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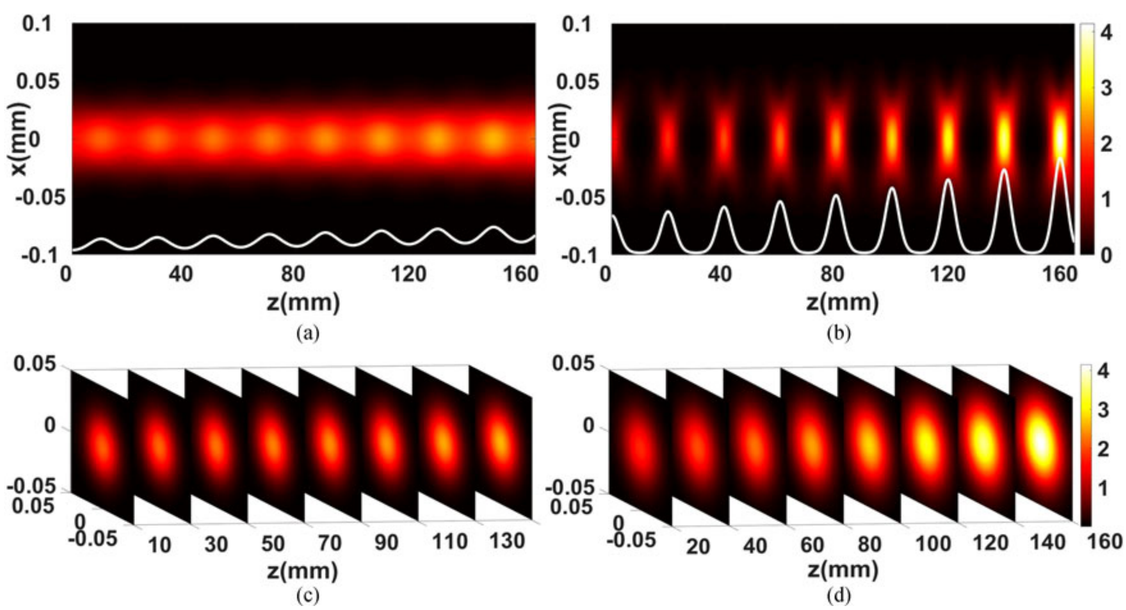

(a) (b) (c) (d)

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Propagation of a Vector Cosine-Gaussian Correlated Beam Through an Active GRIN Medium

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Abstract: This paper presents a convenient approach that uses the tensor method to study the propagation of a nonuniformly correlated electromagnetic Cosine-Gaussian Schell-model (ECGSM) beam through an active gradient-index (GRIN) medium. It is shown that the initial correlation structure has a significant influence on the intensity gain; an ECGSM beam ($n = 1$) experiences more gain than a conventional electromagnetic Gaussian Schell-model (EGSM) beam ($n = 0$). The inhomogeneous gain induced by the medium leads to a nonlinear modulation of beam parameters such as the spectral intensity, coherence, and polarization. Although the gain leads to a broadening of the beam width, the coherence gradually decreases, which yields a useful guideline for manipulating active GRIN materials to generate numerous partially coherent beams. It is found that the polarization ellipse in the center of a conventional EGSM beam gradually evolves into circular polarization. However, the state of polarization of a nonuniformly correlated ECGSM beam gradually changes into linear polarization. The ability to manipulate light beams using active GRIN materials opens an alternative avenue for the generation of complex vector beams, which promises important supports in the fields of waveguide amplifiers, beam shaping, optical sensors, and beam transformer devices.

Index Terms: Coherence and statistical optics, propagation.

1. Introduction

Coherence is an important fundamental of modern optics and exhibits extraordinary influence in both classical and quantum optics [1]. The interpretation of the coherence of a light field and its effects on the interaction between light and matter are important subjects in modern optics. It has been shown that coherence is intrinsically connected with the statistical properties and the statistical moments in terms of the state of polarization (SOP), beam quality and orbital angular momentum [2]–[9]. Gaussian correlated sources (i.e., Shell-model sources) have performed a very important role in optical coherence theory since they were introduced in 1961 [10]. The propagation properties of partially coherent beams generated from Shell-model sources are fairly clear [11]–[19]. Recently, Gori and Santarsiero introduced a sufficient condition for devising genuine spatial correlation functions based on the non-negative definiteness constraint of the cross-spectral density (CSD) [20]. A wide range of scalar partially coherent beams with non-Gaussian correlation functions are gaining popularity because of their nontrivial propagation features, such as their self-focusing and self-accelerating behavior, ring-shaped profile and creation of optical gratings/lattices [21]–[32]. Meanwhile, electromagnetic nonconventional correlation sources have been derived as a vector extension of the scalar random sources. The propagation properties of light beams generated from electromagnetic nonconventional correlation sources through optical systems and random media have been investigated [33]–[38]. Our recent study demonstrated that one can simultaneously manipulate the amplitude and polarization of light beams by modulating the source correlation structure [39].

For many years, the propagation of light beams through gradient-index (GRIN) media has attracted considerable attention. GRIN media have a refractive index that varies quadratically along the transverse direction. The extraordinary waveguide and imaging features of GRIN media have been closely connected to the enormous development of optical communications systems, integrated optics, and micro-optics [40]–[42]. Since Kogelnik first discussed the propagation properties of Gaussian beams in GRIN media, different methods to compute ray trajectories based on geometrical and wave optics have been explored [43]–[47]. Yariv analyzed the modal propagation of a Gaussian beam in GRIN fibers and applied it to the problem of image transmission [44]. Gomez-Reino investigated paraxial imaging, transforming transmission, and modal propagation in a GRIN rod lens, and found that the media can be represented by a transmittance function equivalent to the conventional lens transmittance function [46]–[48]. Ponomarenko demonstrated that partially coherent light beams of arbitrary intensity and spectral degree of coherence profiles can self-image in linear graded-index media [49]. A material that has both a quadratic gain or loss and a refractive index profile along the radial direction is known as an active GRIN medium. Propagation of light beams in this kind of media can be studied from expressions for passive GRIN media using a complex refractive index instead of a real one [43], [46]–[54]. Active GRIN media with gain or loss have been a subject of interest, because of their common existence in lasers material, laser damage material and Kerr media [48]. Propagation of light beams through active media has various applications in waveguide amplifiers, optical sensors, beam shaping and beam transformer devices [40], [48].

In the past decade, great efforts have been devoted to generating and propagating random electromagnetic beams based on the unified theory of coherence and polarization introduced by Wolf [2]. The nontrivial modulation characteristics of electromagnetic beams have important applications in free-space optical communications, optical imaging, optical tweezers, and remote sensing [55]–[59]. To the best of our knowledge, the transmission of electromagnetic beams with nonconventional correlation structures through an active GRIN medium with gain or loss has not been examined. In this work, we analyzed the vector characteristics of an electromagnetic Cosine-Gaussian Schell-model (ECGSM) beam with rectangular symmetry in an active GRIN medium with a complex refractive index using a tensor method. Propagation characteristics have been investigated in detail.

2. Propagation of an ECGSM Beam Through an Active GRIN Medium

In this section, our primary purpose is to characterize the transmission of an ECGSM beam in an active GRIN medium. Let us consider an active GRIN medium with rotational symmetry around the z-axis, limited by plane-parallel faces of thickness $z = d$, whose refractive index is given by a parabolic transverse gain or loss profile:

$$n(x, y) = n_0 \left(1 - \frac{g_0^2}{2}\mathbf{r^2}\right), \quad \mathbf{r^2} = x^2 + y^2, \tag{1}$$

where n_0 is the complex refractive index along the z-axis and g_0 is the complex transverse refractive index distribution parameter of the GRIN medium. Examples of such a medium include inhomogeneous crystals and SELFOC GRIN rod lenses. For an active GRIN medium with gain or loss, the refractive index is expressed in a complex form:

$$n_0 = n_{0r} + i n_{0i}, \quad g_0 = g_{0r} + i g_{0i}, \tag{2}$$

where n_{0r}, n_{0i}, g_r and g_i are real constants.

By submitting Eq. (2) into Eq. (1), the real and the imaginary parts of the complex refractive index can be written as:

$$n_r = n_{0r} - \Re \mathbf{r}^2, \; n_i = n_{0i} - \Im \mathbf{r}^2, \tag{3}$$

where

$$\Re = (n_{0r}/2)\left(g_{0r}^2 - g_{0i}^2\right) - n_{0i} g_{0r} g_{0i}, \tag{4a}$$

$$\Im = (n_{0i}/2)\left(g_{0r}^2 - g_{0i}^2\right) + n_{0r} g_{0r} g_{0i}. \tag{4b}$$

Based on Eqs. (3) and (4), an active GRIN medium with gain or loss can be classified[48]. The real part of the refractive index determines the guidance behavior of the active medium and the gain or loss is determined by the sign of the imaginary part of the refractive index. A medium with $n_i > 0$ corresponds to loss and $n_i < 0$ represents gain. On-axis loss or gain is obtained as $n_{0i} > 0$ and $n_i < 0$, respectively. Likewise, the sign of $\Im$ determines the stability condition. $\Im > 0$ and $\Im < 0$ are called the "unstable" and "stable" cases, respectively.

It is shown that any complex ray in an active GRIN medium can be written as a liner combination of complex and axial rays. The paraxial ray propagation is represented by the ABCD ray-transfer matrix as follows [54]:

$$\begin{pmatrix} \mathbf{r}(z) \\ n_0 \dot{\mathbf{r}}(z) \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{r}(0) \\ n_0 \dot{\mathbf{r}}(0) \end{pmatrix} = \begin{pmatrix} H_A(z) \cdot \mathbf{I} & \frac{H_B(z)}{n_0} \cdot \mathbf{I} \\ n_0 \dot{H}_A(z) \cdot \mathbf{I} & \dot{H}_B(z) \cdot \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{r}(0) \\ n_0 \dot{\mathbf{r}}(0) \end{pmatrix}, \tag{5}$$

where

$$H_A(z) = \cos(g_0 z), \; H_B(z) = \sin(g_0 z)/g_0, \tag{6a}$$

$$\dot{H}_A(z) = -g_0 \sin(g_0 z), \; \dot{H}_B(z) = \cos(g_0 z), \tag{6b}$$

and $\mathbf{I}$ is a 2×2 unit matrix. $H_A(z)$ and $H_B(z)$ are the positions of the complex field and the axial rays, respectively. $\dot{H}_A(z)$ and $\dot{H}_B(z)$ represent the slopes of the complex axial and the field rays, respectively.

In our case, the real and imaginary parts of the position and the slope of the complex axial and the field rays are written in terms of analytic continuation functions as:

$$H_{Ar}(z) = \dot{H}_{Br}(z) = \cos(g_{0r} z) \cos h(g_{0i} z), \tag{7a}$$

$$H_{Ai}(z) = \dot{H}_{Bi}(z) = -\sin(g_{0r} z) \sin h(g_{0i} z), \tag{7b}$$

$$\dot{H}_{Ar}(z) = -g_{0r} \sin(g_{0r} z) \cos h(g_{0i} z) + g_{0i} \cos(g_{0r} z) \sin h(g_{0i} z), \tag{7c}$$

$$\dot{H}_{Ai}(z) = -g_{0r} \cos(g_{0r} z) \sin h(g_{0i} z) - g_{0i} \sin(g_{0r} z) \cos h(g_{0i} z). \tag{7d}$$

$$H_{Br}(z) = \frac{1}{g_{0r}^2 + g_{0i}^2} \left[g_{0r} \sin(g_{0r} z) \cos h(g_{0i} z) + g_{0i} \cos(g_{0r} z) \sin h(g_{0i} z)\right], \tag{7e}$$

$$H_{Bi}(z) = \frac{1}{g_{0r}^2 + g_{0i}^2} \left[g_{0r} \cos(g_{0r} z) \sin h(g_{0i} z) - g_{0i} \sin(g_{0r} z) \cos h(g_{0i} z)\right]. \tag{7f}$$

Let us now consider the propagation of an ECGSM beam through an active GRIN medium. For a partially coherent electromagnetic beam in the space-frequency domain, the second-order spatial coherence properties of a statistically stationary light beam at any two points $\mathbf{r}_1$ and $\mathbf{r}_2$ may be characterized by the 2×2 CSD matrix $\overleftrightarrow{\mathbf{W}}(\mathbf{r}_1, \mathbf{r}_2)$ with elements $W^0_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) = \langle E^*_\alpha(\mathbf{r}_1) E_\beta(\mathbf{r}_2)\rangle$, $(\alpha, \beta = x, y)$. The angular brackets denote the ensemble average and the asterisk represents the complex conjugate. The elements of an ECGSM beam are given by:

$$W^0_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) = A_\alpha A_\beta B_{\alpha\beta} \exp\left[-\frac{\mathbf{r}_1^2}{4\sigma_\alpha^2} - \frac{\mathbf{r}_2^2}{4\sigma_\beta^2}\right] \exp\left[-\frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{2\delta^2_{\alpha\beta}}\right]$$
$$\times \cos\left[\frac{\sqrt{2\pi}n\,(x_1 - x_2)}{\delta_{\alpha\beta}}\right] \cos\left[\frac{\sqrt{2\pi}n\,(y_1 - y_2)}{\delta_{\alpha\beta}}\right], (\alpha, \beta = x, y), \qquad (8)$$

where A_α is the square root of the spectral density of the electric field component E_α; $B_{\alpha\beta} = |B_{\alpha\beta}| \exp(i\varphi_{\alpha\beta}) = B^*_{\beta\alpha}$ is the correlation coefficient between the E_α and E_β field components; both A_α and $B_{\alpha\beta}$ are independent of position but, in general, depend on the frequency; σ_a is the transverse beam size of the spectral density along the α direction; and $\delta_{\alpha\beta}$ is the spatial coherence width. $S = \sqrt{2\pi}n/\delta_{\alpha\beta}$ where n denotes the non-negative order parameter. For an electromagnetic beam, $A_\alpha, B_{\alpha\beta}, \varphi_{\alpha\beta}, \sigma_\alpha, \delta_{\alpha\beta}$ and n are shown to satisfy several intrinsic constraints (e.g., $\delta_{xy} = \delta_{yx}$, $B_{xx} = B_{yy} = 1$) [2]. For brevity, we do not show the dependence of the CSD on frequency.

For convenience of the theoretical analysis, we rewrite the elements of an ECGSM beam in the following tensor form:

$$W^0_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{S_1=\pm S} \sum_{S_2=\pm S} \frac{A_\alpha A_\beta B_{\alpha\beta}}{4} \exp\left[-\frac{ik}{2}\tilde{\mathbf{r}}^T \mathbf{M}^{-1}_{0\alpha\beta}\tilde{\mathbf{r}} + \frac{ik}{2}\tilde{\mathbf{r}}^T \mathbf{S}\right], \qquad (9)$$

with

$$\mathbf{M}^{-1}_{0\alpha\beta} = \begin{pmatrix} \left(-\frac{i}{2k\sigma_\alpha^2} - \frac{i}{k\delta^2_{\alpha\beta}}\right)\mathbf{I} & \frac{i}{k\delta^2_{\alpha\beta}}\mathbf{I} \\ \frac{i}{k\delta^2_{\alpha\beta}}\mathbf{I} & \left(-\frac{i}{2k\sigma_\beta^2} - \frac{i}{k\delta^2_{\alpha\beta}}\right)\mathbf{I} \end{pmatrix}, \mathbf{S} = \frac{2}{k}\left(S_1 \;\; S_2 \;\; -S_1 \;\; -S_2\right)^T. \qquad (10)$$

Here, T stands for the matrix transpose; $\tilde{\mathbf{r}}^T \equiv (\mathbf{r}_1^T \mathbf{r}_2^T) = (x_1 y_1 x_2 y_2)$; k is the wavenumber in free space; and $\mathbf{M}^{-1}_{0\alpha\beta}$ stands for the 4×4 generalized partially coherent complex curvature tensor.

Since the active GRIN medium in the paraxial domain can be regarded as an active first optical system, it is convenient to study the diffraction integral in terms of the elements of the ray matrix. For this medium, the paraxial propagation can be evaluated by means of a generalized Huygens-Fresnel integral. The elements of the CSD matrix in the output plane are expressed in the following form:

$$W^z_{\alpha\beta}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \frac{k^2}{4\pi^2[\det(\mathbf{B})]^{1/2}[\det(\mathbf{B}^*)]^{1/2}} \iint \iint W^0_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2)$$
$$\times \exp\left[-\frac{ik}{2}\left(\mathbf{r}_1^T \mathbf{B}^{-1}\mathbf{A}\mathbf{r}_1 - 2\mathbf{r}_1^T \mathbf{B}^{-1}\boldsymbol{\rho}_1 + \boldsymbol{\rho}_1^T \mathbf{D}\mathbf{B}^{-1}\boldsymbol{\rho}_1\right)\right]$$
$$\times \exp\left[\frac{ik}{2}\left(\mathbf{r}_2^T (\mathbf{B}^*)^{-1}\mathbf{A}^*\mathbf{r}_2 - 2\mathbf{r}_2^T (\mathbf{B}^*)^{-1}\boldsymbol{\rho}_2 + \boldsymbol{\rho}_2^T \mathbf{D}^*(\mathbf{B}^*)^{-1}\boldsymbol{\rho}_2\right)\right] d\mathbf{r}_1 d\mathbf{r}_2, \qquad (11)$$

where det stands for the determinant of a matrix; $W^z_{\alpha\beta}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ are the elements of the CSD matrix in the output plane; $\boldsymbol{\rho}^T \equiv (\boldsymbol{\rho}_1^T \boldsymbol{\rho}_2^T) = (u_1 v_1 u_2 v_2)$ is the position vector in the output plane; and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are the 2×2 sub-matrices of the optical system.

On substituting from Eqs. (5)–(10) into Eq. (11), one can obtain the following expressions for the elements of the CSD matrix of an ECGSM beam at the output plane:

$$W_{\alpha\beta}^{z}(\tilde{\boldsymbol{\rho}}) = \sum_{S_1=\pm S}\sum_{S_2=\pm S} \frac{A_\alpha A_\beta B_{\alpha\beta}}{4\left[\det\left(\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{A}}\right)\right]^{1/2}} exp\left[\frac{ik}{8}\mathbf{S}^T\left(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}\right)^{-1}\mathbf{S}\right]$$
$$\times\ \exp\left[-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\mathbf{M}_{z\alpha\beta}^{-1}\tilde{\boldsymbol{\rho}}+\frac{ik}{2}\mathbf{S}^T\left(\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{A}}\right)^{-1}\tilde{\boldsymbol{\rho}}\right], \tag{12}$$

where $\mathbf{M}_{z\alpha\beta}^{-1}$ represents the generalized partially coherent complex curvature tensor in the output plane

$$\mathbf{M}_{z\alpha\beta}^{-1} = \left(\tilde{\mathbf{C}}+\tilde{\mathbf{D}}\mathbf{M}_{0\alpha\beta}^{-1}\right)\left(\tilde{\mathbf{A}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}\right)^{-1}, \tag{13}$$

and $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$, $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{D}}$ are the 4×4 ray-transfer matrices of the optical system

$$\tilde{\mathbf{A}} = \begin{pmatrix}\mathbf{A} & 0\mathbf{I}\\ 0\mathbf{I} & \mathbf{A}^*\end{pmatrix},\quad \tilde{\mathbf{B}} = \begin{pmatrix}\mathbf{B} & 0\mathbf{I}\\ 0\mathbf{I} & -\mathbf{B}^*\end{pmatrix},\quad \tilde{\mathbf{C}} = \begin{pmatrix}\mathbf{C} & 0\mathbf{I}\\ 0\mathbf{I} & -\mathbf{C}^*\end{pmatrix},\quad \tilde{\mathbf{D}} = \begin{pmatrix}\mathbf{D} & 0\mathbf{I}\\ 0\mathbf{I} & \mathbf{D}^*\end{pmatrix}. \tag{14}$$

Equation (12) is the main result in this paper and can be used for evaluating the propagation characteristics of an ECGSM beam through an active GRIN medium. The spectral intensity and the degree of polarization (DOP) at point $\boldsymbol{\rho}$ in the output plane are defined as follows:

$$I^z(\boldsymbol{\rho}) = W_{xx}^z(\boldsymbol{\rho},\boldsymbol{\rho}) + W_{yy}^z(\boldsymbol{\rho},\boldsymbol{\rho}), \tag{15}$$

$$P(\boldsymbol{\rho}) = \sqrt{1-\frac{4\det\overleftrightarrow{\mathbf{W}}(\boldsymbol{\rho},\boldsymbol{\rho})}{\left\{\mathrm{Tr}\left[\overleftrightarrow{\mathbf{W}}(\boldsymbol{\rho},\boldsymbol{\rho})\right]\right\}^2}}, \tag{16}$$

where Tr denotes the trace of the matrix. The degree of coherence (DOC) of a random electromagnetic beam in any transverse plane can be evaluated by the equation [2]

$$\mu(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \frac{\mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)}{\sqrt{\mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_1)\,\mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho}_2,\boldsymbol{\rho}_2)}}. \tag{17}$$

The behavior of the polarization ellipse of a beam is an alternative representation of the SOP. It is shown that a partially coherent electromagnetic beam can be decomposed into a superposition of a completely polarized portion and a completely unpolarized portion. The polarization ellipse is a parameter characterizing the fully polarized portion of an electromagnetic beam and can be measured with the help of the degree of ellipticity ε and orientation angle θ [7]

$$\theta(\boldsymbol{\rho}) = \frac{1}{2}\arctan\left[\frac{2\mathrm{Re}W_{xy}(\boldsymbol{\rho},\boldsymbol{\rho})}{W_{xx}(\boldsymbol{\rho},\boldsymbol{\rho})-W_{yy}(\boldsymbol{\rho},\boldsymbol{\rho})}\right],\ -\pi/2\le\theta\le\pi/2 \tag{18a}$$

$$\varepsilon = A_{\min}(\boldsymbol{\rho},\boldsymbol{\rho})/A_{\mathrm{maj}}(\boldsymbol{\rho},\boldsymbol{\rho}),\ 0\le\varepsilon\le 1, \tag{18b}$$

where A_{maj} and $A_{\min}$ stand for the major and minor semi-axes, respectively, of the polarization ellipse, and take the form:

$$A_{\substack{\mathrm{maj}\\ \mathrm{min}}}(\boldsymbol{\rho}) = \frac{1}{\sqrt{2}}\left\{\sqrt{\left[W_{xx}(\boldsymbol{\rho},\boldsymbol{\rho})-W_{yy}(\boldsymbol{\rho},\boldsymbol{\rho})\right]^2+4\left|W_{xy}(\boldsymbol{\rho},\boldsymbol{\rho})\right|^2}\right.$$
$$\left.\pm\sqrt{\left[W_{xx}(\boldsymbol{\rho},\boldsymbol{\rho})-W_{yy}(\boldsymbol{\rho},\boldsymbol{\rho})\right]^2+4\left(\mathrm{Re}\left[W_{xy}(\boldsymbol{\rho},\boldsymbol{\rho})\right]\right)^2}\right\}^{\frac{1}{2}}. \tag{19}$$

In Eq. (19), signs "+" and "−" between the two square roots correspond to A_{maj} and $A_{\min}$, respectively. By substituting Eq. (11) into Eqs. (17) and (18), one can determine the behaviors of the parameters of the polarization ellipse through an active GRIN medium. Equation (18) shows that

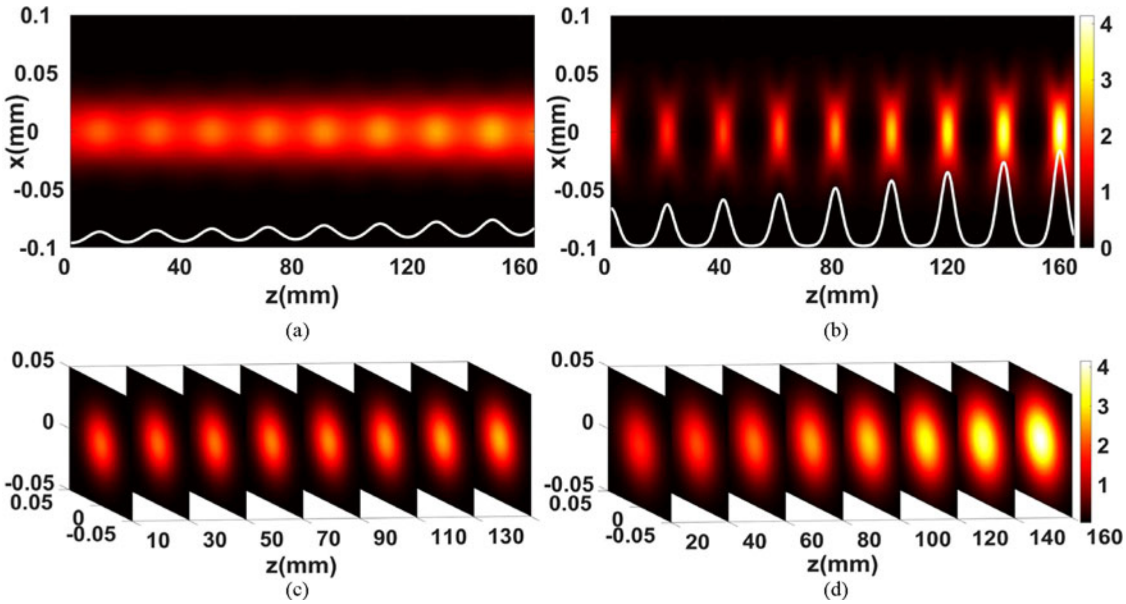

Fig. 1. Normalized intensity distributions of an ECGSM beam propagating through an active GRIN medium for different values of the order parameter n. (a), (c): $n = 0$, (b), (d) $n = 1$. The transverse intensity distributions shown in (c) and (d) at different distances correspond to (a) and (b), respectively.

the minimum value $\varepsilon = 0$ denotes linear polarization and the maximum value $\varepsilon = 1$ corresponds to circular polarization; otherwise denotes elliptical polarization. Under certain conditions, the degree of ellipticity remains invariant under propagation [2], [7].

3. Numerical Examples

In this section, we provide an intuitive numerical analysis of an ECGSM beam propagating through an active GRIN medium. The initial beam parameters are set as: $A_x = 1$, $A_y = 0.8$, $|B_{xy}| = 0.3$, $\varphi_{xy} = \pi/3$, $\lambda = 632.8$ nm, $\sigma_x = \sigma_y = 0.02$ mm, $\delta_{xx} = 0.025$ mm, $\delta_{yy} = 0.03$ mm, $\delta_{xy} = 0.035$ mm. Since the gain or loss is determined by the sign of the imaginary part of the refractive index, let us consider an active medium with gain. The relative values of the parameters are given as: $n_{0\mathrm{r}} = 1.62$, $n_{0\mathrm{i}} = -10^{-5}$, $g_{0\mathrm{r}} = 0.1571352$, $g_{0\mathrm{i}} = -0.000554$. In order to clearly show the gain characteristics of the ECGSM beam on propagation through an active GRIN medium, the intensity distributions for different n along the x-z plane are depicted in Fig. 1(a) and (b). Meanwhile, the lateral or transverse intensity profiles at different propagation distances are plotted in Fig. 1(c) and (d), which correspond to Fig. 1(a) and (b), respectively. For the case of $n = 0$ in Fig. 1(a) and (c), the ECGSM beam reduces to a conventional electromagnetic Gaussian Schell-model (EGSM) beam [36]. One finds that the intensity gain is greatly affected by the order parameter n, and as n increases, the gain of the intensity is significantly improved. Thus, one can infer that higher gain of the output intensity can be achieved by modulating the initial correlation structure. Moreover, it is seen that the medium not only leads to a significant gain in light intensity but also broadens the beam width upon propagation. Furthermore, the gain on-axis is not linear, and a higher gain is obtained on-axis. It is clear that the distance between the adjacent maximum intensity peaks is invariant in Fig. 1(a) and (b), which implies the period still exists and is independent of the beam parameters and the gain induced by the medium [48].

In order to further illustrate the gain characteristics of an ECGSM beam propagating through an active GRIN medium, Fig. 2(a) and (b) display the modulus of the DOC along the x-z plane compared with an on-axis point for different order parameters n. Obviously, the periodicity of the DOC still exists. In order to clearly show the DOC, the modulus of the lateral DOC at different

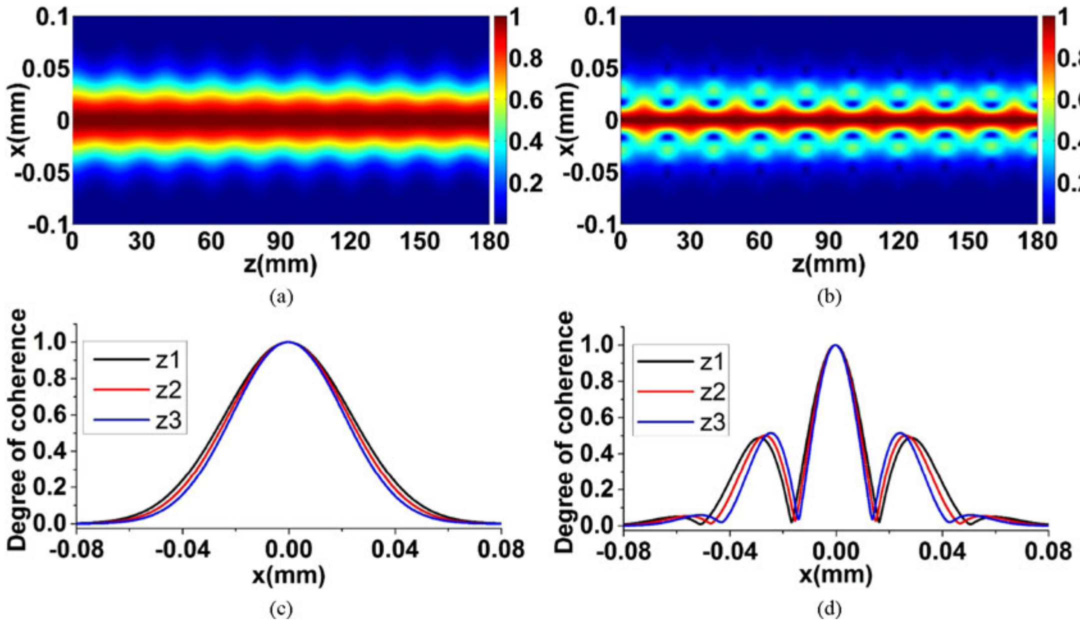

Fig. 2. Modulus of the DOC distributions of an ECGSM beam propagating through an active GRIN medium for different values of order parameter n. (a), (c): $n = 0$, (b), (d) $n = 1$. The modulus of the transverse DOC distributions shown in (c) and (d) at different propagation distances correspond to (a) and (b), respectively. (c), (d): z1 = 10.4 mm, z2 = 50.4 mm, z3 = 90.4 mm.

propagation distances is depicted in Fig. 2(c) and (d). It can be observed that the profile of the DOC of an ECGSM beam maintains its initial shape on propagation through an active GRIN medium, and gradually narrows as the transmission distance increases. Moreover, it is worth noting that the gain of the coherence is nonuniform and the coherence of the beam decreases as the propagation distance increases. The reason is that the lateral gain of the medium is nonuniform and a lower gain is observed on the edge of the medium. Comparing Figs. 1 and 2, although the beam width is spreading during the propagation, the coherence gradually decreases, which is quite a different response compared to the normal diffraction of partially coherent beams [2]. This may be useful for generating various partially coherent beams by taking advantage of an active GRIN medium. For further study of the coherence, Fig. 3 illustrates the contour plots of the modulus of the lateral DOC at different propagation distances in one period for different n. For the case of $n = 0$, the modulus of the DOC of a conventional EGSM beam always has circular symmetry when propagating, although its shape changes during propagation as expected [36]. However, for the case of $n = 1$, the initial DOC of an ECGSM beam displays quite different propagation properties; the initial Gaussian profile gradually disappears during propagation and evolves into different shapes such as a rhombus profile and an array profile at different distances. This demonstrates that the DOC of an ECGSM beam in an active GRIN medium is influenced the most by the initial correlation function.

For vector beams, it is important to know the evolution of the polarization properties including the DOP and the SOP. Fig. 4(a) and (b) depict the contour plots of the DOP of an ECGSM beam in an active GRIN medium along the x-z plane for different order parameter n. Because of the very weak edge intensity, we have neglected the edge DOP to show a fine valuable polarization structure. Different from the on-axis DOC, which is a constant, one finds that the on-axis DOP of a conventional EGSM gradually increases as the transmission distance increases. However, for a nonuniformly correlated ECGSM beam, the on-axis DOP slowly decreases. The depicted lateral DOPs in Fig. 4(c) and (d) illustrate that the influence of the gain medium on the DOP grows as the transverse distance increases. By comparing Fig. 4(c) and (d), one finds that the gain medium has greater influence on a nonuniformly correlated ECGSM beam than on a conventional ECGSM beam. To further illustrate the properties of polarization, the SOP distributions along the

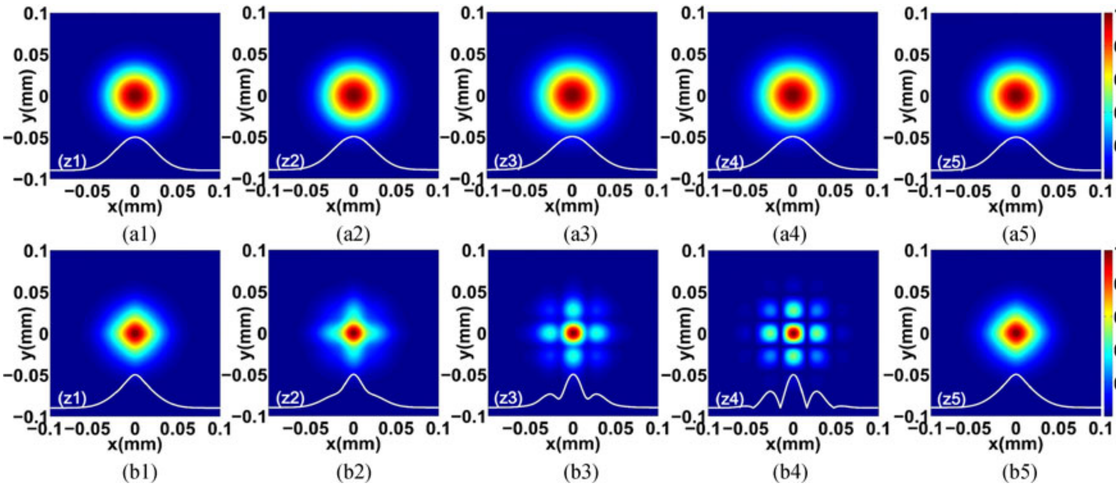

Fig. 3. Contour plots of the modulus of the DOC distributions of an ECGSM beam propagating through an active GRIN medium along the x-y plane for different values of parameter n. (a1)–(a5): $n = 0$, (b1)–(b5) $n = 1$. z1 = 30 mm, z2 = 34 mm, z3 = 37 mm, z4 = 40 mm, and z5 = 50 mm.

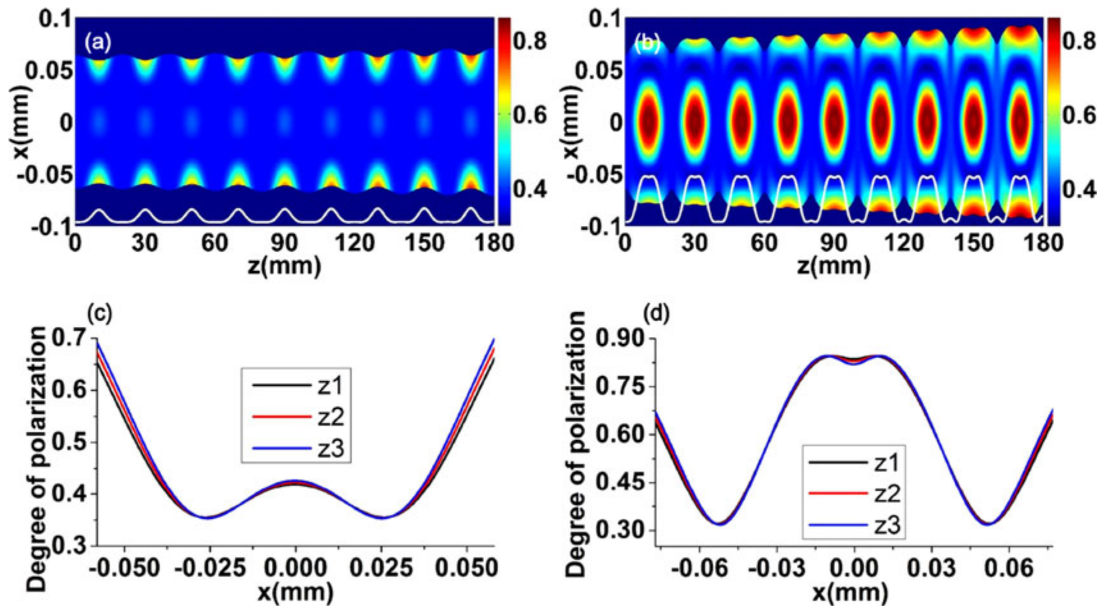

Fig. 4. DOP distributions of an ECGSM beam during propagation through an active GRIN medium for different values of order parameter n. (a), (c): $n = 0$, (b), (d) $n = 1$. The modulus of the transverse DOP distributions shown in (c) and (d) at different propagation distances correspond to (a) and (b), respectively.

transverse plane at different propagation distances for different values of order parameter n are depicted in Fig. 5. It is shown that the elliptical polarization in the center of a conventional EGSM beam gradually evolves into circular polarization (see Figs. 5(a1)–(a4)). However, for a nonuniformly correlated ECGSM beam, the elliptical polarization gradually changes into linear polarization, and the distance between each beamlet increases (see Fig. 5(b1)–5(b4)). This implies that we can modulate the amplitude and the SOP not only by using a correlation structure design [36], [39] but also by using an active GRIN medium.

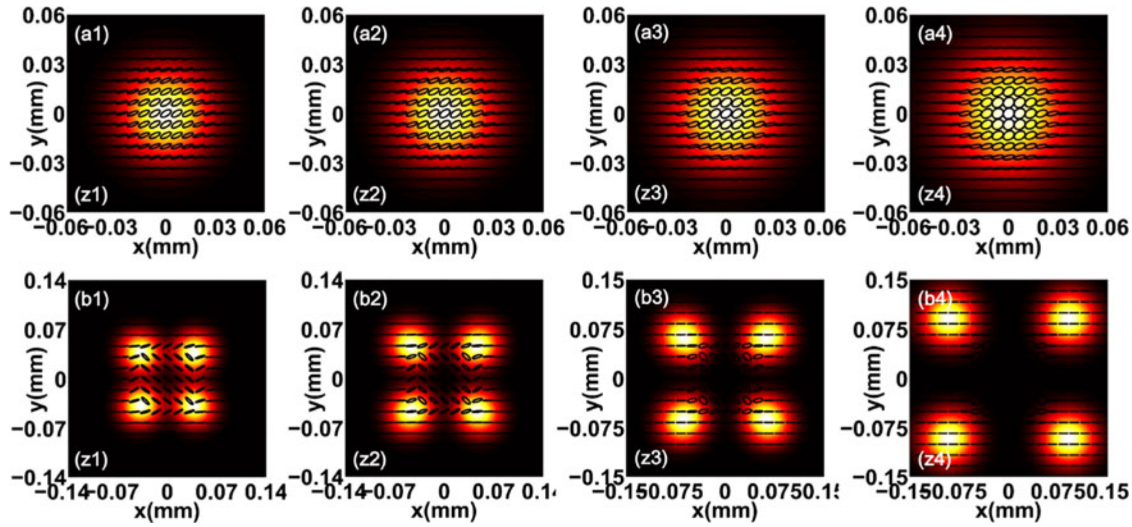

Fig. 5. Contour plots of the SOP of an ECGSM beam during propagation through an active GRIN medium along the x-y plane at different propagation distances for different values of order parameter n with parameter values z1 = 30 mm, z2 = 230 mm, z3 = 430 mm, and z4 = 630 mm.

4. Conclusion

In conclusion, we have demonstrated a convenient approach for studying the propagation of an ECGSM beam through a stigmatic ABCD optical system by means of a tensor method. The statistical properties in an active GRIN medium with a complex refractive index have been studied numerically. One finds that the intensity gain is greatly affected by the source correlation structure and a nonuniformly correlated ECGSM beam experiences more gain than a conventional EGSM beam. Although the gain broadens the beam width during propagation, the coherence gradually decreases, which yields a useful guideline for using an adjustable active GRIN medium to generate various partially coherent beams. Moreover, due to the inhomogeneous lateral complex refractive index distribution, the intensity gain is nonlinear, and lower gain is observed at the edge of the medium. Since the lateral gain is not linear and gradually decreases as the off-axis distance increases, the lateral profile of the DOC gradually narrows as the transmission distance increases. Furthermore, the elliptical polarization in the center of a conventional EGSM beam gradually evolves into circular polarization. However, the SOP of a nonuniformly correlated ECGSM beam gradually changes into linear polarization, and the distance between each beamlet increases. The ability to modulate light beams using an active medium might open an alternative avenue for generating complex vector beams such as high-order vector beams or Airy vector beams, which promises important supports in the fields of waveguide amplifiers, beam shaping, optical sensors, and beam transformer devices.

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