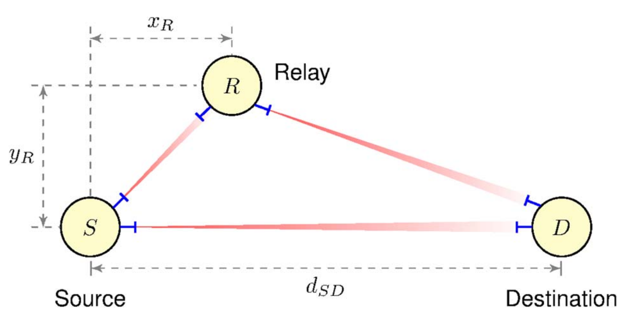
IEEE Photonics Journal
An IEEE Photonics Society Publication

Open Access

Ergodic Capacity Analysis of Decode-and-Forward Relay-Assisted FSO Systems Over Alpha–Mu Fading Channels Considering Pointing Errors

Volume 8, Number 1, February 2016

Rubén Boluda-Ruiz
Antonio García-Zambrana
Beatriz Castillo-Vázquez
Carmen Castillo-Vázquez



DOI: 10.1109/JPHOT.2015.2513203
1943-0655 © 2015 IEEE

Ergodic Capacity Analysis of Decode-and-Forward Relay-Assisted FSO Systems Over Alpha–Mu Fading Channels Considering Pointing Errors

Rubén Boluda-Ruiz,[1] Antonio García-Zambrana,[1] Beatriz Castillo-Vázquez,[1] and Carmen Castillo-Vázquez[2]

[1]Department of Communications Engineering, University of Málaga, 29071 Málaga, Spain
[2]Department of Mathematical Analysis, Statistics and Operations Research and Applied Mathematics, University of Málaga, 29071 Málaga, Spain

DOI: 10.1109/JPHOT.2015.2513203

Manuscript received November 10, 2015; revised December 21, 2015; accepted December 24, 2015. Date of publication December 29, 2015; date of current version January 13, 2016. This work was supported by the Spanish MINECO Project TEC2012-32606. This paper was presented in part at the 2015 Fall IEEE 82nd Vehicular Technology Conference, Boston, MA, USA, September 2015. Corresponding author: R. Boluda-Ruiz (e-mail: rbr@ic.uma.es).

Abstract: The ergodic capacity of decode-and-forward (DF) relay-assisted free-space optical (FSO) communication systems when line of sight is available is analyzed over gamma–gamma fading channels with pointing errors. A novel closed-form approximate ergodic capacity expression is obtained in terms of the H-Fox function for a three-way FSO communications system when the $\alpha - \mu$ distribution to efficiently approximate the probability density function (PDF) of the sum of gamma–gamma with pointing-error variates is considered. Moreover, we present novel asymptotic expressions at high signal-to-noise ratio (SNR), as well as low SNR for the ergodic capacity of DF relay-assisted FSO systems. The main contribution in this paper lies in an in-depth analysis about the impact of pointing errors on the ergodic capacity for cooperative FSO systems. In order to maintain the same performance in terms of capacity, it is corroborated that the presence of pointing errors requires an increase in SNR, which is related to the fraction of the collected power at the receive aperture, i.e., A_0. Simulation results are further demonstrated to confirm the accuracy and usefulness of the derived results.

Index Terms: Free-space optical (FSO), cooperative communications, decode-and-forward (DF), ergodic capacity.

1. Introduction

Free-space optical (FSO) communication systems using intensity modulation and direct detection (IM/DD) can provide high-speed links for a number of applications. Unlimited bandwidth, unlicensed spectrum, excellent security, and low cost are some of the most notable characteristics of FSO systems [1]. However, atmospheric turbulence produces fluctuations in the irradiance of the received optical beam, which is known as *atmospheric scintillation*, severely degrading the link performance of FSO communication systems [2]. Additionally, thermal expansion, dynamic wind loads, as well as weak earthquakes result in the building sway that causes vibrations in the transmitter beam, leading to a misalignment between the transmitter and receiver known as pointing error.

In the last decade, several works have investigated the adoption of cooperative communications in the context of FSO communication systems in order to solve these inconveniences [3]–[7], which have proved that cooperative communications are quite a efficient technique to satisfy the typical bit error-rate (BER) targets for FSO applications without much increase in hardware. The performance of cooperative FSO communication systems over atmospheric turbulence channels with pointing errors has been extensively analyzed in terms of the BER and outage probability. Lately, there have been several studies on ergodic capacity for cooperative FSO systems which have demonstrated that cooperative communications are also able to increase the channel capacity [8]–[12]. Ergodic capacity, also known as average channel capacity, defines the maximum data rate that can be sent over the channel with asymptotically small error probability, without any delay or complexity constraints [13], [14]. Obtained results in ergodic capacity are also applicable to slowly varying (block-fading) channels, i.e., FSO links, when the message is long enough to reveal long-term ergodic properties of the turbulence process [15]. This fact was taken into account in [16] in the context of FSO communication systems. In [8], the end-to-end ergodic capacity of dual-hop FSO system employing amplify-and-forward (AF) relaying is evaluated over gamma-gamma fading channels with pointing errors, by approximating the probability density function (PDF) of the end-to-end signal to noise ratio (SNR), by the $\alpha-\mu$ distribution proposed in [17]. The alpha-mu distribution was already used for evaluating the performance of multiple-input/multiple-output (MIMO) FSO systems by the same authors in [18] and [19]. Furthermore, this distribution was recently used for studying the ergodic capacity of MIMO FSO communication systems with equal gain combining (EGC) reception without considering pointing errors [20]. In [9], [10], the capacity performance of dual-hop subcarrier intensity modulation (SIM)-based FSO system with decode-and-forward (DF) and AF relay is evaluated, respectively, over gamma-gamma fading channels with pointing errors. The derived results are obtained in terms of special function known as generalized bivariate Meijers G-function (GBMGF).

However, to the best of the author's knowledge, the study of the ergodic capacity for cooperative FSO systems over alpha-mu fading channels considering pointing errors wherein the line of sight is taken into account has not been studied yet. Motivated by this issue, the purpose of this paper is to study the ergodic capacity for the bit-detect-and-forward (BDF) cooperative protocol presented in [21]. As concluded in [21], the BDF cooperative protocol is able to achieve a higher diversity order, strongly dependent not only on the relay location but also on pointing errors. In this paper, an approximate closed-form ergodic capacity expression is obtained in terms of the H-Fox function for a 3-way FSO communications system when the irradiance of the transmitted optical beam is susceptible to moderate-to-strong turbulence conditions, following a gamma-gamma distribution of parameters a and b, or pointing error effects, following a misalignment fading model where the effect of beam width, detector size and jitter variance is considered. Here, as proposed in [17], the $\alpha-\mu$ distribution is used for deriving a closed-form expression for the distribution of the sum of gamma-gamma with pointing errors variates. Moreover, an asymptotic analysis for the ergodic capacity is carried out in order to obtain asymptotic expressions at high signal-to-noise ratio (SNR) as well as at low SNR. It is demonstrated that DF relay-assisted FSO systems are able to achieve a greater and robust capacity when line of sight is available compared to a direct transmission without cooperative communication, as well as the two-transmitter case. In addition to this, a numerical observation of the $\alpha-\mu$ parameters is included in order to evaluate how these parameters are affected by atmospheric turbulence and pointing errors. The main contribution in this paper lies in an in-depth analysis about the impact of pointing errors on the ergodic capacity for cooperative FSO systems. In order to maintain the same performance in terms of capacity, it is corroborated that the presence of pointing errors requires an increase in SNR, which is related to the fraction of the collected power at the receive aperture, i.e., A_0 and, hence, not being dependent on the relay location and atmospheric turbulence conditions.

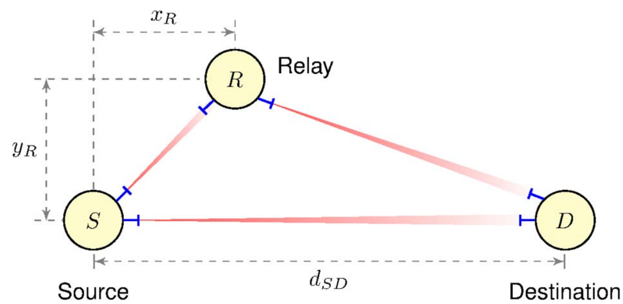


Fig. 1. Block diagram of the considered three-way FSO system.

2. System and Channel Model

We adopt a three-node cooperative FSO system based on three separate FSO links, as shown in Fig. 1, assuming laser sources intensity-modulated and ideal non-coherent (direct-detection) receivers. The cooperative strategy works in two phases. In the first phase, the source node S sends its own data to the relay node R and the destination node D. In the second phase, the relay node R sends the received data from the source node S in the first phase to the destination node D. In this fashion, the relay node R detects each code bit to "0" or "1" and sends the bit with the new power to the destination node D regardless of these bits are detected correctly or incorrectly. For each link of this cooperative FSO communications system, the instantaneous current $y_m(t)$ in the receiving photodetector corresponding to the information signal transmitted from each laser can be written as

$$y_m(t) = \eta i_m(t) x_m(t) + z_m(t) \tag{1}$$

where η is the detector responsivity, assumed hereinafter to be the unity; $X \triangleq x_m(t)$ represents the optical power supplied by the source; $I_m \triangleq i_m(t)$ is the irradiance through the optical channel between the laser and the receive aperture; and $Z_m \triangleq z_m(t)$ is additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_m^2 = N_0/2$, i.e., $Z_m \sim N(0, N_0/2)$, which is independent of the on/off state of the received bit. We use X, Y_m, I_m, and Z_m to denote random variables and $x_m(t)$, $y_m(t)$, $i_m(t)$, and $z_m(t)$ their corresponding realizations. Here, X is either 0 or $2P$, where P is the average transmitted optical power from each node. The received instantaneous electrical SNR can be written as

$$\gamma = \frac{4P^2 I_m^2}{N_0} = 4\gamma_0 I_m^2 \tag{2}$$

where γ_0 represents the received electrical SNR in absence of turbulence. The irradiance I_m is considered to be a product of three factors i.e., $I_m = L_m I_m^a I_m^p$, where L_m is the deterministic propagation loss, I_m^a is the attenuation due to atmospheric turbulence, and I_m^p the attenuation due to geometric spread and pointing errors. L_m is determined by the exponential Beers-Lambert law as $L_m = e^{-\Phi d}$, where d is the link distance, and Φ is the atmospheric attenuation coefficient. It is given by $\Phi = (3.91/V(\text{km}))(\lambda(\text{nm})/550)^{-q}$, where V is the visibility in kilometers, λ is the wavelength in nanometers, and q is the size distribution of the scattering particles, where $q = 1.3$ for average visibility ($6\text{ km} < V < 50\text{ km}$), and $q = 0.16V + 0.34$ for haze visibility ($1\text{ km} < V < 6\text{ km}$). To consider a wide range of turbulence conditions, the gamma-gamma turbulence model proposed in [2] is assumed here. Regarding the impact of pointing errors, we use the general model of misalignment fading given in [22], wherein the effect of beam width, detector size and jitter variance is considered. In this way, assuming a Gaussian spatial intensity profile of beam waist radius, ω_z, on the receiver plane at distance z from the transmitter and a circular receive aperture of radius r, $\varphi = \omega_{z_{eq}}/2\sigma_s$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter) at the receiver,

$\omega_{z_{eq}}^2 = \omega_z^2 \sqrt{\pi}\mathrm{erf}(v)/2v\exp(-v^2)$, $v = \sqrt{\pi}r/\sqrt{2}\omega_z$, $A_0 = [\mathrm{erf}(v)]^2$, and $\mathrm{erf}(\cdot)$ is the error function [23, eqn. (8.250)]. A closed-form expression of the combined PDF of I_m was derived in [24] as

$$f_{I_m}(i) = \frac{\varphi_m^2 i^{-1} G_{1,3}^{3,0}\left(\frac{a_m b_m}{A_0 L_m} i \middle| \begin{matrix} \varphi_m^2 + 1 \\ \varphi_m^2, a_m, b_m \end{matrix}\right)}{\Gamma(a_m)\Gamma(b_m)} \tag{3}$$

where $G_{p,q}^{m,n}[\cdot]$ is the Meijer's G-function [23, eqn. (9.301)]. It must be mentioned that the parameters a and b can be directly linked to physical parameters through the following expressions [25]:

$$a = \left[\exp\left(\frac{0.49\sigma_R^2}{\left(1 + 1.11\sigma_R^{\frac{12}{5}}\right)^{\frac{7}{6}}}\right) - 1\right]^{-1} \tag{4a}$$

$$b = \left[\exp\left(\frac{0.51\sigma_R^2}{\left(1 + 0.69\sigma_R^{\frac{12}{5}}\right)^{\frac{5}{6}}}\right) - 1\right]^{-1} \tag{4b}$$

where $\sigma_R^2 = 1.23 C_n^2 \kappa^{7/6} d^{11/6}$ is the Rytov variance, which is a measure of optical turbulence strength. Here, $\kappa = 2\pi/\lambda$ is the optical wave number and d is the link distance in meters. C_n^2 is the refractive index structure parameter, which is the most significant parameter that determines the turbulence strength. Clearly, C_n^2 not only depends on the altitude but on the local conditions such as terrain type, geographic location, cloud cover, and time of day [26], as well. In addition, C_n^2 is typically within the range $10^{-13} - 10^{-17}$ $m^{-2/3}$ [2]. It must be emphasized that parameters a and b cannot be arbitrarily chosen in FSO applications, being related through the Rytov variance. It can be shown that the relationship $a > b$ always holds, and the parameter b is lower bounded above 1 as the Rytov variance approaches ∞. It is mentioned that the fading coefficient I_m for the paths source-destination (S-D), source-relay (S-R) and relay-destination (R-D) are indicated by I_{SD}, I_{SR} and I_{RD}, respectively. Here, it is further assumed that all coefficients are statistically independent.

3. Ergodic Capacity Analysis

First, we analyze the ergodic capacity of the cooperative FSO system under study. Here, two cases can be considered to evaluate the ergodic capacity corresponding to the BDF relaying scheme, depending on the bit is detected correctly or incorrectly at the relay node. Assuming a statistical channel model as follows:

$$Y_{\mathrm{BDF}} = \frac{1}{2} X I_{\mathrm{SD}} + X^* I_{\mathrm{RD}} + Z_{\mathrm{SD}} + Z_{\mathrm{RD}}, \qquad X \in \{0, 2P\}, \quad Z_{\mathrm{SD}}, Z_{\mathrm{RD}} \sim N\left(0, \frac{N_0}{2}\right) \tag{5}$$

where X^* represents the random variable corresponding to the information detected at node R and, hence, $X^* = X$ when the bit has been detected correctly at node R and $X^* = d_E - X$ when the bit has been detected incorrectly. The division by 2 in (5) is considered to maintain the average optical power in the air at a constant level of P, being that it is transmitted by each node an average optical power P. In this manner, the source node transmits by each laser an average optical power $P/2$ as well as the relay node transmits an average optical power P because only one laser is available. Hence, the ergodic capacity corresponding to the BDF cooperative protocol is given by

$$C_{\mathrm{BDF}} = C_0 \cdot \left(1 - P_b^{\mathrm{SR}}\right) + C_1 \cdot P_b^{\mathrm{SR}} = C_0 + (C_1 - C_0) \cdot P_b^{\mathrm{SR}} \tag{6}$$

where P_b^{SR} denotes the BER corresponding to the S-R link and, C_0 and C_1 are the ergodic capacity when the bit is detected correctly and incorrectly at node R, respectively. The resulting electrical SNR when $X^* = X$, can be defined as $\gamma_{BDF}^0 = (\gamma_0/2)(I_{SD} + 2I_{RD})^2$ and, when $X^* = d_E - X$, the resulting received electrical SNR can be defined as $\gamma_{BDF}^1 = (\gamma_0/2)(I_{SD} - 2I_{RD})^2$ [21]. It must be noted that the ergodic capacity corresponding to the BDF relaying scheme in (6) can be accurately approximated as follows $C_{DF} \approx C_0$ as SNR increases, since the term P_b^{SR} tends to zero as SNR increases. This approximation has been numerically corroborated by Monte Carlo simulation and, it will be checked in following sections. Hence, the ergodic capacity of BDF cooperative protocol can be written as

$$C_{BDF} \approx \frac{B}{2\ln(2)} \int_0^\infty \ln\left(1 + \frac{\gamma_0}{2} i^2\right) f_{I_T}(i)\, di \tag{7}$$

where B represents the channel bandwidth, $\ln(\cdot)$ represents the natural logarithm [23, eqn. (1.511)], and $I_T = I_{SD} + 2I_{RD}$. It should be noted that the factor 1/2 in (16) is because of the source node S is assumed to operate in half-duplex mode. It should be also mentioned that obtaining the corresponding PDF of I_T is remarkably tedious and complicated. In order to solve the integral in (7), we approximate the PDF $f_{I_T}(i)$ by the $\alpha-\mu$ PDF as proposed in [17]

$$f_{I_T}(i) \approx \frac{\alpha \mu^\mu i^{\alpha\mu - 1}}{\hat{i}^{\alpha\mu}\Gamma(\mu)} \exp\left(-\mu \frac{i^\alpha}{\hat{i}^\alpha}\right). \tag{8}$$

The $\alpha-\mu$ distribution is characterized by the α and μ parameters, as well as by the α-root mean value $\hat{i}$ of the random variable I_T. The use of this approximate PDF is suitable in order to study the ergodic capacity of cooperative FSO systems due to the fact that this PDF contains information regarding the mean, the variance, and the fourth moment of I_T. These parameters are obtained as the solution of the system of transcendental equations derived in [17, eqn. (24) and (25)]. The required solution for the system of transcendental equations has been numerically solved in an efficient manner. The parameter $\hat{i}$ can be obtained as $\hat{i} = \mu^{1/\alpha}\Gamma(\mu)\mathbb{E}[I_T]/\Gamma(\mu + 1/\alpha)$, where $\mathbb{E}[\cdot]$ denotes the expectation operator. Therefore, the nth moment of I_T can be determined as follows:

$$\mathbb{E}[I_T^n] = \int_0^\infty \int_0^\infty (i_1 + 2i_2)^n f_{I_{SD}}(i_1) f_{I_{RD}}(i_2)\, di_1\, di_2. \tag{9}$$

According to the binomial theorem, it is possible to expand the power $(i_1 + 2i_2)^n$ into a sum and, after performing some straightforward manipulations in (9), we can express $\mathbb{E}[I_T^n]$ as

$$\mathbb{E}[I_T^n] = \sum_{k=0}^{n} \frac{n!2^k}{k!(n-k)!} \int_0^\infty i_1^{n-k} f_{I_{SD}}(i_1)\, di_1 \cdot \int_0^\infty i_2^k f_{I_{RD}}(i_2)\, di_2. \tag{10}$$

Both integrals in (10) can be solved with the help of [27, eqn. (2.24.2.1)], and then, performing some algebraic manipulations, the corresponding closed-form solution for the nth moment of I_T can be written as

$$\mathbb{E}[I_T^n] = \varphi_{SD}^2 \varphi_{RD}^2 \sum_{k=0}^{n} \frac{n!2^k}{k!(n-k)!} \left(\frac{a_{SD} b_{SD}}{A_{SD} L_{SD}}\right)^{k-n} \frac{\Gamma(n-k+a_{SD})\Gamma(n-k+b_{SD})}{\Gamma(a_{SD})\Gamma(b_{SD})(n-k+\varphi_{SD}^2)} \times \left(\frac{a_{RD} b_{RD}}{A_{RD} L_{RD}}\right)^{-k} \frac{\Gamma(k+a_{RD})\Gamma(k+b_{RD})}{\Gamma(a_{RD})\Gamma(b_{RD})(k+\varphi_{RD}^2)}. \tag{11}$$

For $n=1$, we can obtain the mean of I_T, i.e., $\mathbb{E}[I_T]$. Substituting (8) into (7) and, after making the change of variables $i' = i^\alpha/\hat{i}^\alpha$, the ergodic capacity for BDF relaying scheme can be accurately approximated as follows:

$$C_{\text{BDF}} \approx \frac{B\mu^\mu}{2\ln(2)\Gamma(\mu)} \int_0^\infty \ln\left(1 + \frac{\hat{i}\gamma_0}{2} i'^{\frac{2}{\alpha}}\right) \frac{i'^\mu}{i'} \exp(-\mu i')\, di'. \tag{12}$$

The integral in (12) can be solved using [27, eqn. (8.4.6.5)] and [27, eqn. (8.4.3.1)] in order to express the natural logarithm in terms of the Meijer's G-function as $\ln(1+x) = G_{2,2}^{1,2}(x|_{1,0}^{1,1})$ and the exponential in terms of the Meijer's G-function as $e^x = G_{0,1}^{1,0}(-x|_0^-)$, respectively. Afterwards using [28, eqn. (07.34.21.0012.01)], we can obtain the approximate closed-form solution for the ergodic capacity corresponding to the BDF cooperative protocol C_{BDF} as follows:

$$C_{\text{BDF}} \approx \frac{B}{2\ln(2)\Gamma(\mu)} H_{1,3}^{3,2}\left(\frac{\gamma_0\Gamma(\mu)^2\mathbb{E}[I_T]^2}{2\Gamma(\mu+1/\alpha)^2}\middle| \begin{matrix}(1,1),(1,1),(1-\mu,\frac{2}{\alpha})\\(1,1),(0,1)\end{matrix}\right) \tag{13}$$

where $H_{p,q}^{m;n}[\cdot]$ is the H-Fox function [27, eqn. (8.3.1)]. A computer program in Mathematica for the efficient implementation of the H-Fox function is given in [29, App. A]. An asymptotic expression for the ergodic capacity corresponding to the BDF relaying scheme at high SNR can be readily and accurately lower-bounded as in [30, (8) and (9)] as follows:

$$C_{\text{BDF}}^H \doteq \frac{B}{2\ln(2)} \frac{\partial\mathbb{E}\left[\left(\frac{\gamma_0}{2} I_T^2\right)^n\right]}{\partial n}\Bigg|_{n=0} \tag{14}$$

where $\mathbb{E}[((\gamma_0/2)I_T^2)^n]$ denotes the nth moment of instantaneous electrical SNR γ_{BDF}^0. Hence, the ergodic capacity corresponding to the BDF cooperative protocol at high SNR C_{BDF}^H can be asymptotically expressed as follows:

$$C_{\text{BDF}}^H \doteq \frac{B\ln\left(\frac{\gamma_0}{2}\right)}{2\ln(2)} + \frac{B}{\ln(2)}\left(\ln\left(\frac{\Gamma(\mu)\mathbb{E}[I_T]}{\Gamma(\mu+1/\alpha)}\right) + \frac{\psi(\mu)}{\alpha}\right) \tag{15}$$

where $\psi(\cdot)$ is the psi (digamma) function [31, eqn. (6.3.1)]. Next, we study the ergodic capacity corresponding to the direct transmission (DT) without cooperative communication in order to establish the baseline performance. This closed-form expression was obtained in [32], and it is reproduced here for convenience. Assuming channel side information at the receiver, the ergodic capacity C_{DT} can be obtained as

$$C_{\text{DT}} = \frac{B}{2\ln(2)} \int_0^\infty \ln(1 + 4\gamma_0 i^2) f_{I_{\text{SD}}}(i)\, di. \tag{16}$$

The integral in (16) can be solved using [27, eqn. (8.4.6.5)] in order to express the natural logarithm in terms of the Meijer's G-function as in (12), and afterwards using [27, eqn. (2.24.1.1)]. Hence, the closed-form solution for the ergodic capacity corresponding to the direct transmission can be seen in

$$C_{\text{DT}} = \frac{B\varphi_{\text{SD}}^2 2^{a_{\text{SD}}+b_{\text{SD}}-4}}{\pi\ln(2)\Gamma(a_{\text{SD}})\Gamma(b_{\text{SD}})} \times G_{8,4}^{1,8}\left(\left(\frac{8A_{\text{SD}}L_{\text{SD}}}{a_{\text{SD}}b_{\text{SD}}}\right)^2\gamma_0 \middle| \begin{matrix}1,1,\frac{1-a_{\text{SD}}}{2},\frac{2-a_{SD}}{2},\frac{1-b_{\text{SD}}}{2},\frac{2-b_{\text{SD}}}{2},\frac{1-\varphi_{\text{SD}}^2}{2},\frac{2-\varphi_{\text{SD}}^2}{2}\\1,0,-\frac{\varphi_{\text{SD}}^2}{2},\frac{1-\varphi_{\text{SD}}^2}{2}\end{matrix}\right). \tag{17}$$

An asymptotic expression for the ergodic capacity corresponding to the direct transmission at high SNR can be obtained as in (14) as follows:

$$C_{\mathrm{DT}}^{H} \doteq \frac{B}{2\ln(2)} \frac{\partial \mathbb{E}\left[\left(4\gamma_0 I_{\mathrm{SD}}^2\right)^n\right]}{\partial n}\Bigg|_{n=0} \tag{18}$$

where $\mathbb{E}[(4\gamma_0 I_{\mathrm{SD}}^2)^n]$ denotes the nth moment of instantaneous electrical SNR. Performing some algebraic manipulations in (18), the asymptotic closed-form solution for the ergodic capacity corresponding to the direct transmission at high SNR, C_{DT}^{H}, can be accurately estimated as

$$C_{\mathrm{DT}}^{H} \doteq \frac{B\ln(4\gamma_0)}{2\ln(2)} + \frac{B}{\ln(2)}\left(\psi(a_{\mathrm{SD}}) + \psi(b_{\mathrm{SD}}) - \frac{1}{\varphi_{\mathrm{SD}}^2} - \ln\left(\frac{a_{\mathrm{SD}} b_{SD}}{A_{\mathrm{SD}} L_{\mathrm{SD}}}\right)\right). \tag{19}$$

Furthermore, it can be easily shown that the ergodic capacity corresponding to the BDF cooperative protocol at low SNR can be readily and accurately approximated by the first moment because $\ln(1+z) \approx z$ when $|z| \to 0$ [31, eqn. (4.1.24)]. Hence, the ergodic capacity C_{BDF}^{L} at low SNR can be expressed from (7) as

$$C_{\mathrm{BDF}}^{L} \approx \frac{B}{2\ln(2)} \frac{\gamma_0}{2} \mathbb{E}\left[I_T^2\right]. \tag{20}$$

This approximation can be also used for deriving the corresponding asymptotic expression at low SNR for the direct path link.

4. Numerical Results

For the better understanding of the study of the ergodic capacity in cooperative FSO systems when line of sight is taken into account, the ergodic capacity is depicted in Fig. 2 for a source-destination link distance of $d_{\mathrm{SD}} = 3$ km when different relay locations are considered. Different weather conditions are adopted: haze visibility of 4 km with $C_n^2 = 1.7 \times 10^{-14}$ $m^{-2/3}$ and clear visibility of 16 km with $C_n^2 = 8 \times 10^{-14}$ $m^{-2/3}$, corresponding to moderate and strong turbulence, respectively. Here, a and b are calculated from (4) and, a value of $\lambda = 1550$ nm is assumed. Pointing errors are here present assuming values of normalized beam width of $\omega_z/r = \{5, 10\}$ and a value of normalized jitter of $\sigma_s/r = 1$ for each link. It is clear to observe that the obtained ergodic capacity in (13) is very accurate in the entire SNR regime, i.e., from low to high SNR. A relevant improvement in terms of the capacity has been achieved under different turbulence conditions and pointing error effects. Additionally, we also consider the performance analysis for the direct transmission (non-cooperative link S-D) to establish the baseline performance as well as ergodic capacity performance corresponding to the non-cooperative case with two transmitters following the repetition coding scheme as a benchmark of the FSO scenario [11, (11) and (16)]. As expected, the ergodic capacity of the considered cooperative FSO system is strongly dependent not only on the relay location but also on the pointing error effects. Monte Carlo simulations results are also included as a reference [see (6) for BDF cooperative protocol and (16) for direct transmission], confirming the accuracy of the proposed α–μ approximation, and usefulness of the derived results. There is quite a match between simulated and analytical results as well as between simulated and asymptotic results at high SNR as well as at low SNR. This analysis can be extended in order to obtain a point where the asymptotic ergodic capacity at high SNR intersects with the γ_0-axis. This point can be understood as a SNR threshold, i.e., $\gamma_{\mathrm{BDF}}^{\mathrm{th}}$, in which the ergodic capacity is significantly increased. From (15), it is easy to derive the corresponding expression of $\gamma_{\mathrm{BDF}}^{\mathrm{th}}$ in terms of the α–μ parameters, and it is given by

$$\gamma_{\mathrm{BDF}}^{\mathrm{th}}[\mathrm{dB}] = \frac{20}{\ln(10)}\left(\ln\left(\frac{\sqrt{2}\Gamma(\mu + 1/\alpha)}{\mathbb{E}[I_T]\Gamma(\mu)}\right) - \frac{\psi(\mu)}{\alpha}\right). \tag{21}$$

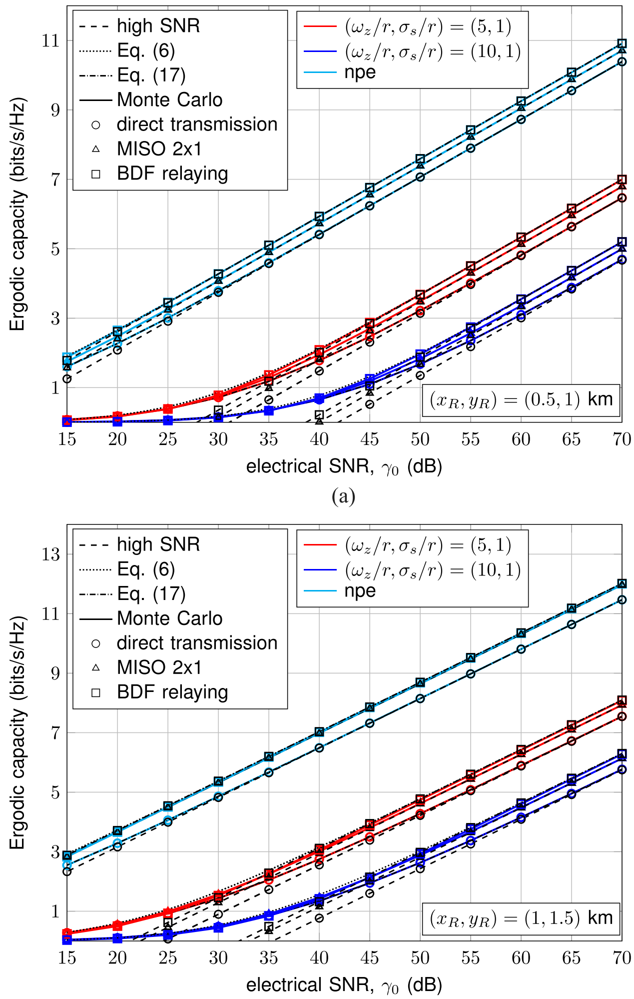


Fig. 2. Ergodic capacity of BDF relaying scheme at high SNR for a source-destination link distance of $d_{\mathrm{SD}} = 3$ km when different weather conditions are assumed. (a) Moderate turbulence. (b) Strong turbulence.

Similar to (23), we can obtain the corresponding SNR threshold, i.e., $\gamma_{\mathrm{DT}}^{\mathrm{th}}$, for the direct transmission without cooperative communication, and it is given by

$$\gamma_{\mathrm{DT}}^{\mathrm{th}}[\mathrm{dB}] = -\frac{20}{\ln(10)}\left(-\ln\left(\frac{a_{\mathrm{SD}} b_{\mathrm{SD}}}{2A_{\mathrm{SD}} L_{\mathrm{SD}}}\right) + \psi(a_{\mathrm{SD}}) + \psi(b_{\mathrm{SD}}) - \frac{1}{\varphi_{\mathrm{SD}}^2}\right). \tag{22}$$

It can be observed from this asymptotic analysis at high SNR that the shift of the ergodic capacity versus SNR is more relevant than the slope of the curve in SNR compared to other performance metric such as BER and outage probability. This shift can be interpreted as an improvement on ergodic capacity. From (21) and (22), we can obtain this improvement or gain, i.e., G[dB], as $G[\mathrm{dB}] = \gamma_{\mathrm{DT}}^{\mathrm{th}}[\mathrm{dB}] - \gamma_{\mathrm{BDF}}^{\mathrm{th}}[\mathrm{dB}]$. It can be seen in Fig. 2 gain values of 3.21 and 3.18 dB for moderate turbulence, as well as gain values of 3.34 and 3.31 dB for strong turbulence, when values of normalized beam width and normalized jitter of $(\omega_z/r, \sigma_s/r) = (5, 1)$ and $(\omega_z/r, \sigma_s/r) = (10, 1)$ are considered, respectively. Now, the error-rate performance analysis in [21] is taken into account, in which was demonstrated that the diversity order gain does not depend on pointing errors when the relation $\varphi^2 > \beta$ is satisfied. Under this desirable

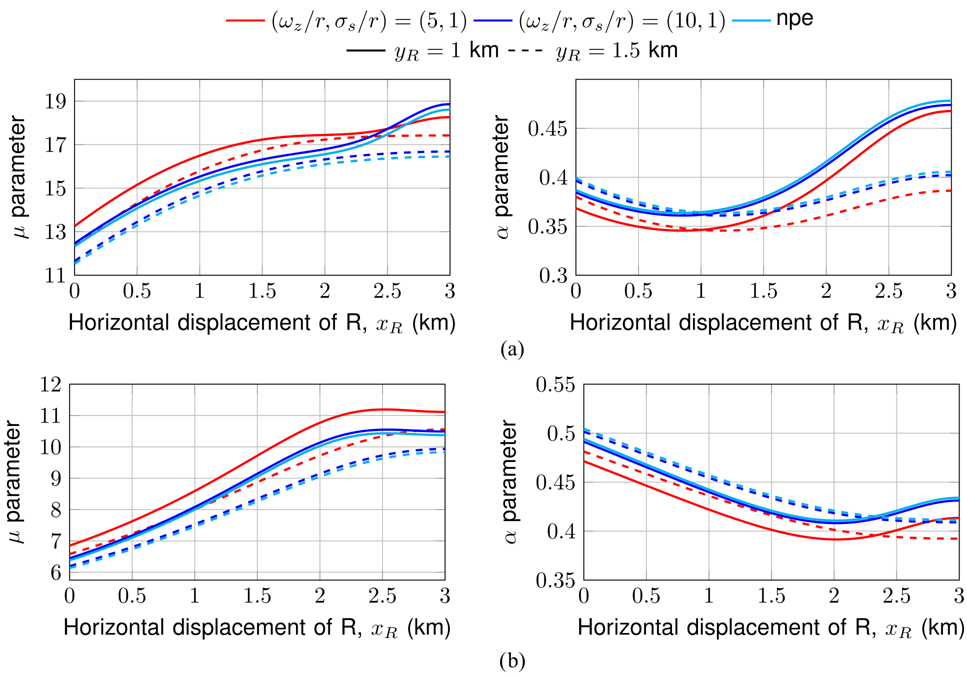


Fig. 3. α–μ parameters as a function of the relay location for a source-destination link distance of $d_{SD} = 3$ km when different weather conditions are assumed. (a) Moderate turbulence. (b) Strong turbulence.

scenario, the effect of misalignment on the ergodic capacity in this cooperative FSO system is analyzed. Knowing that the impact of pointing errors in our analysis can be suppressed by assuming $A_0 \to 1$ and $\varphi^2 \to \infty$ [22], the corresponding gain disadvantage $D_{pe}^{\rm DF}$[dB] relative to this three-way cooperative FSO system without misalignment fading can be derived from (21) as

$$D_{pe}^{\rm BDF}[\mathrm{dB}] = \gamma_{\rm BDF}^{\rm th}[\mathrm{dB}] - \gamma_{\mathrm{BDF}_{npe}}^{\rm th}[\mathrm{dB}]. \tag{23}$$

At this point, it must be noted that the parameters α and μ tend to remain at a constant level as the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation at the receiver increases, i.e., $\varphi^2 \to \infty$. This conclusion has been carefully checked by numerical observation and, hence, the expression in (23) can be approximated as

$$D_{pe}^{\rm BDF}[\mathrm{dB}] \approx \frac{20}{\ln(10)} \ln\left(\frac{\mathbb{E}[I_T^{npe}]}{\mathbb{E}[I_T]}\right) \tag{24}$$

where $\mathbb{E}[I_T^{npe}] = L_{\rm SD} + 2L_{\rm RD}$. In order to validate the previous statement, the parameters α and μ are illustrated in Fig. 3 as a function of the horizontal displacement of the relay node R when different relay locations $y_R = \{1, 1.5\}$ km are assumed. The depicted curves in Fig. 3 have been obtained by using a numerical approach due to the fact that finding the relation between $\alpha\mu$ and a–b parameters can be time-consuming and are technically difficult to perform. Note that the obtained results in Fig. 3 both α and μ when pointing errors are suppressed can be considered negligible as the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation at the receiver increases. Therefore, the expression in (26) can be simplified under the assumption that all links are affected by the same values of normalized beam width and normalized jitter as follows:

$$D_{pe}^{\rm BDF}[\mathrm{dB}] \approx \frac{20}{\ln(10)} \ln\left(\frac{1+\varphi^2}{A_0\varphi^2}\right). \tag{25}$$

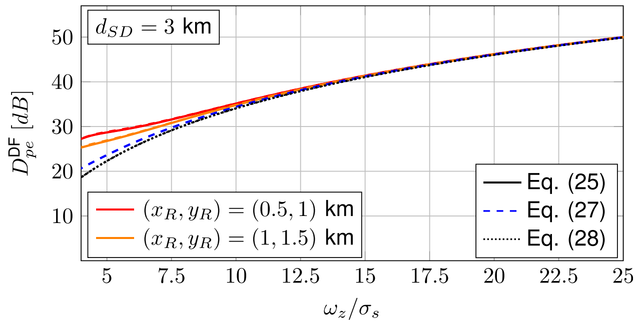


Fig. 4. Gain disadvantage, D_{pe}^{BDF}[dB] for a source-destination link distance of $d_{\mathrm{SD}} = 3$ km.

The expression in (25) is the gain disadvantage corresponding to the direct transmission and, hence, this can be only used when the same values of normalized beam width and normalized jitter are assumed for each link. Moreover, it can be easily deduced that this gain disadvantage depends neither on the relay location nor on the atmospheric turbulence conditions as φ^2 increases. The expression in (25) can be simplified even further as $\varphi^2 \to \infty$, obtaining an expression only dependent on the value of normalized beam width, i.e., A_0. Hence, the gain disadvantage corresponding to the BDF cooperative protocol can be accurately approximated by

$$D_{pe}^{\mathrm{BDF}}[\mathrm{dB}] \approx -\frac{20\ln(A_0)}{\ln(10)}. \tag{26}$$

The gain disadvantage D_{pe}^{BDF}[dB] is depicted in Fig. 4 as a function of the ratio between ω_z and σ_s. It can be observed that there is a perfect match between the exact results and obtained results for greater values of ω_z/σ_s than 7 by using the approximate expression for the gain disadvantage of BDF relaying. According to the expression in (26), it can be seen in Fig. 2 gain disadvantages of 22.3 and 34.07 dB when values of normalized beam width and normalized jitter of $(\omega_z/r, \sigma_s/r) = (5, 1)$ and $(\omega_z/r, \sigma_s/r) = (10, 1)$ are considered, respectively.

5. Conclusion

The ergodic capacity of BDF cooperative protocol has been analyzed over gamma-gamma fading channels with pointing errors when line of sight is available. A novel closed-form approximate ergodic capacity expression has been obtained in terms of the H-Fox function for a three-way FSO communication system when the $\alpha-\mu$ distribution to efficiently approximate the PDF of the sum of gamma-gamma with pointing errors variates is considered. Simple asymptotic expressions at high SNR as well as low SNR for the ergodic capacity of BDF cooperative protocol have been obtained providing a perfect match between simulated and analytical results. It can be concluded that cooperative protocols such as BDF relaying are able to achieve a greater ergodic capacity than a direct transmission without cooperative communication as well as the non-cooperative case with two transmitters following the repetition coding scheme for specific relay locations [11]. In addition, it is demonstrated that the ergodic capacity is strongly dependent on the relay location as well as pointing error effects. Finally, the impact of the pointing errors on the ergodic capacity is deeply analyzed, which corroborates that the presence of pointing errors requires an increase in SNR in order to maintain the same performance in terms of capacity. This increase is related to the parameter A_0.

References

[1] M. A. Khalighi and M. Uysal, “Survey on free space optical communication: A communication theory perspective,” *IEEE Commun. Surveys Tuts.*, vol. 16, no. 4, pp. 2231–2258, 4th Quart. 2014.
[2] L. Andrews, R. Phillips, and C. Hopen, *Laser Beam Scintillation With Applications*, vol. 99. Washington, DC, USA: SPIE, 2001.

[3] M. Karimi and M. Nasiri-Kenari, "BER analysis of cooperative systems in free-space optical networks," *J. Lightw. Technol.*, vol. 27, no. 24, pp. 5639–5647, Dec. 15, 2009.
[4] M. Bhatnagar, "Performance analysis of decode-and-forward relaying in gamma-gamma fading channels," *IEEE Photon. Technol. Lett.*, vol. 24, no. 7, pp. 545–547, Apr. 2012.
[5] M. R. Bhatnagar, "Average BER analysis of differential modulation in DF cooperative communication system over gamma-gamma fading FSO links," *IEEE Commun. Lett.*, vol. 16, no. 8, pp. 1228–1231, Aug. 2012.
[6] C. del Castillo-Vazquez, R. Boluda-Ruiz, B. del Castillo-Vazquez, and A. Garcia-Zambrana, "Outage performance of DF relay-assisted FSO communications using time-diversity," *IEEE Photon. Technol. Lett.*, vol. 27, no. 11, pp. 1149–1152, Jun. 2015.
[7] R. Boluda-Ruiz, A. García-Zambrana, B. Castillo-Vázquez, and C. Castillo-Vázquez, "Impact of relay placement on diversity order in adaptive selective DF relay-assisted FSO communications," *Opt. Exp.*, vol. 23, no. 3, pp. 2600–2617, Feb. 2015.
[8] K. P. Peppas, A. N. Stassinakis, H. E. Nistazakis, and G. S. Tombras, "Capacity analysis of dual amplify-and-forward relayed free-space optical communication systems over turbulence channels with pointing errors," *J. Opt. Commun. Netw.*, vol. 5, no. 9, pp. 1032–1042, Sep. 2013.
[9] M. Aggarwal, P. Garg, and P. Puri, "Ergodic capacity of SIM based DF relayed optical wireless communication systems," *IEEE Photon. Technol. Lett.*, vol. 27, no. 10, pp. 1104–1107, May 2015.
[10] M. Aggarwal, P. Garg, and P. Puri, "Exact capacity of amplify-and-forward relayed optical wireless communication systems," *IEEE Photon. Technol. Lett.*, vol. 27, no. 8, pp. 903–906, Apr. 2015.
[11] R. Boluda-Ruiz, A. García-Zambrana, B. Castillo-Vázquez, and C. Castillo-Vázquez, "On the capacity of MISO FSO systems over gamma-gamma and misalignment fading channels," *Opt. Exp.*, vol. 23, no. 17, pp. 22371–22385, Aug. 2015.
[12] I. S. Ansari, F. Yilmaz, and M.-S. Alouini, "Performance analysis of free-space optical links over Malaga (M) turbulence channels with pointing errors," *IEEE Trans. Wireless Commun.*, DOI: 10.1109/TWC.2015.2467386, to be published.
[13] M. K. Simon and M.-S. Alouini, *Digital Communications Over Fading Channels*, 2nd ed. Piscataway, NJ, USA: Wiley-IEEE Press, 2005.
[14] A. Goldsmith, *Wireless Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2005.
[15] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: Information-theoretic and communications aspects," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2619–2692, Oct. 1998.
[16] S. Z. Denic, I. Djordjevic, J. Anguita, B. Vasic, and M. A. Neifeld, "Information theoretic limits for free-space optical channels with and without memory," *IEEE/OSA J. Lightw. Technol.*, vol. 26, no. 19, pp. 3376–3384, Oct. 2008.
[17] M. D. Yacoub, "The $\alpha - \mu$ distribution: A general fading distribution," in *Proc. IEEE Int. Symp. Pers., Indoor Mobile Radio Commun.*, 2002, vol. 2, pp. 629–633.
[18] K. P. Peppas, "A simple, accurate approximation to the sum of gamma-gamma variates and applications in MIMO free-space optical systems," *IEEE Photon. Technol. Lett.*, vol. 23, no. 13, pp. 839–841, Jul. 2011.
[19] K. P. Peppas, F. Lazarakis, A. Alexandridis, and K. Dangakis, "Simple, accurate formula for the average bit error probability of multiple-input multiple-output free-space optical links over negative exponential turbulence channels," *Opt. Lett.*, vol. 37, no. 15, pp. 3243–3245, Aug. 2012.
[20] J. Zhang, L. Dai, Y. Han, Y. Zhang, and Z. Wang, "On the ergodic capacity of MIMO free-space optical systems over turbulence channels," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 9, pp. 1925–1934, Sep. 2015.
[21] A. Garcia-Zambrana, C. Castillo-Vazquez, B. Castillo-Vazquez, and R. Boluda-Ruiz, "Bit detect and forward relaying for FSO links using equal gain combining over gamma-gamma atmospheric turbulence channels with pointing errors," *Opt. Exp.*, vol. 20, no. 15, pp. 16394–16409, Jul. 2012.
[22] A. A. Farid and S. Hranilovic, "Outage capacity optimization for free-space optical links with pointing errors," *J. Lightw. Technol.*, vol. 25, no. 7, pp. 1702–1710, Jul. 2007.
[23] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 7th ed. New York, NY, USA: Academic, 2007.
[24] H. G. Sandalidis, T. A. Tsiftsis, and G. K. Karagiannidis, "Optical wireless communications with heterodyne detection over turbulence channels with pointing errors," *J. Lightw. Technol.*, vol. 27, no. 20, pp. 4440–4445, Oct. 2009.
[25] M. A. Al-Habash, L. C. Andrews, and R. L. Phillips, "Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media," *Opt. Eng.*, vol. 40, no. 8, pp. 1554–1562, Aug. 2001.
[26] A. K. Majumdar and J. C. Ricklin, *Free-Space Laser Communications: Principles and Advances*, vol. 2. Berlin, Germany: Springer Science + Business Media, 2010.
[27] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series Volume 3: More Special Functions*, vol. 3. New York, NY, USA: Gordon and Breach Science, 1999.
[28] "The Wolfram functions site," Wolfram Res. Inc., Champaign, IL, USA. [Online]. Available: http://functions.wolfram.com.
[29] F. Yilmaz and M.-S. Alouini, "Product of the powers of generalized Nakagami-m variates and performance of cascaded fading channels," in *Proc. IEEE GLOBECOM*, 2009, pp. 1–8.
[30] F. Yilmaz and M.-S. Alouini, "Novel asymptotic results on the high-order statistics of the channel capacity over generalized fading channels," in *Proc. IEEE 13th Int. Workshop SPAWC*, 2012, pp. 389–393.
[31] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, 9th ed. New York, NY, USA: Dover, 1970.
[32] C. Liu, Y. Yao, Y. Sun, and X. Zhao, "Analysis of average capacity for free-space optical links with pointing errors over gamma-gamma turbulence channels," *Chin. Opt. Lett.*, vol. 8, no. 6, pp. 537–540, 2010.