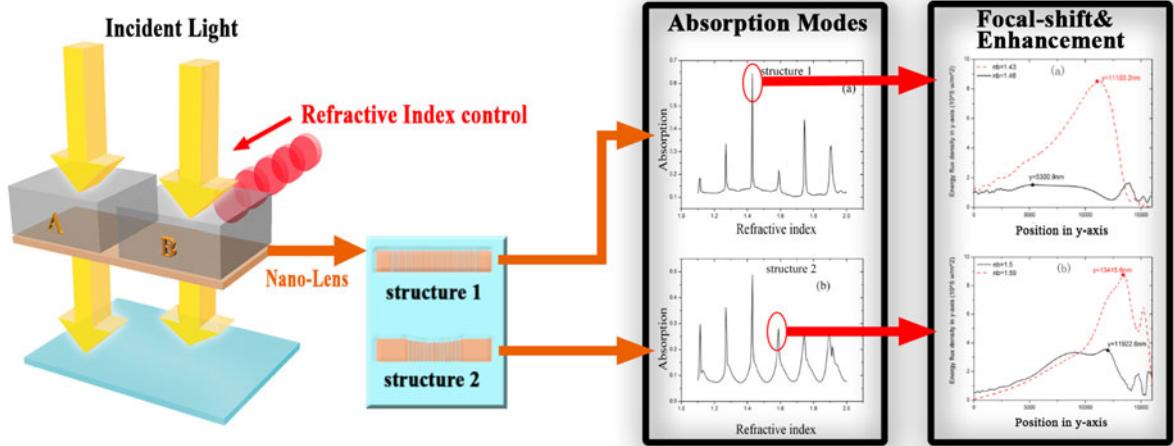


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# Focal Shift of Nano-Optical Lens Affected by Periodic Resonance With Substrate

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**Abstract:** A method of focal-shift resonance with substrate based on two commonly used structures of nano-optical lens is presented with numerical simulations. This resonance exhibits periodicity with the variation of substrate refractive index. Both types of lenses are based on gradual changed nanoslits and analyzed by the extraordinary optical transmission theory. This displacement of focal point can reach the order of microns. A periodic intensity change of focal light is observed between two absorption peaks in metal nanolens. The origin of this focal shift is also discussed with the analysis of the Fresnel number in diffraction. The finite element method is utilized in this paper.

**Index Terms:** Diffractive optics, plasmonics, sub-wavelength structure.

## 1. Introduction

Surface plasmon (SP), as a kind of charge-density oscillation excited by an evanescent wave at optical frequencies, gives rise to strongly enhanced electric fields which are confined near the metal surface [1]. A study on SP and its applications have been widely employed over the past few decades [2], [3]. As a crucial research of SP, the extraordinary optical transmission (EOT) on nano slits array, which can generate abnormally extreme high transmittance peak in the spectrum, was first presented in 1999 by Porto *et al.* [4]. The principle of EOT is considered as the interaction of the SP mode and the Fabry-Perot (F-P) mode [5]–[7], which both can bring high energy absorption in the metal.

It has been confirmed that the characteristic of EOT is decided mainly by the ratio of depth and width of slits [6]. With the adjustment of parameters, modes in the slits can cause expected phase delay. Many researches on nano focusing lens, which can be fabricated by now available technique as a substitution of traditional optical lens, are conducted according to this phase delay phenomenon of EOT [8]–[10]. In order to obtain desired properties of the lens, the theory of focal-shift is summarized thoroughly by introducing Fresnel number which can indicate the focusing process [10]. As explained, if the diffraction effect at the boundary of the slits becomes significant, the focal-shift occurs (i.e., the position of the maximum intensity moves closer to lens from preset focal point). The analysis of Fresnel number, as an instructive method, guides us to explore the

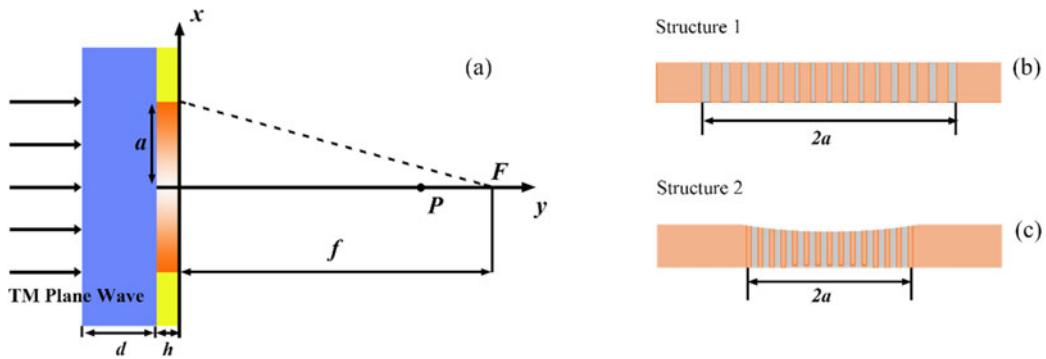


Fig. 1. (a) Two-dimensional schematic diagram of simulation, where  $d = 8 \mu\text{m}$  and  $h = 0.4 \mu\text{m}$ . (b) Lens with arrangement of increasing width of slit, where  $a = 2.22 \mu\text{m}$ . (c) Lens with arrangement of increasing height of wall, where  $a = 1.44 \mu\text{m}$ .

approach to adjusting focal length. Accordingly, the lens which varies depth or width of slits gradually is investigated. Such a lens, with the ability to focus light, has subwavelength structure to integrate, and can be utilized in optical components and lithography. The latest studied tends to reduce the full width at half maximum (FWHM) and the focal length smaller than the operating wavelength, which is known as superfocusing with the benefit of integration on chip applications [11]–[14]. However, most of simulations or experiments are structurally fixed with unchanged focal point [15]–[19], i.e. the lens with different focal length is mainly achieved by different structural parameters. A fast and convenient way for focal-shift is needed in practical application.

In this paper, the influence of refractive index on focal-shift is analyzed by introducing Fresnel number, and the focal-shift of nano lens affected by resonance with substrate is proposed on two commonly-used structures of lens. This resonance exhibits periodicity with the variation of substrate refractive index. The comparison between the focal features of these two types of lens are emphasized. A discussion of focal-shift application in lithography is given with some conventional techniques of refractive index control [20]–[24]. The numerical simulations are based on the finite element method (FEM), and commercial software COMSOL Multiphysics is used.

## 2. Structure and Theory

The 2-D schematic diagram of simulation is shown in Fig. 1(a). The capital letters F and P are used to denote the preset geometrical focal point and observed focal point of incident wave, respectively. A TM plane wave which wavelength is 637 nm incidents along y-axis, and traverses a substrate with thickness of  $d$  and a nano lens with thickness of  $h$  successively, focusing in the air at last. The diameter of the lens is  $2a$ . The refractive index of air and substrate is set to be 1 and  $n_b$ . In Fig. 1(b) and (c), two conventional structures of lens are illustrated. They are designed by arrangement of increasing width of slit and increasing depth of slit, respectively. The first structure refers from Ref. [8], which has constant depth of slit, while the width values of slit are 80 nm, 80 nm, 80 nm, 80 nm, 120 nm, 130 nm, 140 nm, and 150 nm from middle to both sides. The second structure has constant width of slit, and the height values of metal strip are 340 nm, 340 nm, 340 nm, 370 nm, 380 nm, 390 nm, 400 nm, and 400 nm from middle to both sides. The similar focus effect, as the original intention of structural selection, can be obtained on both structures.

The Fresnel number is introduced here:

$$N = \frac{a^2}{\lambda f} \quad (1)$$

where  $\lambda$  is the wavelength of incident monochromatic wave, and  $f$  is the preset focal length. The Fresnel number is a crucial parameter to explain the effect of lens size on focal-shift. There are three methods given about the relationship between focal length and Fresnel number [9]. Here, we

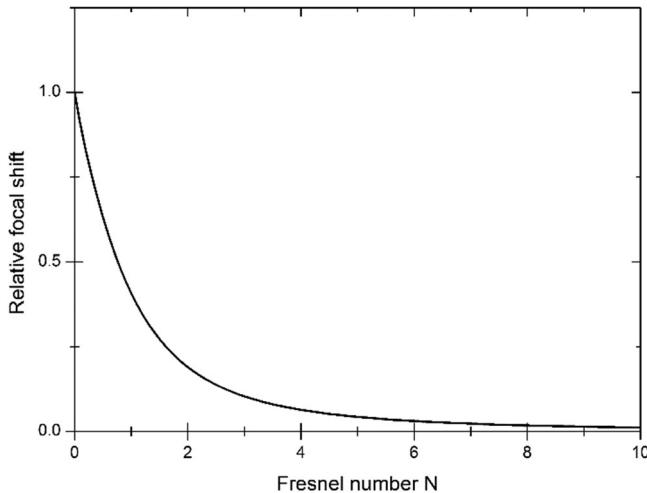


Fig. 2. Relative focal shift as a function of Fresnel number N.

choose the third formula (2), which has only one variable N. If we assume the shift distance is  $\Delta f$ , the trend of relative focal shift changing is shown as

$$\frac{\Delta f}{f} = \left( 1 + N \times \sqrt{1 + \left( \frac{\pi^2 N}{12} \right)^c} \right)^{-1}, \quad c = 1.51. \quad (2)$$

The functional image is presented in Fig. 2. As we can see, the focal point has larger shift distance with smaller Fresnel number. Considering the definition (1), such shift distance is totally depends on the effective radius  $a$  of lens with monochromatic light incidence.

With Fresnel approximation, the phase function  $\varphi(x)$  along the subsurface of lens can be expressed as [10]

$$\varphi(x) = -\frac{k_0}{2f}x^2 = \pi/N|_{x=a} \quad (3)$$

where  $k_0$  is the wave number of light in vacuum. The focal-shift is accompanied by decreasing of  $N$ , and therefore, to move the focal point towards lens is to improve the maximum of  $\varphi(x)$ . When the shift happens, there is a great phase difference between middle and edge of the lens. In plasmonic focal lens, the fundamental modes inside the slits are extremely sensitive with their nano structures, and this characteristic is utilized to achieve the same function of the traditional optical lens working with refraction. We use  $\beta$  to represent the propagation constant of plasmonic modes, and therefore, the phase delay of the light transmitted through the slits can be approximated by [10]

$$\varphi(x) = \text{Re}(\beta(x))h. \quad (4)$$

Surface plasmon resonance brings extra transverse propagation constant [1]. When the condition of excitation SPPs is satisfied on the surface of lens aperture, the phase difference  $\varphi$  increases much larger, and the focal-shift emerges. The nano lens with slits can be regarded as a specific extension of EOT with periodic gratings, which are highly sensitive with modes excited by diffraction when the structure has sub-wavelength slits.

### 3. Results and analysis

The material of lens is set to be gold, and the permittivity is  $-11.04 + 0.78i$ . The absorption of two metal lenses is calculated at first, which is shown in Fig. 3. The refractive index of substrate changes from 1.1 to 2. Each absorption peak corresponds to a plasmonic resonance mode. The resonance

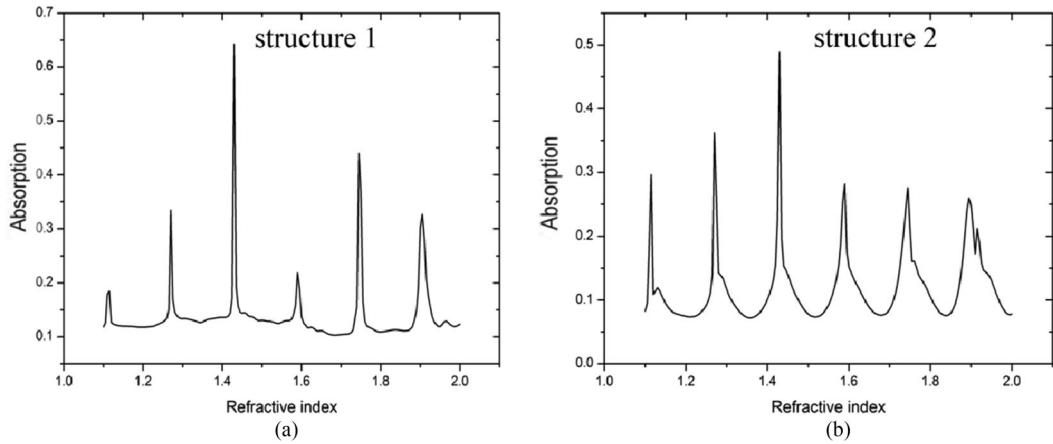


Fig. 3. Relationship between absorption and refractive index of substrate. (a) With structure 1.  
(b) With structure 2.

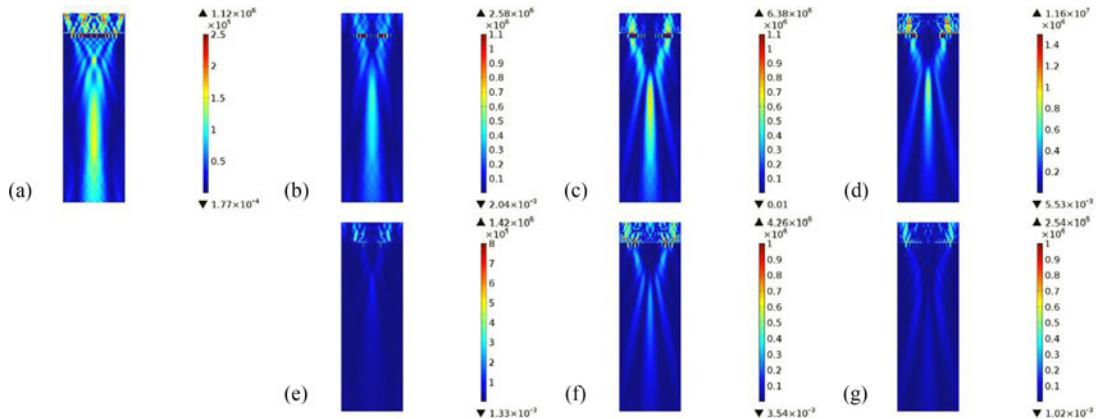


Fig. 4. Absolute value of Poynting vector distribution of structure 1, where the refractive index is 1.46, 1.115, 1.27, 1.43, 1.59, 1.745, and 1.9 from (a) to (g).

modes of both structures occur at the same refractive index  $n_b = 1.115, 1.27, 1.43, 1.59, 1.745$ , and 1.9. If the width of slit is much smaller than the radius of lens, the condition of resonance can be expressed approximately as

$$k_0 n_{eff} a = 2\pi m, \quad m = 1, 2, 3 \dots \quad (5)$$

where  $n_{eff}$  is the effective mode index of metal lens. As we can see in Fig. 3, structure 1 has higher absorption maximum, while structure 2 has more regular spectrum. The average interval of refractive index between two modes is 0.157, which is small enough to be controlled by available techniques of refractive index adjustment, such as temperature sensitive medium, electro-optical effect, voltage-controlled LC, and applied pump light.

In the simulation, the initial refractive index of substrate for the situation of preset focal point is set to be 1.46 and 1.5 in structure 1 and 2, respectively. The absolute value of y-direction Poynting vector distribution of structure 1 and 2 is shown in Fig. 4 and Fig. 5, respectively. The preset focus feature with initial refractive index is described in Fig. 4(a) and Fig. 5(a), while other six resonance modes are described in Fig. 4(b)-(g) and Fig. 5(b)-(g). With the changing of  $n_b$ , it is equivalent to periodically decreasing the effective radius  $a$ , and also the focal-shift occurs. The calculation

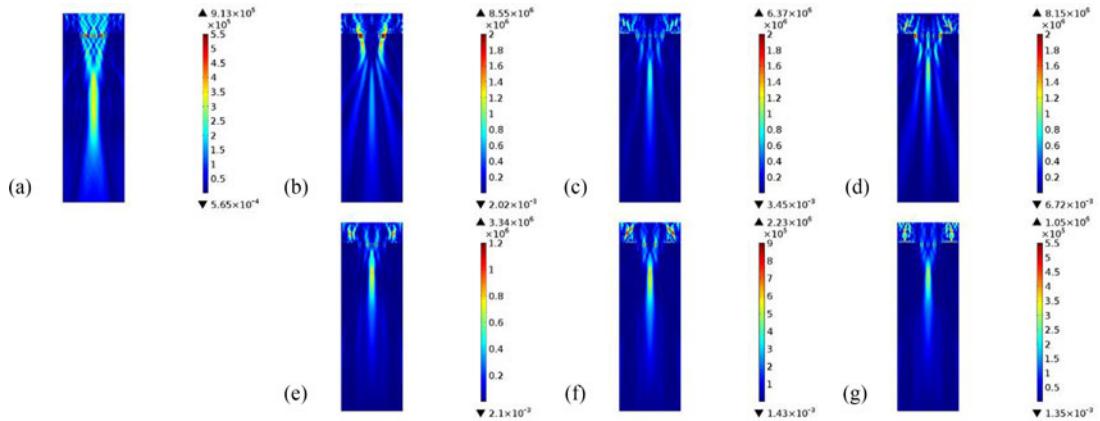


Fig. 5. Absolute value of Poynting vector distribution of structure 2, where the refractive is 1.5, 1.115, 1.27, 1.43, 1.59, 1.745, and 1.9 from (a) to (g).

TABLE 1  
Focusing Characteristics of Structure 1 with Preset  $f = 10699$  nm

Parameters	$f$	$\Delta f$	$\frac{\Delta f}{f}$	N	FWHM (nm)	Normalized Intensity
$n = 1.115$	5675	5024	0.47	0.84	906	3.1
$n = 1.27$	5903	4796	0.45	0.89	648	4.8
$n = 1.43$	4912	5787	0.54	0.67	539	5.5
$n = 1.59$	—	—	—	—	—	—
$n = 1.745$	—	—	—	—	—	—
$n = 1.9$	—	—	—	—	—	—
$n = 1.46$	10699	0	0	1	1236	1

of focusing characteristics on structure 1 and 2 is shown in Tables 1 and 2, respectively. Considering structure 1 with different width of slits, when  $n_b$  is small than the initial refractive index 1.46, the focal-shift towards lens is conspicuous. However, it is not every resonance modes that bring focal-shift to lens. When  $n_b$  exceeds 1.46, the average focal intensity cannot maintain a high level as the former three modes, and the light though slits is difficult to focus persistently. This is obviously shown in Fig. 4(g). The last three modes are considered as inappropriate for focal-shift application. In structure 2, the focal-shift occurs with all six modes. Although the extreme absorption is weaker, the focal intensity is stable in every modes. The focal-shift distance  $\Delta f$  (see Table 2) also remains constant in Fig. 5(c)-(g), which shows better regularity than structure 1.

The FWHM and the normalized intensity of focal point are also given in Table 1 and 2. The intensity of preset focal point is set to be 1. As is shown evidently in both tables, besides the focal-shift in each resonance mode, the FWHM also becomes narrower and the normalized intensity is enhanced. In Fig. 6, the modes which is with  $n = 1.43$  and  $n = 1.59$  are chosen as the most remarkable cases in structure 1 and 2, respectively, and compared with corresponding preset focal points by illustrated the energy flux density along y-direction in x-axis. In structure 1, the intensity is enhanced more than five times, while it is enhanced nearly three times in structure 2. Both modes reduce the FWHM more than a half. Compared with latest studies of superfocusing [11]–[14], although this

TABLE 2  
Focusing Characteristics of Structure 2 with Preset  $f = 4077$  nm

Parameters	$f$	$\Delta f$	$\frac{\Delta f}{f}$	N	FWHM (nm)	Normalized Intensity
$n = 1.115$	2353	1724	0.42	0.96	895	1.5
$n = 1.27$	1754	2323	0.57	0.62	826	1.5
$n = 1.43$	1690	2387	0.59	0.58	742	1.7
$n = 1.59$	1429	2648	0.65	0.47	674	2.7
$n = 1.745$	1429	2648	0.65	0.47	935	1.4
$n = 1.9$	1429	2648	0.65	0.47	1019	1.4
$n = 1.5$	4077	0	0	1	1592	1

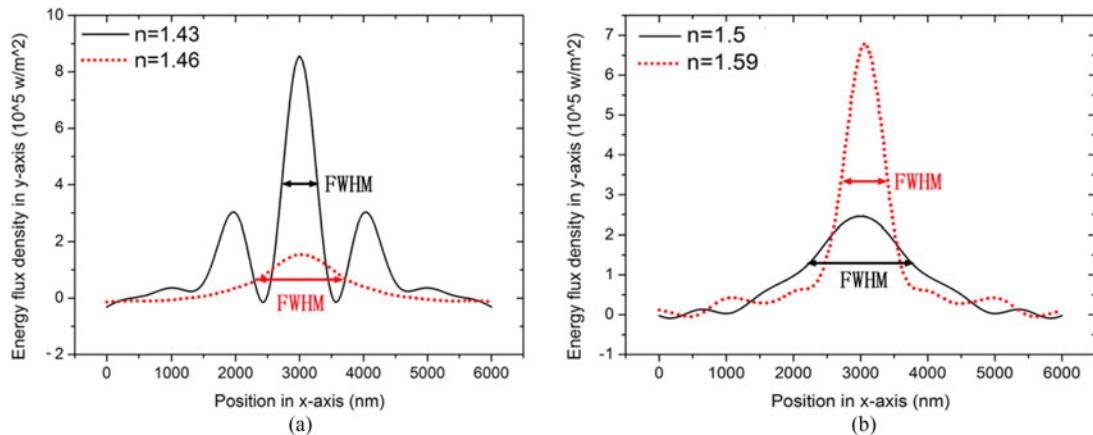


Fig. 6. (a) Comparison of FWHM and intensity between preset and shifted focal point in structure 1.  
(b) Comparison of FWHM and intensity between preset and shifted focal point in structure 2.

focal-shift is still with higher FWHM than operating wavelength, it does have potential application on superfocusing for the reduction of focal length and FWHM. The advantages of narrower FWHM and stronger intensity is benefit for the application of compact devices and improving the efficiency of focusing.

The energy flux density distribution along central y-axis is shown in Fig. 7. We set the coordinate of lens center in  $y = 16 \mu\text{m}$ . The black solid line represents the initial intensity of focusing, and the red dashed line represents the intensity of shift focusing. The most remarkable mode result is also chosen to make contrast with preset focus feature in each structure. In structure 1, the focal-shift distance is 5802.3 nm, which is about four times than the result of structure 2, which is 1493.0 nm. As we can see in both structures that not only the focus point is moved but also the intensity of focal light is greatly enhanced by plasmon resonance. The maximum of enhanced energy flux density is close in two situations, and the first one has higher increment with lower initial intensity of focusing.

The energy flux intensity along y-axis in preset focal point F with the increasing of substrate refractive index is shown in Fig. 8. These 12 modes in Fig. 8(a) and (b), which correspond the absorption peaks in Fig. 3 well, are observed distinctly, even if the maximum of focusing is no longer located in point F. Irrespective of resonance peaks, the energy flux density also shows a periodic increase and decrease with refractive index changing in structure 2.

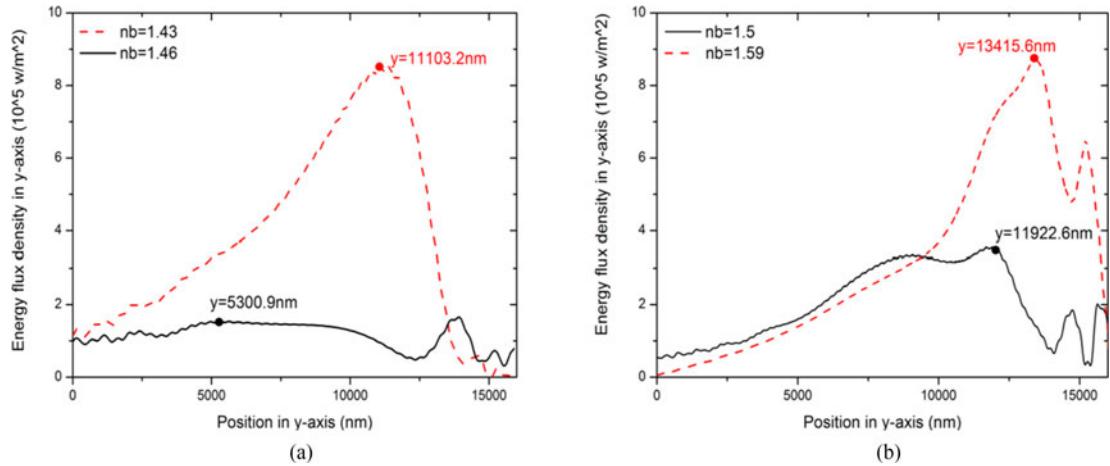


Fig. 7. (a) Energy flux density distribution along central y-axis in structure 1. (b) Energy flux density distribution along central y-axis in structure 2.

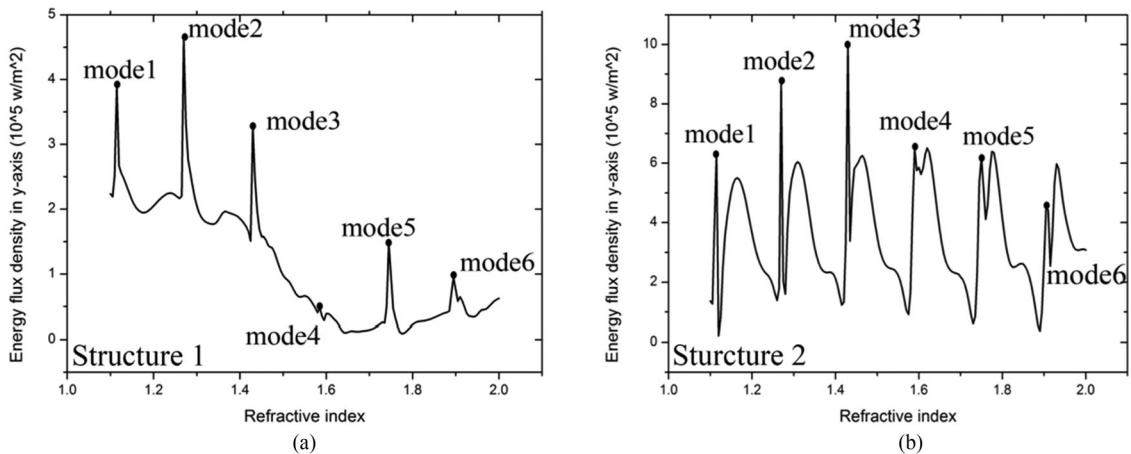


Fig. 8. (a) Energy flux intensity in preset focal point F with the increasing of substrate refractive index in structure 1. (b) Energy flux intensity in preset focal point F with the increasing of substrate refractive index in structure 2.

#### 4. Application Discussion

We have explained the substrate refractive index effect on focal-shift of nano lens. As an example of application, optical lithography has been used widely in integrated circuit manufacturing. For refractive index control, a voltage-controlled plasmonic (VCP) filter based on the index-sensitive properties is used in recent nano plasmonic structure, and the control range of refractive index is from 1.48 to 1.65 with the LC [20]. Another suitable choice is the refractive index control by pump light utilized by some studies of electromagnetically induced transparency (EIT) or EIT-like [21]–[24]. With or without pump light, the refractive index shifts from 1.43 to 1.5 [18] or 1.53 to 1.58 [24]. Both of the index ranges are close to one resonance period we have demonstrated before. Besides, the respond speed is as fast as less than 4.24 ms. The basic schematic of lithography design is shown in Fig. 9. Two planar monochromatic lights incident into the substrate with pump controlled polymer A and B, respectively. The exit light from substrate is focused by lens on bottom wafer. As the pump incidents into substrate B, the change of refractive index motivates resonance in nano lens, and the focal point shifts towards lens. The adjustment can be achieved.

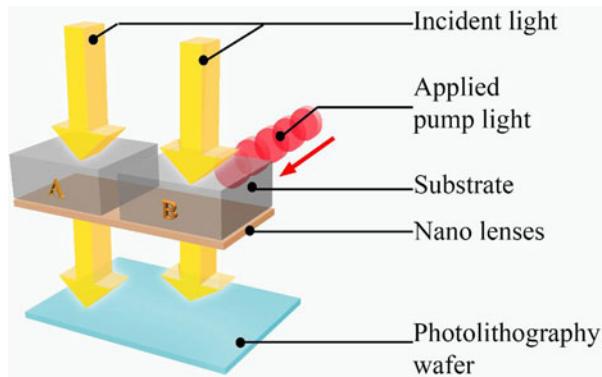


Fig. 9. Basic schematic of lithography design, where the refractive index of polymer substrate is controlled by pump light.

## 5. Conclusion

In conclusion, we have proposed a method of periodic focal-shift affected by resonance with substrate on two typical nano lenses. By analyzing this focal-shift with Fresnel number, the variation of refractive index is regarded as the origin of the periodic resonance with substrate. Besides, the comparison of the resonance feature on these two typical nano lenses is presented. The lens with increasing width of slit has more prominent focal-shift and stronger enhancement of resonance, while the lens with increasing height of metal strip has more regular resonant changing period with refractive index of substrate. What's more, through comparing both two structures, the more regular repetitive characteristic is helpful to periodic production, and the more prominent result can be employed in extreme conditions. This work can be utilized in optical components and lithography. An example of optical lithography design thinking is given to discuss the feasibility of practical use. We hope this work can provide a new thought of shifting the focal point without structural change, and a better detailed comprehension of focal-shift on nano lenses.

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