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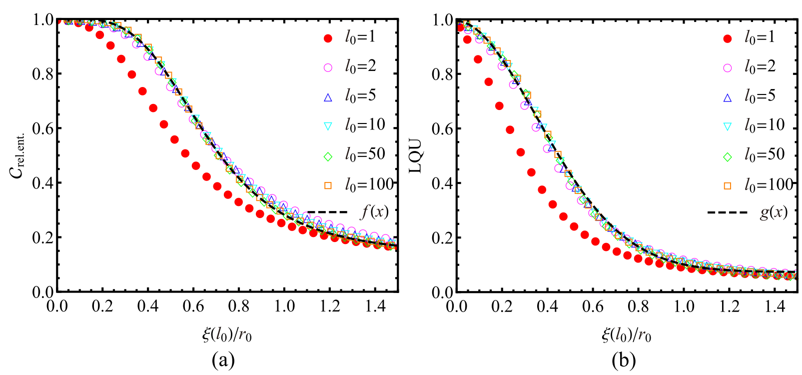

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Universal Decay of Quantumness for Photonic Qubits Carrying Orbital Angular Momentum Through Atmospheric Turbulence

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Abstract: We investigate the decay properties of the quantumness including quantum entanglement, quantum discord, and quantum coherence for two photonic qubits, which are partially entangled in their orbital angular momenta, through Kolmogorov turbulent atmosphere. It is found that the decay of quantum coherence and quantum discord may be qualitatively different from that of quantum entanglement when the initial state of two photons is not maximally entangled. We also derive two universal decay laws for quantum coherence and quantum discord, respectively, and show that the decay of quantum coherence is more robust than nonclassical correlations.

Index Terms: Atmospheric propagation, atmospheric turbulence, quantum information and processing.

1. Introduction

The fact that photons can carry orbital angular momentum (OAM) for encoding quantum states makes them very useful for quantum information science (QIS) [1]–[6]. However, the decay of quantumness will be unavoidable when the encoded photons with OAM transmit through turbulent atmosphere. Photons' wave front will be distorted due to refractive index fluctuations of the turbulent atmosphere, which may be characterized as random phase aberrations on a transferring optical wave [7]. A large amount of efforts, both theoretical [8]–[12] and experimental [13]–[16], have been devoted to exploring the impacts of atmospheric turbulence on the propagation of photons carrying OAM and protecting their entanglement from decoherence due to turbulent atmosphere.

In QIS, it is typically critical to understand the behaviors of nonclassically correlated photons traveling in the turbulent atmosphere since the quantumness contained in the encoded states are usually fragile and can be easily destroyed. Quantum entanglement, a fundamental quantum resource in QIS [17], is a typical kind of quantumness which is usually considered. For instance, the decay of entanglement for photonic OAM qubit states in turbulent atmosphere has been reported [18]–[21] via Wootters' concurrence [22]. However, it has been proved that entanglement

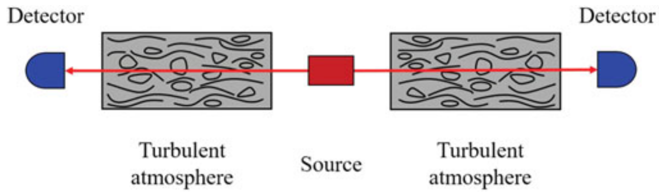

Fig. 1. A pair of OAM-entangled photons is produced by the source, sent through turbulent atmosphere, and finally received by two detectors.

is not the unique resource which can be employed in QIS. There exist other resources such as quantum discord [23]–[25] and quantum coherence [26]–[28] responsible for certain quantum communication protocols. Therefore, the understanding of propagation properties for quantum discord and quantum coherence of OAM photons is also crucial. Recently, Girolami *et al.* [29] proposed a measure of discord-like correlation, called as local quantum uncertainty (LQU), based on the Wigner-Yanase skew information [30]. The LQU is a genuine measure of quantum discord, which is easily computable for any bipartite quantum state. More importantly, it is observable in experiments with single local measurements on quantum states, quantifying the minimal uncertainty as concerned in quantum metrology [29]. On the other hand, Baumgratz *et al.* [26] put forward a measure of coherence, based on the relative entropy. As Streltsov *et al.* [28] suggested, the relative entropy of coherence is a nice coherence quantifier, which not only fulfills all the conditions of coherence measure but also is exactly computable for bipartite quantum state. It may be interesting to analytically explore the impacts of turbulent atmosphere on these resources other than entanglement.

Following [21], in this paper, we consider the case of propagation of a twin-photon state sharing only partial entanglement and investigate the effects of atmospheric turbulence on the quantumness including quantum entanglement, quantum discord and quantum coherence. This situation is more close to realistic excrements, since the preparation of quantum state is usually not determined but probabilistic, yielding a probability distribution (or ensemble) of states. It is shown that the decay of quantum coherence and quantum discord may be qualitatively different from that of quantum entanglement when the initial state of two photons is not maximally entangled. It is also found that the decay of quantum discord obeys an universal exponential law similar to that of entanglement already reported in [21] but with asymptotic vanishing. By contrast, the universal decay of quantum coherence is merely polynomial, indicating that quantum coherence is more robust against atmospheric turbulence.

The paper is organized as the following. In Section 2, we explore the photonic OAM state influenced by the turbulent atmosphere and discuss the evolution of the entanglement of the initial extended Werner-like state. In Section 3, the evolutions of the LQU and relative entropy of coherence as well as their decay laws are discussed. Conclusions are presented in Section 4.

2. Partially Entangled OAM State Through Atmospheric Turbulence

In this work, we use two Laguerre-Gaussian (LG) beams [31] to generate a twin-photon state [6]. As is shown in Fig. 1, two correlated LG beams, produced by the source, propagate through the turbulent atmosphere horizontally and then are received by the two detectors. The two beams carry a pair of photons which may share only partial entanglement of OAM due to impurity and imperfection, and the non-maximal OAM entanglement can be encoded by a spatial light modulator [6]. The entangled LG beams have the same beam waist ω_0 and radial quantum number $p_0 = 0$, but opposite azimuthal quantum numbers l_0 and $-l_0$. Initially, the photon pair is created in an extended Werner-like state defined by

$$\rho^{(0)} = \frac{1-\gamma}{4} I + \gamma \left|\Psi_0\right\rangle \left\langle\Psi_0\right|, \tag{1}$$

where $0 \leq \gamma \leq 1$ denotes the purity of the initial state and $|\Psi_0\rangle$ is the Bell-like state given by

$$|\Psi_0\rangle = \cos\left(\frac{\theta}{2}\right)|l_0, -l_0\rangle + e^{j\phi}\sin\left(\frac{\theta}{2}\right)|-l_0, l_0\rangle, \tag{2}$$

with $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. It shall be mentioned that the quantum state (1) recovers the Bell state considered in [21] for purity $\gamma = 1$ and $\theta = \pi/2$. The extended Werner-like states play an crucial role in various applications of QIS [32], [33]. Here, we focus on the atmospheric turbulence effect on OAM quantumness rather than the laser itself. OAM is an important physical quantity of photons which can be encoded by LG laser. If a helically phased laser beam can be expressed with an azimuthal phase dependence of the form $\exp(il\varphi)$, each photon of this beam can carry an OAM with the azimuthal quantum number l [31]. Both of the two detectors should be used to receive the OAM-entangled photons, and the detectors can read out the azimuthal quantum numbers of the received photons by the azimuthal phase. By calculating the probabilities of the initial azimuthal quantum numbers l_0 and $-l_0$, the output state can be reconstructed, the process of which is called quantum state tomography, and the decay of the quantumness is indirectly measured via quantum state tomography. The action of the turbulent atmosphere on the photons can be treated as a linear map Λ, in terms of which the received state at the detectors reads as

$$\rho = (\Lambda_1 \otimes \Lambda_2)\,\rho^{(0)}, \tag{3}$$

where Λ_1 and Λ_2 are the actions of the atmospheric turbulence on the individual photon state. The density matrix of the photonic state (3) is then given by [34]

$$\begin{aligned}
\rho &\propto \sum_{ii',jj'} \rho_{|ij\rangle\langle i'j'|}\,|ij\rangle\langle i'j'| \\
&= \sum_{ii',jj'} \rho_{ii',jj'}\,|ij\rangle\langle i'j'| \\
&= \sum_{ii',jj'}\sum_{ll',mm'} \Lambda_{1ii'}^{ll'}\Lambda_{2jj'}^{mm'}\rho^{(0)}_{ll',mm'}\,|ij\rangle\langle i'j'| \\
&= \sum_{ii',jj'}\sum_{ll',mm'} \Lambda_{1ii'}^{ll'}\Lambda_{2jj'}^{mm'}\rho^{(0)}_{|lm\rangle\langle l'm'|}\,|ij\rangle\langle i'j'|.
\end{aligned} \tag{4}$$

Here, we let $\Lambda_1 = \Lambda_2 = \Lambda$ due to the same effect of turbulent atmosphere on the photons. The elements $\Lambda_{l,l'}^{l_0,l_0'} = \sum_p \Lambda_{pl,pl'}^{p_0l_0,p_0'l_0'}$ of the linear map Λ is given by [21]

$$\Lambda_{l,\pm l'}^{l_0,l_0'} = \frac{\delta_{l_0-l_0',l\mp l}}{2\pi}\int_0^\infty \mathrm{d}r R_{p_0l_0}(r)\,R^*_{p_0l_0}(r)\,r\int_0^{2\pi}\mathrm{d}\vartheta e^{-i\vartheta[l\pm l-(l_0+l_0')]/2}e^{-D_\phi(2r|\sin(\vartheta/2)|)/2}, \tag{5}$$

where

$$R_{p_0l_0}(r) = \frac{2}{\omega_0}\sqrt{\frac{p_0!}{(p_0+|l_0|)!}}\left(\frac{r\sqrt{2}}{\omega_0}\right)^{|l_0|} L_{p_0}^{|l_0|}\left(\frac{2r^2}{\omega_0}\right)\exp\left(-\frac{r^2}{\omega_0^2}\right), \tag{6}$$

is the radial wave function of LG beam at propagation distance $z = 0$ [8] with generalized Laguerre polynomials

$$L_{p_0}^{|l_0|}(x) = \sum_{m=0}^{p_0}(-1)^m\frac{(|l_0|+p_0)!}{(p_0-m)!\,(|l_0|+m)!m!}x^m. \tag{7}$$

Here, we consider the case of Kolmogorov turbulence with the phase structure function $D_\phi = 6.88\,(r/r_0)^{5/3}$ and the Fried parameter

$$r_0 = \left(0.423C_n^2k^2L\right)^{-3/5}, \tag{8}$$

where C_n^2 is the index-of-refraction structure constant, L is the propagation distance, and k is the optical wave number [35]. The density matrix of the input state (1) in the basis $\{|l_0, l_0\rangle, |l_0, -l_0\rangle, |-l_0, l_0\rangle, |-l_0, -l_0\rangle\}$ can be written as an X form

$$\rho^{(0)} = \begin{pmatrix} \rho_{11}^{(0)} & & & \rho_{14}^{(0)} \\ & \rho_{22}^{(0)} & \rho_{23}^{(0)} & \\ & \rho_{32}^{(0)} & \rho_{33}^{(0)} & \\ \rho_{41}^{(0)} & & & \rho_{44}^{(0)} \end{pmatrix}, \tag{9}$$

where

$$\begin{aligned} \rho_{11}^{(0)} &= \frac{1-\gamma}{4}, \rho_{22}^{(0)} = \frac{1-\gamma}{4} + \gamma \cos^2\left(\frac{\theta}{2}\right), \\ \rho_{33}^{(0)} &= \frac{1-\gamma}{4} + \gamma \sin^2\left(\frac{\theta}{2}\right), \rho_{44}^{(0)} = \frac{1-\gamma}{4}, \\ \rho_{14}^{(0)} &= 0, \rho_{23}^{(0)} = \frac{1-\gamma}{4} + \frac{\gamma}{2} e^{-i\phi} \sin\theta, \\ \rho_{41}^{(0)} &= \rho_{14}^{(0)*}, \rho_{32}^{(0)} = \rho_{23}^{(0)*}. \end{aligned} \tag{10}$$

According to (4), the normalized density matrix of the output state (3) can also be expressed in the X form as

$$\begin{aligned} \rho &= \frac{\sum_{ii',jj'} \rho_{|ij\rangle\langle i'j'|} |ij\rangle \langle i'j'|}{\mathrm{Tr}\left(\sum_{ii',jj'} \rho_{|ij\rangle\langle i'j'|} |ij\rangle \langle i'j'|\right)} \\ &= \begin{pmatrix} \rho_{11} & & & \rho_{14} \\ & \rho_{22} & \rho_{23} & \\ & \rho_{32} & \rho_{33} & \\ \rho_{41} & & & \rho_{44} \end{pmatrix}, \end{aligned} \tag{11}$$

with

$$\begin{aligned} \rho_{11} &= \left(a^2 \rho_{11}^{(0)} + ab\rho_{22}^{(0)} + ab\rho_{33}^{(0)} + b^2 \rho_{44}^{(0)}\right) / (a+b)^2, \\ \rho_{22} &= \left(ab\rho_{11}^{(0)} + a^2 \rho_{22}^{(0)} + b^2 \rho_{33}^{(0)} + ab\rho_{44}^{(0)}\right) / (a+b)^2, \\ \rho_{33} &= \left(ab\rho_{11}^{(0)} + b^2 \rho_{22}^{(0)} + a^2 \rho_{33}^{(0)} + ab\rho_{44}^{(0)}\right) / (a+b)^2, \\ \rho_{44} &= \left(b^2 \rho_{11}^{(0)} + ab\rho_{22}^{(0)} + ab\rho_{33}^{(0)} + a^2 \rho_{44}^{(0)}\right) / (a+b)^2, \\ \rho_{14} &= a^2 \rho_{14}^{(0)} / (a+b)^2, \rho_{41} = a^2 \rho_{41}^{(0)} / (a+b)^2, \\ \rho_{23} &= a^2 \rho_{23}^{(0)} / (a+b)^2, \rho_{32} = a^2 \rho_{32}^{(0)} / (a+b)^2, \end{aligned} \tag{12}$$

where

$$\begin{aligned} a &= \Lambda_{l_0,l_0}^{l_0,l_0} = \Lambda_{-l_0,-l_0}^{-l_0,-l_0} = \Lambda_{-l_0,l_0}^{-l_0,l_0} = \Lambda_{l_0,-l_0}^{l_0,-l_0}, \\ b &= \Lambda_{l_0,l_0}^{-l_0,-l_0} = \Lambda_{-l_0,-l_0}^{l_0,l_0}. \end{aligned} \tag{13}$$

In this sense, the output OAM state can be treated as two-qubit. Then, we can express the entanglement by Wootters' concurrence [22] for the X state as

$$C(\rho) = \max\left\{0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\right\}. \tag{14}$$

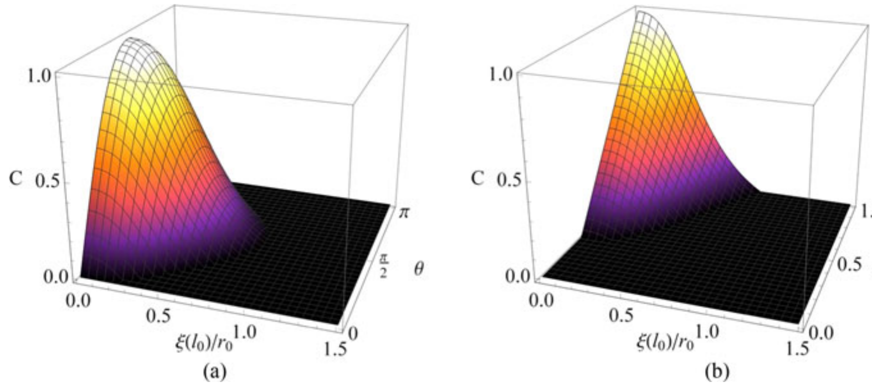

Fig. 2. Concurrence as a function of (a) $\xi(l_0)/r_0$ and θ, and of (b) $\xi(l_0)/r_0$ and γ. The parameters are chosen as $l_0 = 1$, $\omega_0 = 1$ for (a) $\gamma = 1$ and (b) $\theta = \pi/2$.

With (12) and (13), an analytical form of the concurrence can be derived as

$$C(\rho) = \max\left\{0, \frac{a^2\gamma\sin\theta - 2ab\gamma}{(a+b)^2} - \frac{1-\gamma}{2}\right\}. \tag{15}$$

For convenience, one can introduce the phase correlation length $\xi(l_0)$ as [21], defined by

$$\xi(l_0) = \sin\left(\frac{\pi}{2|l_0|}\right)\frac{\omega_0}{2}\frac{\Gamma(|l_0|+3/2)}{\Gamma(|l_0|+1)}, \tag{16}$$

with $\Gamma(x)$ the Gamma function. It should be pointed out that, generally, the entanglement versus the inverse $1/r_0$ of correlation parameter of turbulence r_0 is considered to explore the turbulence effect. Besides, the correlation parameter of turbulence r_0 is usually compared to a specific constant associated with the initial state of beam such as beam waist ω_0 and rms beam radius $r_{p,l}$, i.e., yielding rescaled turbulence parameters ω_0/r_0 [18] and $r_{p,l}/r_0$ [7] for convenience. Here, we want to explore the propagation properties for different l_0 and use the rescaled turbulence parameters $\xi(l_0)/r_0$. For $r_0 \gg \xi(l_0)$, the turbulent atmosphere appears as a homogeneous medium to the OAM biphoton, and its spatial entanglement remains high. As $r_0 \to \xi(l_0)$, the phase errors become sufficiently large to destroy the wave front structure, and the entanglement vanishes, illustrating $\xi(l_0)$ as a characteristic length. By contrast, we will show that the turbulence effect does not completely destroy discord and coherence for $\xi(l_0)/r_0 \approx 1$ in the following. In fact, from (15), the initial concurrence is independent of the phase correlation length $\xi(l_0)$. However, during the transmission process, the phase front exhibits a more rapid oscillation leading to shorter characteristic length $\xi(l_0)$ as the initial quantum numbers l_0 increases. For $\xi(l_0) < r_0$, the atmosphere serves as a more homogeneous medium to OAM photons and the quantumness can be preserved better.

Then, we plot the entanglement $C(\rho)$ versus the ratio $x = \xi(l_0)/r_0$ and the initial state parameter θ in Fig. 2(a). We can see that the concurrence decays with the increase of the ratio $\xi(l_0)/r_0$ and decreases fast to zero with non-asymptotical vanishing, the phenomenon of which is termed as entanglement sudden death (ESD) [36], [37]. Moreover, the entanglement versus the ratio $\xi(l_0)/r_0$ and the purity parameter γ is displayed in Fig. 2(b). It is found that the concurrence is vanishing when the initial state is unentangled for $0 \leq \gamma \leq 1/(1+2\sin\theta)$, since no entanglement can be created between the two photons under independent atmospheric turbulences for initially disentangled states. In the next part, we will investigate the propagation of quantum coherence and quantum correlation via relative entropy of coherence and LQU, which may exhibit qualitative differences from that of entanglement via concurrence.

3. Quantum Coherence and Quantum Correlation of OAM State in Atmospheric Turbulence

Here, the degree of quantum coherence and quantum correlation are quantified via relative entropy of coherence [26] and LQU [29]. The relative entropy of coherence of a quantum state ρ is put forward by Baumgratz *et al.* [26] as

$$\mathcal{C}_{\text{rel.ent.}}(\rho) = S\left(\rho_{\text{diag}}\right) - S(\rho), \tag{17}$$

where $S(\rho) = -\text{Tr}(\rho \log_2 \rho)$ is the von Neumann entropy and ρ_{diag} is the "closest" non-coherent state, which is diagonal by deleting all the off-diagonal elements of ρ. The relative entropy of coherence is shown to be nice coherence quantifier, fulfilling all the conditions proposed in [28] and exactly computable for bipartite quantum state. The LQU, proposed by Girolami *et al.* [29], is a genuine measure of quantum discord, which is easily computable for any bipartite quantum state. More importantly, it is observable in experiments with single local measurements on quantum states, quantifying the minimal uncertainty as concerned in quantum metrology [29]. For a bipartite system AB with the state ρ, a single local measurement on the subsystem A give rise to quantum uncertainty if the state ρ is nonclassical. Then the LQU is defined by the minimal Wigner-Yanase skew information [30] as

$$\begin{aligned} \text{LQU}(\rho) &= \min_{\{K_A\}}\{\mathcal{I}(\rho, K_A \otimes I_B)\} \\ &= -\frac{1}{2}\min_{\{K_A\}}\{\text{Tr}\left([\rho, K_A \otimes I_B]^2\right)\}, \end{aligned} \tag{18}$$

where K_A is a local observable on system A and I_B is the identity operator for subsystem B. For the state considered here, the calculation of LQU can be simplified as

$$\text{LQU}(\rho) = 1 - \lambda_{\max}\{W_{AB}\}, \tag{19}$$

where $\lambda_{\max}$ denotes the maximal eigenvalue of the matrix W_{AB} with

$$(W_{AB})_{ij} = \text{Tr}\left[\sqrt{\rho_{AB}}\,(\sigma_{iA} \otimes I_B)\sqrt{\rho_{AB}}\,(\sigma_{jA} \otimes I_B)\right], \tag{20}$$

with σ_{iA} $(i = x, y, z)$ the Pauli matrixes of subsystem A.

Then, the relative entropy of coherence and LQU as functions of $\xi(l_0)/r_0$ and θ are displayed in Fig. 3(a) and (b). We can see that the relative entropy of coherence and LQU decay fast with the increase of the ratio $\xi(l_0)/r_0$ when θ is close to $\pi/2$ and then decrease slowly in a non-vanishing manner even when the $\xi(l_0)/r_0$ is large enough. Therefore, the phenomenon of sudden vanishing as for the entanglement (ESD) does not occur here. We also show the relative entropy of coherence and LQU as functions of $\xi(l_0)/r_0$ and γ in Fig. 3(c) and (d). When the purity γ of the initial state is close to zero, the quantum coherence and quantum correlation of the two photons are very weak but still be non-vanishing, which is quite different from the entanglement shown in Fig. 2(b). When γ is close to 1, the relative entropy of coherence and LQU seem to decay in a nearly exponential manner with the increase of $\xi(l_0)/r_0$, which is also contrast to that of entanglement with sudden vanishing as shown in Fig. 2(b).

In order to see the evolution in turbulent atmosphere more clearly, the relative entropy of coherence and LQU as functions of $\xi(l_0)/r_0$ for different θ are also demonstrated in Fig. 4. We can clearly see that the relative entropy of coherence and LQU decay more and more slowly with the increase of the ratio $\xi(l_0)/r_0$. For certain initial state, for instance $\theta = \pi/3$ shown in Fig. 4(b), the decay rate of LQU may be non-continuously changed when the $\xi(l_0)/r_0$ is still small, which is termed as the sudden change phenomenon [38], [39] as for discord-like quantum correlations.

Moreover, we would like to explore the precise decay laws of quantum coherence and quantum correlation for different values of phase correlation length which is determined by the azimuthal quantum number l_0 as shown in (16). As can be seen from Fig. 5(a) and (b), both the relative entropy and LQU decay fastest at $l_0 = 1$. When the azimuthal quantum number l_0 increases, the decays of relative entropy of coherence and LQU are slowed down and finally collapse onto two

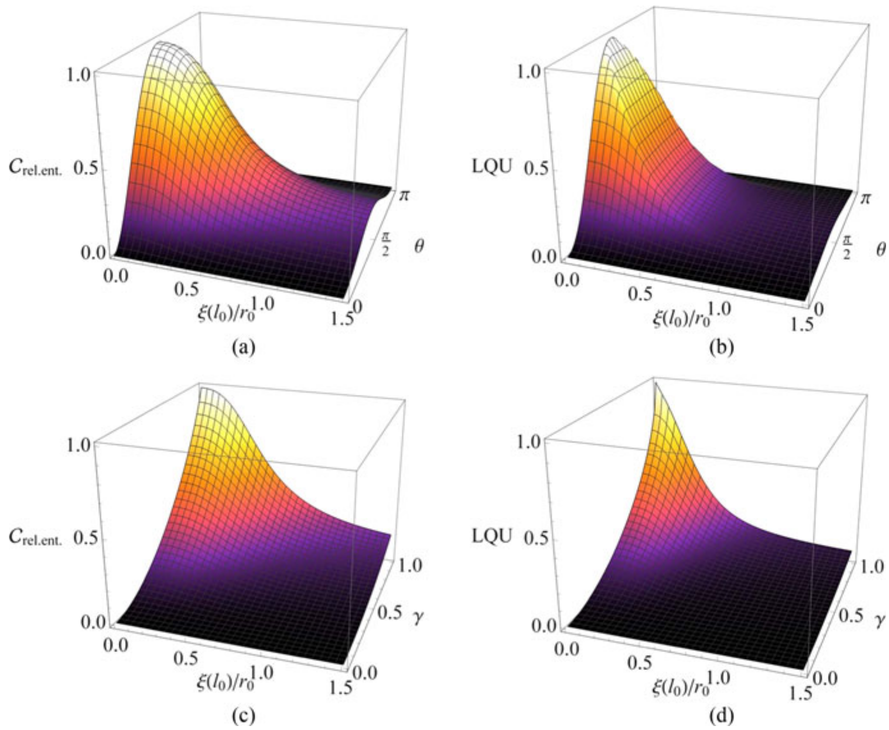

Fig. 3. (a), (c) Relative entropy of coherence and (b), (d) LQU as functions of (a), (b) $\xi(l_0)/r_0$ and θ, and of (c), (d) $\xi(l_0)/r_0$ and γ. The parameters are chosen as $l_0 = 1$, $\omega_0 = 1$ for (a), (b) $\gamma = 1$ and (c), (d) $\theta = \pi/2$.

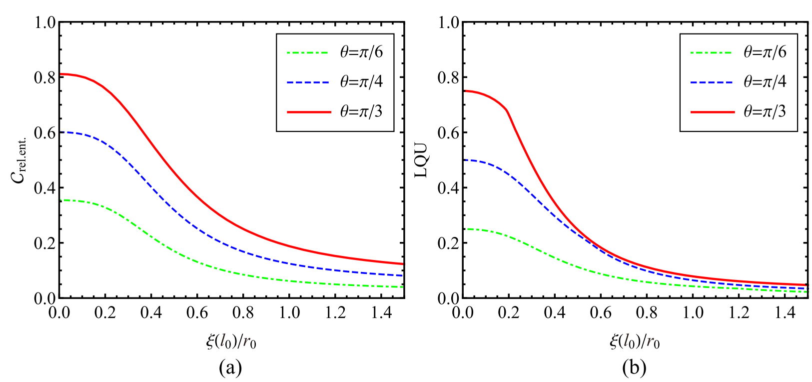

Fig. 4. (a) Relative entropy of coherence and (b) LQU as functions of $\xi(l_0)/r_0$. The parameters are chosen as $l_0 = 1$, $\omega_0 = 1$ and $\gamma = 1$.

universal curves. Their functions of the ratio $x = \xi(l_0)/r_0$ is given by $f(x) = \frac{0.183}{x^{3.78} + 0.21} + 0.131$ and $g(x) = 0.92[\exp(-3.50x^{1.90}) + 0.08]$, which means that the relative entropy of coherence decays in a polynomial manner while the LQU decays in an exact exponential manner. Since the LQU is a kind of nonclassical correlation apart from entanglement, then it is natural to derive the universal exponential decay similar to that of entanglement reported in [21]. It is believed that nonclassical correlations (such as quantum discord and entanglement) reveal only parts of quantumness, while quantum coherence is a more general concept for quantumness [28], which may be the physical mechanism underlined the phenomenon of the more robust decay law for coherence against atmospheric turbulence. Finally, we would like to point out that the decay of quantumness versus

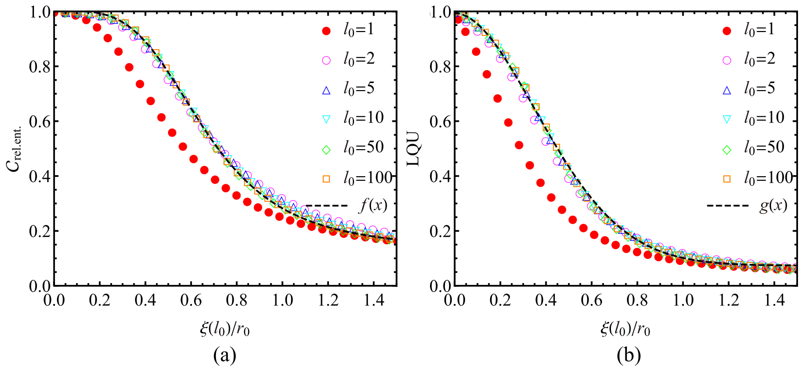

Fig. 5. (a) Relative entropy of coherence and (b) LQU as functions of $\xi(l_0)/r_0$ under different l_0. The parameters are chosen as $\omega_0 = 1$, $\theta = \pi/2$ and $\gamma = 1$.

$1/r_0$ becomes slower as l_0 increases. Meanwhile, the characteristic length $\xi(l_0)$ decreases as l_0 increases, which makes the difference of the results between the OAM-entangled photons with different OAM mode small when we consider quantumness versus the rescaled turbulence parameter $\xi(l_0)/r_0$. This approach is somewhat like performing the finite-size scaling of quantum phase transition in condensed matter physics. In addition, The characteristic length $\xi(l_0)$ decreases with increasing l_0, which keeps almost constant for $l_0 \to \infty$. As a consequence, the decay rate of the entanglement becomes almost the same for large l_0.

4. Conclusion

In conclusion, we have investigated the decay properties of quantum coherence and nonclassical correlations (entanglement and discord) for photonic states carrying orbital angular momentum (OAM) through Kolmogorov turbulent atmosphere via relative entropy of coherence, local quantum uncertainty (LQU), and concurrence, respectively. By considering that the photonic OAM qubits, generated from a source, are initially prepared in an extended Werner-like state (partially entangled), the decay effects of the turbulent atmosphere are explored for the output state received by the detectors. It is shown that the quantumness measured by concurrence, relative entropy of coherence and LQU decays as the increase of the ratio of phase correlation length and the Fried parameter but with different phenomena for different measures. The concurrence decays suddenly to zero with the so-called entanglement sudden death (ESD), while both the relative entropy of coherence and the LQU decay asymptotically. For certain initial state, the LQU may demonstrate an extra sudden change phenomenon when the ratio of phase correlation length to Fried parameter is not large. Moreover, we study the decays of quantum coherence and quantum correlation with different values of phase correlation length (the azimuthal quantum number), and find that two different universal decay laws emerge as the azimuthal quantum number becomes large. The decay of LQU is universally in an exact exponential manner similar to that of entanglement already reported in [21] but with asymptotic vanishing. By contrast, the decay of relative entropy is merely polynomial, which illustrates that the quantum coherence can be more robust against atmospheric turbulence.

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