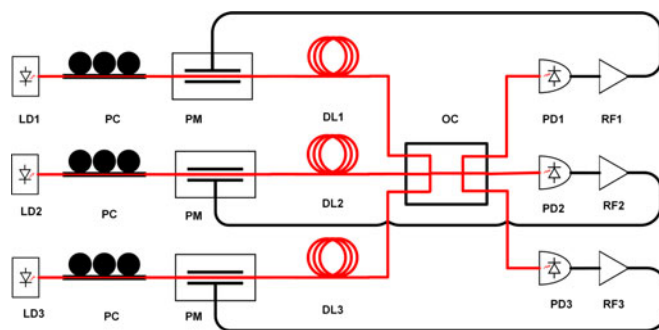


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Mengfan Cheng  
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Deming Liu



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Xinhua Zhu,<sup>1</sup> Mengfan Cheng,<sup>1</sup> Lei Deng,<sup>1</sup> XingXing Jiang,<sup>1</sup>  
Changjian Ke,<sup>1</sup> Minming Zhang,<sup>1</sup> Songnian Fu,<sup>1</sup> Ming Tang,<sup>1</sup>  
Ping Shum,<sup>2</sup> and Deming Liu<sup>1</sup>

<sup>1</sup>Next Generation Internet Access National Engineering Laboratory (NGIA), School of Optical and Electronic Information, Huazhong University of Science and Technology (HUST), Wuhan 430074, China

<sup>2</sup>School of Electrical and Electronic Engineering, College of Engineering, Nanyang Technological University (NTU), Singapore 639798

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**Abstract:** We present an optically coupled chaotic system involving three-phase modulated electro-optic nonlinear loops and an optical coupler. The dynamical properties and the time delay signature (TDS) suppressing performance of the system is analyzed in detail. Numerical results show that the TDS can be suppressed not only under statistical analysis of a single output, but also under mutual statistical analysis of the multiple outputs. Compared with the intensity modulated electrically coupled scheme, the presented system has less interior noise due to the simpler construction.

**Index Terms:** Optical chaos, time delay signature (TDS) suppression, electro-optic nonlinear loop.

## 1. Introduction

Chaotic dynamics of optical system have drawn considerable attention due to their advantages. With the broad bandwidth, large transmission capability and high level of privacy [1]–[5], the optical chaotic system has vast applications in fields such as secure communication [6]–[8], chaotic radar [9], [10] and fast physical random bit generation [11], [12]. Methods with external cavity and feedback loops can provide high-dimensional chaotic signals, which contain time delay signature (TDS) caused by the introduced periodic components. From the perspective of security, the attacker could reconstruct the chaotic carrier with the information of the TDS, and the dimension of the key space could be reduced [13], [14]. For random bit generation, the TDS will also impose restriction on the choices of sampling periods and limit the statistical performance [15]. These facts reveal the truth that the TDS is a vital key for these applications.

However, the TDS can be identified by using statistical methods including the autocorrelation function (ACF) [16], delayed mutual information (DMI) [17], the neural network and the filling factor analysis [18], [19], etc. Therefore, a series of effective schemes have been proposed for the purpose

of concealing the TDS. Modified feedback approaches like distributed time delay feedback by using a fiber grating [20] and polarization-resolved chaos system [21] have been proposed. In [22]–[24], the TDS are eliminated in chaos communication systems by performing a digital phase mask, meanwhile the key space is significantly enlarged. System coupling is another type of effective way to deal with the problem of TDS. In [25], [26], electrical or optical heterodyne are adopted as the coupling strategy between two separated systems. In [27], the nonlinear dynamics in mutually coupled semiconductor lasers is investigated. In [28], [29], an electrically coupled chaos system with three nonlinear feedback loops is demonstrated. These coupled chaos systems can generate multiple TDS free chaos signals simultaneously, which could be used to improve the efficiency of chaotic radar [30] and secure communications under WDM scenario [29]. In these schemes, the TDS cannot be recovered through different statistical analyses under certain parameter range. However, the TDS analyses in the literatures are focused on the statistical feature of one single output, the mutual statistical feature between different signals is ignored. Unfortunately, the TDS can still be extracted by performing statistical analysis between the multiple outputs.

In this paper, an optically coupled chaotic system is proposed. It involves with three phase modulated electro-optic nonlinear loops and an optical coupler (OC). Not only can the TDS be concealed by the system itself under ACF and DMI analyses, but also it cannot be recovered by using mutual statistical methods to analyze two signals from different chains, either. Compared with our former proposed scheme [28], the proposed system has better performance and the system construction is much simpler.

## 2. System Model

The configuration of the optically coupled electro-optic chaos system is illustrated in Fig. 1. The system consists of three mutually coupled nonlinear chains. In each chain, the output of a continuous-wave laser diode (LD) is firstly modulated by a phase modulator (PM), which is used as an external nonlinear component, then detected by a photodetector (PD) and amplified by a radio-frequency (RF) driver. The polarization controller (PC) is used to achieve maximum modulation depth. The transfer function of PM is  $E_{out} = E_{in} \exp(j\pi V(t)/V_\pi)$ ,  $V_\pi$  means the half-wave voltage of PM. And the transfer function of PD is  $V_{PD} = g \cdot |E(t) \cdot E^*(t)|$ ,  $g$  is the conversion gain. The transfer function of RF drivers is  $V_{out} = G \cdot V_{in}$ , where the constant  $G$  denotes the gain. The time delays introduced by the optical fibers are expressed as  $DL_i = 1,2,3$ , and the latency of the other components is ignored. A  $3 \times 3$  OC is adopted to make the three chains couple to each other in optical field.

Ordinarily, a phase-modulation to intensity-modulation converter (like a Mach–Zehnder interferometer [22] or an optical filter [31]) is indispensable in phase chaos system. In our system, such conversion is realized by an OC, which achieves the interferences between the three different chains. Meanwhile the coupling is conducted in this process. Compared with our former proposed intensity coupled scheme in [28], the advantages on system structure lie in two aspects. Firstly, phase modulating avoids the bias controlling which is necessary in intensity modulating. Secondly, an OC takes place of multiple electrical power splitters and electrical power combiners. As a result, the system structure is simpler, and the stability of the system could be better due to the fact that the introduced noise in an OC is much smaller than that in multiple electrical components.

The mathematical model of the system can be derived as follows. The coupling matrix of the  $3 \times 3$  OC is expressed as:

$$\begin{bmatrix} E'_1 \\ E'_2 \\ E'_3 \end{bmatrix} = e^{-j\alpha z} \begin{bmatrix} f & c & c \\ c & f & c \\ c & c & f \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (1)$$

where  $k$ ,  $z$  and  $\alpha$  mean coupling factor, coupling length and propagation constant of the OC respectively. And

$$f = [\exp(i2kz) + 2\exp(-ikz)]/3,$$

$$c = [\exp(i2kz) - 2\exp(-ikz)]/3.$$

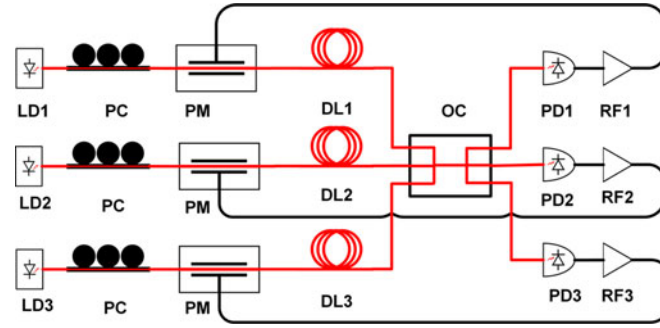


Fig. 1. The configuration of the optically coupled electro-optic chaos system.

In (1),  $E_i = E_{i0} \exp(j(w_i t - x_i(t))) = E_{i0} \exp(j(w_i t - \pi V_i(t)/2V_\pi))$  means the electric field of the transmitted signal after PM.  $x_i(t) = \pi V_i(t)/2V_\pi$  represents the dimensionless variable describing the system.  $V_i(t)$  is the electrical voltage for PM electrode.  $E'_i$ ,  $i = 1, 2, 3$  means the electric field after OC. Then we get:

$$\begin{bmatrix} E'_1 \\ E'_2 \\ E'_3 \end{bmatrix} = n \begin{bmatrix} 1 & m & m \\ m & 1 & m \\ m & m & 1 \end{bmatrix} \begin{bmatrix} E_{10} \exp(j(w_1 t - x_1(t))) \\ E_{20} \exp(j(w_2 t - x_2(t))) \\ E_{30} \exp(j(w_3 t - x_3(t))) \end{bmatrix} \quad (2)$$

where  $m = \frac{e^{3jkz} - 1}{e^{3jkz} + 2}$ ,  $n = \frac{e^{-j(k+p)z}(e^{3jkz} + 2)}{3}$ , when  $kz = 2\pi/9$ ,  $|m| = 1$ . The input power of the coupler is separated to the three output chains homogeneously. Suppose the frequencies of the three LDs as  $f_0, f_0 + \Delta f, f_0 + 2\Delta f$ ,  $\Delta f$  means the frequency detuning. When the frequency detuning  $\Delta f = 0$ , the three lasers have same wavelength. The dynamical equation of the coupled chaotic system can be described as:

$$\begin{aligned} x_1 + \tau_1 \frac{dx_1}{dt} + \frac{1}{\theta_1} \int_0^t (x_1(s) ds) &= \beta_1 \frac{5 + \cos 3kz}{9} \\ &\cdot |\exp(-jx_1(t)) + m \exp(j(2\pi\Delta f - x_2(t))) + m \exp(j(4\pi\Delta f - x_3(t)))|^2. \\ x_2 + \tau_2 \frac{dx_2}{dt} + \frac{1}{\theta_2} \int_0^t (x_2(s) ds) &= \beta_2 \frac{5 + \cos 3kz}{9} \\ &\cdot |m \exp(-jx_1(t)) + \exp(j(2\pi\Delta f - x_2(t))) + m \exp(j(4\pi\Delta f - x_3(t)))|^2. \\ x_3 + \tau_3 \frac{dx_3}{dt} + \frac{1}{\theta_3} \int_0^t (x_3(s) ds) &= \beta_3 \frac{5 + \cos 3kz}{9} \\ &\cdot |m \exp(-jx_1(t)) + m \exp(j(2\pi\Delta f - x_2(t))) + \exp(j(4\pi\Delta f - x_3(t)))|^2. \end{aligned} \quad (3)$$

In (3),  $\beta_i = \beta = \pi g A G P / 2V_\pi$  is defined as the loop gain, which is also considered as the bifurcate parameter, where  $A$  means the overall attenuation of this feedback loop,  $P$  means the output power of the LD. The bandwidth of the feedback loop is supposed in first approximation to result from a bandpass filter, with low and high cutoff frequencies  $f_L$  and  $f_H$ , respectively, where  $\theta = 1/2\pi f_L$ ,  $\tau = 1/2\pi f_H$ . Parameters in the system are:  $\tau_i = \tau = 25$  ps and  $\theta_i = \theta = 5$  us. The time delay in the three chains are set as  $T_i = 30, 25$  and  $20$  ns respectively,  $i = 1, 2, 3$ .

The simulations in our work are conducted by using the MATLAB tool. Equation (3) is calculated with a fourth-order Runge-Kutta algorithm. The variable-step is  $1/60$  ns. Fig. 2(a)–(c) show the time series while  $\beta = 0.7, 1.0, 3$  respectively. When  $\beta < 0.6$ , the oscillation is hard to start due to the small loop gain, so the output attenuates to zero. While  $\beta$  is about  $0.7$ , the electrical input of PM is small and the transfer function of PM can be treated as linear approximately. The feedback loop can be seen as a linear oscillator, and the output is periodic. When  $\beta$  increases, the nonlinearity

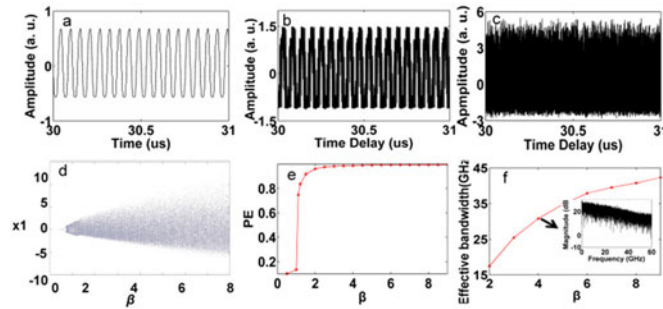


Fig. 2. (a) Time series while  $\beta = 0.7$ ; (b) time series while  $\beta = 1.0$ ; (c) time series while  $\beta = 3$ ; (d) Bifurcation diagram of system; (e) PE as a function of  $\beta$  for the ordinal pattern length  $L = 6$  and the embedding delay  $D = 2$ ; (f) effective bandwidth of the transmitted signal as a function of  $\beta$ , the insets shows the spectrum of the transmitted signal with  $\beta = 4$ ;

of the loop is emerging, quasi-periodic oscillation can be observed. Continuing to increase  $\beta$ , the system evolves into chaotic state due to the strong nonlinearity of the loop. These process can also be observed in the bifurcation diagram, as shown in Fig. 2(d). Permutation entropy (PE) [32] is used to map the dynamical complexity. We choose a time series  $x_1$  with length of  $4 \times 10^4$ . PE is calculated for the ordinal pattern length  $L = 6$  and the embedding delay  $D = 2$ . The embedding delay corresponds to the sampling time of the signal. The time series is partitioned into subsets of ordinal pattern length with embedding delay. Result is shown in Fig. 2(e). The curve of PE shows that the system can generate the most complex time series (PE = 0.99) in the chaotic zone.

The spectrum characteristic of the optical chaos signal is also analyzed. Due to the drastic fluctuations in the spectrum, the bandwidth may not be very easy to estimate by using the concept of “3 dB bandwidth”. Here we use the principle of effective bandwidth [33]. The signal energy is calculated from 0 to 60 GHz. The effective bandwidths of the generated optical chaos signals are demonstrated in Fig. 2(f) while  $\beta$  ranges from 2 to 9. Since the numerical value of the effective bandwidth relies on the calculating range, this result is only used to demonstrate a qualitative relationship between  $\beta$  and the spectrum. The complexity and the bandwidth performance of the presented system are similar to the intensity coupled chaotic system. In real world applications,  $\beta$  should be as large as possible for large bandwidth and high dynamical complexity, but the value of  $\beta$  is restricted by the half wave voltage of the modulator and the gain of the RF driver.

### 3. Main Results

#### 3.1 Statistical Analysis With Single Time Series

In this work, we discuss the concealment of the TDS, which plays an important role in chaos-based applications. We define  $T = (T_1, T_2, T_3)$ , and time delay difference  $\Delta T = T_1 - T_2 = T_2 - T_3$ . Time series  $x_1$  with length of  $10^6$  (from  $1 \times 10^6$  to  $2 \times 10^6$ ) are used to perform ACF and  $\text{DMI}_s$  (which means DMI analysis of a single output) analysis for  $T = (25, 25, 25)\text{ns}$ ,  $T = (27, 25, 23)\text{ns}$  and  $T = (30, 25, 20)\text{ns}$ . The formula of  $\text{ACF}(v)$  and  $\text{DMI}_s(v)$  for a single time series  $v(t)$  can be expressed as

$$C_s(\Delta t) = \frac{\langle (v(t+\Delta t) - \langle v(t) \rangle) (v(t) - \langle v(t) \rangle) \rangle}{\left( \langle (v(t) - \langle v(t) \rangle)^2 \rangle \langle (v(t+\Delta t) - \langle v(t) \rangle)^2 \rangle \right)^{1/2}} \quad (4)$$

$$I_s(\Delta t) = \sum p(v(t), v(t+\Delta t)) \ln \frac{p(v(t), v(t+\Delta t))}{p(v(t)) p(v(t+\Delta t))} \quad (5)$$

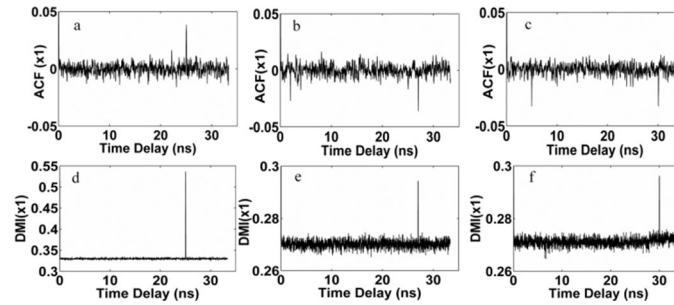


Fig. 3.  $ACF(x_1)$  and  $DMI_s(x_1)$  curve while the parameter is given as  $\beta = 4$ . (a)  $ACF(x_1)$  while  $T = (25, 25, 25)$ ns; (b)  $ACF(x_1)$  while  $T = (27, 25, 23)$ ns; (c)  $ACF(x_1)$  while  $T = (30, 25, 20)$ ns; (d)  $DMI_s(x_1)$  while  $T = (25, 25, 25)$ ns; (e)  $DMI_s(x_1)$  while  $T = (27, 25, 23)$ ns; (f)  $DMI_s(x_1)$  while  $T = (30, 25, 20)$ ns.

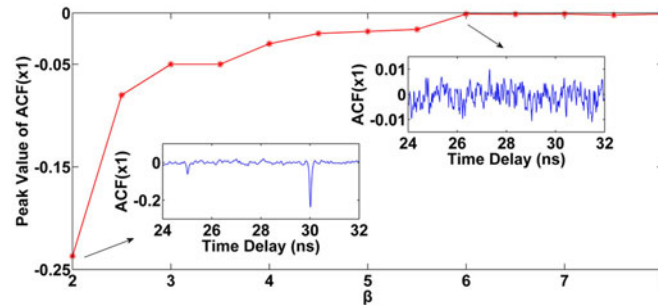


Fig. 4. Value of the peaks at  $T = 30$  ns in  $ACF(x_1)$ . The insets show the  $ACF(x_1)$  curves while  $\beta = 2$  and 6.

where  $v(t)$  represents single chaotic time series  $x_1$ ,  $\Delta t$  means the time shift.  $\langle \cdot \rangle$  means time average.  $p(v(t))$  and  $p(v(t), v(t + \Delta t))$  are mean probability distribution of marginal and joint fault probability distribution respectively. The results of  $ACF(x_1)$  and the  $DMI_s(x_1)$  while  $\beta = 4$  are shown in Fig. 3.

As it can be seen, clear peaks appeared at time shift  $T_1$ , which reveal the TDS. When  $\Delta T$  increases, the peak values in  $ACF(x_1)$  barely have no change. But the peak values in  $DMI_s(x_1)$  decrease while  $\Delta T$  increases from 0 to 5ns; Continue to increase  $\Delta T$ , the peak values in  $DMI_s(x_1)$  remain static. So in the following parts, the time delay in three chains are chosen as  $T = (30, 25, 20)$  ns.

The influence of the bifurcate parameter  $\beta$  on the TDS concealment is also discussed in detail.  $ACF$  and  $DMI_s$  analyses have been conducted while  $\beta$  ranges from 2 to 8. As it can be seen in Fig. 4, which displays the peak value at  $T = 30$  ns from the background in  $ACF(x_1)$ , the peak is distinguishable for  $\beta = 2.5 \sim 3.5$ ; When  $\beta$  increases to 4.5, the peak becomes unobvious; And when  $\beta$  continues to rise to a critical large value like  $\beta = 6$ , the peak is entirely buried in the background. Similar phenomena can be observed in the results of  $DMI_s(x_1)$ , as shown in Fig. 5. Therefore, the TDS will be concealed successfully in  $ACF(x_1)$  and  $DMI_s(x_1)$  while  $\beta > 6$ . It is also suitable for time series  $x_2$  and  $x_3$  due to the symmetry of the system structure.

### 3.2 Mutual Statistical Analysis With Two Different Time Series

According to the system setup and theoretical model, three sets of chaotic signal transmitted in separate chains can be obtained. The aforementioned results show that the TDS can be eliminated under statistical analysis based on one single time series. However, for coupled systems, the mutual statistical feature between different sub-systems could cause the information leak. It was proposed in [34] that cross correlation function (CCF) could be used to analyze TDS of the rings of



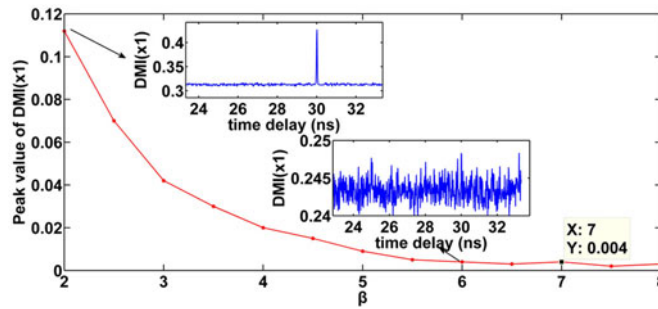


Fig. 5. Value of the peaks at  $T = 30$  ns in DMI ( $x_1$ ). The insets show the DMI<sub>s</sub> ( $x_1$ ) curves while  $\beta = 2$  and 6.

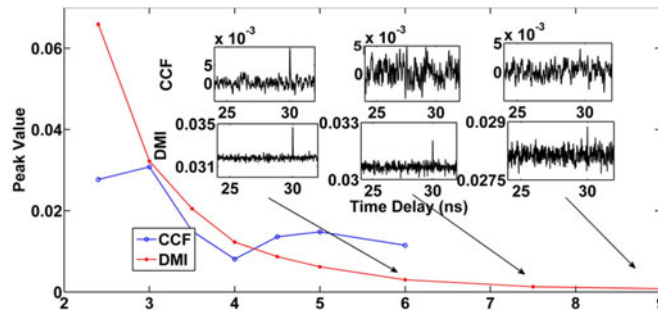


Fig. 6. The peak values of the CCF( $x_1, x_2$ ) and DMI<sub>d</sub> ( $x_1, x_2$ ) curves of the electrically coupled electro-optic chaos system while  $\beta$  ranges from 2 to 9. The insets show the CCF and DMI curves while  $\beta = 6, 7.5$  and 9.

delay-coupled elements. In this paper, mutual statistical methods CCF and DMI<sub>d</sub> (which means DMI analysis of double outputs) are performed between double outputs of different chains. The formula of CCF( $v_1, v_2$ ) and DMI<sub>d</sub>( $v_1, v_2$ ) for two different time series  $v_1(t)$  and  $v_2(t)$  can be expressed as:

$$C_d(\Delta t) = \frac{\langle (v_1(t) - \langle v_1(t) \rangle) (v_2(t+\Delta t) - \langle v_2(t) \rangle) \rangle}{\left( \langle (v_1(t) - \langle v_1(t) \rangle)^2 \rangle \langle (v_2(t+\Delta t) - \langle v_2(t) \rangle)^2 \rangle \right)^{1/2}} \quad (6)$$

$$I_d(\Delta t) = \sum p(v_1(t), v_2(t+\Delta t)) \ln \frac{p(v_1(t), v_2(t+\Delta t))}{p(v_1(t)) p(v_2(t+\Delta t))}. \quad (7)$$

First, we consider our former scheme, the electrically coupled electro-optic chaos system. According to [28], we can conclude that the TDS will be concealed in ACF ( $x_1$ ) and DMI<sub>s</sub> ( $x_1$ ) if the bifurcate parameter  $\beta > 4$  under single time series analysis. However, the statistical property between two time series  $x_1$  and  $x_2$  can be revealed by CCF ( $x_1, x_2$ ) and DMI<sub>d</sub> ( $x_1, x_2$ ). As shown in Fig. 6, the peak values of CCF ( $x_1, x_2$ ) and DMI<sub>d</sub> ( $x_1, x_2$ ) exist while  $\beta > 4$ , the peaks are apparent though the absolute value decreases if we increase  $\beta$ . When  $\beta = 6$ , the TDS can still be extracted under DMI<sub>d</sub> ( $x_1, x_2$ ). These results indicate that the electrically coupled electro-optic chaotic system is not safe enough under mutual statistical analysis.

In the same way, CCF ( $x_1, x_2$ ) and DMI<sub>d</sub> ( $x_1, x_2$ ) are used to analyze the system presented in this article. As shown in Fig. 7, the peaks in CCF and DMI<sub>d</sub> are both entirely buried in the background when  $\beta = 6$ , which means that the TDS cannot be revealed even under mutual analysis.

### 3.3 The Influence of Frequency Detuning

The three nonlinear loops are coupled in optical field by an OC. This fact makes the wavelength of the light source a new degree of freedom to investigate the system. The frequency detuning between

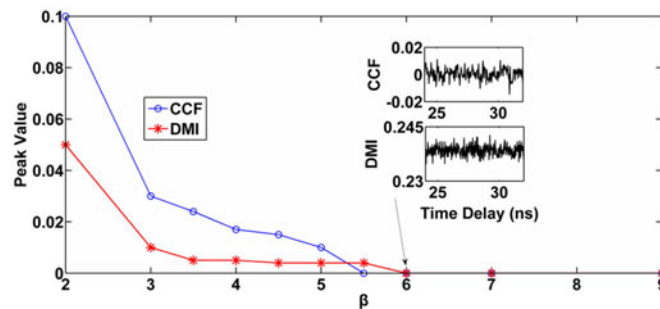


Fig. 7. The peak values of the CCF ( $x_1, x_2$ ) and  $\text{DMI}_d(x_1, x_2)$  curves of the presented system while  $\beta$  ranges from 2 to 9. The insets show the CCF and DMI curves while  $\beta = 6$  and 7.

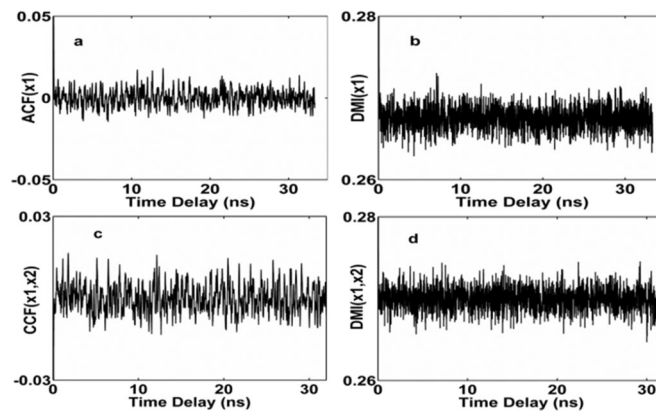


Fig. 8. The  $\text{ACF}(x_1)$ ,  $\text{DMI}_s(x_1)$  and  $\text{CCF}(x_1, x_2)$ ,  $\text{DMI}_d(x_1, x_2)$  curves while  $\Delta f = 1$  GHz,  $\beta = 4$ . (a)  $\text{ACF}(x_1)$ ; (b)  $\text{DMI}_s(x_1)$ ; (c)  $\text{CCF}(x_1, x_2)$ ; (d)  $\text{DMI}_d(x_1, x_2)$ .

the optical sources could affect the performance of the chaotic system. When  $\Delta f = 1$  GHz, the single chain analysis of the transmitted signal  $x_1$  are shown in Fig. 8(a) and (b). The mutual property between the different chains is also analyzed by  $\text{CCF}(x_1, x_2)$  and  $\text{DMI}_d(x_1, x_2)$ , which are shown in Fig. 8(c) and (d). These results indicate that the TDS are concealed under ACF and  $\text{DMI}_s$  analysis and the system can also resist the CCF and  $\text{DMI}_d$  attack with two signals in the different chains.

Similar conclusions can be obtained while the frequency detuning ranges from 0.5 to 15 GHz, the TDS are totally suppressed while  $\beta > 4$ . Compared with the results in Figs. 4–6, the frequency detuning between the optical sources play a positive role in TDS suppression. Smaller  $\beta$  is needed to achieve the TDS suppression, which means the system is easier to implement. Considering that large  $\beta$  has a high demand for the half-wave voltage of PM and the gain of the RF, the presented system has advantages in the implementation cost and the performance of TDS suppression.

In real world systems, even with the same wavelength, the lasers will have phase noise due to a finite linewidth. We have analyzed the performance of the TDS suppression when the lasers in our model have typical linewidth like 100 KHz–1 MHz. Similar results could be obtained and no significant influence is observed. Those mean that the laser linewidth has few influences on the performance of TDS suppression. The reason could be attributed to the fact that the beat interference caused by the phase noise is limited in low frequency region of a few hundred kHz or several MHz. However, the generated chaotic signals are broadband signals. The energy is distributed in a wide frequency range of several GHz. As a result, the low frequency noise will not affect the system performance significantly.



## 4. Conclusion

In summary, we have present an optically coupled chaotic system based on electro-optic loops and OC. The OC introduces interference between the different loops of chaotic dynamics. We have investigated the performances of the chaotic dynamic and analyses the statistical properties of the system. The results show that the TDS information can't be recovered by ACF,  $DMI_s$  analysis of one single output under proper range. And the system can also resist mutual statistical analysis like CCF and  $DMI_d$  between multiple outputs. With the introducing of frequency detuning, the proposed system can conceal the TDS by itself in a larger range. What's more, the system has less interior noise than the intensity modulated electrically coupled scheme due to the introducing of the phase modulator and the OC. The presented system has advantage in the implementation cost and the performance of the TDS suppression, which make the presented system gain a promising expectation in secure communications and fast physical random number generation in the future.

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