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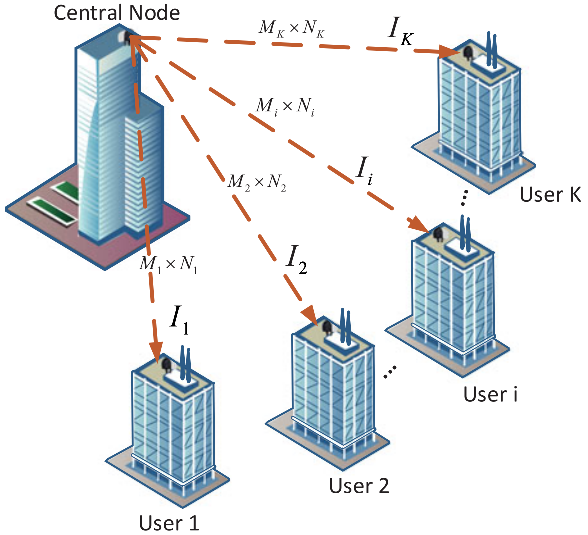

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Generalized Selection Multiuser Scheduling for the MIMO FSO Communication System and Its Performance Analysis

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Abstract: This paper analyzes the impact of generalized selection multiuser scheduling on the performance of the multiple-input–multiple-output (MIMO) free-space optical (FSO) communication system. We consider a general scenario that the point-to-multipoint system consists of a central transmitter equipped with K aperture groups containing M transmit apertures and K users equipped with N receive apertures communicating over gamma–gamma turbulence FSO channels. For the MIMO links between the central node and user, we derive the combining signal-to-noise ratio (SNR) distributions of MIMO repetition coding and MIMO transmit laser selection. Then, a statistics analysis is presented for the multiuser diversity scheme in which the user with the kth best combining SNR is selected. Based on the analysis, the closed-form expressions of the outage probability are derived, and the diversity benefit provided by the channel fluctuations is further discussed. Numerical results verify the accuracy of these analytical results.

Index Terms: Atmospheric turbulence, free-space optics, multiple-input-multiple-output (MIMO), multiuser diversity.

1. Introduction

As a practical alternative to break the broadband connectivity bottleneck, free-space optical (FSO) communication has drawn considerable attention. Offering a high capacity with low cost and excellent security over unlicensed optical spectrum, FSO links can be widely distributed to connect the "last mile" from the fiber backbone to the client building or cellular tower [1], [2]. Furthermore, as a part of FSO communication, visible laser light communication has achieved a surprising high data rate [3], [4], which can be applied for short distance communication among cloud servers in data center. However, the performance of FSO communication system is heavily impacted by atmospheric turbulence that occurs due to random fluctuations of the air mass along the channel path caused by inhomogeneities in temperature, pressure, humidity, etc. [5].

To combat the degrading impact of atmospheric turbulence, several fading mitigation techniques, especially spatial diversity technique, have been proposed in FSO communication systems. Spatial diversity technique, for example, multiple-input-multiple-output (MIMO) FSO systems in which multiple apertures are equipped at both the transmitter and receiver sides can effectively mitigate the atmospheric turbulence fading by averaging out it and make the FSO links more reliable. The space diversity technique used in FSO systems has been first proposed in [6]. Shin *et al.* [7] have investigated the outage probability of MIMO FSO systems over log-normal fading channels. [8] and [9] have studied the bit-error rate (BER) performance of MIMO FSO links over weak and strong atmospheric turbulence channels, respectively. While fading mitigation techniques is considered in point-to-point FSO systems, [10] and [11] utilized the turbulence fading to provide multiuser diversity gain, which can increase the communication rate and enhance the link reliability significantly in point-to-multipoint FSO systems. Also, multiuser diversity over parallel and hybird FSO and radio-frequency (RF) links has been discussed in [12]. They were both motivated by the time-varying characteristic of the atmospheric turbulence fading and viewed the turbulence fluctuations as a randomization resource that can be exploited.

In this paper, we investigate the impact of generalized selection multiuser scheduling (GSMuS) on the performance of MIMO FSO system over strong turbulence channels. Multiuser schedule provides the diversity benefit by exploring the different fading channel to multiple users. Since different user channel likely experience independent fading, there is highly probable that at least one user has acceptable fading condition. Higher system performance will be achieved if we always choose the best user channel to access at any time instant. Meanwhile, multiuser selective schedule avoids the channel interference for RF system. Distinguished from RF communication, FSO communication has strong directivity and there is no interference among users. Multiuser schedule can still improve the system capacity when the system power is given. Previous works [10]–[12] on the multiuser diversity focused on SISO point-to-multipoint FSO system. To the best of our knowledge, there is no analysis for the multiuser diversity in MIMO FSO links. Here, we first derive the outage probability and the diversity order expressions of GSMuS for MIMO FSO System.

The rest of the paper is organized as follows. We will start by describing the proposed FSO system model in Section 2. Section 3 presents the statistical analysis for the MIMO combining and the multiuser diversity (MD) scheme, and Section 4 gives the outage probability and the diversity order expressions, and discusses the special cases. Section 5 validates the analytical results by the numerical results. Finally, a conclusion of our work follows in the last section.

2. System Model

We consider a MIMO FSO communication system consisting of a central transmitter (namely, the central node) equipped with K aperture groups containing M_i transmit apertures, respectively, and K users equipped with N_i receive apertures, respectively, where $i \in \{1, 2, ..., K\}$. We assume that the transmit apertures of the ith aperture group at the central node are directed to the apertures of the ith user, whose receive apertures are directed to the transmit apertures of the ith aperture group as well. Hence, the central transmitter communicates with ith user through $M_i \times N_i$ MIMO FSO links as shown in Fig 1. Therefore, we have a point-to-multipoint MIMO FSO system. In this system, the K aperture groups are alternated in a time division multiplexing schedule and only the transmit apertures in one group will work in each time slot. We assume that each FSO channel can be modeled by gamma-gamma turbulence distribution. It is also is assumed that the system employs on-off keying (OOK) modulation with the typical intensity-modulation direct-detection (IM/DD).

Considering the signal transmission between the mth transmit aperture and the nth receive aperture in the MIMO FSO links of ith user, where $m \in \{1, 2, ..., M_i\}$, and $n \in \{1, 2, ..., N_i\}$, the received signal $y_i^{(m,n)}$ can be written as

$$y_i^{(m,n)} = \eta G_i I_i^{(m,n)} x_i + n_i^{(m,n)} \tag{1}$$

where η is the effective optical-to-electrical conversion ratio, G_i is a gain factor denoting the path-loss, $I_i^{(m,n)}$ is the normalized irradiance, x_i is the modulated optical signal (0 or 1 for OOK modulation), and $n_i^{(m,n)}$ is white Gaussian noise with zero mean and variance $N_0/2$. We assume that the distance between ith user and the central node is d_i and that the average distance between all the users

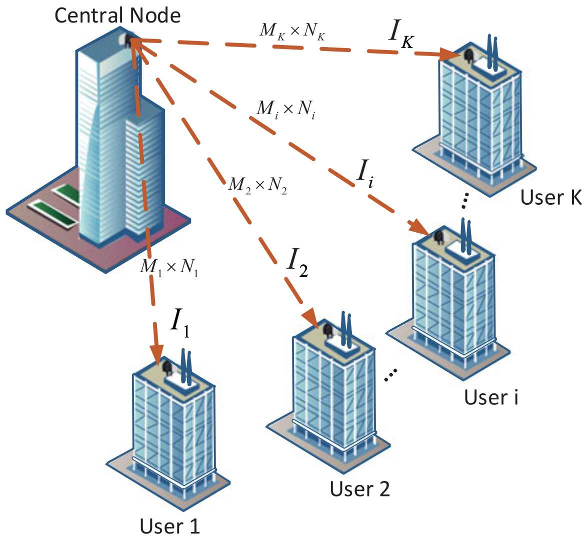

Fig. 1. MIMO FSO multiuser system model.

and the central node is $\bar{d}$. The path gain factor can be written as

$$G_i = \left(\frac{\bar{d}}{d_i}\right)^2 e^{-\sigma(d_i-\bar{d})} \tag{2}$$

where σ is the attenuation coefficients.

As stated at the beginning of this section, we utilize the gamma-gamma distribution to model the FSO channel fading. Hence, the probability distribution function (PDF) of the irradiance $I_i^{(m,n)}$ can be expressed as [13]

$$f_{I_i^{(m,n)}}(I) = f^{GG}(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta)/2-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta I}) \tag{3}$$

where $\Gamma(\cdot)$ is the gamma function, and $K_\nu(\cdot)$ is the modified Bessel function of the second kind of order ν. The parameters α and β are related to the atmospheric conditions and can be written as [14]

$$\alpha = \left[\exp\left(0.49\delta^2/\left(1+1.11\delta^{12/5}\right)^{7/6}\right)-1\right]^{-1} \tag{4}$$

$$\beta = \left[\exp\left(0.51\delta^2/\left(1+0.69\delta^{12/5}\right)^{5/6}\right)-1\right]^{-1} \tag{5}$$

where δ^2 is the Rytov variance, which is the function of the link distance d, the optical wave number k, and the turbulence strength factor C_n^2 and can be given by $\delta^2 = 1.23C_n^2 k^{7/6} d^{11/6}$.

Focusing on the MIMO links for ith user, we consider two practical diversity combining and selection strategies: MIMO repetition coding (RC) and MIMO transmit laser selection (TLS). For RC, the same information symbol is transmitted from all transmit apertures and the transmitter do not require any channel state information (CSI) feedback. In sharp contrast to RC, the TLS solution selectively activates the laser to ensure the maximum irradiance at the receiver, hence, a feedback link as well as the CSI is required. Then, the combining irradiance of ith user can be expressed as

$$\tilde{I}_i = \begin{cases} \frac{1}{M_i}\sum_{m=1}^{M_i}\sum_{n=1}^{N_i} I_i^{(m,n)}, & \text{MIMO-RC} \\ \max_{m=1,\ldots,M_i}\sum_{n=1}^{N_i} I_i^{(m,n)}, & \text{MIMO-TLS.} \end{cases} \tag{6}$$

Hence, according to (1), the combining received signal y_i of ith user from the MIMO links can be expressed as

$$y_i = \eta G_i \tilde{I}_i x_i + n_i \tag{7}$$

where n_i is the sum of $n_i^{(m,n)}$, and its variance is equal to $N_i N_0/2$. By the model of (7), the combining SNR between ith user and the central node can be expressed as

$$\gamma_i = \bar{\gamma}_i \tilde{I}_i^2 \tag{8}$$

where $\bar{\gamma}_i = \frac{\eta^2 G_i{}^2 x_i{}^2}{N_i N_0}$ is the average SNR of ith user.

3. Statistics Analysis

3.1. Statistics Analysis for Diversity Combining

We apply the gamma-gamma distribution to describing the irradiance $I_i^{(m,n)}$ affected by the atmospheric turbulence. Traditionally, the cumulative distribution function (CDF) of the gamma-gamma distribution, which is named the Meijer's G-function, is used to analyze the system performance. However, the function is too complex to get the closed-form performance expression when applied to the MIMO diversity situation such as RC and TLS. To simplify the analysis, the following approximation of gamma-gamma distribution is made.

Lemma 1: The PDF and CDF of the gamma-gamma distribution can be approximated as [14]

$$f_{I_i^{(m,n)}}(I) \approx C_i f^G(I, \beta_i, A_i) \tag{9}$$

$$F_{I_i^{(m,n)}}(I) \approx \frac{C_i}{\Gamma(\beta_i)} \gamma(\beta_i, I/A_i) \tag{10}$$

where $f^G(I, k, \theta) = \frac{1}{\Gamma(k)\theta^k} I^{k-1} e^{-x/\theta}$ is the PDF of gamma distribution, $\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$ is the lower incomplete gamma function, and

$$C_i = \frac{(\alpha_i - \beta_i - 1)^{\beta_i} \Gamma(\alpha_i - \beta_i)}{\Gamma(\alpha_i)} \tag{11}$$

$$A_i = \frac{\alpha_i - \beta_i - 1}{\alpha_i \beta_i} \tag{12}$$

are two constants related to FSO link parameters α_i and β_i.

Proof 1: See the approximation method in [14, Sec. III.A].

Lemma 2: Based on Lemma 1, the CDF of MIMO combining irradiance $\tilde{I}_i$ can be approximated as

$$F_{\sum_{m=1}^{M_i} \sum_{n=1}^{N_i} I_i^{(m,n)}}(I) \approx \frac{C_i^{M_i N_i}}{\Gamma(M_i N_i \beta_i)} \gamma(M_i N_i \beta_i, M_i I/A_i), \quad \text{MIMO-RC} \tag{13}$$

$$F_{\max\limits_{m=1,\dots,M_i} \sum_{n=1}^{N_i} I_i^{(m,n)}}(I) \approx \left[\frac{C_i^{N_i}}{\Gamma(N_i \beta_i)} \gamma(N_i \beta_i, I/A_i) \right]^{M_i}, \quad \text{MIMO-TLS.} \tag{14}$$

Proof 2: The sum of independent gamma random variables with the same scale parameter is a gamma random variable having the same scale parameter and a shape parameter equal to the sum of the shape parameters [15], and the maximal value of M independent random variables with the same CDF $F(x)$ is a random variable with the CDF $(F(x))^M$, according to the theory of order statistics [16].

Lemma 3: Let x be an absolutely continuous random variable with support R_x, PDF $f_x(x)$ and CDF $F_x(x)$. Let $g : \mathbb{R} \to \mathbb{R}$ be strictly increasing and differentiable on the support of x. The support of $y = g(x)$ is $R_y = \{y = g(x) : x \in R_y\}$, and its PDF and CDF is

$$f_y(y) = f_x(h(y))\frac{dh(y)}{dy} \tag{15}$$

$$F_y(y) = F_x(h(y)) \tag{16}$$

where $h(y)$ is the inverse function of $g(x)$.

Proof 3: See the function of one random variable in [17, ch. 5.1].

According to the Lemma 2, Lemma 3 and the equation (8), the CDF of the FSO MIMO combining SNR γ_i can be approximated as

$$F_{\gamma_i}(\gamma_i) \approx \begin{cases} \frac{C^{M_i N_i}}{\Gamma(M_i N_i \beta)}\gamma(M_i N_i \beta, M_i\sqrt{\gamma_i \bar{\gamma}_i^{-1}}/A), & \text{MIMO-RC} \\ \left[\frac{C^{N_i}}{\Gamma(N_i\beta)}\gamma(N_i\beta, \sqrt{\gamma_i\bar{\gamma}_i^{-1}}/A)\right]^{M_i}, & \text{MIMO-TLS.} \end{cases} \tag{17}$$

3.2. Statistics Analysis for Multiuser Scheduling

In this subsection, we analyze the statistics of GSMuS. We assume that central node select the kth best user to serve per time slot with the precondition knowing all of the channel state information. Such a multiuser diversity scheme has been applied to provide a near-optimal low-complexity solution for RF systems [18], [19]. The random variables γ_i is arranged in an increasing order of feedback value, $\gamma_{1:K} \geq \cdots \geq \gamma_{k:K} \geq \cdots \geq \gamma_{K:K}$, and $\gamma_{k:K}$ denote the kth best user SNR. By the order statistics of random variables, the combining SNR distribution of the kth user can be expressed as [20, Eq. (6)]

$$f_{\gamma_{k:K}}(\gamma) = \sum_{\substack{\gamma_1,\cdots,\gamma_{k-1} \\ \gamma_1<\gamma_2<\cdots<\gamma_{k-1}}} \sum_{\gamma_k} f_{\gamma_k}(\gamma)\left[\prod_{l=1}^{k-1}[1 - F_{\gamma_l}(\gamma)]\right]\left[\prod_{l'=k+1}^{K} F_{\gamma'_l}(\gamma)\right] \tag{18}$$

where $f_{\gamma_k}(\gamma)$ is the PDF of the kth best user SNR, and $F_{\gamma_l}(\gamma)$ is the CDF of the lth largest user SNR.

In order to simplify the calculation, we assume that each MIMO link has the same number of the transmit apertures and the receive apertures, i.e. $M_i = M$, $N_i = N$, for $i = \{1, \cdots, K\}$ and has, as well the same average link distance $\bar{d}$. Based on the assumptions, the above SNRs is i.i.d. We can reduce equation (18) to obtain the PDF and CDF of the kth best user SNR

$$f_{\gamma_{k:K}}(\gamma) = \frac{K!}{(K-k)!(k-1)!}[F(\gamma)]^{k-1}[1 - F(\gamma)]^{K-k} f(\gamma) \tag{19}$$

$$F_{\gamma_{k:K}}(\gamma) = \sum_{i=K-k+1}^{K}\binom{K}{i}[F(\gamma)]^i[1 - F(\gamma)]^{K-i} \tag{20}$$

where $f(\cdot)$ and $F(\cdot)$ is the PDF and CDF of a single user SNR, respectively. Applying (17) to (20), the CDF of the kth best user SNR of the FSO MIMO multiuser system with GSMuS can be given by

$$F_{\gamma_{k:K}}^{\text{MIMO-RC}}(\gamma) \approx \sum_{i=K-k+1}^{K}\sum_{j=0}^{K-i}\binom{K}{i}\binom{K-i}{j}(-1)^j \times \left[\frac{C^{MN}}{\Gamma(MN\beta)}\gamma\left(MN\beta, M\sqrt{\gamma\bar{\gamma}^{-1}}/A\right)\right]^{i+j} \tag{21}$$

$$F_{\gamma_{k:K}}^{\text{MIMO-TLS}}(\gamma) \approx \sum_{i=K-k+1}^{K}\sum_{j=0}^{K-i}\binom{K}{i}\binom{K-i}{j}(-1)^j \times \left[\frac{C^{N}}{\Gamma(N\beta)}\gamma\left(N\beta, \sqrt{\gamma\bar{\gamma}^{-1}}/A\right)\right]^{M(i+j)}. \tag{22}$$

4. Performance Analysis

In this section, we derive the outage probability for the proposed FSO MIMO multiuser diversity scheme. The diversity order are given via further approximation at high SNR and two special cases are additionally discussed.

4.1. Outage Probability

An outage happens when the instantaneous SNR γ falls below a given threshold γ_{th}. Then, the outage probability can be written as

$$P_{out} = \Pr(\gamma < \gamma_{th}). \tag{23}$$

As we have already derived the CDF expressions (21) and (22) of the instantaneous SNR for the proposed scheme, the outage probability can be rewritten as $P_{out} = F_{\gamma_{k:K}}(\gamma_{th})$.

4.2. Diversity Order

In order to analyze the diversity order of the proposed scheme, the outage probability should be written in the following form [21], [22]:

$$P_{out} = (g_c\bar{\gamma})^{-\nu} \tag{24}$$

where g_c is the coding gain, and ν is the diversity order. In the high SNR regime, we can have the approximation $\gamma(s, x) \approx \frac{x^s}{s}$, and (21) and (22) can be written as

$$P_{out}^{\text{MIMO-RC}} \approx \sum_{i=K-k+1}^{K} \sum_{j=0}^{K-i} \binom{K}{i}\binom{K-i}{j}(-1)^j \times \left[\frac{C^{MN}(M/A)^{MN\beta}}{\Gamma(MN\beta+1)}\right]^{i+j}\left[\sqrt{\gamma_{th}\bar{\gamma}^{-1}}\right]^{MN\beta(i+j)} \tag{25}$$

$$P_{out}^{\text{MIMO-TLS}} \approx \sum_{i=K-k+1}^{K} \sum_{j=0}^{K-i} \binom{K}{i}\binom{K-i}{j}(-1)^j \times \left[\frac{C^{N}A^{-N\beta}}{\Gamma(N\beta+1)}\right]^{M(i+j)}\left[\sqrt{\gamma_{th}\bar{\gamma}^{-1}}\right]^{MN\beta(i+j)}. \tag{26}$$

When $\bar{\gamma} \to \infty$, we can conclude that the diversity order of the proposed multiuser selective scheme is $(K-k+1)MN\beta/2$ for either MIMO-RC or MIMO-TLS. If $k=1$, i.e., always selecting the best user, the diversity order is $KMN\beta/2$. The above conclusions reveal that the MIMO combining diversity and the multiuser selective diversity are independent, and they can be achieved simultaneously.

4.3. Special Cases

The derived performance expressions are general results for the proposed system. In this section, we analyze the system performance in two special cases: The MIMO FSO system degenerates into SISO FSO system, and the best user is selected to serve in the schedule, respectively.

4.3.1. Multiuser Scheduling for SISO FSO Links: When $M = N = 1$, the MIMO FSO system degenerates into SISO FSO system. We can find that the CDFs of the two MIMO diversity strategies are degraded into the same form

$$F_{\gamma_{k:K}} = \sum_{i=k}^{K}\sum_{j=0}^{K-i}\binom{K}{i}\binom{K-i}{j}(-1)^j\left[\frac{C}{\Gamma(\beta)}\gamma\left(\beta, \sqrt{\gamma\bar{\gamma}^{-1}}/A\right)\right]^{i+j} \tag{27}$$

$$\approx \sum_{i=K-k+1}^{K}\sum_{j=0}^{K-i}\binom{K}{i}\binom{K-i}{j}(-1)^j\left[\frac{CA^{-\beta}}{\Gamma(\beta+1)}\right]^{i+j}\left[\sqrt{\gamma_{th}\bar{\gamma}^{-1}}\right]^{\beta(i+j)}. \tag{28}$$

The diversity order of the SISO FSO system is $(K-k+1)\beta/2$, which is similar to the result $k'\min(\alpha,\beta)/2$ in the literature [11] where $k' = K-k+1$. Since β is in general smaller than α in practical scenarios, our result for the SISO FSO system and the literature result match well.

4.3.2. Selecting the Best User: Consider the scenario that the central node always severs the user with the best SNR, *i.e.* $k = 1$. Statistically the outage happens when the best SNR is smaller than the threshold γ_{th}. In other word, we should calculate the probability that all the SNRs is below γ_{th}. It can be expressed as

$$F_{\gamma_k}(\gamma) = \prod_{i=1}^{K} F_{\gamma_i}(\gamma). \tag{29}$$

If all the users have the same link distance and the same number of apertures, we can find that the outage probability is

$$P_{out}(\gamma_{th}) = [F_{MIMO}(\gamma_{th})]^K \tag{30}$$

and the diversity order is $KMN\beta/2$, which matches the derivation in Section 4.2.

5. Numerical Results

In this section, we present numerical simulations to validate our analysis for the proposed multiuser selective scheme under the MIMO combining consideration: MIMO-RC and MIMO-TLS. Through the analysis in Section 2, we can know that the key channel fading parameters α and β are related to the Rytov variance, which depends on the link distance and the turbulence strength. We have assumed that all the users have the same link distances in above analysis. Here, we reference the parameters in [13], where the link distance $d = 5$ km, the wavelength $\lambda = 1.55\ \mu$m, the aperture diameter $D_R = D_T = 0.1$ m, the atmospheric attenuation $\sigma = 0.01$ km^{-1}, and the turbulence strength factor $C_n^2 = 2\times 10^{-14} m^{-2/3}$. For all the simulations, we take the outage probability as the performance metric, where the SNR threshold $\gamma_{th} = 5$ d B. We use an even distributed variable as the Gamma-Gamma CDF result, and then utilize the inverse function of the Gamma-Gamma CDF to simulate the gamma-gamma distributed variable.

Figs. 2 and 3 show the outage probability for the proposed MIMO FSO system with GSMuS. In Fig. 2, $\bar{\gamma} = 0 \sim 20$ dB, and SISO, MIMO ($M = 2, N = 2$ and $M = 2, N = 3$) for RC and TLS are considered, respectively. We provide both the analytical results and the simulation for outage probability performance. As we can see from the figure, the curves match well in the high SNR regime, which proves the correctness of our analysis. In the low SNR regime, the analytical results deviate from the simulation obviously. It is caused by the approximation method in Lemma 1, where only the first two terms in the power series expansion of the modified Bessel function are considered to simplify the gamma-gamma distribution. And the operation in Lemma 2 amplifies the error. A asymptotic analysis which is directly correlated with the diversity order is presented as well. However, in Fig. 3, we evaluate the outage probability versus different K (the number of users), where $\bar{\gamma} = 10$ dB, and SISO, MIMO ($M = 2, N = 1, k = 1, 2$ and $M = 2, N = 2, k = 1, 2$) for RC and TLS are considered, respectively. With the increase of K, the logarithmic value of outage probability decreases linearly, which indicates the linear correlation between diversity order and K.

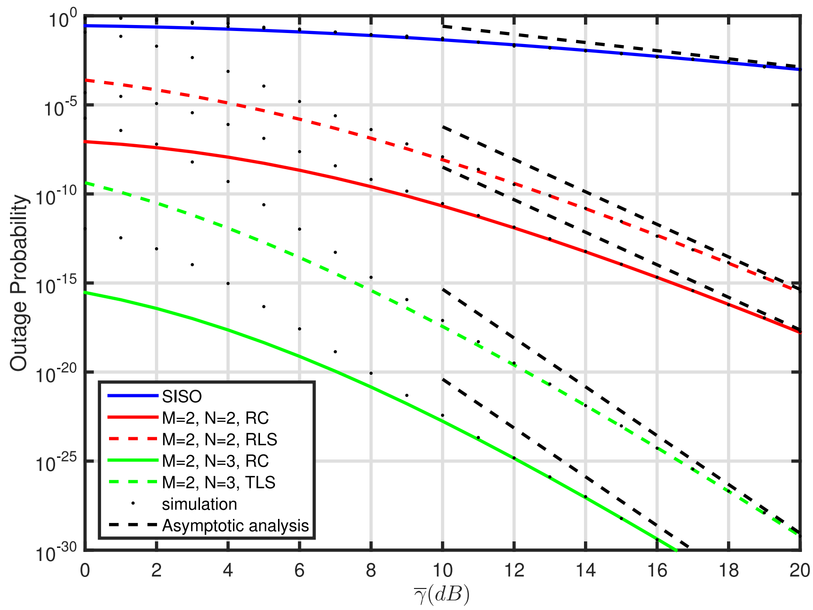

Fig. 2. Outage probability of the MIMO FSO system versus different $\bar{\gamma}$ for different number of apertures.

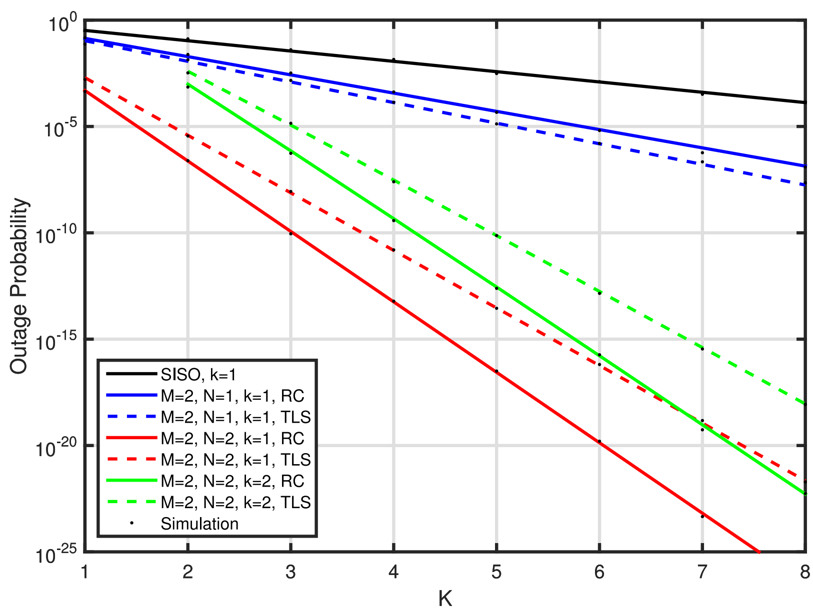

Fig. 3. Outage probability of the MIMO FSO system versus different K.

6. Conclusion

In this work, we have analyzed the impact of GSMuS on the performance of the MIMO FSO system. We have derived the closed-form expression of the outage probability and the diversity order for this system over gamma-gamma fading channel. The numerical results validate the correctness of our analysis. Specially, our analysis shows that the MIMO combining diversity and the multiuser selective diversity are independent, and can be achieved simultaneously for FSO communication system.

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