mathrm{CEO}}}(f)$ and $C_{f_{\mathrm{rep}}}(f)$ are the transfer functions of f_{CEO} and f_{rep} for a modulation of the PZT shown in Fig. 5. All transfer functions in Eq. (1) are complex numbers that incorporate both the amplitude and phase information.

The stabilization loop of f_{rep} might have a significant effect on f_{CEO} in our laser only in the case where $C_{f_{\mathrm{CEO}}}/C_{f_{\mathrm{rep}}} \approx N$, i.e., if the ratio of the transfer functions of f_{CEO} and f_{rep} ($C_{f_{\mathrm{CEO}}}/C_{f_{\mathrm{rep}}}$) was approximately equal to the ratio of the free-running frequency fluctuations of f_{CEO} and f_{rep} (i.e., if $\Delta f_{\mathrm{CEO}}/\Delta f_{\mathrm{rep}} \approx -N$). In other words, the fixed points corresponding to the effect of the PZT and to the dominant noise source in the free-running laser should coincide. In the present case, $(C_{f_{\mathrm{CEO}}}/C_{f_{\mathrm{rep}}})/N \approx 0.3$, so that the FN-PSD of f_{CEO} is reduced in the best case by a factor of $(1-0.3)^{-2} \approx 2$ within the repetition rate loop bandwidth (i.e., for $|H| \gg 1$), which is too small to be visible on the noise spectrum of f_{CEO} displayed in Fig. 4(c).

The effect of the repetition rate stabilization on the optical line at ν_{N} is even more detrimental than for f_{CEO} and its FN-PSD is significantly degraded when the repetition rate is stabilized. Whereas the fluctuations of $N \cdot f_{\mathrm{rep}}$ and f_{CEO} did compensate each other to a large extent in the noise of the optical line ν_{N} in the free-running laser (as ν_{N} is located close to the fixed point), this situation no longer stands when the repetition rate is locked. The noise of f_{rep} is strongly reduced by the stabilization loop, while f_{CEO} is not affected. Therefore, the FN-PSD of the optical line mainly corresponds to the noise of f_{CEO} when f_{rep} is stabilized (within the loop bandwidth) and is thus higher than in the free-running case. Beyond the loop bandwidth, the frequency noise of the optical mode is still enhanced in the frequency range between ∼1 kHz and ∼10 kHz in comparison to the unstabilized case. This is believed to result from a slight difference in the noise amplitude of f_{CEO} and $N \cdot f_{\mathrm{rep}}$ and from their partially uncorrelated part, which are both introduced by the loop. The parasitic noise peak at ∼14-kHz that is coupled onto f_{rep} and ν_{N} by the servo-controller is strongly enhanced when the repetition rate loop is closed. As this peak arises from a spurious technical noise of the servo-controller and is not inherent to the laser itself, it is not taken into account in the estimation of the optical mode linewidth.

Therefore, the linewidth of the optical line obtained by integrating the FN-PSD between 10 Hz and 10 kHz (i.e., without considering the 14-kHz excess noise peak) amounts to ∼600 kHz, which is slightly more than a factor 2 broader than the free-running linewidth.

4. Conclusion

We have presented a detailed analysis of the frequency noise of a 25-GHz ERGO DPSSL, including an indirect assessment of the frequency noise of the CEO signal. This signal was indirectly obtained using an electrical scheme that does not require the detection of the CEO beat using a nonlinear interferometry scheme (such as f-to-$2f$ or $2f$-to-$3f$ [29]). Instead, we assessed the frequency noise of f_{CEO} using an appropriate combination of electrical signals made of the laser repetition rate and of a frequency-divided heterodyne beat with a narrow linewidth laser. A relatively high frequency noise of the CEO signal was estimated from these measurements. The regular $1/f$ noise observed at low frequency is higher by only a factor ∼10 compared to a 1-GHz DPSSL at 1 μm that we recently fully stabilized with CEO self-referencing [23], which appears relatively moderate owing to the 25-fold higher repetition rate of the ERGO laser. However, the white frequency noise plateau occurring at Fourier frequencies above 10 kHz has a significant contribution to the large CEO linewidth of ∼5 MHz (at 100-ms integration time) assessed from its extrapolated frequency noise.

Despite the frequency noise assessed for the CEO signal, the $1/f$ noise of an optical mode of the laser was lower by at least 1-2 orders of magnitude, leading to a much narrower linewidth $<$300 kHz at the same integration time of 100 ms. The reason behind this observation is a partial compensation of the noise of $N \cdot f_{\mathrm{rep}}$ and f_{CEO} in the optical mode of the laser that arises from their similar amplitude and strong anti-correlation, leading to a fixed point in the spectrum of the laser that is located in the vicinity of the optical carrier. Amplitude noise of the pump diode is believed to be the dominant noise source responsible for this behavior. The noise compensation no longer holds when the repetition rate is stabilized by active feedback to the cavity length, which strongly reduces the fluctuations of f_{rep} but lets the noise of f_{CEO} almost unchanged within the locking bandwidth. Therefore, the frequency noise and linewidth of an optical mode of the laser are degraded when f_{rep} is stabilized. This degradation is not related to the quality of the stabilization of f_{rep}, but is a direct consequence of the different fixed points corresponding to the major noise source in the free-running laser and to the PZT actuator used for the stabilization, as shown by our detailed noise analysis. The laser performance, i.e., the optical linewidth, may be improved when the repetition rate is stabilized by additionally phase-locking one mode of the laser to an optical reference. The optical reference can be either a free-running low frequency noise laser, such as a planar-waveguide external cavity laser [30] with a linewidth in the range of a few kilohertz, or a cavity-stabilized laser with a narrower linewidth. In order to faithfully transfer the noise properties of the optical reference to the optical modes of the ERGO laser, i.e., to achieve a tight phase-lock, a feedback bandwidth of less than 50 kHz is estimated to be sufficient from the crossing-point of the FN-PSD of the optical line ν_{N} with the β-separation line [24]. If a modulation of the current of the ERGO laser pump diode was not fast enough to achieve such bandwidth as a result of the laser cavity dynamics, a faster stabilization could be achieved using a feedforward method using an acousto-optic modulator in the laser output beam [11]. In this a way, a multi-wavelength laser source with narrow-linewidth optical modes compatible with the requirements of 16-ary quadrature amplitude modulation (16-QAM) coherent optical telecommunication systems can be obtained [13].

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