IEEE Photonics Journal
An IEEE Photonics Society Publication

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Volume 9, Number 2, April 2017

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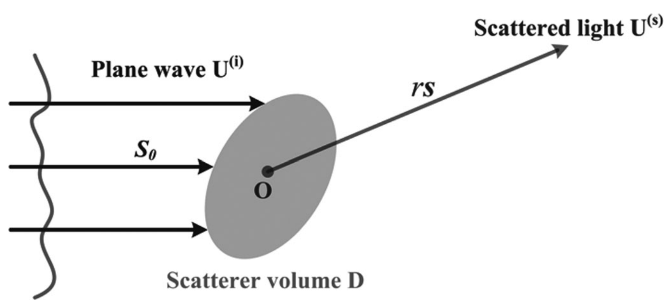

DOI: 10.1109/JPHOT.2017.2683942

Reciprocal Relations for Third-Order Correlation Between Intensity Fluctuations of Light Scattered From a Quasi-Homogeneous Medium

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DOI:10.1109/JPHOT.2017.2683942

Manuscript received February 19, 2017; revised March 10, 2017; accepted March 13, 2017. Date of publication March 16, 2017; date of current version April 5, 2017. Corresponding author: Jia Li (email: jxl1180@miami.edu).

Abstract: Within the validity of the first-order Born approximation, we introduce the third-order correlation between intensity fluctuation (CIF) of light scattered from a quasi-homogeneous medium. The third-order CIF is defined as the degree of intensity correlations specified at three position vectors in scattered field. Expressions are derived for the normalized CIF and variance in intensity fluctuations (VIF) of the scattered field. Two reciprocal relations are generalized for the third-order CIF of the scattered field: 1) The normalized CIF is proportional to the real portion of the Fourier transform products of the strength of the scattering potential; 2) the third-order VIF of the scattered field is proportional to the cube of the Fourier transform of the normalized correlation coefficient of the scattering potential.

Index Terms: Reciprocal relation, intensity fluctuation, scattering potential.

1. Introduction

It has been well known that statistical properties of a medium, of which the refractive index is either random or deterministic in distribution, can be quantitatively determined from scattering experiments where the irradiance statistics of scattered fields were measured [1], [2]. Two evident tendencies can be noticed in developments of the weak scattering theory. One intended to solve the inverse scattering problem, more specifically, it determined the correlation functions of a spatially random medium [3]–[5]. Another established intrinsic relations between irradiance properties of scattered fields and correlation statistics of the incident beams [6]–[8]. Regarding the studies performed on solving the inverse scattering problem, reciprocal relations were crucial for reconstructing correlation functions of a 3-D random, quasi-homogeneous (QH) scatterer [9]. Extensions of the reciprocal relations were done for the case where a stochastic electromagnetic beam scatters from a QH medium was considered [10]. It was shown that a reciprocal relation was satisfied for the correlation between intensity fluctuations (CIF) of a far-zone scattered field [11]: The normalized CIF and variance in intensity fluctuation (VIF) of a scattered field were proportional to the Fourier transform of the scattering potential of the medium. The reciprocal relations for weak scattering of light from a QH, anisotropic medium were obtained in [12]. In particular, it was shown that anisotropy of a QH medium leads substantial influences on analytic forms of reciprocal relations for a far-zone scattered field. In addition, it was reported that a square law holds true between the spectrum of the scattered field and spatial frequency of correlation function of scattering potential of tissue [13]. Based upon these results, the intensity-intensity correlations of a far-zone scattered field governed by Gaussian statistics from a particle were derived in [14], which further generalized the results in [15] to the case of an arbitrary correlated incident field. Even though, higher-order CIF of a weakly scattered field, to the best our knowledge, has not been fully addressed in any current literature.

In this paper, we introduce the third-order CIF of a scattered field by defining it as the degree of intensity correlations specified at three position vectors in scattered field. In reaching the results, we assume that the interaction between the incident field and the 3-D QH scatterer is extremely weak, hence the first-order Born approximation can be adopted to treat the scattering process. Expressions are derived for the normalized CIF and VIF of the scattered field by employing the Gaussian Moment Theorem (GMT). We show that two classes of reciprocal relations are satisfied for the third-order CIF of the scattered field, and the results provide a prospective approach to determine the correlation function of a spatially QH medium.

2. Third-Order Intensity Correlations of a Weakly-Scattered Field

To begin, let us assume a monochromatic plane wave carries frequency ω and a space-dependent electric field

$$U^{(i)}\left(\mathbf{r}';\omega\right) = a\left(\omega\right)\exp\left(ik\mathbf{u_0}\cdot\mathbf{r}'\right) \tag{1}$$

where $a(\omega)$ is the complex-valued spectral amplitude, $\mathbf{r}'$ is a position vector, $\mathbf{u}_0$ is the unit vector that represents the direction of the plane wave, $k = \omega/c$ is the wave number, and c denotes the speed of light in vacuum. In the far field, the scattered light from the medium is characterized within the validity of the first-order Born approximation [9]–[11]

$$U^{(s)}\left(r\mathbf{u};\omega\right) = \frac{\exp\left(ikr\right)}{r}\int_D U^{(i)}\left(\mathbf{r}';\omega\right)F\left(\mathbf{r}';\omega\right)\exp\left(-ik\mathbf{u}\cdot\mathbf{r}'\right)d^3r' \tag{2}$$

where the subscript "D" represents the scatterer volume, $F(\mathbf{r}',\omega)$ denotes the scattering potential of the medium, and $\mathbf{u}$ is the unit vector that represents the scattered direction. Equation (2) provides a fundamental basis for deducing higher-order intensity correlations of the scattered field, as long as the statistical properties of the scattering potential is known. In the following discussion, we primary concentrate on the third-order CIF of scattered field from a QH medium. To start, let us briefly review the definition of the two-point CIF of a statistically stationary optical field [16]

$$U^{(s)}\left(r\mathbf{u};\omega\right) = \frac{\exp\left(ikr\right)}{r}\int_D U^{(i)}\left(\mathbf{r}';\omega\right)F\left(\mathbf{r}';\omega\right)\exp\left(-ik\mathbf{u}\cdot\mathbf{r}'\right)d^3r' \tag{3}$$

where frequency dependence for all quantities has been omitted for the sake of simplicity, unless elsewhere specified. The intensity fluctuation specified at a single point of the scattered field can be given by

$$\triangle I^{(s)}\left(\mathbf{r}_\alpha\right) = I^{(s)}\left(\mathbf{r}_\alpha\right) - \left\langle I^{(s)}\left(\mathbf{r}_\alpha\right)\right\rangle, \qquad \left(\alpha = 1, 2, \ldots n\right). \tag{4}$$

Based on (3) and (4), one can further obtain higher order CIFs (generally higher than two) of the scattered field. Typically, the third-order CIF, which is defined as the degree of intensity correlations specified at three position vectors in scattered field, can be derived from (3) and (4):

$$\left\langle \Delta I^{(s)}(\mathbf{r_1})\,\Delta I^{(s)}(\mathbf{r_2})\,\Delta I^{(s)}(\mathbf{r_3})\right\rangle = \left\langle I^{(s)}(\mathbf{r_1})\,I^{(s)}(\mathbf{r_2})\,I^{(s)}(\mathbf{r_3})\right\rangle$$
$$- \left\langle I^{(s)}(\mathbf{r_1})\right\rangle\left\langle I^{(s)}(\mathbf{r_2})\right\rangle\left\langle I^{(s)}(\mathbf{r_3})\right\rangle - \left\langle I^{(s)}(\mathbf{r_1})\right\rangle\left\langle \Delta I^{(s)}(\mathbf{r_2})\,\Delta I^{(s)}(\mathbf{r_3})\right\rangle$$
$$- \left\langle I^{(s)}(\mathbf{r}_2)\right\rangle\left\langle \Delta I^{(s)}(\mathbf{r}_1)\,\Delta I^{(s)}(\mathbf{r_3})\right\rangle - \left\langle I^{(s)}(\mathbf{r}_3)\right\rangle\left\langle \Delta I^{(s)}(\mathbf{r}_1)\,\Delta I^{(s)}(\mathbf{r}_2)\right\rangle. \quad (5)$$

By using (2), the third-order intensity correlation of the scattered field can be obtained by taking the ensemble average over the products of instantaneous scattered intensities specified at three position vectors $r\mathbf{u}_1$, $r\mathbf{u}_2$, and $r\mathbf{u}_3$

$$\left\langle I^{(s)}(r\mathbf{u_1})\,I^{(s)}(r\mathbf{u_2})\,I^{(s)}(r\mathbf{u_3})\right\rangle = \frac{a^6}{r^6}\int_D\int_D\int_D \left\langle F^*(\mathbf{r_1}')\,F(\mathbf{r_2}')\,F^*(\mathbf{r_3}')\,F(\mathbf{r_4}')\,F^*(\mathbf{r_5}')\,F(\mathbf{r_6}')\right\rangle$$
$$\times\ \exp\left[-ik(\mathbf{u_0}-\mathbf{u_1})\cdot(\mathbf{r_1}'-\mathbf{r_2}') - ik(\mathbf{u_0}-\mathbf{u_2})\cdot(\mathbf{r_3}'-\mathbf{r_4}')\right]$$
$$\times\ \exp\left[-ik(\mathbf{u_0}-\mathbf{u}_3)\cdot(\mathbf{r}_5'-\mathbf{r}_6')\right] d^3r_1'd^3r_2'd^3r_3'd^3r_4'd^3r_5'd^3r_6'. \quad (6)$$

Equation (6) contains the six-folder integrations over spatial variables within the scatterer volume. We notice that the ensemble average over the sixth-order correlation function of the scattering potential, as shown in the integrand of (6), can be expanded into a sum of second-order correlation functions of the medium, if assuming that the scattering potential obeys the Gaussian statistics [16]

$$\left\langle F^*(\mathbf{r_1}')\,F(\mathbf{r_2}')\,F^*(\mathbf{r_3}')\,F(\mathbf{r_4}')\,F^*(\mathbf{r_5}')\,F(\mathbf{r_6}')\right\rangle = \Gamma_{123456} + \Gamma_{123654}$$
$$+ \Gamma_{143256} + \Gamma_{143652} + \Gamma_{163254} + \Gamma_{163452} \quad (7)$$

where

$$\Gamma_{\alpha\beta\gamma\zeta\tau\upsilon} = \left\langle F^*(\mathbf{r}_\alpha')\,F(\mathbf{r}_\beta')\right\rangle\cdot\left\langle F^*(\mathbf{r}_\gamma')\,F(\mathbf{r}_\zeta')\right\rangle\cdot\left\langle F^*(\mathbf{r}_\tau')\,F(\mathbf{r}_\upsilon')\right\rangle, \qquad (\alpha,\beta,\gamma,\zeta,\tau,\upsilon = 1,2,3,4,5,6). \quad (8)$$

For QH medium, the correlation function of the scattering potential can be represented as the product form [8]–[10]

$$\left\langle F^*(\mathbf{r}'_\alpha)F(\mathbf{r}'_\beta)\right\rangle = S_F\left[(\mathbf{r}'_\alpha + \mathbf{r}'_\beta)/2\right]\eta_F(\mathbf{r}'_\beta - \mathbf{r}'_\alpha) \quad (9)$$

where S_F and η_F are the strength and normalized correlation coefficient (NCC) of the scattering potential of the medium, respectively. Upon substituting from (7)-(9) into (6), the third-order intensity correlation of the scattered field can be obtained as

$$\left\langle I^{(s)}(r\mathbf{u_1})\,I^{(s)}(r\mathbf{u_2})\,I^{(s)}(r\mathbf{u_3})\right\rangle = \frac{a^6}{r^6}\left\{\left[\tilde{S}_F(0)\right]^3\tilde{\eta}_F\left[k(\mathbf{u_0}-\mathbf{u_1})\right]\right.$$
$$\times\ \tilde{\eta}_F\left[k(\mathbf{u_0}-\mathbf{u_2})\right]\tilde{\eta}_F\left[k(\mathbf{u_0}-\mathbf{u_3})\right] + \tilde{S}_F(0)\,\tilde{\eta}_F\left[k(\mathbf{u_0}-\mathbf{u_1})\right]$$
$$\times\ \left|\tilde{S}_F\left[k(\mathbf{u_3}-\mathbf{u_2})\right]\right|^2\tilde{\eta}_F^2\left[k\left(\mathbf{u_0}-\frac{\mathbf{u_2}+\mathbf{u_3}}{2}\right)\right] + \tilde{S}_F(0)\,\tilde{\eta}_F\left[k(\mathbf{u_0}-\mathbf{u_3})\right]$$
$$\times\ \left|\tilde{S}_F\left[k(\mathbf{u_2}-\mathbf{u_1})\right]\right|^2\tilde{\eta}_F^2\left[k\left(\mathbf{u_0}-\frac{\mathbf{u_1}+\mathbf{u_2}}{2}\right)\right] + \tilde{S}_F(0)\,\tilde{\eta}_F\left[k(\mathbf{u_0}-\mathbf{u_2})\right]$$
$$\times\ \left|\tilde{S}_F\left[k(\mathbf{u_3}-\mathbf{u_1})\right]\right|^2\tilde{\eta}_F^2\left[k\left(\mathbf{u_0}-\frac{\mathbf{u_1}+\mathbf{u_3}}{2}\right)\right] + 2\mathrm{Re}\left\{\tilde{S}_F\left[k(\mathbf{u_2}-\mathbf{u_1})\right]\right\}$$
$$\times\ \tilde{S}_F\left[k(\mathbf{u_3}-\mathbf{u_2})\right]\tilde{S}_F\left[k(\mathbf{u_1}-\mathbf{u_3})\right]\tilde{\eta}_F\left[k\left(\mathbf{u_0}-\frac{\mathbf{u_1}+\mathbf{u_2}}{2}\right)\right]$$
$$\left.\times\ \tilde{\eta}_F\left[k\left(\mathbf{u_0}-\frac{\mathbf{u_2}+\mathbf{u_3}}{2}\right)\right]\tilde{\eta}_F\left[k\left(\mathbf{u_0}-\frac{\mathbf{u_1}+\mathbf{u_3}}{2}\right)\right]\right\} \quad (10)$$

where

$$\tilde{S}_F(k\mathbf{u}) = \int_D S_F(\mathbf{r}')\exp(-ik\mathbf{u}\cdot\mathbf{r}')\,d^3r' \tag{11}$$

$$\tilde{\eta}_F(k\mathbf{u}) = \int_D \eta_F(\mathbf{r}')\exp(-ik\mathbf{u}\cdot\mathbf{r}')\,d^3r' \tag{12}$$

are the Fourier transform of the strength and NCC of the scattering potential of the medium, respectively. Equation (10) shows that the third-order intensity correlation of the scattered field is dependent on both the strength and NCC of the scattering potential. One may notice that (10) is evaluated at three position vectors in scattered field, that is to say the third-order intensity correlation of a scattered field takes an averaged measure of mutual degrees of correlation between three position vectors in scattered field. In this following section, we derive formulas for the normalized CIF and the third-order VIF of the scattered field, respectively, and further obtain the reciprocal relations for the third-order CIF of the scattered field.

3. Reciprocal Relations for the Third-Order CIF of a Weakly-Scattered Field

Prior to deriving the reciprocal relations for the third-order CIF of a scattered field, let us first recall the average irradiance of a scattered field from a QH medium, which can be indexed by reviewing [9, Eq. (24)]

$$\left\langle I^{(s)}(r\mathbf{u})\right\rangle = \frac{a^2}{r^2}\tilde{S}_F(0)\,\tilde{\eta}_F[k(\mathbf{u_0}-\mathbf{u})]. \tag{13}$$

Also reviewed is the second-order CIF of the scattered field (see [15, Eq. (15)])

$$\left\langle \Delta I^{(s)}(r\mathbf{u}_\alpha)\,\Delta I^{(s)}(r\mathbf{u}_\beta)\right\rangle = \frac{a^4}{r^4}\left|\tilde{S}_F[k(\mathbf{u}_\beta-\mathbf{u}_\alpha)]\right|^2\tilde{\eta}_F^2\left[k\left(\mathbf{u_0}-\frac{\mathbf{u}_\alpha+\mathbf{u}_\beta}{2}\right)\right]. \tag{14}$$

Upon substituting from (10), (13), and (14) into (5), the third-order CIF of the scattered field can be expressed as

$$D^{(3)}(r\mathbf{u_1}, r\mathbf{u_2}, r\mathbf{u_3}) = \frac{2a^6}{r^6}\mathrm{Re}\left\{\tilde{S}_F[k(\mathbf{u_2}-\mathbf{u_1})]\,\tilde{S}_F[k(\mathbf{u_3}-\mathbf{u_2})]\right.$$
$$\left.\times\,\tilde{S}_F[k(\mathbf{u_1}-\mathbf{u_3})]\right\}\tilde{\eta}_F\left[k\left(\mathbf{u_0}-\frac{\mathbf{u_1}+\mathbf{u_2}}{2}\right)\right]\tilde{\eta}_F\left[k\left(\mathbf{u_0}-\frac{\mathbf{u_2}+\mathbf{u_3}}{2}\right)\right]\tilde{\eta}_F\left[k\left(\mathbf{u_0}-\frac{\mathbf{u_1}+\mathbf{u_3}}{2}\right)\right]. \tag{15}$$

Since (15) is dependent on three position vectors rather than two in scattered field, one cannot retrieve the NCC of the scattering potential from the third-order CIF of the scattered field. However, we can achieve this goal by introducing the normalized third-order CIF of the scattered field, which is defined by normalizing Eq. (15). In reaching the result, we recall that the Fourier transform of the NCC of the scattering potential is a slowly varying function of the variable. A reliable approximation, therefore, is attained [9]–[11] as

$$\tilde{\eta}_F\left[k\left(\mathbf{u}_0-\frac{\mathbf{u}_1+\mathbf{u}_2}{2}\right)\right]\approx\tilde{\eta}_F\left[k\left(\mathbf{u}_0-\frac{\mathbf{u}_2+\mathbf{u}_3}{2}\right)\right]\approx\tilde{\eta}_F\left[k\left(\mathbf{u}_0-\frac{\mathbf{u}_1+\mathbf{u}_3}{2}\right)\right]\approx\tilde{\eta}_F. \tag{16}$$

In (16), $\tilde{\eta}_F$ is a constant that represents the approximated value of the Fourier transform of the NCC. We also notice that $\tilde{S}_F$ rapidly reduces when increasing the argument. This implies that (15) has the maximum value

$$D^{(3)}_{\max}(r\mathbf{u}_1, r\mathbf{u}_2, r\mathbf{u}_3) = \frac{2a^6}{r^6}\tilde{S}_F(0)\,\tilde{\eta}_F^3. \tag{17}$$

With the help of (16), we can obtain the normalized third-order CIF of the scattered field by performing the ratio of (15) to (17), i.e.,

$$N^{(3)}(r\mathbf{u}_1, r\mathbf{u}_2, r\mathbf{u}_3) = \frac{\mathrm{Re}\left\{\tilde{S}_F\left[k(\mathbf{u}_2 - \mathbf{u}_1)\right]\tilde{S}_F\left[k(\mathbf{u}_3 - \mathbf{u}_2)\right]\tilde{S}_F\left[k(\mathbf{u}_1 - \mathbf{u}_3)\right]\right\}}{\tilde{S}_F^3(0)}. \tag{18}$$

Equation (18) is the first reciprocal relation for the third-order CIF of a far-zone scattered field. It establishes the connection between the normalized third-order CIF of the scattered field and the strength of the scattering potential of the medium. It indicates that the normalized third-order CIF of the scattered field is proportional to the real portion of the Fourier transform products of the strength of the scattering potential. In terms of the reciprocal relation for the second-order degree of correlation of a scattered field (see [11, Eq. (27)])

$$\mu_s\left(r\mathbf{u}_\alpha, r\mathbf{u}_\beta\right) = \frac{\tilde{S}_F\left[k\left(\mathbf{u}_\alpha - \mathbf{u}_\beta\right)\right]}{\tilde{S}_F\left(0\right)} \tag{19}$$

(18) then can be rewritten as the alternative form

$$N^{(3)}(r\mathbf{u}_1, r\mathbf{u}_2, r\mathbf{u}_3) = \mathrm{Re}\left[\mu^{(s)}(r\mathbf{u}_1, r\mathbf{u}_2)\mu^{(s)}(r\mathbf{u}_2, r\mathbf{u}_3)\mu^{(s)}(r\mathbf{u}_3, r\mathbf{u}_1)\right]. \tag{20}$$

Equation (20) shows that the normalized, third-order CIF of the scattered field is proportional to the product of the mutual degrees of correlation specified at three position vectors in scattered field. Since the modulus of the degree of correlation of a medium is bounded between zero and one, it follows from (20) that

$$0 \leq N^{(3)}(r\mathbf{u}_1, r\mathbf{u}_2, r\mathbf{u}_3) \leq 1. \tag{21}$$

Equation (21) indicates that the normalized third-order CIF can, in general, reach the lowest value (zero) when scattered fields at any pair of position vectors are uncorrelated, i.e. $\mu^{(s)}(r\mathbf{u}_\alpha, r\mathbf{u}_\beta) = 0$. The upper limited value of the normalized CIF can be reached if, and only if, when $\mathbf{u}_1 = \mathbf{u}_2 = \mathbf{u}_3$ is satisfied. Particularly, the normalized third-order CIF specified at three position vectors, is then equal to the normalized second-order CIF specified at two position vectors if we consider $\mathbf{u}_1 = \mathbf{u}_3$ in (18) (also see [11, Eq. (19)])

$$N^{(3)}\left(r\mathbf{u_1}, r\mathbf{u_2}, r\mathbf{u_1}\right) = N^{(2)}\left(r\mathbf{u_1}, r\mathbf{u_2}\right) = \frac{\left|\tilde{S}_F\left[k\left(\mathbf{u_1} - \mathbf{u_2}\right)\right]\right|^2}{\tilde{S}_F^2\left(0\right)}. \tag{22}$$

Compared (22) with (19), one can further obtain that the normalized third-order CIF is equal to the modulus of the squared second-order degree of correlation of the scattered field, if two points out of three coincide with each other. Equation (22) confirms the validity of the reciprocal relation in (18). In addition, the third-order VIF, i.e. $V^{(3)}(r\mathbf{u})$ which is specified at a single position vector in scattered field, can be obtained by substituting from $\mathbf{u_1} = \mathbf{u_2} = \mathbf{u_3} = \mathbf{u}$ into (15) such that

$$V^{(3)}\left(r\mathbf{u}\right) = D^{(3)}\left(r\mathbf{u}, r\mathbf{u}, r\mathbf{u}\right) = \frac{2a^6}{r^6}\tilde{S}_F^3\left(0\right)\tilde{\eta}_F^3\left[k\left(\mathbf{u_0} - \mathbf{u}\right)\right]. \tag{23}$$

Equation (23) is the second reciprocal relation, revealing that the third-order VIF of a scattered field strongly depends on the NCC of the scattering potential of the medium. Such reciprocal relation can be interpreted as follows: the third-order VIF of a scattered field from a QH medium is proportional to the cube of the Fourier transform of the NCC of the scattering potential.

Our results can be compared with those previously derived for the second-order CIF of a scattered field. Evident discrepancies can be distinguished from comparing our results with [10, Eqs. (19) and (24)], which derived the normalized second-order CIF and VIF of a scattered field. For comparisons, (18) and (23) in this paper have entirely different forms and physical meanings. Defined as the degree of mutual intensity correlations specified at three position vectors of a scattered field, the third-order CIF plays an essential role as a more "generalized" degree of intensity correlation of the scattered field and describes the "average" CIF which is specified at three position vectors in scattered field. Compared with the second-order CIF of a scattered field which was obtained from

[10], [11], and [16], our results provide an essential approach to study the mutual degree of CIFs specified at three or even more points of a scattered field. The reciprocal relations (18) and (23) have more degrees of freedom than those relations derived in [10], [11] and [16] for characterizing CIF properties. Moreover, analytic forms of (18) and (23) also differ from those derived in previous literature. For instance, [11, Eqs. (19) and (24)] showed the second-order normalized CIF and VIF of a scattered field, which have similar analytic forms as our relations. However, the physical meaning of our results are entirely different from those obtained in [11]. The reciprocal relations in our paper can be helpful to determine the scattering potential parameters of a QH medium. Specifically, (18) indicates that the normalized third-order CIF of the scattered field only contains the information of the strength of the scattering potential, hence it can be applied to obtain the potential's strength parameter once the far-zone CIF properties are known. Furthermore, (23) implies that the third-order VIF of the scattered field is only related to the NCC of the scattering potential. As a result, a significant aspect of our results for applications can be concluded as follows: the NCC can be obtained from the inverse scattering experiment where the far-zone VIF of the scattered field should be measured by optical detectors. Our findings provide benefits for studying higher order CIF properties of a weakly-scattered field. In future, extension work can be done to introduce definitions of generalized higher-order CIF of scattered field and, meanwhile consider non-Gaussian statistics for characterizing the scattering potential of a QH medium.

4. Conclusion

In summary, the third-order CIF, which is specified at three position vectors in a weakly-scattered field, is introduced by assuming that the scattering potential of the QH medium obeys the Gaussian statistics. Two reciprocal relations are obtained for the scattered field. The first reciprocal relation indicates that the normalized third-order CIF of the scattered field is proportional to the real portion of the Fourier transform products of the strength of the scattering potential, while the second relation stipulates that the third-order VIF of the scattered field is proportional to the cube of the Fourier transform of the NCC of the scattering potential. The obtained results, i.e., (18) and (23) provide a flexible approach to get access to higher-order scattered properties from a 3-D medium, and are helpful to determine the scattering potential of the medium from knowing the CIF and VIF of the far-zone scattered field.

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