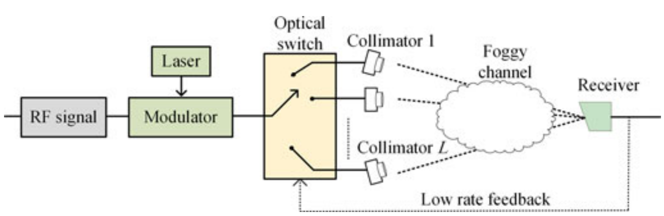
IEEE Photonics Journal
An IEEE Photonics Society Publication

Outage Probability Analysis of FSO Links Over Foggy Channel

Volume 9, Number 2, April 2017

Maged Abdullah Esmail, ***Student Member, IEEE***
Habib Fathallah, ***Senior Member, IEEE***
Mohamed-Slim Alouini, ***Fellow, IEEE***



DOI: 10.1109/JPHOT.2017.2672641
1943-0655 © 2017 IEEE

Outage Probability Analysis of FSO Links Over Foggy Channel

Maged Abdullah Esmail,[1] ***Student Member, IEEE*,**
Habib Fathallah,[1,2] ***Senior Member, IEEE*,**
and Mohamed-Slim Alouini,[3] ***Fellow, IEEE***

[1]Electrical Engineering Department, King Saud University, Riyadh 11421, Saudi Arabia, and also with the KACST Technology Innovation Center in Radio Frequency and Photonics Center, Riyadh 11421, Saudi Arabia
[2]Computer Department, College of Science of Bizerte, University of Carthage, Carthage 1054, Tunisia
[3]Computer, Electrical, and Mathematical Science and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955, Saudi Arabia

DOI:10.1109/JPHOT.2017.2672641

Manuscript received January 23, 2017; revised February 15, 2017; accepted February 16, 2017. Date of publication February 22, 2017; date of current version March 9, 2017. This work was supported by the College of Engineering Research Center and Deanship of Scientific Research at King Saud University, Riyadh, Saudi Arabia. Corresponding author: M. A. Esmail (e-mail: abdullahmaged@gmail.com).

Abstract: Outdoor free-space optic (FSO) communication systems are sensitive to atmospheric impairments, such as turbulence and fog, in addition to being subject to pointing errors. Fog is particularly severe because it induces an attenuation that may vary from few decibels up to few hundreds of decibels per kilometer. Pointing errors also distort the link alignment and cause signal fading. In this paper, we investigate and analyze the FSO systems performance under fog conditions and pointing errors in terms of outage probability. We then study the impact of several effective communication mitigation techniques that can improve the system performance including multi-hop, transmit laser selection, and hybrid radio-frequency (RF)/FSO transmission. Closed-form expressions for the outage probability are derived and practical and comprehensive numerical examples are suggested to assess the obtained results. We found that the FSO system has limited performance that prevents applying FSO in wireless microcells that have a 500-m minimum cell radius. The performance degrades more when pointing errors appear. Increasing the transmitted power can improve the performance under light to moderate fog. However, under thick and dense fog, the improvement is negligible. Using mitigation techniques can play a major role in improving the range and outage probability.

Index Terms: Free-space optic (FSO), fog, outage probability, mitigation techniques, transmit laser selection.

1. Introduction

Free space optic (FSO) communication systems have gained increasing interest during the last decade due to the advances in optical devices and the decrease of their fabrication cost. FSO has a huge bandwidth which makes it very attractive as a solution for last mile backhaul problem in current and next generation of wireless networks [1]. In fact, it has unregulated limitless spectrum straightforwardly suitable for backhaul access bottleneck. Moreover, it reduces the capital expenditure (Capex) and time because it avoids the ground digging and all other civilian engineering works. The overall network operational expenditure (Opex) can also be reduced compared to fiber based links, because the network infrastructure becomes much reduced to discrete small high bandwidth wireless nodes. Other advantages include reduced interference, high security, and system simplicity.

In spite of these advantages, FSO technology has many challenges mainly in outdoor environment due to atmospheric conditions. Fog introduces high signal attenuation and therefore is considered as a limiting factor for the spreading of the FSO technology [2]. It degrades the system performance and limits the reach distance. Most of the work done in foggy channel modeling during the last decade assumes the channel to be deterministic [3]. However, recently the fog attenuation effect is shown to follow random behavior [2], [4]. In addition to fog, building thermal expansion, dynamic winds loads, and mechanical misalignment can cause pointing errors that disrupt the link alignment [1]. The sway issue can also be generalized to the base station towers placed at the nodes of wireless cellular systems.

In [4], a channel model under fog condition has been proposed. In this work, we use this model to analyze the FSO system performance under fog conditions in terms of outage probability. We first study the outage probability due to fog conditions only. Then we combine the effect of fog with pointing errors. Closed-form expressions of the outage probability are derived for both scenarios. Furthermore, to improve the system performance, we propose some mitigation techniques including 1) multi-hop FSO system employing decode and forward (DF) relay transmissions, 2) transmit laser selection (TLS), and 3) hybrid radio-frequency (RF)/FSO transmission. For each technique, we derive closed-form expression for the outage probability and give numerical examples that show the system performance as a function of the link length and average transmitted power.

Our results indicate that under dense fog conditions, the possibility of making a communication link is practically inexistent. This is because the signal attenuation at this density beats hundreds of dBs per km. However, using mitigation techniques can reduce this effect and allows working over short range distance that cover picocells. Under moderate and light fog, the system can achieve reasonable outage probability when mitigation techniques are used. Hence, that the system can be installed in microcells that have 500 m-1000 m size. When the link simultaneously suffers from fog attenuation and pointing errors, the system performance undergoes more disruption. Note that the cell size in next generation of wireless networks is decreasing down to few hundred meters which make FSO technology attractive in such short range cells.

The remaining of the paper is organized as follows. In Section 2, we study and derive closed form expressions for the outage probability under fog conditions only and when combined with pointing errors. In Section 3, we investigate some mitigation techniques that can help in reducing the fog effect. In Section 4, we discuss and analyze the results based on some selected numerical examples, and finally, we conclude in Section 5.

2. System Model

We consider an FSO system with intensity modulation/ direct detection (IM/DD) link that exploits on-off keying (OOK) modulation. The received signal undergoes fluctuations in signal intensity due to outdoor atmospheric losses and pointing errors in addition to additive noise. The received signal is modeled as [5], [6]

$$y = R\,h\,x + n \tag{1}$$

where R is the receiver responsivity in A/W, h is the channel state, x is the signal intensity, and n is the signal additive white Gaussian noise (AWGN) with variance σ_n^2. In our model, the channel state incorporates atmospheric loss due to fog and pointing errors, which is formulated as

$$h = h_a \times h_p \tag{2}$$

where h_{a} and h_{p} are the channel states due to fog atmospheric loss, and pointing errors, respectively. The turbulence effect is ignored because it has inverse correlation with fog [5]. For slow fading channel, and equal-probability symbols drawn from OOK constellation, the received electrical SNR is defined as

TABLE 1
Channel Parameters and System Configuration

Parameter	Symbol	Value
Dense fog	k	36.05
	β	11.91
Thick fog	k	6.00
	β	23.00
Moderate fog	k	5.49
	β	12.06
Light fog	k	2.32
	β	13.12
Receiver responsivity	R	0.75 A/W
Noise standard deviation	σ_n	10^{-7} A/Hz
SNR threshold	γ_{th}	6 dB
Beam radius at 1 km	w_z	2.5 m
Pointing error displacement standard deviation	σ_s	30 cm
Receiver radius	a	10 cm
Ratio between the equivalent beam radius and σ_s	ρ	2.5

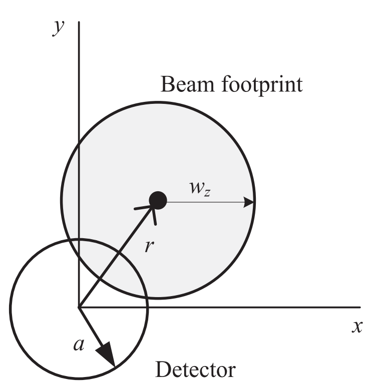


Fig. 1. Detector and beam footprint with misalignment on the detector plane.

$$\gamma = \frac{2P_t^2 R^2 h^2}{\sigma_n^2} \tag{3}$$

where P_t is the average optical transmitted power. The foggy channel state has a probability distribution function (PDF) defined in [4] as

$$f_{h_a}(h_a) = \frac{z^k}{\Gamma(k)} \left[\ln\left(\frac{1}{h_a}\right)\right]^{k-1} {h_a}^{z-1} \tag{4}$$

where $0 < h_a \leq 1$, $z = 4.343/\beta l$, and $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ is the Gamma function. The parameter l represents the transmission link length in km. The parameters k and β represents the shape and scale parameters of the signal attenuation PDF as defined in [4] and their associated values for different fog densities are listed in Table I. The average value of signal attenuation due to fog is determined by $k\beta$. The channel severity, i.e. fog density increases as the average value of signal attenuation increase. For example, under light fog, $k\beta = 30.4$ dB/ km and under moderate fog, $k\beta = 66.2$ dB/ km. Therefore, the channel gain is affected by the transmission link length and the fog density. High gain is achieved over short distances or under light fog density.

In the following, we will study and analyze the FSO system performance analytically in terms of outage probability.

2.1 Single-Hop Foggy Channel

In single-hop FSO system, the signal is transmitted over the link without using repeaters or relays. Under fog condition, the probability that the SNR falls below a certain threshold is given by

$$P_{\text{out}} = P\left(\gamma \leq \gamma_{\text{th}}\right) = P\left(h_a \leq \sqrt{\gamma_{\text{th}}/\gamma_o}\right) \tag{5}$$

where $\gamma_o = 2P_t^2 R^2/\sigma_n^2$ is the Gaussian SNR, and γ_{th} is the SNR threshold. Substituting (4) in (5) yields

$$P_{\text{out}} = \frac{1}{\Gamma(k)}\Gamma\left(k, z\ln\left(\sqrt{\gamma_o/\gamma_{\text{th}}}\right)\right) \tag{6}$$

where $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$ is the incomplete Gamma function. We can notice that the outage probability improves when the transmission link length and/or the fog density reduce(s) and *vice versa*.

2.2 Foggy Channel With Pointing Errors

Because of thermal expansion and wind load, building and telecom towers are subject to sway. Therefore, the reliability and performance of the line-of-sight (LOS) FSO system are degraded. This effect is random and cause signal fading at the receiver. The pointing error effect is described by a PDF given by [5]

$$f_{h_p}(h_p) = \frac{\rho^2}{A_o^{\rho^2}} h_p{}^{\rho^2-1}, \quad 0 \leq h_p \leq A_o \tag{7}$$

where $\rho = w_{z_{eq}}/2\sigma_s$ is the ratio between the equivalent beam radius and the standard deviation of the pointing error displacement, σ_s at the receiver. At pointing error $r = 0$, the fraction of collected power is $A_o = (erf(v))^2$ where $v = \sqrt{\pi/2}\, a/w_z$, and a is the receiver radius as shown in Fig. 1. The beam waist w_z is related to the equivalent beam radius by

$$w_{z_{eq}}^2 = w_z^2 \frac{\sqrt{\pi}erf(v)}{2v\exp(-v^2)}. \tag{8}$$

The probability distribution of the channel that includes atmospheric loss and pointing errors is defined as

$$f_h(h) = \int f_{h|h_a}(h|h_a) f_{h_a}(h_a)\, dh_a \tag{9}$$

where $\int f_{h|h_a}(h|h_a)$ is the conditional probability which is given by

$$f_{h\,h_a}(h|h_a) = \frac{\rho^2}{A_o{}^{\rho^2} h_a}\left(\frac{h}{h_a}\right)^{\rho^2-1}, 0 \leq h \leq A_o h_a. \tag{10}$$

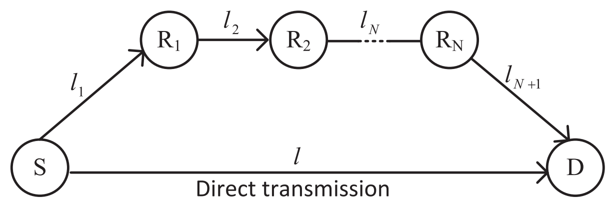


Fig. 2. Multi-hop FSO communication system with N relays. S: source, D: Destination, Ri: i^{th} relay, l: link length.

Substituting (4) and (10) in (9) yields

$$f_h(h) = \frac{z^k \rho^2 h^{\rho^2-1}}{\Gamma(k) A_o{}^{\rho^2}} \int_{h/A_o}^{1} \left[\ln\left(\frac{1}{h_a}\right)\right]^{k-1} h_a{}^{m-1} dh_a \tag{11}$$

where $m = z - \rho^2$. Let $x = ln(1/h_a)$, then using the following identity [[7, p. 348]

$$\int_0^u x^{\nu-1} e^{-\mu x} dx = \mu^{-\nu} [\Gamma(\nu) - \Gamma(\nu, \mu u)], \quad [\text{Re}(\nu) > 0] \tag{12}$$

the integration in (11) is solved to yield

$$f_h(h) = \frac{z^k \rho^2}{m^k \Gamma(k) A_o{}^{\rho^2}} h^{\rho^2-1} [\Gamma(k) - \Gamma(k, m \ln(A_o/h))]. \tag{13}$$

Substituting (13) in (5) and assuming $x = ln(A_o/h)$, the outage probability under fog and pointing errors is found to be

$$P_{\text{out}} = \frac{z^k}{m^k q^{\rho^2}} - \frac{z^k \rho^2}{m^k \Gamma(k)} \int_{\ln(q)}^{\infty} e^{-\rho^2 x} \Gamma(k, mx)\, dx \tag{14}$$

where $q = A_o/\sqrt{\gamma_{\text{th}}/\gamma_o}$. The first term has value $<<1$ and can be neglected. The integral in the second term can be solved using FullSimplification function in Mathematica software [8] and the outage probability is found to be

$$P_{\text{out}} = \frac{z^k}{m^k \Gamma(k)} \Big[-e^{-\rho^2 \ln q} \Gamma(k, m \ln q) + w^{-1} (m \ln q)^k \\ \left(e^{-(\rho^2+m) \ln q} + (k-1) En(2-k, w) \right) \Big] \tag{15}$$

where $w = (\rho^2 + m) \ln q$, and E_n (z) is the exponential integral.

3. Mitigation Techniques of Fog Effect

In this section, we extend our study in previous section to cover some techniques that can improve the FSO system performance under fog conditions. For each technique, we derive close-form expression for the outage probability. Such techniques can improve the outage probability and extend the reach distance.

3.1 Multi-Hop FSO Communication System

In order to reduce the effect of channel in power limited systems such as FSO and extend the reach distance, multi-hop technique is proposed. Since fog introduces power loss, multi-hop technique is an attractive for FSO systems. We consider a multi-hop FSO system with DF relay transmissions as shown in Fig. 2 where the transmitted signal propagates from the source to the destination over N relays. Each relay detects, decodes, modulates, and retransmits the signal to the next relay. The received signal at the i^{th} node is defined as

$$y_i = R_i h_a^i x_i + n_i \tag{16}$$

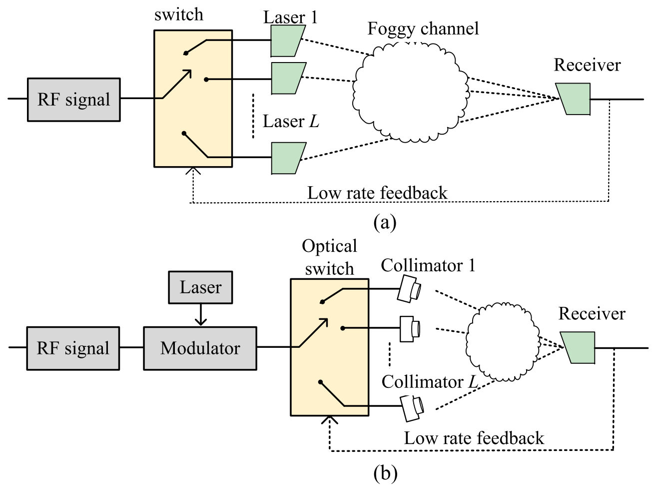


Fig. 3. FSO communication system using transmit laser selection. (a) Proposed configuration in literature. (b) Our proposed configuration.

where R_i, h_a^i, x_i and n_i are associated to the i^{th} segment of the multi-hop FSO system. The total average transmitted power of the system is constrained to P_t and hence the average transmitted optical power of the i^{th} node is given by

$$P_t^i = P_t/(N+1). \tag{17}$$

From (6), the outage probability of the i^{th} path can be expressed as

$$P_{out}^i = \frac{1}{\Gamma(k_i)}\Gamma\left(k_i, z_i \ln\left(\sqrt{\gamma_o^i/\gamma_{\text{th}}}\right)\right) \tag{18}$$

where $z_i = 4.343/\beta_i l_i$, $l_i = l/(N+1)$ is the length of the i^{th} path and $\gamma_o^i = 2(P_t^i R_i/\sigma_n^i)^2$. The overall outage performance of the FSO system depends on the outage probability of each hop which is expressed as [9]

$$P_{out} = 1 - \prod_{i=1}^{N}[1 - P_{out}^i]. \tag{19}$$

For symmetrical paths and relays, the outage probability of multi-hop FSO system under fog condition can be expressed as

$$P_{out} = 1 - \left[1 - \frac{1}{\Gamma(k)}\Gamma\left(k, z\ln(\sqrt{\gamma_o/\gamma_{\text{th}}}\right)\right]^N. \tag{20}$$

3.2 Transmit Laser Selection Diversity (TLS)

One of the effective techniques to combat the effect of channel is spatial diversity. It can enhance the system performance and achieve longer distance under bad weather conditions. The TLS diversity technique has been proposed for RF systems and recently for FSO systems where the best path is selected and processed [10], [11]. This reduces the hardware complexity and cost by reducing the number of chains at the transmitter side. Fig. 3(a) shows one configuration for TLS system that uses L laser sources in the transmitter side. The cost is reduced using the same RF chain. Instead of using laser sources at the front end of the system, we propose using the TLS configuration shown in Fig. 3(b). In this configuration, the optical modulated signal is generated using an optical modulator and single laser source. The front end of the system uses optical collimators that use

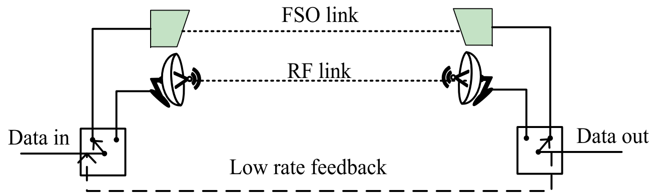


Fig. 4. RF/FSO hybrid communication system.

lenses to focus the beam toward the receiver. These collimators are passive components. Hence, the system cost is reduced much as compared to the configuration in Fig. 3(a) that uses a laser source for each channel.

Path selection is based on availability of channel state information (CSI) at the receiver and also at the transmitter. Feeding back CSI from the receiver to the transmitter is not an issue in terms of bandwidth as FSO has huge bandwidth [10], [11].

Consider L laser transmitters (or collimators) pointed simultaneously toward a single photodetector receiver. Only one transmitter is selected that has high path fading gain. The channel states $\{h_a^1, h_a^2, \ldots\ldots, h_a^L\}$ are assumed independent and identically distributed (i.i.d.) with a distribution function given by (4). The channel state corresponding to the best selected path is given by

$$h_a^m = \max_{j=1,2,\ldots,L} h_a^j. \tag{21}$$

For i.i.d. random variables, the cumulative distribution function (CDF) corresponding to the channel state in (21) is expressed as

$$F_{h_a^m}(h_a^m) = \left[F_{h_a}(h_a)\right]^L. \tag{22}$$

This CDF can be utilized to obtain the outage probability of the TLS system at certain threshold. Substituting (6) in (22), the outage probability is given by

$$P_{\text{out}} = \left[\frac{1}{\Gamma(k)}\Gamma\left(k, z\ln\left(\sqrt{\gamma_o/\gamma_{\text{th}}}\right)\right)\right]^L. \tag{23}$$

3.3 Hybrid RF/FSO Transmission System

Hybrid RF/FSO transmission is a practical solution that is currently used in wireless market which exploits a millimeter wave (MMW) RF link as a backup [12]. FSO uses infrared band that has short wavelengths while RF uses long wavelengths. Therefore, FSO and MMW RF exhibits complementary behavior under different weather conditions [13]. The outage probability in hybrid RF/FSO transmission depends on the switching scheme between both links. To reduce power consumption in the transmitter, only one link is used while the other is off as shown in Fig. 4. Such scheme is widely used in commercial hybrid products [14]. In this scheme, the FSO is used as long as the link quality is above a certain threshold. The RF link is used when the quality of FSO link drops below that threshold. If the quality of both links drops below a specific threshold, an outage is declared. Switching between two links requires CSI at the transmitter which can be offered using low rate link between the receiver and the transmitter.

The outage probability using the switching scheme discussed above is given by [14]

$$P_{\text{out}} = P_{\text{out}}^{\text{FSO}}(\gamma_{\text{th}}^{\text{FSO}}) \times P_{\text{out}}^{\text{RF}}(\gamma_{\text{th}}^{\text{RF}}) \tag{24}$$

where $\gamma_{\text{th}}^{\text{FSO}}$ and $\gamma_{\text{th}}^{\text{RF}}$ are respectively the SNR threshold for the FSO and RF links. The outage probability for the FSO link under fog conditions is given in (6). For the RF link, we assume the Nakagami-m fading channel with outage probability given by [14]

$$P_{\text{out}}^{\text{RF}} = \frac{1}{\Gamma(m)}\gamma\left(m, \frac{\gamma_{\text{th}}^{\text{RF}}}{\gamma_{\text{RF}}}m\right) \tag{25}$$

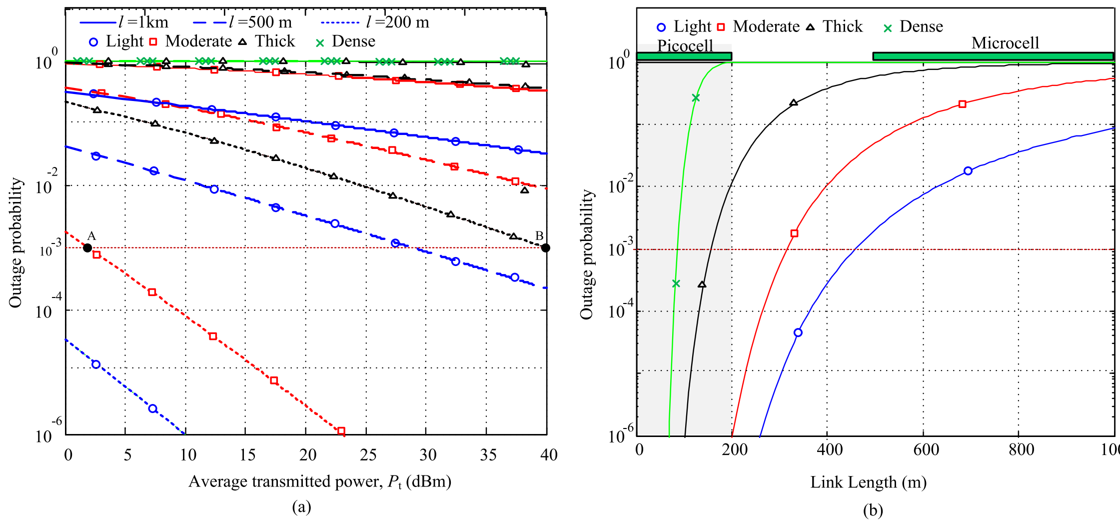


Fig. 5. Outage probability versus (a) average transmitted power for different link lengths and different fog types, (b) transmission link length with $P_t = 22\,\text{dBm}$.

where $\overline{\gamma_{RF}}$ is the average SNR for the RF link and m is a parameter that indicates the fading severity. Substituting (6) and (25) in (24) yields

$$P_{out} = \frac{1}{\Gamma(k)\Gamma(m)}\gamma\left(m, \frac{\gamma_{th}^{RF}}{\overline{\gamma_{RF}}}m\right)\Gamma\left(k, z\ln\left(\sqrt{\gamma_o/\gamma_{th}}\right)\right). \tag{26}$$

4. Results and Discussions

We consider in this section an FSO system that is subject to fog conditions and pointing errors. The system configuration and channel parameters' typical values are listed in Table I. The performance of the system will be studied using the outage of probability models that have been derived in previous sections. We determine the system performance as a function of the average transmitted power and the transmission distance.

4.1 Analysis of a Single-Hop FSO Communication System

The outage probability of the FSO system versus the average transmitted power and transmission link length is shown in Fig. 5. In can be noticed that under dense fog with link length greater than 100 m, the system performance is poor with $P_{out} \asymp 1$ even if the transmitted power increased. For lower fog densities, the performance improves as we reduce the distance or increase the transmitted power. However, this improvement is proportional with the fog density. For 200 m link length and 10^{-3} outage probability, the required average transmitted power under moderate and thick fog is 1.7 dBm (Point A) and 40 dBm (Point B), respectively as shown in Fig. 5(a). This means that about 38.3 dBm more power is required to maintain the same performance as the fog changes from moderate to thick. For light fog, less than 0 dBm is needed to achieve 10^{-3} outage probability over the same distance. As the link length increases, the outage probability drops below 10^{-3}.

Fig. 5(b) shows exponential increase of the outage probability with the transmission link length at 22 dBm average transmitted power. For picocells that have 200 m maximum size, less than 10^{-6} outage proability can be achieved under moderate and light fog. For thick and dense fog, the outage probability decreases to 10^{-2} and 1, respectively. For microcells that have size more than 500 m, the FSO system cannot achieved 10^{-3} outage probability even under light fog. Therefore, exploiting fog mitigation techniques is necessary to improve the system performance.

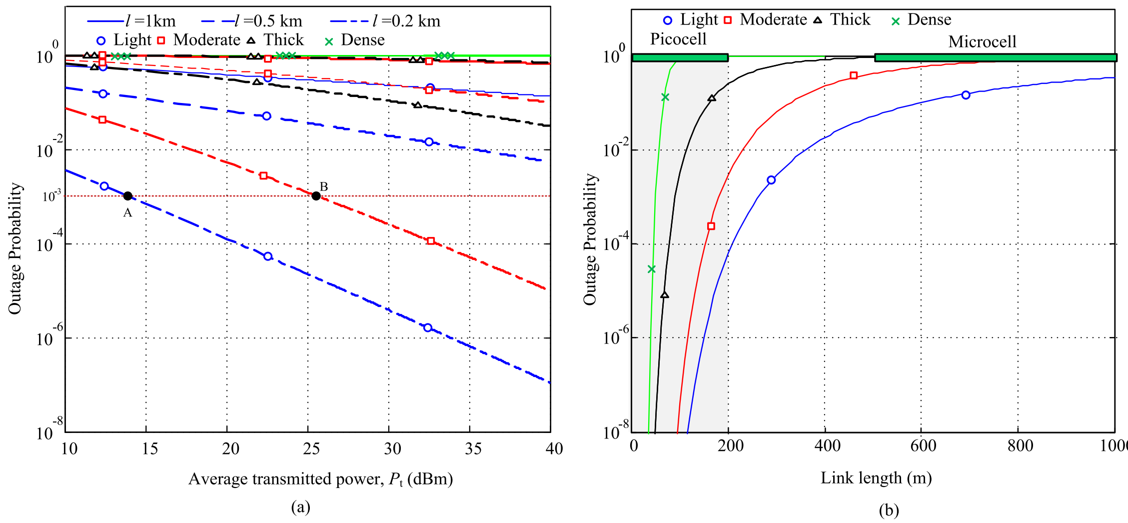


Fig. 6. Outage probability of FSO system under fog conditions and pointing errors versus (a) average transmitted power and (b) link length with $P_t = 22$ dBm.

4.2 Analysis of Foggy Channel With Pointing Errors

The performance of FSO system when pointing errors and fog atmospheric losses are combined together is shown in Fig. 6. The results clearly show an important degradation in FSO system performance in comparison to the case in Fig. 5(a) where only atmospheric loss due to fog is considered. Only short links (200 m) can provide outage probability less than 10^{-3} under light and moderate fog with 13 dBm (Point A) and 25 dBm (Point B) transmitted power, respectively. For dense and thick fog, the outage probability is very poor even when high transmission power, $P_t = 40$ dBm is considered.

Fig. 6(b) illustrates how the outage probability changes as a function of the link length at 22 dBm transmitted power. We see high degradation as the link length increases. Only short link lengths less than 200 m can support acceptable outage probability. In general, the results show that precise pointing is required to improve the outage probability.

4.3 Analysis of a Multi-Hop FSO Communication System

The effect of using relays on the FSO system is shown in Fig. 7(a) as a function of the average transmitted power over 1 km link length, and in Fig. 7(b) as a function of the link length at 22 dBm average transmitted power. We can notice the high improvement in outage probability when the number or relays increases. However, this improvement is achieved when the fog density is low and/or the transmission distance is short. The worst outage probability is achieved under direct transmission without using relays. For example, to achieve $P_{out} \leq 10^{-3}$ (Line H), with $P_t \leq 22$ dBm (Line V), we need $N = 2$ for light fog (Point A), $N = 3$ for moderate fog (Point B). For thick and dense fog, this performance level is unachievable, where more relays are required.

In Fig. 7(b), we notice that the outage probability increases as the link length increases. However, more relay nodes can be used to improve he outage probability significantly. For example, to achieve outage probability less than 10^{-3} (Line H), with 500 m link length (Line V), we need $N = 1$ for light fog (Point A), $N = 1$ for moderate fog (Point B), and $N = 3$ for thick fog (Point C). For dense fog, it is impossible to achieve this performance level. Furthermore, under direct transmission, 10^{-3} outage probability is obtained over 450 m, 310 m, 150 m, and 85 m, for light, moderate, thick, and dense fog, respectively. If $N = 3$ (three nodes), then we obtain the same outage probability over 1400 m, 1000 m, 500 m, and 290 m, for light, moderate, thick and dense fog, respectively. Hence, the distance has increased more than three times as compared to direct transmission.

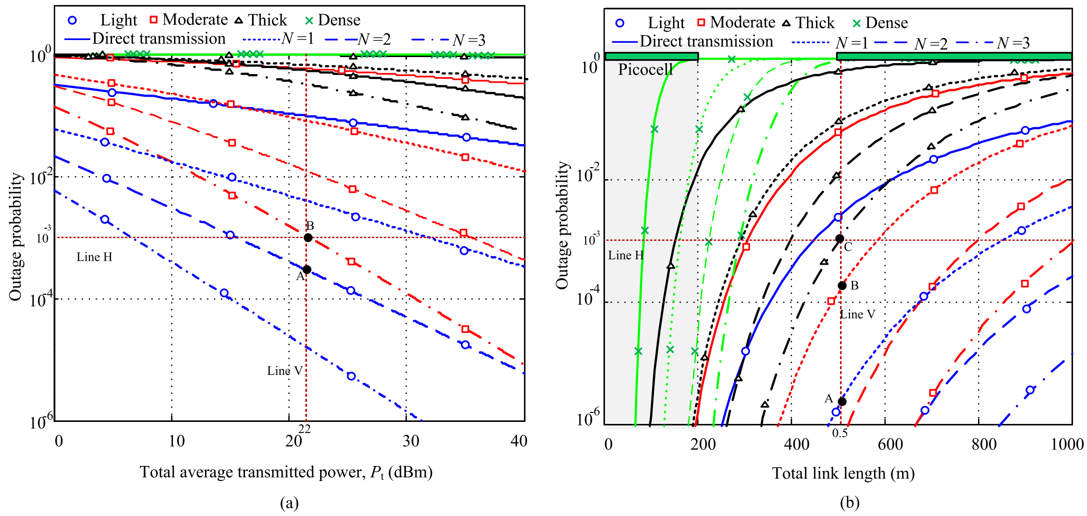


Fig. 7. Outage probability of multi-hop FSO system versus (a) the total average transmitted power with $l = 1$ km and (b) total link length with $P_t = 22$ dBm.

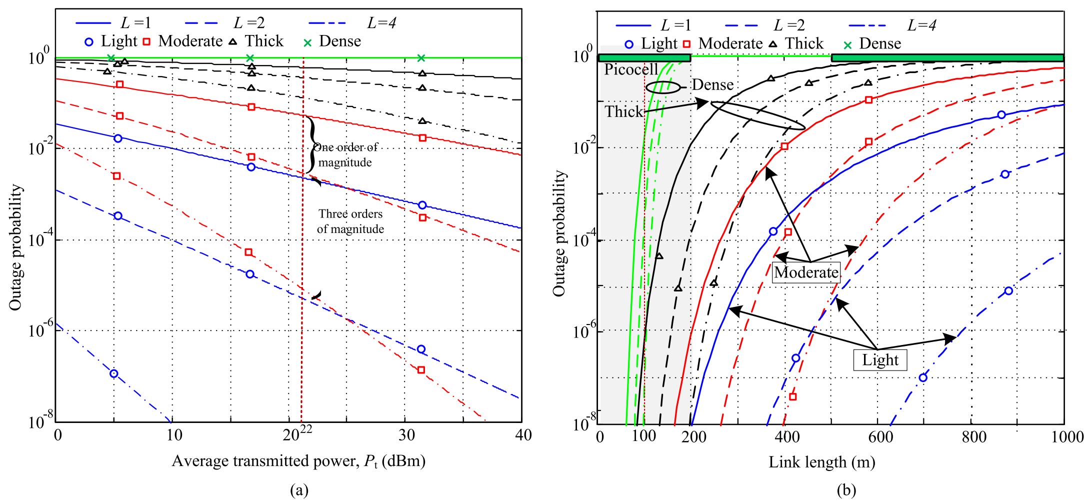


Fig. 8. Outage probability of FSO system employing TLS diversity technique versus (a) average transmitted power over 500 m link length and (b) transmission link length, $P_t = 22$ dBm.

4.4 Analysis of Transmitter Laser Selection Diversity

The performance of FSO link using TLS diversity technique in terms of outage probability versus the average transmitted power is shown in Fig. 8(a) for 500 m link length. We notice good impact of TLS system on the outage probability. However, this effect is reduced under thick fog and vanishes under dense fog. For example, at 22 dBm average transmitted power, the outage probability is reduced by three orders of magnitude for light fog when we used $L = 2$. Under moderate fog, this improvement reduced in to one order of magnitude. For thick fog, we notice small improvement and higher number of transmitters is needed. Under dense fog we see no improvement in system performance even if we use large number of transmitters.

In Fig. 8(b), we plot the outage probability of TLS FSO system as a function of the transmission link length at 22 dBm average transmitted power. The curves in this figure show higher improvement as the link length decreases. For example, under dense fog and for 100 m link length which is very short distance, we achieve 1.8×10^{-2}, 3.2×10^{-4}, and 1.06×10^{-7} outage probability using single, double, and quadruple transmitters, respectively.

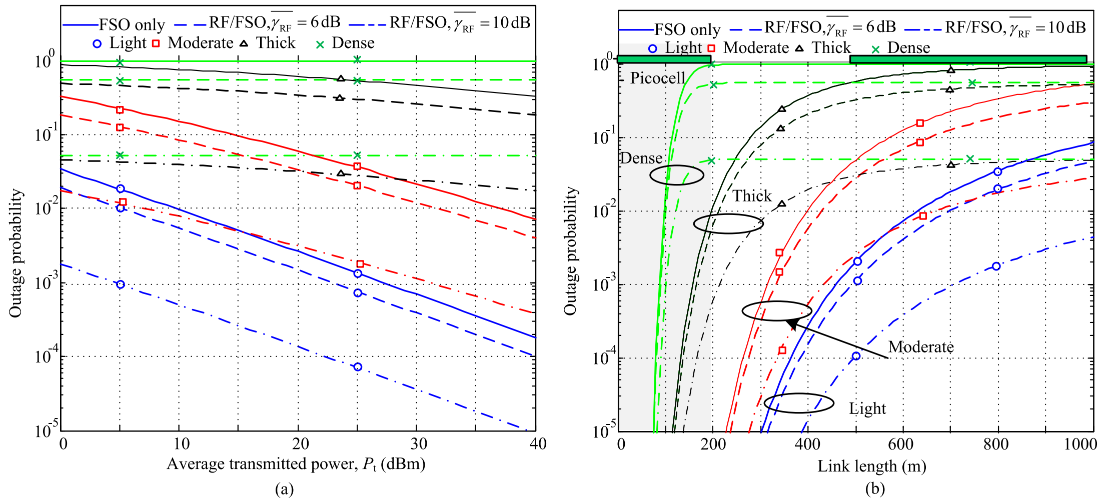


Fig. 9. Outage probability of hybrid RF/FSO system versus (a) the average transmitter power of the FSO link over 500 m link length and (b) transmission link length with $P_t = 22$ dBm. The system has $\gamma_{th}^{FSO} = \gamma_{th}^{RF} = 6$ dB, $m = 5$.

4.5 Analysis of a Hybrid RF/FSO Communication System

Fig. 9(a) shows the outage probability of the hybrid RF/FSO transmission versus the average transmitted power under different fog densities. For each fog density, we plot three curves correspond to FSO only, hybrid RF/FSO where RF link has 6 dB average SNR, and hybrid RF/FSO where RF link has 10 dB average SNR. We notice small improvement in outage probability for low quality RF link, i.e. low SNR. As the RF link average SNR increases, we notice more than one order of magnitude improvement in outage probability. Moreover, the results illustrate that with dense fog the outage probability approaches one for FSO link only. Using the hybrid link, the outage probability improved to the outage probability of the RF link.

In Fig. 9(b), we show the outage probability versus the transmission link length using 22 dBm average transmitted power. In general, we notice higher improvement in outage probability as the transmission link length decreases. Moreover, we notice that under dense fog, the outage probability of the hybrid system is that of the RF link for link length longer than 200 m. For shorter length, we notice high improvement where the outage probability is reduced by one order of magnitude when the link length reduced from 200 m to 100 m for 10 dB average RF SNR.

5. Conclusion

In this paper, we investigated the performance of FSO communication system under foggy conditions and pointing errors. We found that fog affects FSO link severely especially under dense fog. Pointing errors degrade more the link performance when combined with fog effect. Therefore, using mitigation techniques can enhance the power budget and the reach distance. The results show that such techniques can play a major role in combating fog effect and improving the overall system performance.

References

[1] M. A. Khalighi and M. Uysal, "Survey on free space optical communication: A communication theory perspective," *IEEE Commun. Surv. Tuts.*, vol. 16, no. 4, pp. 2231–2258, Jun. 2014.
[2] M. S. Khan, S. S. Muhammad, M. S. Awan, V. Kvicera, M. Grabner, and E. Leitgeb, "Further results on fog modeling for terrestrial free-space optical links," *Opt. Eng.*, vol. 51, no. 3, pp. 031207-1–031207-9, Apr. 2012.

[3] M. Ijaz, Z. Ghassemlooy, J. Pesek, O. Fiser, H. Le Minh, and E. Bentley, “Modeling of fog and smoke attenuation in free space optical communications link under controlled laboratory conditions,” *IEEE/OSA J. Lightw. Technol.*, vol. 31, no. 11, pp. 1720–1726, Jun. 2013.

[4] M. A. Esmail, H. Fathallah, and M.-S. Alouini, “Channel modeling and performance evaluation of FSO communication systems in fog,” in *Proc. 23rd Int. Conf. Telecommun.*, Thessaloniki, Greece, May 2016, pp. 1–5.

[5] A. A. Farid and S. Hranilovic, “Outage capacity optimization for free-space optical links with pointing errors,” *IEEE/OSA J. Lightw. Technol.*, vol. 25, no. 7, pp. 1702–1710, Jul. 2007.

[6] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*. Hoboken, NJ, USA: Wiley, 2005.

[7] D. Zwillinger, *Table of Integrals, Series, and Products*. New York, NY, USA: Elsevier, 2014.

[8] Wolfarm Research. (2015). [Online]. Available: http://www.wolfram.com/mathematica/

[9] Q.-S. Hu, J.-B. Wang, J.-Y. Wang, M. Chen, and X. Song, “Outage probability analysis of multi-hop free space optical communications over strong turbulence channels,” in *Proc. Int. Conf. Wireless Commun. Signal Process.*, Hangzhou, China, Oct. 2013, pp. 1–6.

[10] H. Samimi, “Performance analysis of free-space optical links with transmit laser selection diversity over strong turbulence channels,” *IET Commun.*, vol. 5, no. 8, pp. 1039–1043, May 2011.

[11] A. García-Zambrana, B. Beatriz Castillo-Vázquez, and C. Castillo-Vázquez, “Asymptotic error-rate analysis of FSO links using transmit laser selection over gamma-gamma atmospheric turbulence channels with pointing errors,” *Opt. Exp.*, vol. 20, no. 3, pp. 2096–2109, 2012.

[12] H. Dahrouj, A. Douik, F. Rayal, T. Y. Al-Naffouri, and M.-S. Alouini, “Cost-effective hybrid RF/FSO backhaul solution for next generation wireless systems,” *IEEE Wireless Commun.*, vol. 22, no. 5, pp. 98–104, Oct. 2015.

[13] F. Nadeem, V. Kvicera, M. Saleem Awan, E. Leitgeb, S. S. Muhammad, and G. Kandus, “Weather effects on hybrid FSO/RF communication link,” *IEEE J. Sel. Areas Commun.*, vol. 27, no. 9 pp. 1687–1697, Dec. 2009.

[14] M. Usman, H. C. Yang, and M.-S. Alouini, “Practical switching-based hybrid FSO/RF transmission and its performance analysis,” *IEEE Photon. J.*, vol. 6, no. 5, pp. 1–13, Oct. 2014.