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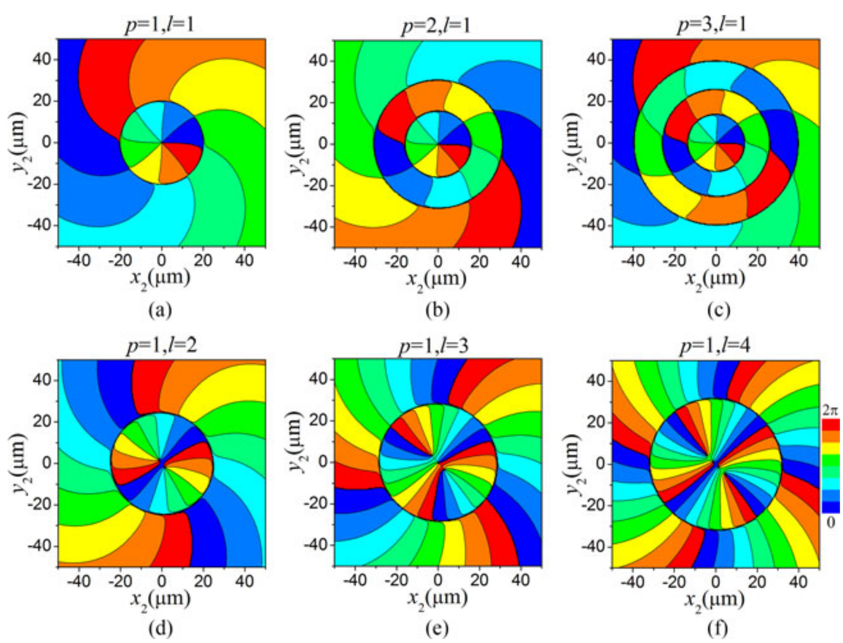

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Propagation of Correlation Singularities of a Partially Coherent Laguerre–Gaussian Electromagnetic Beam in a Uniaxial Crystal

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Abstract: The correlation singularities of a partially coherent Laguerre–Gaussian (LG) electromagnetic beam in a uniaxial crystal is shown to possess interesting evolution behavior. The analytical formula for the cross-spectral density matrix of a partially coherent LG electromagnetic beam propagating in a uniaxial crystal orthogonal to the optical axis is derived. It is found that the correlation singularities of a partially coherent LG electromagnetic beam in a uniaxial crystal is closely determined by the spatial coherence and mode orders of the beam, and the refractive indexes of the crystal. For a partially coherent LG electromagnetic beam, the correlation singularities can propagate further in a uniaxial crystal than that in isotropic medium. For a fully coherent LG electromagnetic beam, in isotropic medium the central correlation singularities and circular edge dislocations persist upon propagation; in a uniaxial crystal the circular edge dislocations become imperfect, and the central correlation singularities with topological charges $l \neq 1$ split up into correlation singularities with topological charges equaling 1.

Index Terms: Coherence and statistical optics, singular optics, crystal optics, propagation.

1. Introduction

In recent years, phase singularities in optical fields have attracted much interest since the pioneering work of Nye and Berry [1]. Phase singularities occur at positions where the field amplitude is identically zero and hence the phase becomes indeterminate. Along a closed counterclockwise path enclosing a phase singularity, the phase varies from zero to an integer multiple of 2π, where the integer is called topological charge. A typical example of phase singularities presents in a LG beam with azimuthal mode index $l \neq 0$. The vast majority of the research on phase singularities

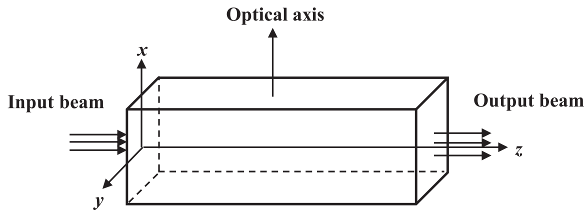

Fig. 1. Geometry of the propagation of a laser beam in a uniaxial crystal orthogonal to the optical axis.

has been restricted to fully coherent beams where phase is well defined [2]–[4]. For partially coherent beams where phase is ill defined, no phase singularities of intensity exist [5]–[8]. In 2003 Schouten *et al.* first extended the field of singular optics to the realm of correlation functions [9]. Correlation singularities occur at pairs of points for which the spectral degree of coherence of the field vanishes. The phase of the spectral degree of coherence is shown to posses a vortex structure around correlation singularities, and a new term coherence vortices has been introduced to refer to them [7]. Much research on the behavior of correlation singularities has been done analytically and experimentally [7]–[29]. However, these studies are all restricted to correlation singularities in scalar fields. More recently, the research on correlation singularities was extended from scalar field to vector field. Raghunathan *et al.* demonstrated that correlation singularities generally occur in partially coherent electromagnetic beams, and studied the evolution behavior of correlation singularities of an electromagnetic Gaussian Schell-model beam in free space [30], [31]. The evolution behavior of correlation singularities of a partially coherent electromagnetic beam with initially radial polarization in free space and turbulent atmosphere has also been explored in [32], [33]. Though the correlation singularities of scalar partially coherent LG beams have been studied extensively [8], [10], [14]–[19], [27], [28] (In fact, partially coherent vortex beams can be regarded as special cases of partially coherent LG beams.), the existence of correlation singularities in partially coherent LG electromagnetic beams has not been studied.

On the other hand, investigation of the propagation of optical fields in anisotropic medium plays an important role for designing polarizers, compensators, amplitude- and phase-modulation devices [34]–[48]. Recently, the statistical properties such as the spectral density, the degree of coherence, and the degree of polarization of partially coherent electromagnetic beams propagating in a uniaxial crystal orthogonal to the optical axis have attracted much attention [42]–[48]. However, correlation singularities of partially coherent electromagnetic beams in a uniaxial crystal have not been reported. In this paper, our aim is to study the propagation of correlation singularities of a partially coherent LG electromagnetic beam in a uniaxial crystal. The evolution behavior of correlation singularities of a partially coherent LG electromagnetic beam propagating in a uniaxial crystal orthogonal to the optical axis is illustrated in detail. Some interesting and useful results are found.

2. Analytical Formula for the Cross-Spectral Density Matrix of a Partially Coherent LG Electromagnetic Beam in a Uniaxial Crystal

The geometry of the propagation of a laser beam in a uniaxial crystal orthogonal to the optical axis is shown in Fig. 1. The optical axis of the crystal coincides with the x axis, and the dielectric tensor of the crystal can be expressed as

$$\varepsilon = \begin{pmatrix} n_e^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_o^2 \end{pmatrix}, \tag{1}$$

where n_e and n_o are the ordinary and extraordinary refractive indexes, respectively. We assume that a partially coherent electromagnetic beam propagating along the z axis is incident on a uniaxial crystal at the plane $z = 0$. Within the validity of the paraxial approximation, the elements of the electric field at the output plane z inside the uniaxial crystal have the form [36]

$$\begin{aligned}
E_x(x,y,z) &= \frac{kn_o \exp(ikn_e z)}{2\pi iz}\int dx_0 dy_0 E_x(x_0,y_0,0) \\
&\quad\times \exp\left\{-\frac{k}{2izn_e}\left[n_o^2(x-x_0)^2+n_e^2(y-y_0)^2\right]\right\}, \\
E_y(x,y,z) &= \frac{kn_o \exp(ikn_o z)}{2\pi iz}\int dx_0 dy_0 E_y(x_0,y_0,0) \\
&\quad\times \exp\left\{-\frac{kn_o}{2iz}\left[(x-x_0)^2+(y-y_0)^2\right]\right\}, \qquad (2)
\end{aligned}$$

where $E_x(x_0, y_0, 0)$ and $E_y(x_0, y_0, 0)$ denotes the elements of the electric field at the input plane $z = 0$. (x_0, y_0) and (x, y) are the position coordinates at the input and output planes, respectively. $k = 2\pi/\lambda$ is the wave number with λ being the wavelength.

Based on the unified theory of coherence and polarization, the second order correlation properties of a paraxial partially coherent electromagnetic beam in space-frequency domain can be characterized by the cross-spectral density matrix of the electric field. We may write the elements of the cross-spectral density matrix at the input and output planes as [5]

$$\begin{aligned}
W_{\alpha\beta}(x_{01},y_{01},x_{02},y_{02},0) &= \left\langle E_\alpha^*(x_{01},y_{01},0)E_\beta(x_{02},y_{02},0)\right\rangle, \\
W_{\alpha\beta}(x_1,y_1,x_2,y_2,z) &= \left\langle E_\alpha^*(x_1,y_1,z)E_\beta(x_2,y_2,z)\right\rangle \ (\alpha,\beta=x,y), \qquad (3)
\end{aligned}$$

where $\langle\rangle$ denotes the ensemble average and "$*$" is the complex conjugate. Using (2) and (3), the main diagonal elements of the cross-spectral density matrix of a partially coherent electromagnetic beam in a uniaxial crystal are given by the expressions [42]–[48]

$$\begin{aligned}
W_{xx}(x_1,y_1,x_2,y_2,z) &= \frac{k^2n_o^2}{4\pi^2z^2}\int dx_{01}dx_{02}dy_{01}dy_{02}W_{xx}(x_{01},y_{01},x_{02},y_{02},0) \\
&\quad\times \exp\left\{\frac{k}{2izn_e}\left[n_o^2(x_1-x_{01})^2-n_o^2(x_2-x_{02})^2\right.\right. \\
&\quad\left.\left.+\ n_e^2(y_1-y_{01})^2-n_e^2(y_2-y_{02})^2\right]\right\},
\end{aligned}$$

$$\begin{aligned}
W_{yy}(x_1,y_1,x_2,y_2,z) &= \frac{k^2n_o^2}{4\pi^2z^2}\int dx_{01}dx_{02}dy_{01}dy_{02}W_{yy}(x_{01},y_{01},x_{02},y_{02},0) \\
&\quad\times \exp\left\{\frac{kn_o}{2iz}\left[(x_1-x_{01})^2-(x_2-x_{02})^2\right.\right. \\
&\quad\left.\left.+\ (y_1-y_{01})^2-(y_2-y_{02})^2\right]\right\}. \qquad (4)
\end{aligned}$$

The electric field of a fully coherent LG electromagnetic beam at the plane $z = 0$ is expressed as [49], [50]

$$\begin{aligned}
\vec{E} &= E_\alpha(r,\varphi,0)\vec{e}_\alpha + E_\beta(r,\varphi,0)\vec{e}_\beta, \\
E_\alpha(r,\varphi,0) &= A_\alpha\left(\frac{\sqrt{2}r}{w_0}\right)^l L_p^l\left(\frac{2r^2}{w_0^2}\right)\exp\left(-\frac{r^2}{w_0^2}\right)\exp(il\varphi), \\
E_\beta(r,\varphi,0) &= A_\beta\left(\frac{\sqrt{2}r}{w_0}\right)^l L_p^l\left(\frac{2r^2}{w_0^2}\right)\exp\left(-\frac{r^2}{w_0^2}\right)\exp(il\varphi). \qquad (5)
\end{aligned}$$

Now we extend the coherent LG electromagnetic beam to the partially coherent case. For a partially coherent beam which is generated by a Schell-model source, the spectral degree of coherence depends only on the difference in transverse coordinates. Thus the elements of the cross-spectral density matrix of a partially coherent LG electromagnetic beam generated by a Schell-model source can be defined in the following form

$$W_{\alpha\beta}(x_{01}, y_{01}, x_{02}, y_{02}, 0) = \sqrt{I_{\alpha\beta}(x_{01}, y_{01}, 0)}\sqrt{I_{\alpha\beta}(x_{02}, y_{02}, 0)} \times B_{\alpha\beta}\exp\{-[(x_{01}-x_{02})^2+(y_{01}-y_{02})^2]/2\delta_{\alpha\beta}^2\}\ (\alpha,\beta = x, y), \quad (6)$$

where $I_{\alpha\beta}(x_{01}, y_{01}, 0)$ and $I_{\alpha\beta}(x_{02}, y_{02}, 0)$ are the intensities, and $B_{\alpha\beta}\exp\{-[(x_{01}-x_{02})^2+(y_{01}-y_{02})^2]/2\delta_{\alpha\beta}^2\}$ is the spectral degree of coherence. Substituting (5) into (6), and applying the expansion formula $\exp(il\varphi)\rho^l L_p^l(\rho^2) = \frac{(-1)^p}{2^{2p+l}p!}\sum_{m=0}^{p}\sum_{n=0}^{l} i^n \binom{p}{m}\binom{l}{n} H_{2m+l-n}(x)H_{2p-2m+n}(y)$ [51], the elements of the cross-spectral density matrix of a partially coherent LG electromagnetic beam at $Z = 0$ can be expressed as

$$\begin{aligned} W_{\alpha\beta}(x_{01}, y_{01}, x_{02}, y_{02}, 0) &= \frac{A_\alpha A_\beta B_{\alpha\beta}}{2^{4p+2l}(p!)^2}\sum_{m=0}^{p}\sum_{n=0}^{l}\sum_{h=0}^{p}\sum_{s=0}^{l}(i^n)^* i^s\binom{p}{m}\binom{l}{n}\binom{p}{h}\binom{l}{s} \\ &\times H_{2m+l-n}\left(\frac{\sqrt{2}x_{01}}{w_0}\right)H_{2h+l-s}\left(\frac{\sqrt{2}x_{02}}{w_0}\right)\exp\left(-\frac{x_{01}^2+x_{02}^2}{w_0^2}\right)\exp\left(-\frac{(x_{01}-x_{02})^2}{2\delta_{\alpha\beta}^2}\right) \\ &\times H_{2p-2m+n}\left(\frac{\sqrt{2}y_{01}}{w_0}\right)H_{2p-2h+s}\left(\frac{\sqrt{2}y_{02}}{w_0}\right)\exp\left(-\frac{y_{01}^2+y_{02}^2}{w_0^2}\right)\exp\left(-\frac{(y_{01}-y_{02})^2}{2\delta_{\alpha\beta}^2}\right) \\ &(\alpha,\beta = x, y), \end{aligned} \quad (7)$$

where A_α is the square root of the spectral density of electric field components E_α, $B_{\alpha\beta} = |B_{\alpha\beta}|\exp(i\theta)$ is the correlation coefficient between E_x and E_y field components, θ is the phase difference between x and y components of the field and is removable in most cases. L_p^l denotes the Laguerre polynomial with mode orders p and l, H_m denotes the Hermite polynomial of order m, and $\binom{p}{m}$ and $\binom{l}{n}$ are binomial coefficients. w_0 is the waist size of the fundamental Gaussian mode, δ_{xx}, δ_{yy} and δ_{xy} are the widths of autocorrelation functions of x component of the field, of y component of the field, and of the mutual correlation function of x and y field components, respectively.

Substituting (7) into (4), the main diagonal elements of the cross-spectral density matrix of a partially coherent LG electromagnetic beam in a uniaxial crystal have the form

$$\begin{aligned} W_{xx}(x_1, y_1, x_2, y_2, z) &= \frac{k^2 n_o^2}{4\pi^2 z^2}\frac{A_x^2}{2^{5p+5l/2}(p!)^2}\frac{\pi^2}{\sqrt{M_1 M_2 N_1 N_2}} \\ &\left(1-\frac{2}{w_0^2 M_1}\right)^{l/2}\left(1-\frac{2}{w_0^2 N_1}\right)^{p}\left(\frac{\sqrt{2}}{iw_0\sqrt{M_2}}\right)^{l}\left(\frac{\sqrt{2}}{iw_0\sqrt{N_2}}\right)^{2p} \\ &\times \exp\left[\frac{ik}{2B_1}x_2^2-\frac{ik}{2B_1}x_1^2\right]\exp\left(-\frac{k^2 x_2^2}{4M_1 B_1^2}\right)\exp\left[-\frac{k^2}{4M_2}\left(\frac{x_1}{B_1}-\frac{x_2}{2M_1 B_1 \sigma_{xx}^2}\right)^2\right] \\ &\times \exp\left[-\frac{ik}{2B_2}y_1^2+\frac{ik}{2B_2}y_2^2\right]\exp\left[-\frac{k^2 y_2^2}{4N_1 B_2^2}\right]\exp\left[\frac{-k^2}{4N_2}\left(\frac{y_1}{B_2}-\frac{y_2}{2N_1 B_2 \sigma_{xx}^2}\right)^2\right] \end{aligned}$$

$$\times \sum_{m=0}^{p}\sum_{n=0}^{l}\sum_{h=0}^{p}\sum_{s=0}^{l}\sum_{d=0}^{[(2m+l-n)/2]}\sum_{e=0}^{[(2p-2m+n)/2]}\sum_{f=0}^{2h+l-s}\sum_{f_1=0}^{[f/2]}\sum_{g=0}^{2p-2h+s}\sum_{g_1=0}^{[g/2]}(-1)^{n+d+e+f_1+g_1}$$

$$\times 2^{2d+2e-f-g+2f_1+2g_1} i^{n+s+2d+2e-f-g+2f_1+2g_1}\binom{p}{m}\binom{l}{n}\binom{p}{h}\binom{l}{s}$$

$$\times \left(\frac{1}{\sqrt{M_2}}\right)^{f-2f_1}\left(\frac{2\sqrt{2}}{\sqrt{M_2}w_0}\right)^{2m-n-2d}\frac{(2m+l-n)!}{d!(2m+l-n-2d)!}\frac{(2h+l-s)!}{f!(2h+l-s-f)!}$$

$$\times \left(\frac{1}{\sqrt{N_2}}\right)^{g-2g_1}\left(\frac{2\sqrt{2}}{\sqrt{N_2}w_0}\right)^{-2m+n-2e}\frac{(2p-2m+n)!}{e!(2p-2m+n-2e)!}\frac{(2p-2h+s)!}{g!(2p-2h+s-g)!}$$

$$\times \left(1-\frac{2}{w_0^2M_1}\right)^{(2h-s)/2}\left(1-\frac{2}{w_0^2N_1}\right)^{(-2h+s)/2}\frac{f!}{f_1!(f-2f_1)!}\frac{g!}{g_1!(g-2g_1)!}$$

$$\times H_{2h+l-s-f}\left(\frac{-i\sqrt{2}kx_2}{\sqrt{2}B_1\left(M_1^2w_0^2-2M_1\right)^{1/2}}\right)H_{2p-2h+s-g}\left(\frac{-i\sqrt{2}ky_2}{\sqrt{2}B_2\left(N_1^2w_0^2-2N_1\right)^{1/2}}\right)$$

$$\times \left(\frac{2}{\sigma_{xx}^2\left(M_1^2w_0^2-2M_1\right)^{1/2}}\right)^{f-2f_1} H_{2m+l-n-2d+f-2f_1}\left(\frac{kx_2}{4M_1\sqrt{M_2}\sigma_{xx}^2B_1}-\frac{kx_1}{2\sqrt{M_2}B_1}\right)$$

$$\times \left(\frac{\sqrt{2}q}{\sigma_{xx}^2\left(N_1^2w_0^2-2N_1\right)^{1/2}}\right)^{g-2g_1} H_{2p-2m+n-2e+g-2g_1}\left(\frac{ky_2}{4N_1\sqrt{N_2}B_2\sigma_{xx}^2}-\frac{ky_1}{2\sqrt{N_2}B_2}\right)$$

$$W_{yy}(x_1,y_1,x_2,y_2,z)=\frac{k^2n_0^2}{4\pi^2z^2}\frac{A_y^2}{2^{5p+5l/2}(p!)^2}\frac{\pi^2}{L_1L_2}\left(1-\frac{2}{w_0^2L_1}\right)^{(2p+l)/2}\left(\frac{\sqrt{2}}{iw_0\sqrt{L_2}}\right)^{2p+l}$$

$$\times \exp\left[\frac{ik}{2B_3}x_2^2-\frac{ik}{2B_3}x_1^2\right]\exp\left(-\frac{k^2x_2^2}{4L_1B_3^2}\right)\exp\left[-\frac{k^2}{4L_2}\left(\frac{x_1}{B_3}-\frac{x_2}{2L_1B_3\sigma_{yy}^2}\right)^2\right]$$

$$\times \exp\left[-\frac{ik}{2B_3}y_1^2+\frac{ik}{2B_3}y_2^2\right]\exp\left[-\frac{k^2y_2^2}{4L_1B_3^2}\right]\exp\left[\frac{-k^2}{4L_2}\left(\frac{y_1}{B_3}-\frac{y_2}{2L_1B_3\sigma_{yy}^2}\right)^2\right]$$

$$\times \sum_{m=0}^{p}\sum_{n=0}^{l}\sum_{h=0}^{p}\sum_{s=0}^{l}\sum_{d=0}^{[(2m+l-n)/2]}\sum_{e=0}^{[(2p-2m+n)/2]}\sum_{f=0}^{2h+l-s}\sum_{f_1=0}^{[f/2]}\sum_{g=0}^{2p-2h+s}\sum_{g_1=0}^{[g/2]}(-1)^{n+d+e+f_1+g_1}$$

$$\times 2^{2d+2e-f-g+2f_1+2g_1} i^{n+s+2d+2e-f-g+2f_1+2g_1}\binom{p}{m}\binom{l}{n}\binom{p}{h}\binom{l}{s}\left(\frac{2\sqrt{2}}{\sqrt{L_2}w_0}\right)^{-2d-2e}$$

$$\times \frac{f!}{f_1!(f-2f_1)!}\frac{g!}{g_1!(g-2g_1)!}\frac{(2m+l-n)!}{d!(2m+l-n-2d)!}\frac{(2p-2m+n)!}{e!(2p-2m+n-2e)!}$$

$$\left(\frac{2}{\sigma_{yy}^2\sqrt{L_2}\left(L_1^2\omega_0^2-L_1q^2\right)^{1/2}}\right)^{f+g-2f_1-2g_1}\frac{(2h+l-s)!}{f!(2h+l-s-f)!}\frac{(2p-2h+s)!}{g!(2p-2h+s-g)!}$$

$$\times H_{2h+l-s-f}\left(\frac{-i\sqrt{2}kx_2}{\sqrt{2}B_3\left(L_1^2w_0^2-2L_1\right)^{1/2}}\right)H_{2m+l-n-2d+f-2f_1}\left(\frac{kx_2}{4L_1\sqrt{L_2}\sigma_{yy}^2B_3}-\frac{kx_1}{2\sqrt{L_2}B_3}\right)$$

$$\times H_{2p-2h+s-g}\left(\frac{-i\sqrt{2}ky_2}{\sqrt{2}B_3\left(L_1^2w_0^2-2L_1\right)^{1/2}}\right)H_{2p-2m+n-2e+g-2g_1}\left(\frac{ky_2}{4L_1\sqrt{L_2}B_3\sigma_{yy}^2}-\frac{ky_1}{2\sqrt{L_2}B_3}\right) \tag{8}$$

where

$$M_1=\frac{1}{w_0^2}+\frac{1}{2\delta_{xx}^2}-\frac{ikn_o^2}{2zn_e}$$
$$M_2=\frac{1}{w_0^2}+\frac{1}{2\delta_{xx}^2}+\frac{ikn_o^2}{2zn_e}-\frac{1}{4M_1\delta_{xx}^4}$$
$$N_1=\frac{1}{w_0^2}+\frac{1}{2\delta_{xx}^2}-\frac{ikn_e}{2z}$$
$$N_2=\frac{1}{w_0^2}+\frac{1}{2\delta_{xx}^2}+\frac{ikn_e}{2z}-\frac{1}{4N_1\delta_{xx}^4}$$
$$L_1=\frac{1}{w_0^2}+\frac{1}{2\delta_{yy}^2}-\frac{ikn_o}{2z}$$
$$L_2=\frac{1}{w_0^2}+\frac{1}{2\delta_{xx}^2}+\frac{ikn_o}{2z}-\frac{1}{4L_1\delta_{yy}^4} \tag{9}$$

The spectral degree of coherence of a partially coherent electromagnetic beam at pair of points (x_1, y_1, z) and (x_2, y_2, z) is given by the formula [5]

$$\mu(x_1, y_1, x_2, y_2, z)=\frac{Tr\hat{W}(x_1, y_1, x_2, y_2, z)}{\sqrt{Tr\hat{W}(x_1, y_1, x_1, y_1, z)}\sqrt{Tr\hat{W}(x_2, y_2, x_2, y_2, z)}} \tag{10}$$

where Tr stands for the trace of the cross-spectral density matrix. Correlation singularities happen at pair of points for which the spectral degree of coherence $\mu(x_1, y_1, x_2, y_2, z)=0$, at which the phase of the spectral degree of coherence becomes singular. Without loss of generality, we will restrict our discussions to the case for the variation of x_2 and y_2 by keeping x_1 and y_1 fixed in a transverse plane of fixed z.

3. Propagation of Correlation Singularities of a Partially Coherent LG Electromagnetic Beam in a Uniaxial Crystal

In this section, we study the evolution behavior of correlation singularities of a partially coherent LG electromagnetic beam propagating in a uniaxial crystal orthogonal to the optical axis. Numerical results are given to illustrate the influences of the spatial coherence and mode orders of the beam and the refractive indexes of the crystal on the correlation singularities. We assume that a partially coherent LG electromagnetic beam is incident on a uniaxial crystal at the plane $z=0$. On substituting from (7) into (10), the spectral degree of coherence of a partially coherent LG electromagnetic beam at $z=0$ is expressed as

$$\mu(x_{01}, y_{01}, x_{02}, y_{02}, 0)=T(x_{01}, y_{01}, x_{02}, y_{02}, 0)\frac{1}{A_x^2+A_y^2}$$
$$\times\left[A_x^2\exp\left(-\frac{(x_{01}-x_{02})^2+(y_{01}-y_{02})^2}{2\delta_{xx}^2}\right)+A_y^2\exp\left(-\frac{(x_{01}-x_{02})^2+(y_{01}-y_{02})^2}{2\delta_{yy}^2}\right)\right], \tag{11}$$

where $T(x_{01}, y_{01}, x_{02}, y_{02}, 0)$ is a complex function whose expression is omitted here for brevity, and the other part of $\mu(x_{01}, y_{01}, x_{02}, y_{02}, 0)$ is a real function, so the phase of the spectral degree of coherence $\mu(x_{01}, y_{01}, x_{02}, y_{02}, 0)$ depends only on $T(x_{01}, y_{01}, x_{02}, y_{02}, 0)$. $T(x_{01}, y_{01}, x_{02}, y_{02}, 0)$ depends on w_0, p, l, x_{01}, y_{01}, x_{02} and y_{02}, and is independent of A_x, A_y, δ_{xx} and δ_{yy}. Thus the

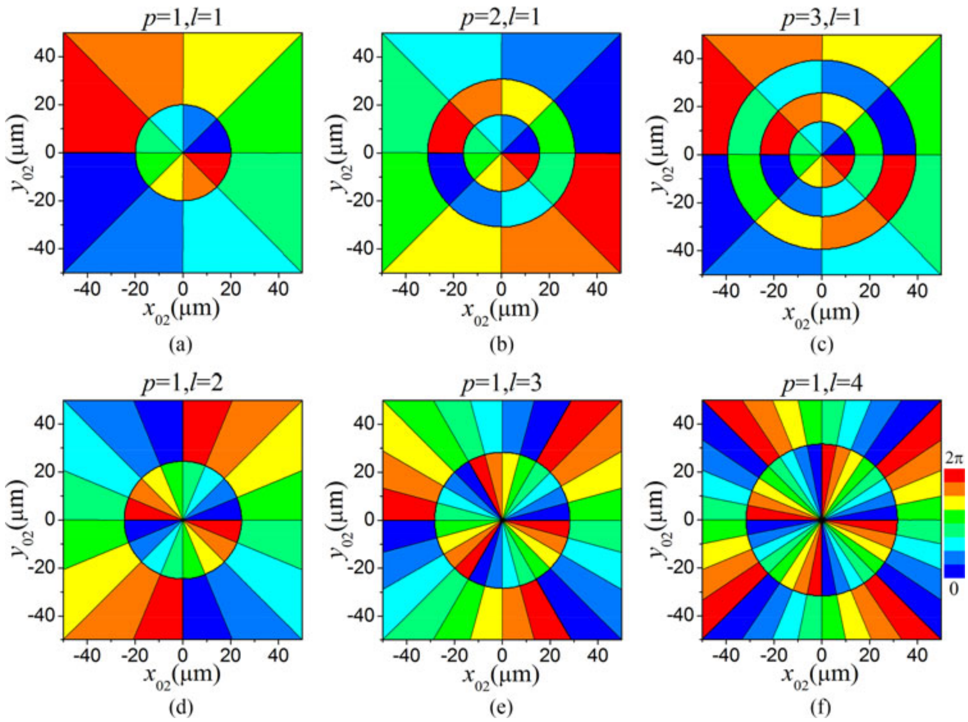

Fig. 2. Phase contours $Arg[\mu(x_{01}, y_{01}, x_{02}, y_{02}, 0)]$ of a partially coherent or fully coherent LG electromagnetic beam for different values of p and l.

phase of the spectral degree of coherence $\mu(x_{01}, y_{01}, x_{02}, y_{02}, 0)$ depends only on w_0, p, l, x_{01}, y_{01}, x_{02} and y_{02}. When $\delta_{xx} = \delta_{yy} = Infinity$, a partially coherent LG electromagnetic beam reduces to a fully coherent LG electromagnetic beam. From the analysis of (11), one sees that the phase of the spectral degree of coherence $\mu(x_{01}, y_{01}, x_{02}, y_{02}, 0)$ of a fully coherent LG electromagnetic beam is the same as that of a partially coherent LG electromagnetic beam, which can also be explained by the fact that the application of Schell-model to the beam in (7) is just a multiplication by Gaussian functions. We calculate in Fig. 2 the phase contours $Arg[\mu(x_{01}, y_{01}, x_{02}, y_{02}, 0)]$ of a partially coherent or fully coherent LG electromagnetic beam for different values of mode orders p and l with $w_0 = 20$ μm, $x_{01} = 1$ μm and $y_{01} = 0$. As can be seen, the phase contours converge at the central correlation singularities, the phase increases by a multiple l of 2π as one follows a counterclockwise closed path around the singularities. From Fig. 2 we can also see that there exist circular edge dislocations whose number equals p, across which the phase has a discontinuity of π.

With the help of (8)–(10), we calculate in Figs. 3–7 the phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a partially coherent LG electromagnetic beam in a uniaxial crystal, the corresponding results for a fully coherent LG electromagnetic beam and in isotropic medium are also given for comparison. Fig. 3 shows the phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a fully coherent LG electromagnetic beam in a uniaxial crystal at several propagation distances for different values of n_e/n_o, with $p = 1$, $l = 1$, $A_x = 1$, $A_y = 1.5$, $\lambda = 0.5$ μm, $w_0 = 20$ μm, $n_o = 2$, $x_1 = 1$ μm and $y_1 = 0$. It can be seen from Fig. 3 that the central correlation singularity with topological charge $l = 1$ of a fully coherent LG electromagnetic beam persists upon propagation, both in isotropic medium and a uniaxial crystal. The circular edge dislocation also persists upon propagation in isotropic medium [see Fig. 3(b)–(d)]. However, the circular edge dislocation becomes imperfect in a uniaxial crystal, and with the increase of n_e/n_o the circular edge dislocation becomes more imperfect [see Fig. 3(f)–(h) and Fig. 3(j)–(l)]. Here imperfect means that the edge is not a pure circular edge dislocation because the differences of the phases inside and outside some parts of the edge are not π.

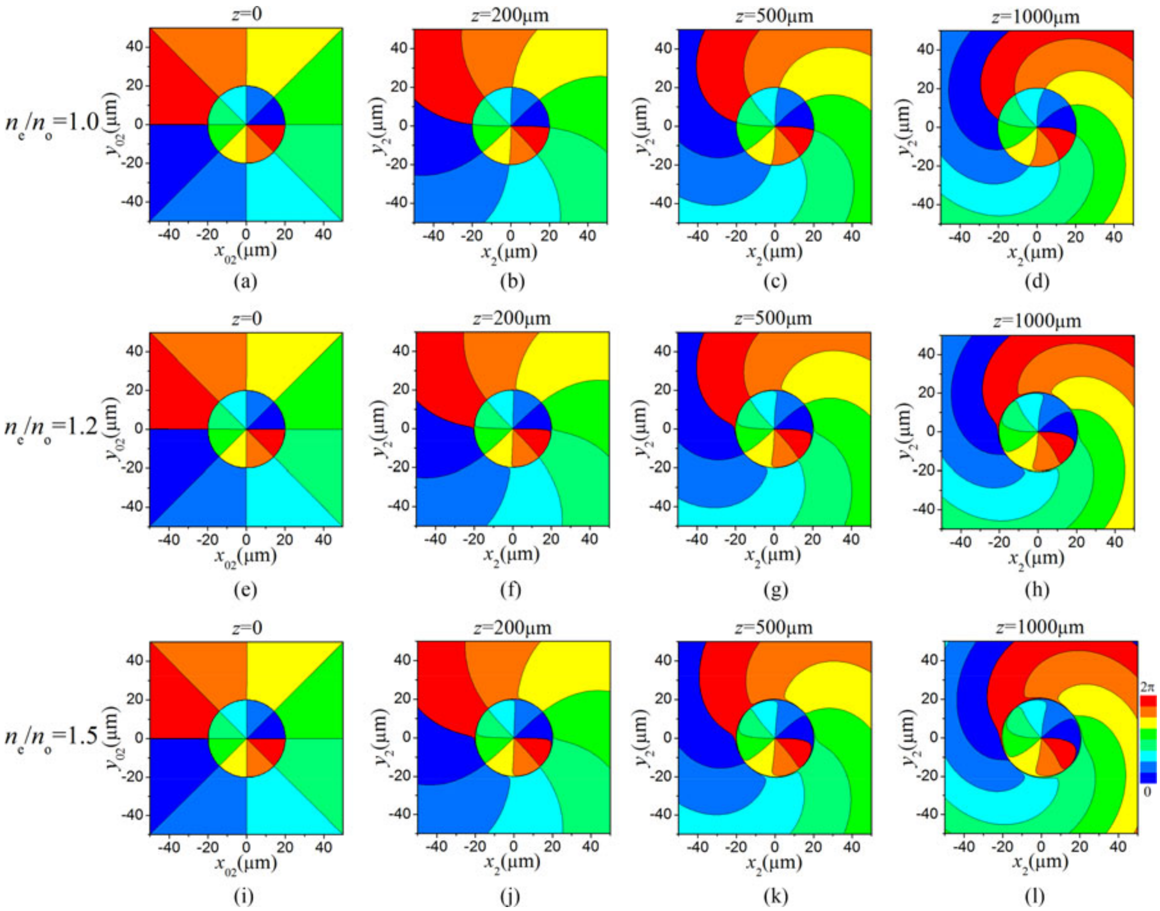

Fig. 3. Phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a fully coherent LG electromagnetic beam in a uniaxial crystal at several propagation distances for different values of n_e/n_o.

Fig. 4 shows the phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a partially coherent LG electromagnetic beam in a uniaxial crystal at several propagation distances for different values of n_e/n_o with $\delta_{xx} = 6\ \mu$m and $\delta_{yy} = 4\ \mu$m, the other calculation parameters are the same as those in Fig. 3. From Fig. 4 we see that for a partially coherent LG electromagnetic beam, after propagation there exist only two correlation singularities labeled A and B, which is quite different from that of a fully coherent LG electromagnetic beam. A comparison of Fig. 4(f) with Fig. 4(g) shows that the correlation singularities A and B approach each other with increasing propagation distance z. A further increase of z to $z = 160\ \mu$m in Fig. 4(h) results in an annihilation of correlation singularities A and B. The analysis of vorticity of phase contours around correlation singularities shows that during the reorganization process the total topological charge remains unchanged, i.e., the charges of A and B in Fig. 4(f) and (g) are $+1$ and -1, respectively. From Fig. 4, we can also find that as n_e/n_o is increased from 1 to 1.5, the propagation distance z at which the pair of correlation singularities annihilate increases, which means that correlation singularities can propagate further in a uniaxial crystal than that in isotropic medium.

Fig. 5 shows the phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a partially coherent LG electromagnetic beam in a uniaxial crystal at several propagation distances for different values of δ_{xx} and δ_{yy} with $n_e/n_o = 1.2$, the other calculation parameters are the same as those in Fig. 3. It is shown in Fig. 5 that with the increase of δ_{xx} and δ_{yy}, the propagation distance z at which the pair of correlation singularities annihilate increases. Fig. 6 shows the phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a partially coherent LG electromagnetic beam in a uniaxial crystal at $z = 200\ \mu$m for different values

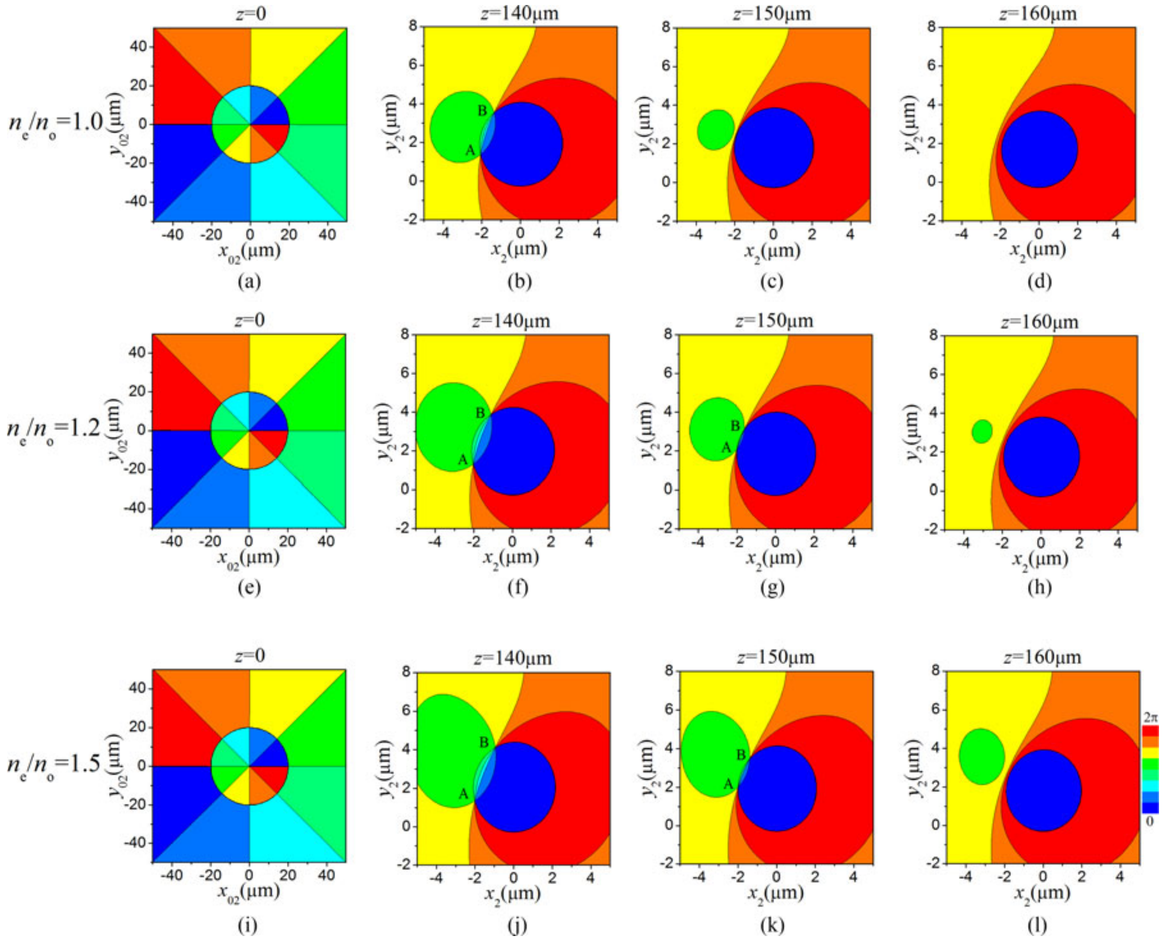

Fig. 4. Phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a partially coherent LG electromagnetic beam in a uniaxial crystal at several propagation distances for different values of n_e/n_o.

of δ_{xx} and δ_{yy} with $n_e/n_o = 1.2$, the other calculation parameters are the same as those in Fig. 3. It can be seen from Fig. 6(a) that for a fully coherent LG electromagnetic beam, there exists a correlation singularity. As δ_{xx} and δ_{yy} are decreased to $\delta_{xx} = \delta_{yy} = 25$ μm, a second correlation singularity moves in from infinity [see Fig. 6(b)]. From Fig. 6(b)–(e), we see that as δ_{xx} and δ_{yy} are decreased from $\delta_{xx} = \delta_{yy} = 25$ μm to $\delta_{xx} = \delta_{yy} = 10$ μm, the two correlation singularities approach each other. A further decrease of δ_{xx} and δ_{yy} to $\delta_{xx} = \delta_{yy} = 5$ μm in Fig. 6(f) results in an annihilation of the two correlation singularities. The annihilation behavior of correlation singularities may be explained by the fact that the distribution of the spectral degree of coherence of the beam will change after propagation, which is determined by the parameters of the beam and the uniaxial crystal.

Fig. 7 gives the phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a fully coherent LG electromagnetic beam in a uniaxial crystal at $z = 500$ μm with $A_x = 1$, $A_y = 1.5$, $\lambda = 0.5$ μm, $n_o = 2$, $n_e/n_o = 1.2$, and the other parameters are the same as those in Fig. 2. Comparing Fig. 7 with Fig. 2, one finds that the circular edge dislocations become imperfect in a uniaxial crystal. The central correlation singularities with topological charges $l \neq 1$ split up into correlation singularities with topological charges equaling 1, whose number is equal to l. In isotropic medium, the central correlation singularities and circular edge dislocations of a fully coherent LG electromagnetic beam with different values of the mode orders p and l persist upon propagation, which is omitted here.

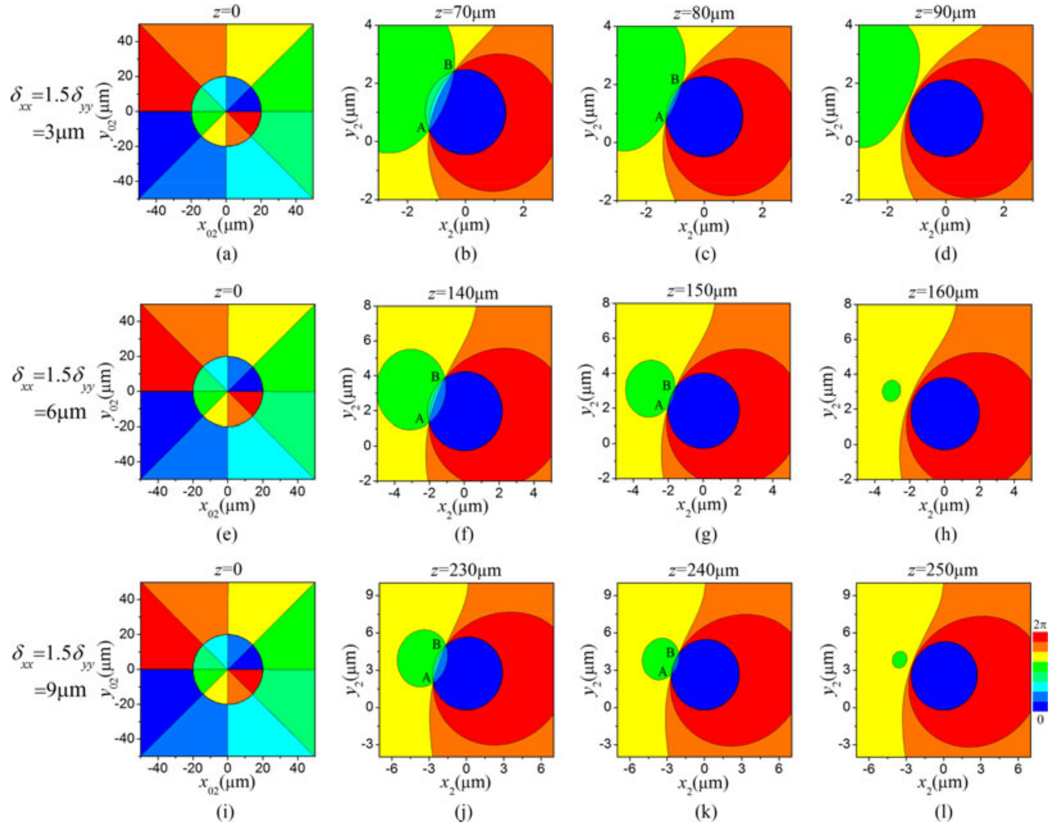

Fig. 5. Phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a partially coherent LG electromagnetic beam in a uniaxial crystal at several propagation distances for different values of δ_{xx} and δ_{yy}.

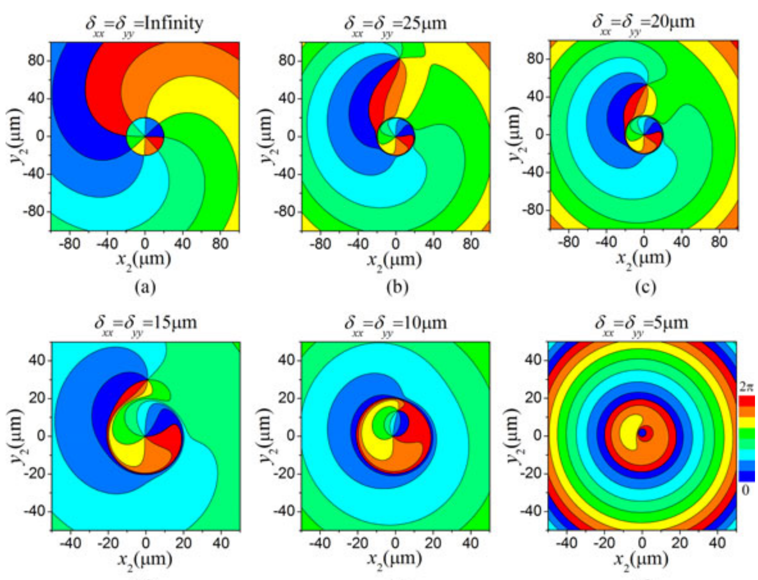

Fig. 6. Phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a partially coherent LG electromagnetic beam in a uniaxial crystal at $z = 200$ μm for different values of δ_{xx} and δ_{yy}.

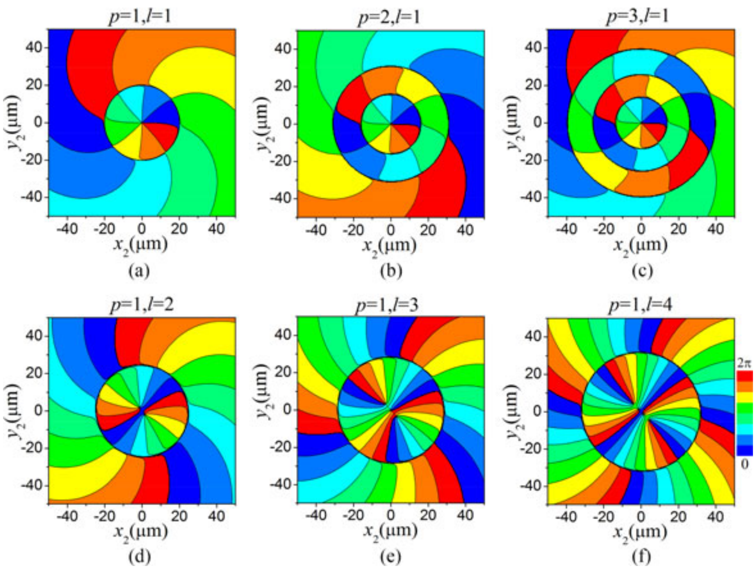

Fig. 7. Phase contours $Arg[\mu(x_1, y_1, x_2, y_2, z)]$ of a fully coherent LG electromagnetic beam in a uniaxial crystal at $z = 500$ μm for different values of p and l.

4. Summary

We have derived the analytical formula for the cross-spectral density matrix of a partially coherent LG electromagnetic beam propagating in a uniaxial crystal orthogonal to the optical axis, and analyzed the influences of the spatial coherence and mode orders of the beam and the refractive indexes of the crystal on the correlation singularities in detail. It is shown that at the source plane the correlation singularities is independent of the spatial coherence, i.e., the distribution of the correlation singularities of a partially coherent LG electromagnetic beam is the same as that of a fully coherent LG electromagnetic beam. After propagation there exist two correlation singularities of a partially coherent LG electromagnetic beam, and with the increase of n_e/n_o and the spatial coherence, the distance at which the correlation singularities annihilate increases. For a fully coherent LG electromagnetic beam, the central correlation singularities and circular edge dislocations persist upon propagation in isotropic medium. While in a uniaxial crystal, the circular edge dislocations become imperfect, and the central correlation singularities with topological charges $l \neq 1$ split up into correlation singularities with topological charges equaling 1. Our results may have potential applications in determining the topological charges of a vortex field after propagating through isotropic or anisotropic medium via the correlation singularities, and one also can determine whether the medium is isotropic or anisotropic through measuring the evolution properties of the correlation singularities.

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