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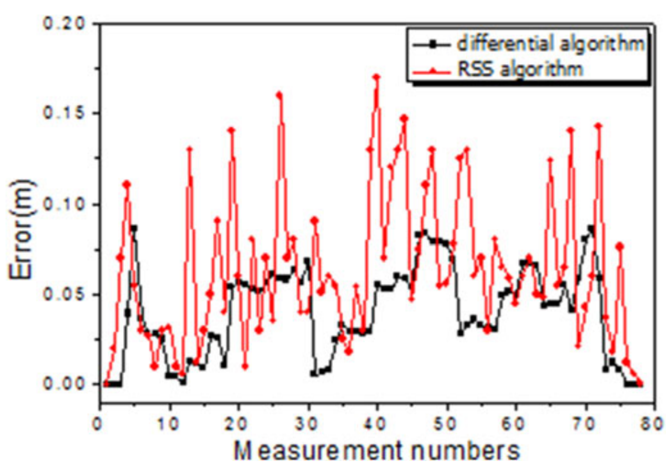

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High Accuracy VLC Indoor Positioning System With Differential Detection

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Abstract: Recently, several positioning systems based on visible light communications (VLC) have been proposed, and most of them are based on received signal strength (RSS) because of its simplicity and high accuracy. In order to improve the accuracy further, a differential detection-based positioning algorithm is proposed. It can reduce positioning instability caused by light intensity fluctuation and the area outside the light-emitting diode (LED) cell can also be positioned. Experimental results show that the proposed method can improve the positioning accuracy from 10.0 to 4.0 cm and reduce the standard deviation from 9.0 to 2.5 cm. Meanwhile, the positioning accuracy outside the cell can reach 10.0 cm or less.

Index Terms: Indoor positioning, received signal strength (RSS), intensity fluctuations, differential detection positioning algorithm.

1. Introduction

With the rapid developments of wireless sensor network and physical networking technology, location-based services become more and more popular recently. Due to the wide application scenarios such as large shopping malls and underground parking lots, indoor positioning is attracting much attention. In order to achieve high-precision indoor navigation, many researchers and engineers have researched a number of indoor positioning technologies, such as those based on assisted GPS (A-GPS), a pseudo-satellite (Pseudo lite), wireless local area network (WLAN), radio frequency tags (RFID), Zigbee, Bluetooth (BT), ultra-wideband radio (UWB), infrared, computer vision, magnetic, ultrasonic, and visible light [1]–[7]. However, when GPS signal is used for indoor positioning, it becomes weak due to the high path loss of building walls. WLAN positioning uses fingerprint library method. However, the collection and establishment of fingerprint library is a difficult work, while positioning accuracy of WLAN is low. Ultrasonic attenuation in the air is large, making UWB based positioning not suitable for large occasions. Other positioning methods, such as infrared, Zigbee, and Bluetooth, are vulnerable to fluctuations in signal sources. As a result, these methods fail in guaranteeing accuracy for the indoor positioning [8]. From the practical point of application, the LED based visible light positioning system has great potential. Cameras or photodetectors are commonly used at the receiver end for visible light localization [9]–[17].Yoshino proposed a positioning system based on visible light image sensor [9]. The positioning accuracy of

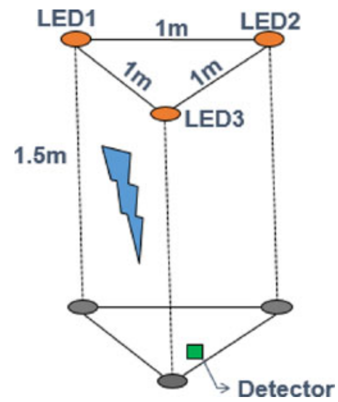


Fig. 1. Principle of the positioning system.

the camera is higher than non-imaging positioning, but the positioning algorithm is complex, and the positioning speed is limited by image processing speed. The research of positioning system based on photodetector is more common [10]. Commonly visible location technologies are based on the triangulation method, and they need to estimate the distance or angle between transmitter and receiver. There are many ways to achieve the distance or angle such as received signal strength (RSS) [11], [12], time of arrival (TOA), time difference of arrival (TDOA) [13], and angle of arrival (AOA) [14]. AOA technology can achieve high accuracy, but a sensor array needs to be deployed, which is very expensive. For indoor environments, the short distance between transmitter and the receiver results in short transmission time, so high accuracy clock and synchronization is required for the transmitter and receiver. Thus, the TOA or TDOA method is difficult to employ in practical applications for these reasons [15]. On the contrary, the RSS based method can calculate the distance between transmitter and receiver by the strength of received signal easily. Aminikashani *et al* have used four modulated LEDs as signal transmitter, and estimated distance between transmitter and receiver by the difference of received power [16]. To take the cost, difficulty and accuracy of indoor positioning into consideration, received signal strength (RSS) based positioning is preferred due to its low cost and high accuracy [17]. However, the performance of RSS positioning is limited by the power fluctuation of light source. In this work, a differential detection positioning method using two detectors to achieve RSS is proposed. The principle of the method is analyzed, a prototype is built, and experiments are made to evaluate the performance of the method.

The remainder of this paper is organized as follows: In Section 2, the principle and theoretical model of the RSS positioning algorithm are introduced, and RSS based system positioning error due to fluctuations of emitting power is measured and analyzed. In Section 3, differential detection positioning algorithm is proposed, and the validity of the algorithm is verified by experiments. Finally, conclusions are drawn in Section 4.

2. Received Signal Strength Positioning Algorithm Principle

2.1 RSS Positioning Algorithm Principle

The principle of the positioning system is shown in Fig. 1. The transmitter of the positioning system is made of three LEDs as mentioned in our previous research [18]. The LEDs are modulated by the asynchronous code. While a detector receives the light signals of three LEDs the signal intensity of each LED can be separated according to the code of each LED.

According to Lambertian radiation model [19], the channel gain between the LED and the detector can be show as

$$H(0)_{\mathrm{LOS}} = \frac{(m + 1)\, A \cos^{m}(\varphi) \cos(\theta)}{2\pi d^2} \tag{1}$$

where φ is the radiation angle between the LED and the detector, d is the distance between the LED and the detector, A is the effective area of the detector, and θ is the angle of light incident to the receiving surface of the detector. $\varphi_{1/2}$ is the half power angle of LED, and $m = -\ln 2/\ln(\cos\varphi_{1/2})$. The received power P of the receiver can be shown as

$$P = P_0 \frac{(m+1)A\cos^m(\varphi)\cos(\theta)}{2\pi d^2} = P_0 \frac{(m+1)A\,h^{(m+1)}}{2\pi d^{(m+3)}} \tag{2}$$

where P_0 is the light power of LED, and h is the vertical distance between the detector and the LED. Thus, the distance from the detector to LED_1 can be expressed as

$$d_1 = \sqrt[(m+3)]{\frac{P_0}{P_1}\left(\frac{(m+1)A\,h^{(m+1)}}{2\pi}\right)} \tag{3}$$

where h is a known constant. Similarly, the distance from the detector to LED_1 or LED_2 can also be expressed obtained. On the other hand, the projection distance r_1, r_2, r_3 between LEDs and receiver can be obtained as

$$r_1 = \sqrt{{d_1}^2 - {h_1}^2}. \tag{4}$$

Thus, the distances between each LED and the receiver are obtained. Assume the coordinates of the LEDs are (X_1, Y_1), (X_2, Y_2), (X_3, Y_3), the coordinate (x, y) of receiver can be calculated by (5) as follows:

$$\left\{\begin{array}{l} (x - X_1)^2 + (y - Y_1)^2 = {r_1}^2 \\ (x - X_2)^2 + (y - Y_2)^2 = {r_2}^2 \\ (x - X_3)^2 + (y - Y_3)^2 = {r_3}^2 \end{array}\right\}. \tag{5}$$

Equations (3)–(5) show that the RSS based positioning method can solve the location of the receiver with the received power of each LED. Actually, the power (P_0) is hard to measure. One method to solve this problem is using the relative light intensity (or the LED illumination value after normalization) for residential location [9]. Its principle is as follows:

To set that $d_1, d_2 \ldots d_N$ is the distance between the receiver and different LEDs, we can get from (3)

$$\left\{\begin{array}{ll} d_1 = \sqrt[(m+3)]{\frac{P_0}{P_1}\left(\frac{(m+1)A\,h^{(m+1)}}{2\pi}\right)} & (6) \\ d_2 = \sqrt[(m+3)]{\frac{P_0}{P_2}\left(\frac{(m+1)A\,h^{(m+1)}}{2\pi}\right)} & \\ \bullet\bullet\bullet & (7) \\ d_N = \sqrt[(m+3)]{\frac{P_0}{P_N}\left(\frac{(m+1)A\,h^{(m+1)}}{2\pi}\right)} & (8). \end{array}\right.$$

To divide (6) by (8), we get

$$\frac{d_1}{d_N} = \sqrt[(m+3)]{\frac{P_1}{P_N}}.$$

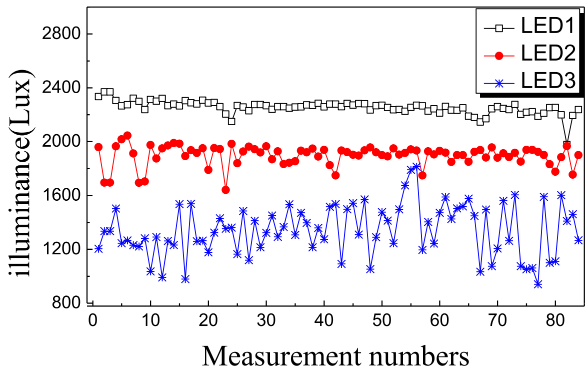


Fig. 2. LEDs' illumination fluctuates over time.

TABLE 1
Experimental Setup

Symbol	Quantity
Φ	60°
Power	1 w
h	1.5 m
dis	1.0 m

The relative distance of the receiver can be obtained by measuring the intensity of different LED light values received by the detector. Then, we have

$$\begin{cases} (x - X_1) + (y - Y_1)^2 = {r_1}^2 \\ \bullet\bullet\bullet \\ (x - X_N)^2 + (y - Y_N)^2 = {r_N}^2 \\ r_1 : r_2 \approx \sqrt[(m+3)]{\dfrac{P_1}{P_2}} \\ \bullet\bullet\bullet \\ r_1 : r_N \approx \sqrt[(m+3)]{\dfrac{P_1}{P_N}} \end{cases} . \tag{9}$$

The (X_A, Y_A) is the projection coordinates of LED A on the measuring plane. The location of receiver can be obtained by solving (9). However, the fluctuation of light intensity will result in the instability of the calculated position. It is known that the influence of background noise is weak [20]. Therefore, the positioning instability caused by the LED power fluctuation is a major problem.

2.2 Positioning Instability Caused by the Intensity Fluctuations of Received Power

Actually, the power of LEDs is not constant. In Fig. 2, three LEDs' light luminance value were measured over a period of time, the average power of LED was 1 w, the receiver is APD120A2 made by THOR LAB, and the height between LEDs and receiver was 1.5 m (see Table 1). The interval between two neighboring measured points was 3 s.

Fig. 2 shows that the received illuminance from LEDs changes randomly with time, and the fluctuation amplitude reached 38% of the average. The illumination fluctuatuation for RSS positing algorithm is fatal, which can be seen from (3), (6), (7), and (8). The LED power fluctuations can

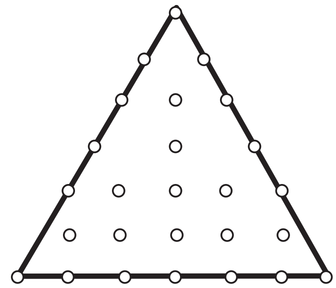


Fig. 3. Measured points.

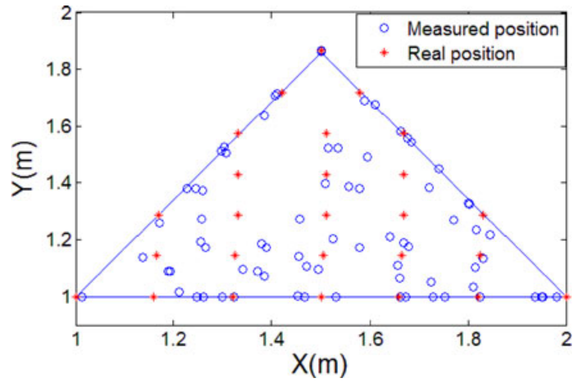


Fig. 4. Measured positions by RSS algorithm.

cause the measured distances between receiver to LEDs changes, making the position (x, y) obtained by (5) and (9) change.

To evaluate the error that the fluctuations caused, a 1.0 m × 1.0 m × 1.5 m (Fig. 1) experimental scene was built, and corresponding experiments were made.

As is shown in Fig. 3, we took measurements with 26 points on the received plane. Each point has been measured three times.

The measurement data is calculated directly by the positioning algorithm mentioned before. The coordinates of the receiver can be obtained as in Fig. 4. In this experiment, the coordinates of three LEDs are (1, 1, 1.5), (2, 1, 1.5), and (1, 1.832, 1.5).

For each measurement, the error is calculated as

$$D = \sqrt{(x_r - x_m)^2 + (y_r - y_m)^2} \quad (10)$$

where (x_r, y_r) is the real coordinate of the receiver, (x_m, y_m) is the measured coordinate of the receiver. And the positioning error of different measurements is shown in Fig. 5.

For each positioning point, the three measurements were averaged, and the error of different points is shown in Fig. 6.

2.3 Fluctuation Analyses of Light Sources

As mentioned previously, the fluctuation of LEDs' illumination will increase the positioning errors. In order to reduce the positioning error, the impact of LEDs' illumination fluctuations should be reduced. As Fig. 2 shows, the fluctuation of LED power is not consistent, so we add a reference detector in the positioning system to detect the real-time intensity fluctuation of light sources. The principle of the system is shown in Fig. 7. The distance between LEDs is 1.0 m, LED power is 1.0 W. The receiver is APD120A2 made by THOR LAB. In this system, two detectors will receive light signal at the same time. The position of the reference detector is known and can be placed

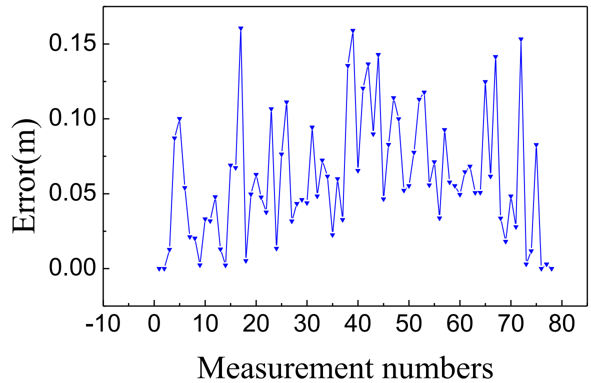


Fig. 5. Positioning error over time.

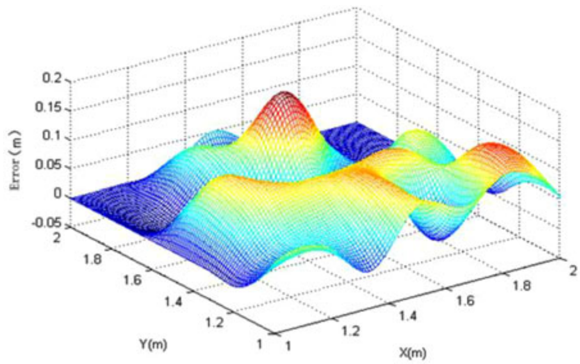


Fig. 6. Distribution of positions errors on the received plane.

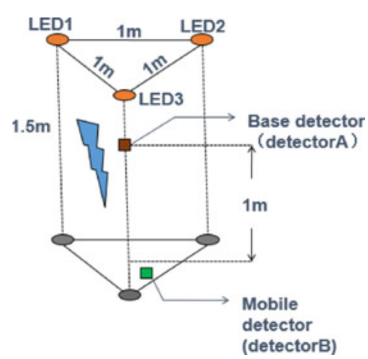


Fig. 7. Scheme setup of a positioning cell with differential detection.

anywhere in the district. In this positioning system, the reference detector is put on the center of three LEDs, and the height of the reference detector is 1.0 m (see Table 2).

The received intensity of three LEDs by the two detectors are shown as Fig. 8. It shows that the three LEDs' illumination fluctuates independently of each other, but the change rate of the LED power received by the two detectors is consistent, regardless of the position of the detector. Therefore, the base station detector can be used to detect the fluctuation of light sources' power, and the mobile station detector can obtain original light intensity by mitigating the power fluctuation.

Note that if the detected illumination of base station is inaccurate, the compensation algorithm cannot improve the positioning accuracy effectively. The closer initial light illumination received by base station to ideal illumination, the better the positioned result will be. Therefore,

TABLE 2
Experimental Setup

Symbol	Quantity
Φ	60.0°
Power	1.0 w
H	1.5 m
Distance	1.0 m
DetectorA Height	1.0 m
DetectorB Height	0.0 m

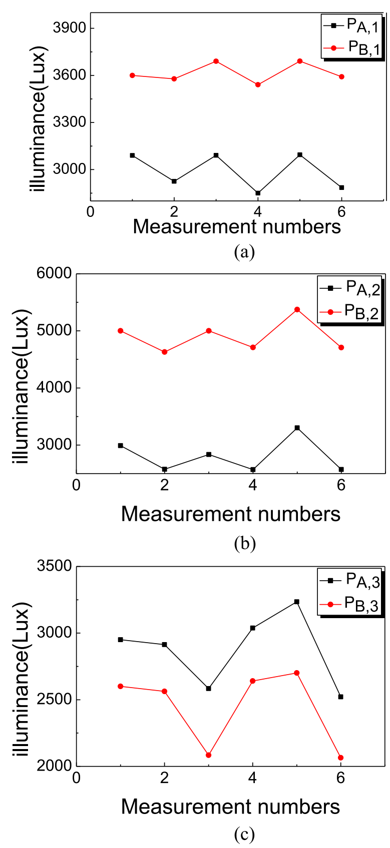


Fig. 8. Light illumination received by two detectors. (a) Detector A, B synchronously receive the light from LED1, represented by $P_{A,1}$ and $P_{B,1}$, respectively. (b) Detector A, B synchronously receive the light from LED2, represented by $P_{A,2}$ and $P_{B,2}$. (c) Detector A, B synchronously receive the light from LED3, represented by $P_{A,2}$ and $P_{B,2}$.

the positioning accuracy depends on the initial illumination received by the base detector. With the exact coordinates of the base detector, the correct initial illumination of it can be given as follows:

1) Two detectors receive the signal synchronously.
2) The base detector calculates its own coordinates.
3) Compared the coordinates calculated in the second step with the known coordinates.
4) Find the corresponding value of the rate at the minimum of the error.

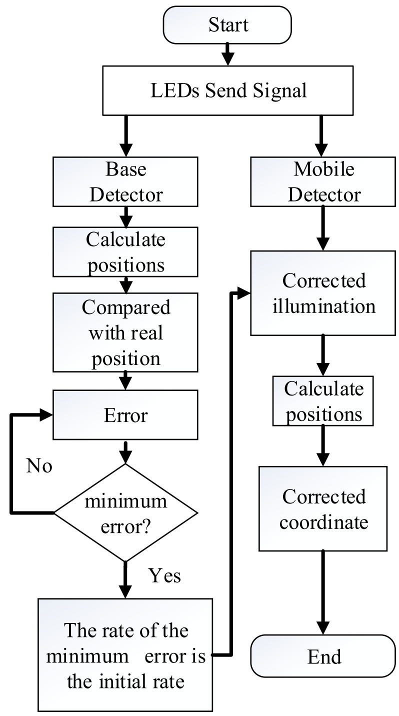


Fig. 9. Flowchart of the differential detection positioning algorithm.

5) Take the rate as the initial value of calculating the coordinates of Mobile detector.
System flowchart is shown in Fig. 9

3. Principle of Differential Detection Positioning Algorithm

3.1 Differential Detection Positioning Algorithm

When the LEDs' illumination fluctuations, light source power P_0 in (2) is not fixed. We use $P_0(t)$ to represent it.

For the same detector, twice samples received LEDs' illumination have a relationship as

$$\frac{P_{A,t_1}}{P_{A,t_2}} = \frac{P_0(t_1)\dfrac{(m+1)A\,h^{(m+1)}}{2\pi d^{(m+3)}}}{P_0(t_2)\dfrac{(m+1)A\,h^{(m+1)}}{2\pi d^{(m+3)}}} = \frac{P_0(t_1)}{P_0(t_2)}. \tag{11}$$

As (11), for the same detector, the ratio of received illumination only related to the light source. Thus, we derive

$$R = \frac{P_0(t_2) - P_0(t_1)}{P_0(t_1)} \tag{12}$$

where R is the rate of light intensity fluctuations (the change rate of two received light intensity of detectors). As (12), R is not related to detector's location; it is only determined by the time interval between two received signals.

For the two detectors, if the received initial light intensity of one detector is known, the detector can be used as base detector. If the two detectors sample simultaneously, the base detector can compensation the fluctuation of the other detector.

Assuming detector A is the base detector, detector B is need to be compensated. The intensity received by detector from a single LED for each sample can be recorded as $A_1, A_2, A_3 \ldots\ldots$ Then, the change rate of signal intensity can be shown as $R_1 = \frac{A_2 - A_1}{A_1}, \ldots\ldots, R_{n-1} = \frac{A_n - A_1}{A_1}$. For detector B, the received signal intensity B_n is the superposition of fluctuating light illumination and original light intensity:

$$B_n = B_1(1 + R_{n-1}). \tag{13}$$

Equation (13) can be obtained from:

$$B_1 = \frac{B_n}{1 + R_{n-1}} \tag{14}$$

Therefore, after the detector B received the n-th light intensity, the original illumination B_1 can be obtained with the detector A.

The discussion above is confined to one single LED that received by two detectors. But in actual positioning system, we used three or more LEDs. The case where used multiple LEDs received by two detectors is discussed below.

It is easy to extend the compensation algorithm from single LED to multiple LEDs: Suppose there are N LEDs, the base detector A receives n samples of each LED. The light intensity can be represented as

$$\begin{bmatrix} A_{1,\mathrm{LED1}} & A_{1,\mathrm{LED2}} & \cdots\cdots & A_{1,\mathrm{LEDN}} \\ A_{2,\mathrm{LED1}} & A_{2,\mathrm{LED2}} & \cdots\cdots & A_{2,\mathrm{LEDN}} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ A_{n,\mathrm{LED1}} & A_{n,\mathrm{LED2}} & \cdots\cdots & A_{n,\mathrm{LEDN}} \end{bmatrix}. \tag{15}$$

For (15), using (12) can obtain the rate of light intensity fluctuation as

$$\begin{bmatrix} R_{1,\mathrm{LED1}} & R_{1,\mathrm{LED2}} & \cdots\cdots & R_{1,\mathrm{LEDN}} \\ R_{2,\mathrm{LED1}} & R_{2,\mathrm{LED2}} & \cdots\cdots & R_{2,\mathrm{LEDN}} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ R_{n-1,\mathrm{LED1}} & R_{n-1,\mathrm{LED2}} & \cdots\cdots & R_{n-1,\mathrm{LEDN}} \end{bmatrix}. \tag{16}$$

The measured illumination of each LED by detector B can be represented as

$$\begin{bmatrix} B_{n,\mathrm{LED1}} & B_{n,\mathrm{LED2}} & \cdots\cdots & B_{n,\mathrm{LEDN}} \end{bmatrix}. \tag{17}$$

Equation (17) contains the power fluctuation of the source LEDs, which can be suppressed by

$$\begin{bmatrix} \frac{B_{n,\mathrm{LED1}}}{1 + R_{n-1,\mathrm{LED1}}} & \frac{B_{n,\mathrm{LED2}}}{1 + R_{n-1,\mathrm{LED2}}} & \cdots\cdots & \frac{B_{n,\mathrm{LEDN}}}{1 + R_{n-1,\mathrm{LEDN}}} \end{bmatrix}. \tag{18}$$

Thus, the original illumination value is obtained. This value is not influenced by the fluctuation of the light source.

3.2 Experimental Results

3.2.1 Positioning Inside the LEDs Cell: The prototype experimental scenario is shown in Fig. 10: length is 1.0 m, width is 1.0 m, and height is 1.5 m. The base detector is placed in the center of the triangle area. The mobile station detector was measured at the position shown in Fig. 10. It was placed at the height of 0.0 m. As in Fig. 3, 26 points were measured, and three measurements were made at each point. Fig. 11 compares the measured positions with traditional RSS algorithm and differential detection algorithm. It can be found that the deviation of the measured results with differential detection algorithm for each point is less than that with traditional RSS algorithm. Fig. 12 compares the measurement error over time. It can be seen from Fig. 12(a) that the error is significantly reduced with differential detection positioning scheme, and the average error of 78 measurements is reduced from 10.0 cm to 4.0 cm. Fig. 12(b) compares the average deviation at each measured point. The standard deviation of 26 points is reduced from 9.0 cm to 2 .5 cm. It can

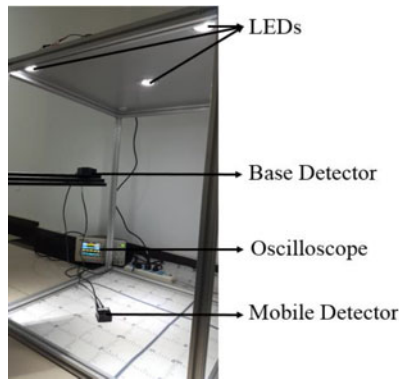


Fig. 10. Experimental setup of the differential detection positioning scheme.

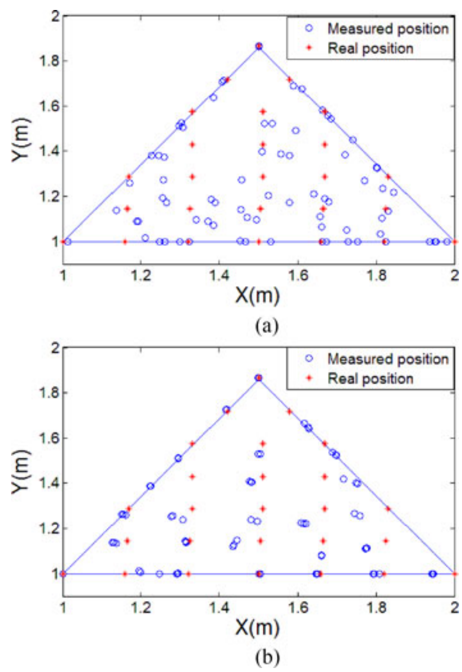


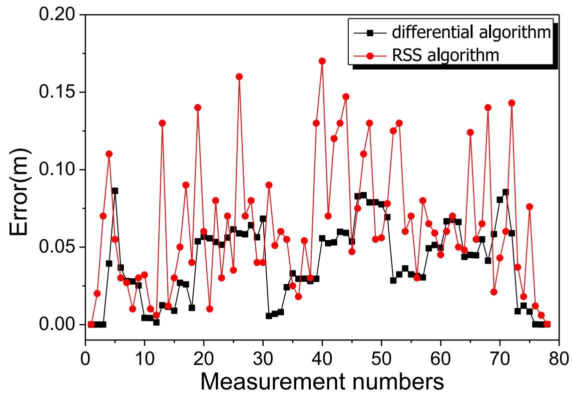
Fig. 11. Positioning error of (a) traditional RSS algorithm based system and (b) differential detection positioning based system.

be concluded that with double detectors, differential detection positioning scheme can efficiently mitigate localization instability caused by the fluctuation of light source power.

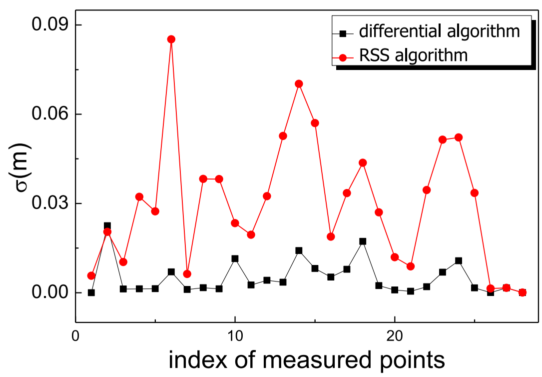
3.2.2 Positioning Outside the LEDs Cell: When differential detection positioning scheme is employed to position outside the LDEs cell, we can get from (3) that

$$d_{\mathrm{A,LED1}} = \sqrt[(m+3)]{\frac{P_{0,\mathrm{LED1}}}{P_{\mathrm{A,LED1}}}\left(\frac{(m+1)A\,{h_A}^{(m+1)}}{2\pi}\right)} \tag{19}$$

$$d_{\mathrm{B,LED1}} = \sqrt[(m+3)]{\frac{P_{0,\mathrm{LED1}}}{P_{\mathrm{B,LED1}}}\left(\frac{(m+1)A\,{h_B}^{(m+1)}}{2\pi}\right)}. \tag{20}$$



(a)



(b)

Fig. 12. (a) Measurement error over 78 times and (b) average deviation at each measured point.

Then, there is

$$\frac{d_{\mathrm{B,LED1}}}{d_{\mathrm{A,LED1}}} = \frac{\sqrt[(m+3)]{\frac{P_{0,\mathrm{LED1}}}{P_{\mathrm{B,LED1}}}\left(\frac{(m+1)A\,{h_A}^{(m+1)}}{2\pi}\right)}}{\sqrt[(m+3)]{\frac{P_{0,\mathrm{LED1}}}{P_{\mathrm{A,LED1}}}\left(\frac{(m+1)A\,{h_B}^{(m+1)}}{2\pi}\right)}} = \sqrt[(m+3)]{\frac{P_{\mathrm{A,LED1}}\,{h_A}^{(m+1)}}{P_{\mathrm{B,LED1}}\,{h_B}^{(m+1)}}} \tag{21}$$

where $d_{\mathrm{A,LED1}}$ is known. $d_{\mathrm{B,LED1}}$ can be solve by (21), and $d_{\mathrm{B,LED2}}$, $d_{\mathrm{B,LED3}}$ can be obtained as

$$\begin{Bmatrix} (x_1-x)^2+(y_1-y)^2+(z_1-z)^2 = d^2{}_{\mathrm{B,LED1}} \\ (x_2-x)^2+(y_2-y)^2+(z_2-z)^2 = d^2{}_{\mathrm{B,LED2}} \\ (x_3-x)^2+(y_3-y)^2+(z_3-z)^2 = d^2{}_{\mathrm{B,LED3}} \end{Bmatrix}. \tag{22}$$

The position of mobile detector is obtained by solving (22).

The experimental scenario is shown in Fig. 13: The space is 0.4 m $\times$ 0.4 m $\times$ 1.5 m in size. The locations of three LEDs are (0.24, 0.3, 1.5), (0.64, 0.3, 1.5), and (0.44, 0.64, 1.5). Base detector is placed in the center of triangle area, and its height is 1.0 m. The height of mobile station detector is 0.0 m. In Fig. 14(a), the triangle marks the projection boundary on the receiver plane of three LEDs. The stars are the real positions and the circles mark the measured results. The positioning error of the experiment is shown in Fig. 14(b). The error of each measurement point is within 10.0 cm. The

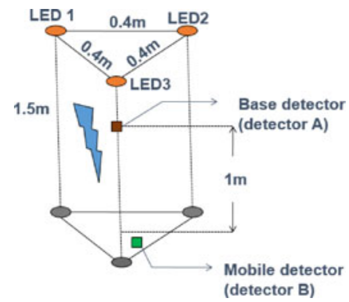


Fig. 13. Schematic diagram of experiment

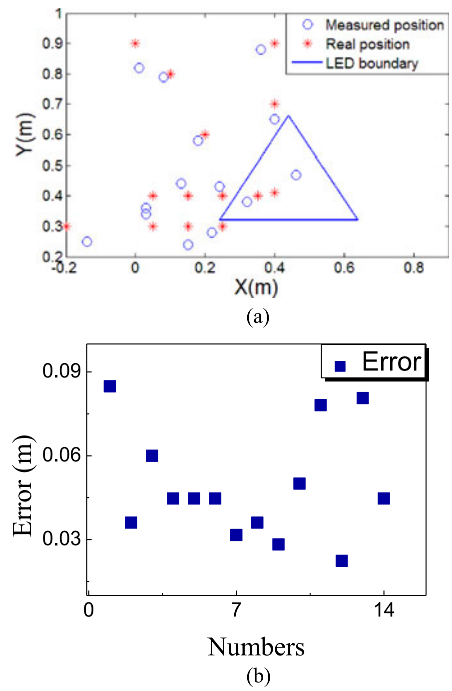


Fig. 14. (a) Measured positions and the actual points. (b) Position error.

coverage of each LED on the ground is less than 2.02 m because of receiver's receiving accuracy limit. From this view of point, the positioning area is increased from 0.068 square meters to 3.14 square meters, i.e., enhanced by 46 times. It proved that the differential detection algorithm is effective in both inside and outside the LEDs cell.

4. Conclusion

In order to improve the performance of received signal strength (RSS) positioning technology, this paper proposed a differential detection positioning based algorithm that uses two detectors. The algorithm can fix the positioning instability caused by LED light intensity fluctuation, and can achieve positioning out of the LEDs' cell. When one detector's position is known, the other detector's positioning result can be corrected. The algorithm can solve the positioning error caused by LED illumination fluctuation effectively and can extend the positioning range. Experimental results show that the method can improve the positioning accuracy from 10.0 cm to 4.0 cm, and the standard deviation can be reduced from 9.0 cm to 2.5 cm. The positioning range is enhanced by 46 times. Thus, the proposed positioning method can be used as a high precision, high stability, and large area positioning in actual LED lighting environment.

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