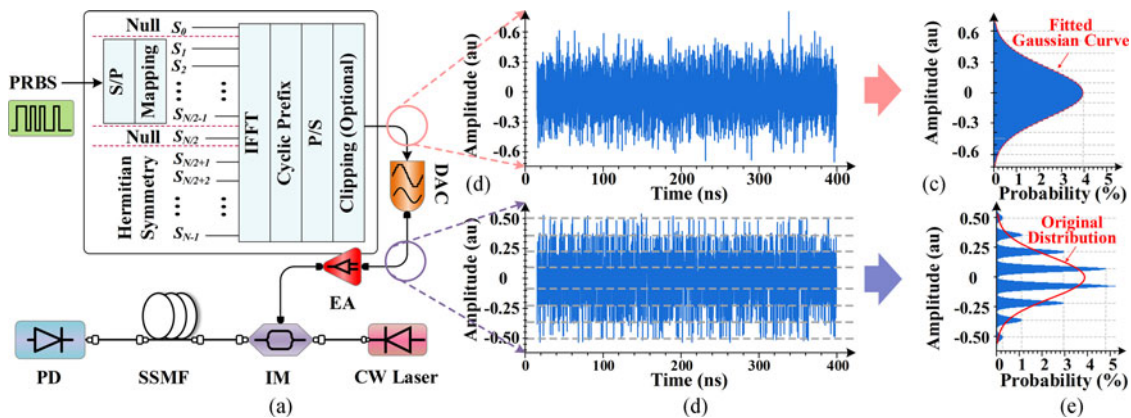


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**Abstract:** We propose and demonstrate via both simulation and experiment a scheme for a low-bit resolution digital-to-analog convertor (DAC) to improve the signal-to-quantization noise ratio (SQNR) of an intensity-modulated direct-detection (IM-DD) orthogonal frequency division multiplexing (OFDM) signal. In this scheme, the discrete output levels of a DAC are nonuniform and properly assigned based on the distribution of the OFDM signal. Such nonuniformity allows a better exploitation of output levels for the OFDM signal so that low-level waveforms that appear much more frequently experience mitigated quantization noise. To calculate these optimized output levels with the distribution of an OFDM signal, a numerical algorithm is proposed and verified. Our proposed scheme is proven effective in the simulations for Gaussian-distributed signals and brings over 3.6-dB and ~2.5-dB improvements on SQNR and receiver sensitivity, respectively. The feasibility is also verified by a proof-of-concept experiment with ~7.4-Gbps quadrature phase shift keying (QPSK)-modulated OFDM signals output by a 3-bit DAC. Our scheme enables transmission of such signals with no further optimization with ~-25-dBm receiver sensitivity, while it is impossible using a conventional way. Both simulation and experimental results validate our scheme as a promising way to be implemented in low-bit resolution DACs for IM-DD OFDM applications.

**Index Terms:** Orthogonal frequency division multiplexing (OFDM), modulation.

## 1. Introduction

Owing to emerging online applications such as cloud computing, HD video streaming, and social networking, both long-haul and short-reach optical networks have witnessed an exponential growth in terms of traffic throughput and bandwidth demand [1]. The next-generation optical network systems should therefore satisfy the requirements of high capacity, improved spectral efficiency, and low cost [1], [2]. With the great success of orthogonal frequency division multiplexing

(OFDM) in electronic communication systems, optical OFDM has attracted great attention and is regarded as one promising technology for optical networks [2]. Apart from being immune to chromatic dispersion, its high spectral efficiency can significantly reduce the electrical bandwidth requirement [3]–[5]. The transmission of an over 100-Gbps intensity-modulated direct-detection (IM-DD) OFDM signal has been realized by using only 10-GHz electronic components [3]. In such implementations, a digital-to-analog convertor (DAC) is employed to convert the OFDM samples to time-domain analog waveforms. Electrical signals are output by generating discrete electrical levels according to digital samples, followed by an anti-alias filter to remove frequency image duplicates [6]–[8]. A high-bit resolution is desirable for DACs implemented in an IM-DD OFDM scenario, but such resolution is limited by both the sampling rate and the digital signal processing (DSP) capability of the transmitter [6], resulting in increased cost and high power consumption [7].

A number of system demonstrations have been presented to validate the possibility of employing a 3-bit or 4-bit resolution DAC for IM-DD OFDM applications [9]–[11]. Such systems enjoy the merits of high sampling rate and low cost [6]. In these cases, quantification noise becomes detrimental and various methods have been proposed to improve the signal's signal-to-quantification noise ratio (SQNR). Effective methods include DSP-based optimized clipping [9], [10], peak-to-average power ratio (PAPR) mitigation [12], and nonuniform analog-to-digital quantizer [13].

In this paper, for the first time to the best of our knowledge, we propose and demonstrate by both simulation and experiment a scheme implemented in a low-bit (3~4-bit) resolution DAC to adaptively improve the SQNR of an IM-DD OFDM signal. As a low-bit resolution DAC can only support a few number of discrete levels, the quantification noise may greatly depend on both the statistic distribution of digital samples and the discrete output levels within the DAC's dynamic range [13]. In our scheme, instead of being uniformly-distributed as in a conventional way, the discrete output levels of the DAC are nonuniform, as used in image processing [14], [15]. These levels are properly assigned based on the Gaussian distribution of IM-DD OFDM signals, which favor small-amplitude waveforms that appear much more frequently to experience less quantization noise, thus leading to an effective reduction of overall distortions. In order to identify these optimized output levels, a numerical algorithm is proposed and the obtained results are verified by system simulations, where over 3.6-dB improvement on SQNR is recorded. In addition, quadrature phase shift keying (QPSK)- and 16-point quadrature amplitude modulation (16-QAM)-modulated OFDM signals are transmitted over an 80-km fiber with 3-bit and 4-bit resolution DACs, respectively. Simulation results show that respective improvements of ~2.1 dB and ~2.5 dB on optical receiver sensitivity are achieved by using our proposed scheme. The above analysis and the feasibility of the scheme are also verified by a proof-of-concept experiment, where our proposed scheme enables the transmission and reception of a ~7.4-Gbps QPSK-modulated OFDM signal output by a 3-bit DAC without any DSP-based optimization. A sensitivity of ~-25 dBm is achieved at a bit-to-error ratio (BER) level of  $2 \times 10^{-3}$ , whereas this threshold cannot be achieved by using a conventional way. In practical systems, a DAC can be utilized to generate only one type of signal with a fixed statistic distribution, optimized discrete output levels can thus be taken into consideration for hardware designing. Our proposed scheme can in this way be implemented in a low-bit resolution DAC, as a promising way to improve the SQNR and the receiver sensitivity for various IM-DD OFDM applications.

## 2. Operation Principle

Fig. 1(a) illustrates the schematic diagram of an IMDD-OFDM optical transmission system. The electrical OFDM signal is generated by DSP chips and output by a high-speed DAC. The incoming bit stream is first packed into  $x$  bits per symbol to form a complex constellation symbol  $S_n$ , where  $x$  is the modulation degree determined by the constellation size. In order to get a real output of an inverse fast Fourier transform (IFFT) operation, Hermitian symmetry is imposed and satisfies the

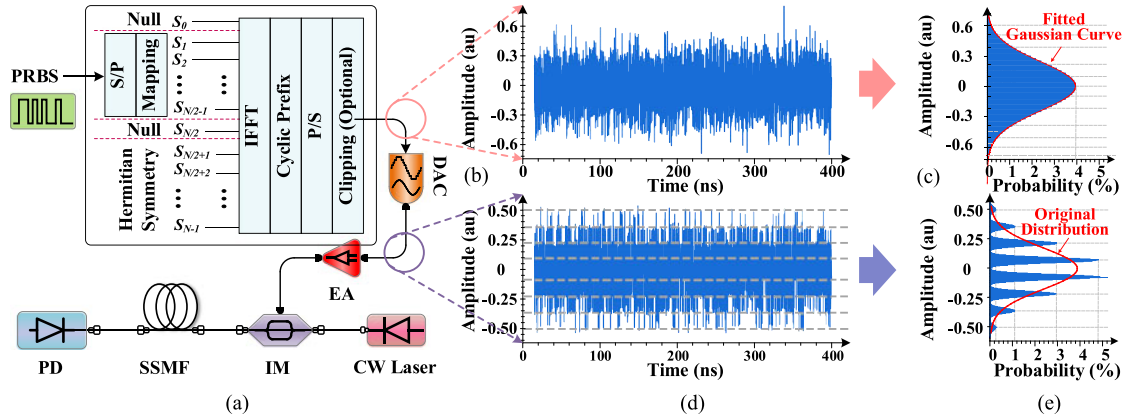


Fig. 1. (a) Schematic diagram of an IM-DD OFDM system and the principle DSP process in the transmitter. (b) and (c) Temporal waveform and distribution histogram of the OFDM samples, respectively. (d) and (e) Electrical waveform and signal distribution with a commercially available 3-bit DAC, respectively. PRBS: pseudo-random binary sequence, S/P: serial-to-parallel, P/S: parallel-to-serial, IM: intensity modulator, SSMF: standard singlemode fiber, PD: photodetector.

following conditions [2]:

$$\begin{cases} S_{N-k} = S_k^*, & \text{for } k = 1, 2, \dots, \frac{N}{2} - 1 \\ S_0 = 0 \\ S_{N/2} = 0 \end{cases} \quad (1)$$

where  $(.)^*$  is the Hermitian conjugate operator,  $N$  is the IFFT size, and  $k$  is the subcarrier index. Assuming the complex symbol  $S_n$  can be expressed as  $a_n - jb_n$ , with  $a_n$  and  $b_n$  being real, and  $j$  denoting the imaginary unit, the output OFDM samples can be written as [16], [17]

$$s(n) = \frac{1}{\sqrt{N}} \sum_{k=1}^{\frac{N}{2}-1} \left( a_k \cos \frac{2\pi kn}{N} + b_k \sin \frac{2\pi kn}{N} \right), n = 0, 1, \dots, N-1. \quad (2)$$

Since the complex symbols are independently mapped and assigned, the time-domain OFDM samples become Gaussian-distributed for large  $N$  values ( $N \geq 256$ ). The probability density function (PDF)  $f(x)$  of the OFDM samples can, therefore, be expressed as [16]

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_s} e^{-\frac{x^2}{2\sigma_s^2}}, \quad (3)$$

Here,  $\sigma_s$  represents the standard derivation of the signal and can be determined from samples' variance. Fig. 1(b) depicts a sampled OFDM temporal waveform with  $N = 256$  and its statistic distribution can be deemed highly matched with a Gaussian curve, as shown in Fig. 1(c). Such distribution means that an OFDM signal has its most waveform samples constrained in a small amplitude region, while some peaks occur with low probabilities. A DAC is then used to bridge between digital samples and electrical output, and can only generate limited number of discrete levels, depending on its bit resolution. For a DAC with a low resolution (3~4 bits), quantification noise can be strong and detrimental. Fig. 1(d) illustrates an example, exhibiting the OFDM signal waveform that is output by a commercially available 3-bit DAC. Eight discrete outputs are clearly observed from its corresponding distribution histogram, as shown in Fig. 1(e).

Typically, commercialized DACs have their discrete output levels and thresholds uniformly-distributed within signal's full range, before being filtered with an anti-alias filter [6]. A DAC with high bit resolution can naturally support a higher modulation degree, whereas in the case of a low bit resolution, the performance depends strongly on signal's statistic distribution and that of the

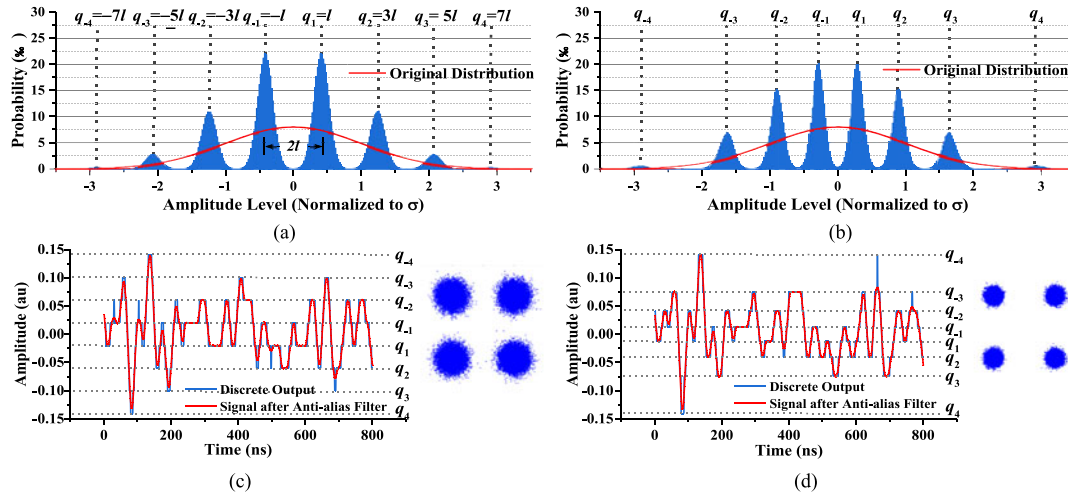


Fig. 2. (a) and (b) Distribution histogram and waveform segment of an IM-DD OFDM signal output by a conventional uniformly-distributed 3-bit DAC, respectively. (c) and (d) Distribution histogram and waveform segment of the same signal output by our proposed nonuniformly-distributed 3-bit DAC, respectively.

output levels of a DAC [13]. For a Gaussian-distributed OFDM signal, a properly designed DAC with output levels that take into account signal's distribution can better exploit all available levels and thus optimize SQNR. We illustrate this idea by providing the statistic distribution histograms as well as the temporal waveforms of an OFDM signal output by a 3-bit DAC. In a conventional case with eight output levels [ $q_{-4}$ ,  $q_{-3}$ ,  $q_{-2}$ ,  $q_{-1}$ ,  $q_1$ ,  $q_2$ ,  $q_3$ ,  $q_4$ ] being uniformly placed, as shown in Fig. 2(a) and (c), different amplitudes of the signal experience the same quantization level, regardless that high peaks occur rarely. By contrast, in a nonuniform case illustrated in Fig. 2(b) and (d), the six output levels [ $q_{-3}$ ,  $q_{-2}$ ,  $q_{-1}$ ,  $q_1$ ,  $q_2$ ,  $q_3$ ] in the middle of the dynamic range are more densely assigned. OFDM signals which have their waveforms mostly concentrated in small amplitude regions can thus benefit from that, resulting in a reduced overall quantization noise for the whole signal. Consequently, the insets of Fig. 2(c) and (d) show that the constellation diagrams, as taken from our simulations, become more concentrated. In the following, we provide the mathematical derivation of the method for a  $n$ -bit DAC.

In our proposed scheme, the quantization noise  $e_q$  is the key factor to be minimized, which statistically speaking is a function of signal's PDF  $f(x)$  [14]:

$$e_q = \int_{-\infty}^{+\infty} (x - x_q)^2 f(x) dx \propto \int_0^{+\infty} (x - x_q)^2 f(x) dx \quad (4)$$

where  $x_q$  is the discrete output of a digital sample point  $x$ . Since a  $n$ -bit DAC has only  $2^n$  discrete output levels, the output voltage of  $x$  is imposed to be the nearest discrete output  $x_q$ , and they follow the following relations:

$$x_q(x) = \begin{cases} q_{2^n-1}, & \text{for } x > \frac{q_{2^n-1} + q_{2^n-2}}{2} \\ \vdots & \vdots \\ q_1, & \text{for } \frac{q_1 + q_2}{2} \geq x > 0 \\ -q_1, & \text{for } 0 \geq x > -\frac{q_1 + q_2}{2} \\ \vdots & \vdots \\ -q_{2^n-1}, & \text{for } x \leq -\frac{q_{2^n-1} + q_{2^n-2}}{2} \end{cases} \quad (5)$$

1	Calculate the results of $h(x)$ and $g(x)$ with a scan step (precision) = $0.001\sigma_s$ .
2	For output levels $[q_1, q_2, \dots, q_{2^{n-1}}]$ , we define their respective scan windows $\{[q_{1\min}, q_{1\max}], [q_{2\min}, q_{2\max}], \dots, [q_{2^{n-1}\min}, q_{2^{n-1}\max}]\}$ . Here $q_{2^{n-1}\min} = q_{2^{n-1}\max} = \text{clipping ratio}$ .
3	We calculate possible results of Eq. (7) with all combinations in scan windows for each output level
4	The smallest result is chosen and we return the corresponding output levels as a candidate for the optimized combination
5	<b>If</b> None of the returned output levels reaches the bound of the scan window Save the combination and corresponding clipping ratio <b>Else</b> Update those scan windows for output levels which have reached their bound. <b>Go to 3</b>

Fig. 3. Pseudocode implemented in MATLAB to identify the optimized discrete output levels for a n-bit resolution DAC.

Here, we assume that the outputs are symmetric for their positive and negative levels and  $[q_1, q_2 \dots q_{2^{n-1}-1}, q_{2^{n-1}}]$  represent the positive levels of a n-bit resolution DAC. They are our target to be optimized. Based on the thresholds in (5), (4) can be split into  $2^{n-1}$  parts:

$$e_q \propto \int_0^{\frac{q_1+q_2}{2}} (x - q_1)^2 f(x) dx + \int_{\frac{q_1+q_2}{2}}^{\frac{q_2+q_3}{2}} (x - q_2)^2 f(x) dx + \dots$$

$$+ \int_{\frac{q_{2^{n-1}-1}+q_{2^{n-1}-2}}{2}}^{\frac{q_{2^{n-1}-1}+q_{2^{n-1}-1}}{2}} (x - q_{2^{n-1}-1})^2 f(x) dx + \int_{\frac{q_{2^{n-1}-1}+q_{2^{n-1}-1}}{2}}^{+\infty} (x - q_{2^{n-1}-1})^2 f(x) dx. \quad (6)$$

After denoting the two functions  $h(x) = \int_0^x f(t) dt$  and  $g(x) = \int_0^x f(t) dt$ , (6) can be simplified as follows:

$$e_q \propto \underbrace{(q_1^2 - q_2^2)g\left(\frac{q_1 + q_2}{2}\right) + \dots + (q_{2^{n-1}-1}^2 - q_{2^{n-1}}^2)g\left(\frac{q_{2^{n-1}-1} + q_{2^{n-1}}}{2}\right) + q_{2^{n-1}}^2 g(+\infty)}_{\text{functions with } g(x)}$$

$$- 2 \times \underbrace{\left[ (q_1 - q_2)h\left(\frac{q_1 + q_2}{2}\right) + \dots + (q_{2^{n-1}-1} - q_{2^{n-1}})h\left(\frac{q_{2^{n-1}-1} + q_{2^{n-1}}}{2}\right) + q_{2^{n-1}}h(+\infty) \right]}_{\text{functions with } h(x)}$$

$$\times \underbrace{C}_{\text{Constant}}. \quad (7)$$

In this way,  $e_q$  turns out to be a function of positive discrete output levels  $[q_1, q_2 \dots q_{2^{n-1}-1}, q_{2^{n-1}}]$ . Instead of resorting to analytical solutions, we use MATLAB to identify the optimized combination of these output levels, which minimizes  $e_q$  for a given PDF, bit resolution, and clipping ratio. Clipping is considered here as it is a practical method to improve signal's SQNR [9], with clipping ratio defined as the ratio between an amplitude clipping threshold and signal standard derivation [10]. We provide in Fig. 3 the pseudocode that has been realized and implemented in MATLAB. In this calculation, the PDF of the signal is assumed to be Gaussian and follows (3) with signal standard derivation being  $\sigma_s$ . In order to lower computation time, we first calculate all results of  $h(x)$  and  $g(x)$  with a high precision of  $0.001\sigma_s$ . By doing this, all integrations can be replaced by subtractions. On the other hand, we set a scan window for each output level to further lower the computing complexity. The scan windows can be updated according to previous calculations of (7), permitting to find out an optimized combination with less computation time. By this means, we obtain the combination of the discrete output levels to minimize  $e_q$  for 3-bit and 4-bit resolutions with clipping ratios varying from 1 to 5. Resolution higher than 4-bit is not provided due to the computing complexity. Tables 1 and 2 extract some results for the 3-bit and 4-bit resolution cases, respectively, which are all normalized to  $\sigma_s$ .

TABLE 1  
Levels For 3-Bit Resolution

$q_1$	$q_2$	$q_3$	$q_4$
0.135	0.409	0.694	1.000
0.159	0.483	0.824	1.200
0.181	0.550	0.946	1.400
0.201	0.613	1.063	1.600
0.218	0.669	1.172	1.800
0.234	0.721	1.273	2.000
0.248	0.766	1.365	2.200
0.261	0.807	1.449	2.400
0.271	0.841	1.523	2.600
0.280	0.871	1.589	2.800
0.287	0.896	1.645	3.000
0.294	0.918	1.694	3.200
0.299	0.935	1.735	3.400
0.303	0.949	1.768	3.600
0.306	0.960	1.796	3.800
0.309	0.970	1.819	4.000
0.311	0.977	1.836	4.200
0.313	0.983	1.851	4.400
0.314	0.987	1.861	4.600
0.315	0.991	1.870	4.800
0.316	0.994	1.877	5.000

In order to verify the validity of our model and calculation, we calculate the SQNRs for IM-DD OFDM signals using the calculated optimized output levels listed in Tables 1 and 2 for each clipping ratio. Then, we compare them with the results obtained by a uniformly-distributed DAC with the same bit resolution [18]. Fig. 4 summarizes these results. One can see that our scheme always provides a better SQNR, no matter what the clipping ratio is. For a high clipping ratio where high peaks of a signal tend to be conserved, our proposed scheme brings over 3.6-dB SQNR improvements, which can be attributed to a better exploitation of discrete output levels: quantification noise for small amplitudes is greatly reduced. When a signal is strongly clipped with a small clipping ratio in Fig. 4, its distribution becomes quasi-uniform and our scheme thus leads to limited improvements in comparison with a uniformly-distributed DAC. It is worthy to point out that even so our proposed scheme still outperforms the conventional ones with 0.36-dB and 0.85-dB improvements at the best clipping ratios for 3-bit and 4-bit resolutions, respectively. Please note that one hundred thousand OFDM symbols were examined for each clipping ratio and the obtained SQNR results in our calculation have a precision  $\leq 0.006$  dB. SQNR analysis verifies our model and calculation to be effective for OFDM signals with a Gaussian distribution.

### 3. Simulation and Experimental Verifications

We then performed a system simulation using the OptiSystem 14 commercial software to investigate the feasibility of our proposed scheme implemented in an optical transmission system.

TABLE 2  
Levels For 4-Bit Resolution

$q_1$	$q_2$	$q_3$	$q_4$	$q_5$	$q_6$	$q_7$	$Q_8$
0.063	0.190	0.318	0.447	0.579	0.714	0.854	1.000
0.074	0.223	0.373	0.526	0.684	0.847	1.018	1.200
0.084	0.253	0.424	0.600	0.782	0.974	1.178	1.400
0.093	0.280	0.471	0.668	0.874	1.094	1.333	1.600
0.101	0.306	0.514	0.730	0.959	1.206	1.481	1.800
0.109	0.328	0.553	0.788	1.038	1.312	1.624	2.000
0.115	0.348	0.588	0.839	1.109	1.409	1.759	2.200
0.121	0.365	0.617	0.883	1.171	1.496	1.884	2.400
0.126	0.380	0.642	0.920	1.224	1.572	1.998	2.600
0.130	0.392	0.664	0.953	1.271	1.640	2.103	2.800
0.133	0.402	0.681	0.979	1.310	1.697	2.195	3.000
0.136	0.411	0.696	1.002	1.343	1.747	2.277	3.200
0.138	0.418	0.709	1.021	1.371	1.788	2.347	3.400
0.140	0.423	0.718	1.036	1.393	1.822	2.407	3.600
0.141	0.427	0.726	1.048	1.411	1.850	2.456	3.800
0.143	0.432	0.733	1.058	1.425	1.871	2.496	4.000
0.143	0.433	0.736	1.064	1.435	1.888	2.528	4.200
0.144	0.436	0.741	1.070	1.444	1.902	2.553	4.400
0.144	0.437	0.743	1.074	1.450	1.911	2.572	4.600
0.145	0.438	0.745	1.077	1.455	1.919	2.587	4.800
0.145	0.439	0.747	1.080	1.459	1.925	2.598	5.000

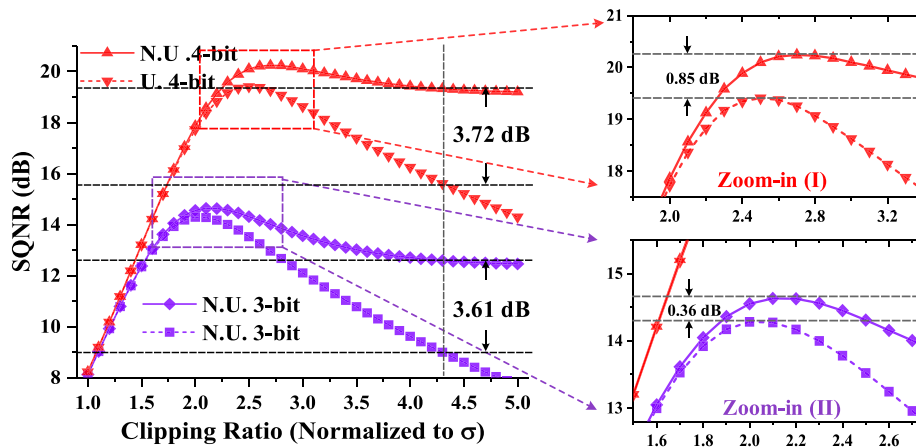


Fig. 4. Calculated SQNR curves with uniformly-distributed and our proposed DACs for 3-bit and 4-bit resolutions. Zoom-in views (I) and (II) show in details the best SQNR conditions for 4-bit and 3-bit DACs, respectively. N.U.: nonuniformly-distributed, U.: uni-formly-distributed.



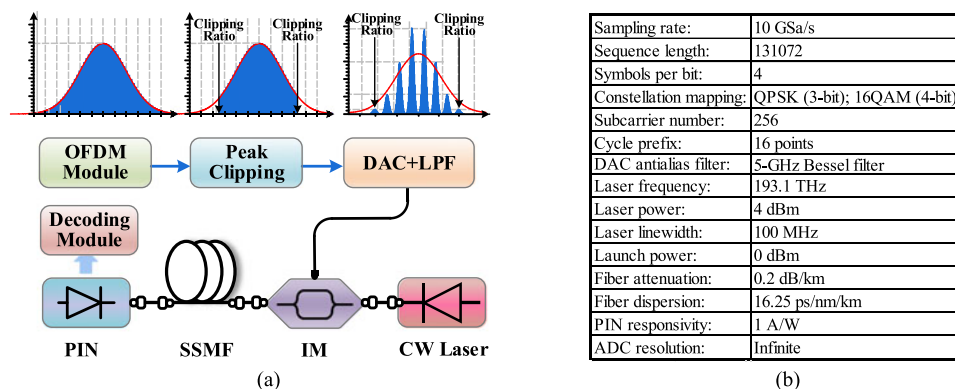


Fig. 5. (a) Schematic diagram of the simulation setup. (b) Main parameters utilized in the simulation.

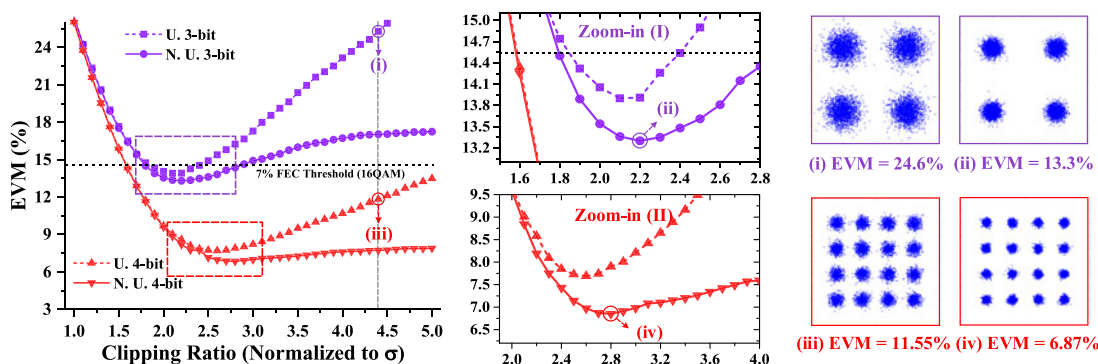


Fig. 6. EVM curves versus clipping ratio (normalized to  $\sigma$ ) for B2B condition. Zoom-in views (I) and (II) show in detail the improvements brought by our scheme at best clipping ratios. Insets (i)–(iv) show corresponding constellation diagrams.

The simulation setup and parameters are depicted in Fig. 5(a) and (b), respectively. OFDM samples are generated by a built-in MATLAB module with  $N = 256$ . Peak-clipping step is applied in the simulation to provide clipping duplicates of the Gaussian-distributed OFDM signal with various clipping ratios. A DAC with 3-bit or 4-bit resolution is used to convert digital samples to electrical levels (zero-holding) before entering into a low-pass filter (LPF) to emulate an anti-alias filter. QPSK and 16-QAM mapping are utilized for the 3-bit and 4-bit resolution output signals, respectively. It is important to note that the clipping ratio can cause a significant impact on the Gaussian distribution of the signal, which limits its amplitude range. For a given clipping ratio, signal's distribution is determined, we calculate and apply the optimized combination of discrete outputs. The output electrical OFDM signal is then modulated onto a continuous-wave (CW) laser by an intensity modulator. The lightwave is sent into an optical fiber with a launch power of 0 dBm to avoid the impact of fiber nonlinearity. After fiber transmission, the modulated light is detected by a PIN photodetector and the recovered electrical signal is used for error vector magnitude (EVM) estimation and BER counting in an OFDM decoding module which is enabled by MATLAB.

The estimated EVM curves versus clipping ratio are presented in Fig. 6 for back-to-back (B2B) condition. Our scheme which optimizes DAC's output levels based on signal's distribution is shown 1) to have constantly smaller EVM values compared with the conventional ones, 2) to be effective for signals whose distributions are nonuniform, and 3) to be insensible to the increase of clipping ratio. For both 3-bit and 4-bit cases, the EVM curves of the proposed scheme are always under the conventional ones and therefore have better performances. In the case of a QPSK-modulated

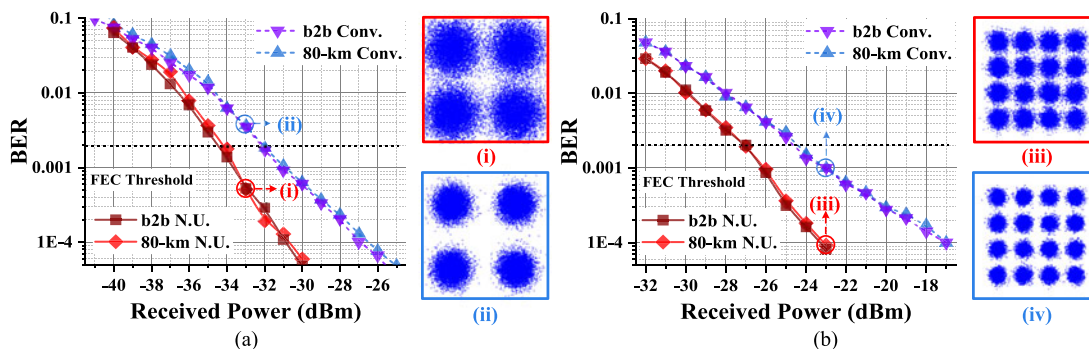


Fig. 7. Simulated BER curves versus received optical power at B2B condition and after 80-km SSMF transmission (a) for 3-bit DACs and (b) for 4-bit DACs. QPSK and 16-QAM are adopted for 3-bit and 4-bit DACs, respectively. Conv.: conventional way.

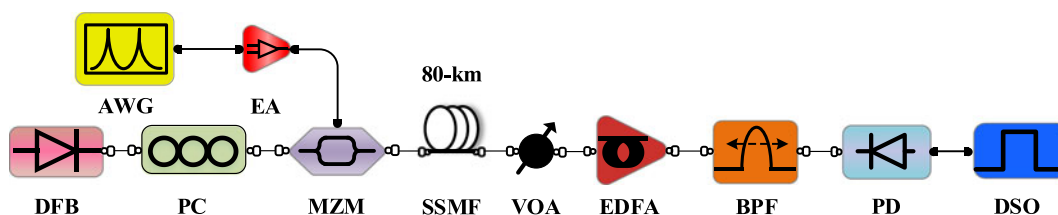


Fig. 8. Experimental setup of our proposed scheme that optimizes DAC's discrete output levels based on OFDM signal distribution. DFB: distributed feedback laser. PC: polarization controller.

OFDM signal output by the 3-bit DAC, only 0.6% improvement in term of EVM is achieved by our proposed scheme at a clipping ratio of 2.2 dB, while that value increases to 7.6% at 4.3-dB clipping ratio. The improvement becomes more significant with an increased clipping ratio. In addition, the EVM levels remain quasi-unchanged with increased clipping ratio for the signals output by DACs using our proposed scheme, revealing the effectiveness of our scheme to minimize the quantification noise over a wide range. Insets of Fig. 6 show constellation diagrams for various clipping ratios.

Fig. 7 plots the BER curves for both B2B and 80-km transmission. The tested OFDM signals here are with a high clipping ratio of 4.3. BER curves for QPSK-modulated (output by a 3-bit DAC) and 16-QAM-modulated (output by a 4-bit DAC) OFDM signals are measured to determine the receiver sensitivity improvement brought by our propose scheme. The estimated power penalty over 80-km fiber transmission is only  $\sim 0.1$  dB, which is due to a small signal bandwidth. It is shown that sensitivity improvements of  $\sim 2.1$  dB and  $\sim 2.5$  dB are achieved for 3-bit output (QPSK) signal and 4-bit output (16QAM) signal, respectively.

We further verified the above analysis by performing a proof-of-concept experiment, whose schematic setup is depicted in Fig. 8. Due to the unavailability of a low-bit resolution DAC capable of varying its discrete output levels, we used a 10-bit resolution arbitrary waveform generator (AWG) (Tektronix 7122C). The used AWG is able to output 1024 electrical discrete levels that are linearly distributed within the dynamic range. Among those levels, only eight were chosen and utilized to perform as a nonuniformly-distributed 3-bit DAC according to the signal's distribution or a uniformly-distributed 3-bit one. This can be realized by a digital quantizer by MATLAB. In comparison with an ideal hardware design, such software-defined non-uniform DAC introduces only a maximal output error of  $\sim 0.78\%$ , which corresponds to the ratio between the maximal quantification noise and minimal discrete level interval of the DAC. OFDM samples are generated offline by MATLAB and the generation DSP processing is the same as that shown in Fig. 1(a). QPSK mapping is adopted for the OFDM signals. No clipping operation is used in the experiment for the OFDM signals so that they can conserve a perfect Gaussian distribution. A Mach-Zehnder modulator (MZM) biased at the

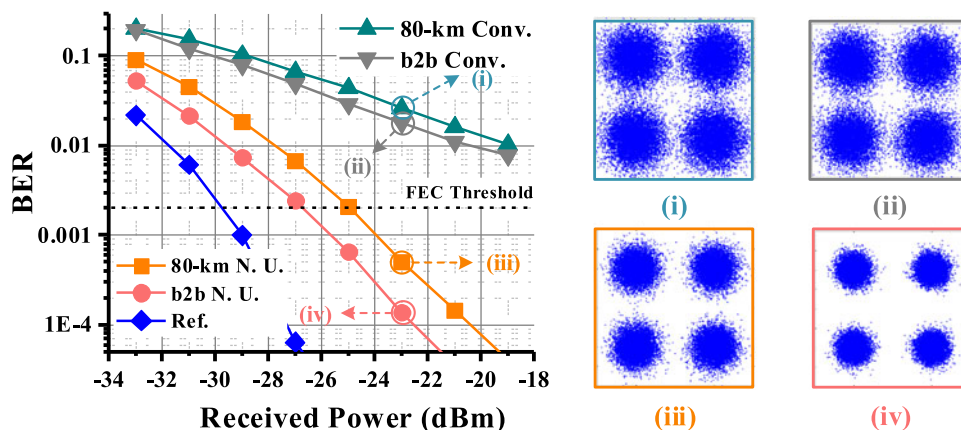


Fig. 9. Experimentally measured BER curves versus received optical power at B2B condition and after 80-km SSMF transmission for a 3-bit DAC. Ref. curve measures the same OFDM signal output by a 10-bit AWG. Insets (i)–(iv) show observed constellation diagrams at a received optical power of  $-23$  dBm.

quadrature point of its transmission curve is employed to modulate the CW lightwave from a DFB laser at 1550 nm. The electrical OFDM signals are output by the AWG with an 8-GSa/s sampling rate and a 3.2-GHz analog bandwidth. Therefore the OFDM signals have a bit rate of  $\sim 7.4$  Gbps, which is determined by the sampling rate, the degree of modulation, as well as the lengths of the cyclic prefix and the training symbols [19]. An electrical amplifier (EA) is used to boost electrical signals to match with the linear region of the MZM. The modulated light is then injected into an 80-km standard singlemode fiber (SSMF) with a 5-dBm launch power. Afterward, the modulated light is attenuated by a variable optical attenuator (VOA) to change the received optical power, prior to a preamplifier receiver comprising of an erbium-doped fiber amplifier (EDFA) with a 4.5-dB noise figure, a 12.5-GHz band-pass filter (BPF), and a PIN photodetector with a 10-GHz bandwidth and a 5-dBm optimized received power. We utilize a digital storage oscilloscope (DSO) (LeCroy 10-100Zi-A) having a 10-bit resolution, a 33-GHz analog bandwidth, and an 80-GSa/s sampling rate to capture the photodetected signals. The demodulation is realized offline by MATLAB to perform down-sampling, time-offset and channel estimation, fast Fourier transform (FFT), one-tap equalization, constellation-symbol demapping, and BER counting. For each measurement, more than 1 million bits are taken into account.

We provide in Fig. 9 the BER curves for the QPSK-modulated signals output by such uniformly-/nonuniformly-distributed 3-bit DAC for both B2B and 80-km transmissions. The reference BER curve for the same signal output using a 10-bit resolution at B2B condition is measured for performance comparison. In our experiment, a BER level ( $\text{BER} = 2 \times 10^{-3}$ ) is set as the 7% software-decision forward error correction (SD-FEC) threshold and this threshold cannot be achieved for the signal output by the conventional 3-bit DAC. Strong quantification noise is the major system impairment. As for the nonuniformly-distributed case, this threshold can easily be achieved at a received optical power of  $\sim -25$  dBm, indicating that the quantification noise has been dramatically reduced. The sensitivity penalty between the 10-bit DAC and our proposed 3-bit scheme is  $\sim 3$ -dB. The EVM comparison is provided with four constellation diagrams shown in the insets of Fig. 9. It is clearly observed that our proposed scheme leads to more concentrated constellation diagrams at a given receiver optical power, compared with a DAC having uniformly-distributed output levels, which is a direct result of effective reduction of quantification noise. Since, in a practical implementation, a DAC may be utilized exclusively for generating only one type of signal having a fixed distribution, the discrete output levels can thus be determined. Our proposed scheme can therefore be implemented in a low bit resolution DAC in various IM-DD applications to improve the signal SQNR and the system performances.

## 4. Conclusion

We have proposed and demonstrated a DAC optimization scheme, which calculates the optimized discrete output levels based on the distribution of its output OFDM signals. In our scheme, the quantification noise for the whole signal can be effectively mitigated and the SQNR can be improved. Both our simulations and a proof-of-concept experimental demonstration have verified its feasibility with Gaussian-distributed OFDM signals. In the simulations, over 3.6-dB and  $\sim$ 2.5-dB improvements in terms of SQNR and receiver sensitivity are achieved, respectively. Experimental results validate our scheme to be effective as it enables the transmission of a QPSK-modulated OFDM signal by a 3-bit DAC, which turns out to be impossible using a conventional way. Our proposed scheme can be implemented, based on above analyses, in a low-bit DAC for IM-DD OFDM applications to improve signal SQNR and transmission performances.

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