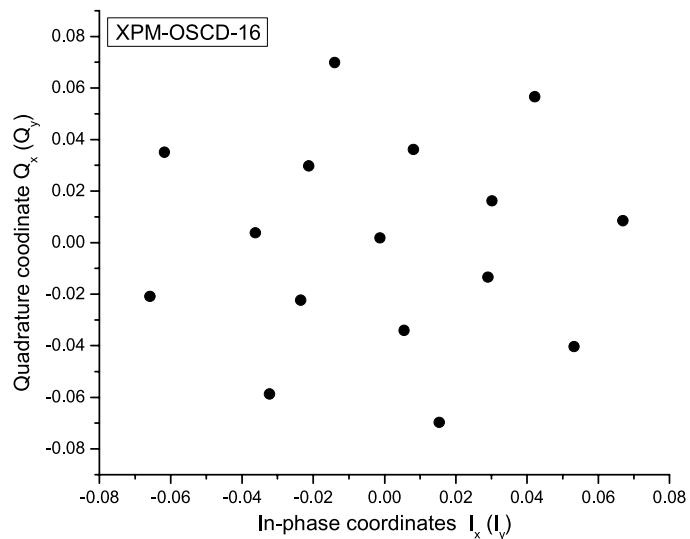


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Tao Liu, Student Member, IEEE  
Ivan B. Djordjevic, Senior Member, IEEE



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# Signal Constellation Design for Cross-Phase Modulation Dominated Channels

Tao Liu, *Student Member, IEEE*, and  
Ivan B. Djordjevic, *Senior Member, IEEE*

Department of Electrical and Computer Engineering, The University of Arizona,  
Tucson, AZ 85721 USA

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**Abstract:** In this paper, we introduce an optimal signal constellation design algorithm suitable for wavelength-division-multiplexed systems dominated by cross-phase modulation (XPM), which is called XPM-OSCD. Additionally, a corresponding demodulation scheme is also proposed in order to better compensate for the nonlinear phase errors. In this scheme, the cumulative log-likelihood function is used as the optimization criterion. The signal constellations obtained by this algorithm significantly outperform conventional quadrature amplitude modulation (QAM).

**Index Terms:** Microwave photonics communications, wavelength-division multiplexing (WDM), optimal signal constellation, demodulator, cross-phase modulation, block-interleaved coded modulation (BICM).

## 1. Introduction

Wavelength-division-multiplexed (WDM) systems have been extensively studied in currently existing literature [1], [2]. As the launch power increases, the nonlinear signal and noise interaction becomes relevant, and in WDM systems, the nonlinear interaction between WDM channels is an important factor in performance degradation. For optical fibers with nonzero chromatic dispersion coefficient, the interchannel nonlinearities between WDM channels are typically due to cross-phase modulation (XPM) arising from the Kerr effect. Different approaches to signal constellation design have been investigated in order to deal with degradation caused by nonlinearities. Given the complexity of the problem, typically the signal constellation design for the self-phase modulation (SPM) dominated scenario has been studied [3]–[6]. Unfortunately, not many research studies have focused on the signal constellation design for the XPM dominated channel, which is the subject of this paper.

In this paper, we first describe a channel model suitable for study of coherent detection WDM systems dominated by the residual nonlinear phase noise introduced by XPM, then a new signal constellation design, obtained by generalization of [7], but now applicable to XPM dominated channel, is also proposed. We define the cumulative log-likelihood function and use it as an optimization criterion instead of using the Euclidian distance as optimization criterion used in [7],

which is optimum only for additive white Gaussian noise (AWGN) channel and has been verified by experiments [8]–[10]. The optimum source distribution is obtained by maximizing the mutual information, based on the Arimoto-Blahut algorithm [11]. Since the proposed signal constellation design algorithm is optimum for the XPM dominated channel, it has been named here as XPM-based optimum signal constellation design (XPM-OSCD) algorithm. Moreover, a collaborative demodulation scheme suitable for WDM systems employing XPM-OSCD constellation is also proposed in this paper. Monte Carlo simulations indicate that the WDM lightwave system based on XPM-OSCD significantly outperforms the corresponding LDPC-coded quadrature amplitude modulation (QAM) by increasing the transmission length by 600 km. We demonstrate by simulations that the proposed demodulation scheme further increases the transmission length by more than 2000 km for 16-ary constellation.

This paper is organized as follows. Section 2 introduces the cross phase modulation (XPM) dominated channel model. Section 3 describes the new signal constellation design (OSCD) suitable for XPM dominated WDM system. Section 4 introduces the collaborative demodulation scheme. The simulation results provided in Section 5 demonstrate the superiority of the proposed LDPC-coded XPM-OSCDs over existing constellations. Some important concluding remarks are provided in Section 6.

## 2. XPM Dominated Channel Model

The XPM-induced channel impairments have been intensively studied in optical fiber transmission, such as in [12]. There are several theoretical XPM dominated channel models, which were verified by both simulations and experiments [13]–[16]. Our channel model is targeting the worst-case scenario, and it is based on the pump-probe model with discrete amplification for the finite number of fiber spans. Meanwhile, the purpose of this channel model is to explore the signal constellation design method for the XPM dominated channels and to analyze how much improvement we can get by only optimizing the signal constellation sets. When the optical signal is periodically amplified by EDFA, the phase error is unavoidably added to the optical signal, which can be simulated by the pump-probe model, and accumulated as the number spans increases. For convenience, we consider the discrete memoryless channel model, which is introduced in [17] and can be described as follows:

$$Y = (X + Z)e^{-j\Phi_{XPM}} \quad (1)$$

where  $X \in \mathcal{X}$  is the channel input,  $Z$  is the total additive noise, and  $Y$  is the channel observation. (In (1),  $j$  denotes the imaginary unit.) For single channel transmission, in each fiber span of length  $L$ , the self-phase modulation phase shift  $\Phi_{SPM}$  is given by

$$\Phi_{SPM} = \int_0^L \gamma P(z) dz = \gamma L_{eff} P \quad (2)$$

where  $P$  is the launch power, and  $\gamma$  is the nonlinear Kerr-parameter. For a fiber span length of  $L$  with attenuation coefficient of  $\alpha$ , the power evolution is described as  $P(z) = Pe^{-\alpha z}$ , and the effective length is defined as

$$L_{eff} = \frac{1 - e^{-\alpha L}}{\alpha}. \quad (3)$$

On the other hand, in WDM systems, for  $N_A$  fiber spans, the overall nonlinear phase noise due to XPM is given by

$$\Phi_{XPM} = \gamma L_{eff} \left\{ \sum_{k=0}^{N_A} \sum_{i=1}^2 \left| E_k + \sum_{j=1}^i n_j^k \right|^2 \right\} \quad (4)$$

TABLE 1

Constants and system parameters

Symbol	Value	Definition
$\gamma$	$1.2 \text{ W}^{-1}\text{km}^{-1}$	Nonlinearity parameter
$n_{sp}$	<b>1.41</b>	Spontaneous emission factor
$h$	$6.626 \times 10^{-44} \text{ J} \cdot \text{s}$	Planck's constant
$\nu$	$1.936 \times 10^{14} \text{ Hz}$	Optical carrier frequency
$\alpha$	$0.0578 \text{ km}^{-1}$	Fiber loss
$\Delta\nu$	<b>42.6 GHz</b>	Optical bandwidth

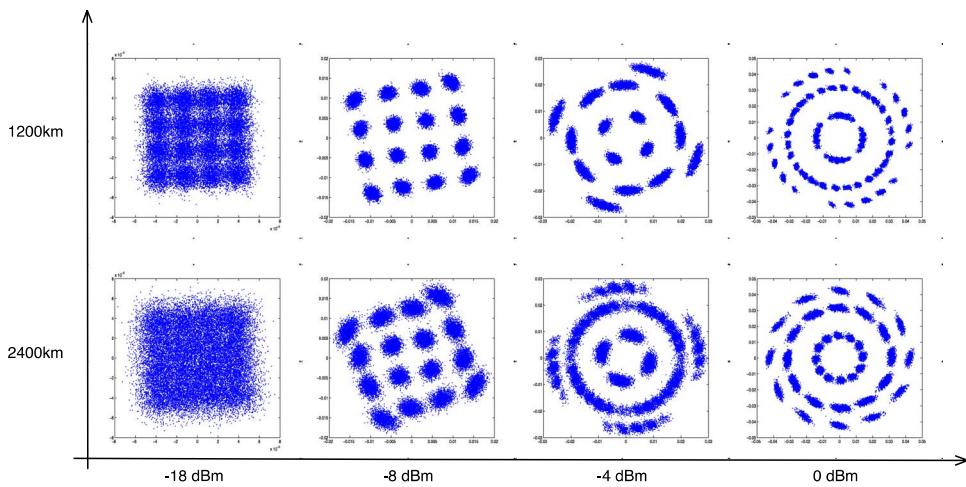


Fig. 1. Received signal constellations for different transmission lengths and launch powers.

where  $E_0$ ,  $E_1$ , and  $E_2$  denote the transmitted electric field of the observed wavelength channel and its two neighboring WDM channels, and  $n_j^k$  denotes the additive noise at the  $j$ th span in the  $k$ th channel, which is independent identically distributed zero-mean circular Gaussian random complex variable with variance  $\delta_k^2$ . The total additive noise for each channel at the end of all fiber segments has the variance  $\delta_{\text{total},k}^2 \triangleq E[Z^2] = 2N_A\delta_k^2$  and can be calculated as [18]:  $\delta_{\text{total},k}^2 = 2n_{sp}h\nu\alpha N_{AL}$ . The introduced channel model can be considered as the worst-case scenario of XPM dominated WDM channel with the signal spacing close to symbol rate.

In this channel model, the variance of the phase error is dependent on the channel input and the channel is decided by the number of spans, transmission length, and also by the launch power. In our case, the span length is set to 80 km, and then, we first fix the transmission length to find the optimal launch power and compare the BER versus transmission distance curve based on the optimal power. The parameters used in the simulation are summarized in Table 1, while an illustration of the received symbols for different transmission lengths and launch powers is shown in Fig. 1. It is evident that the XPM effects become more relevant as the transmission length and/or launch power increase.

### 3. Signal Constellation Sets for XPM Dominated WDM System

For XPM dominated WDM system, we can use an algorithm similar to OSCD algorithm [19] but now changing the optimization criterion from minimizing the mean square error to maximizing

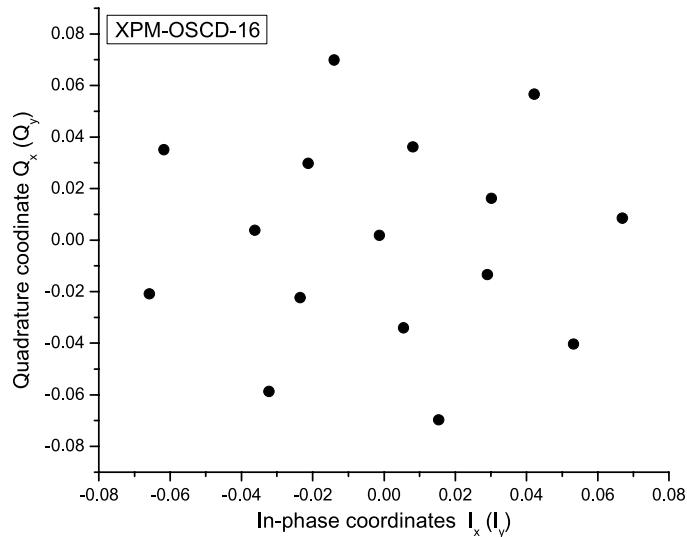


Fig. 2. 16-ary XPM-OSCD constellation.

the cumulative log-likelihood function in order to get make the constellation suitable for XPM dominated channel. The optimum source distribution for the nonlinear phase noise channel can be obtained by Arimoto-Blahut algorithm, and then, we can generate the training samples from this source. In this algorithm, we perform clustering of the constellation points generated by optimum source based on cumulative log-likelihood function. New constellation points will be then obtained by calculating the center of mass of such obtained clusters. This procedure is repeated until convergence or until a predetermined number of iterations has been reached. The cumulative log-likelihood function  $LL(x_j, y)$ , used in optimization, is defined as

$$LL(x_k, y) = \frac{1}{NS} \sum_{i=1}^{NS} -\frac{\{x_{k1} - \text{Re}[(y_1 + y_2j)e^{-j\times PN_i}]\}^2 + \{x_{k2} - \text{Im}[(y_1 + y_2j)e^{-j\times PN_i}]\}^2}{2\delta^2} \quad (5)$$

where NS denotes the number of phase error samples and the corresponding nonlinear phase error sample is denoted as  $PN_i$ , which can be generated using the channel model described in Section 2 [see (4)]. When increasing or decreasing the number of WDM channels in the target scheme, the level of noise samples will also change, which will make the algorithm always suitable for any target scheme. Meanwhile, the algorithm should be repeated for different WDM channels under consideration as every channel might experience a different XPM effects  $x_{k1}$  and  $x_{k2}$  denote the first and second coordinates of the constellation point  $x_k$ . Similarly,  $y_1$  and  $y_2$  denote the coordinates of the received vector (point)  $y$ . The equation above is applicable to 2-D signal constellation designs, but it can straightforwardly be generalized for arbitrary dimensionality.

As an illustration, in Fig. 2 we provide our resulting 16-ary XPM-OSCD constellation, which is obtained for XPM dominated scenario in the presence of ASE noise, optimized at launch power of 1 dBm and transmission distance of 2720 km.

#### 4. Collaborative Demodulation Scheme for XPM Dominated WDM Systems

Our proposed LDPC coded modulation scheme, suitable for WDM systems with polarization-division multiplexing, we used in the simulation, is shown in the Fig. 3(a). (To facilitate explanations only single polarization state is shown.) For each WDM channel, the  $b$  independent data streams are first encoded by the  $(n, k)$  LDPC encoders, whose codewords are written in row-wise fashion into  $b \times n$  block-interleaver. Then XPM-OSCDs have been used for modulation and 2-D (I/Q) modulator performs the electrical-to-optical conversion. The LDPC decoders perform decoding of  $b$  data streams at the same time and the extrinsic information is iterated

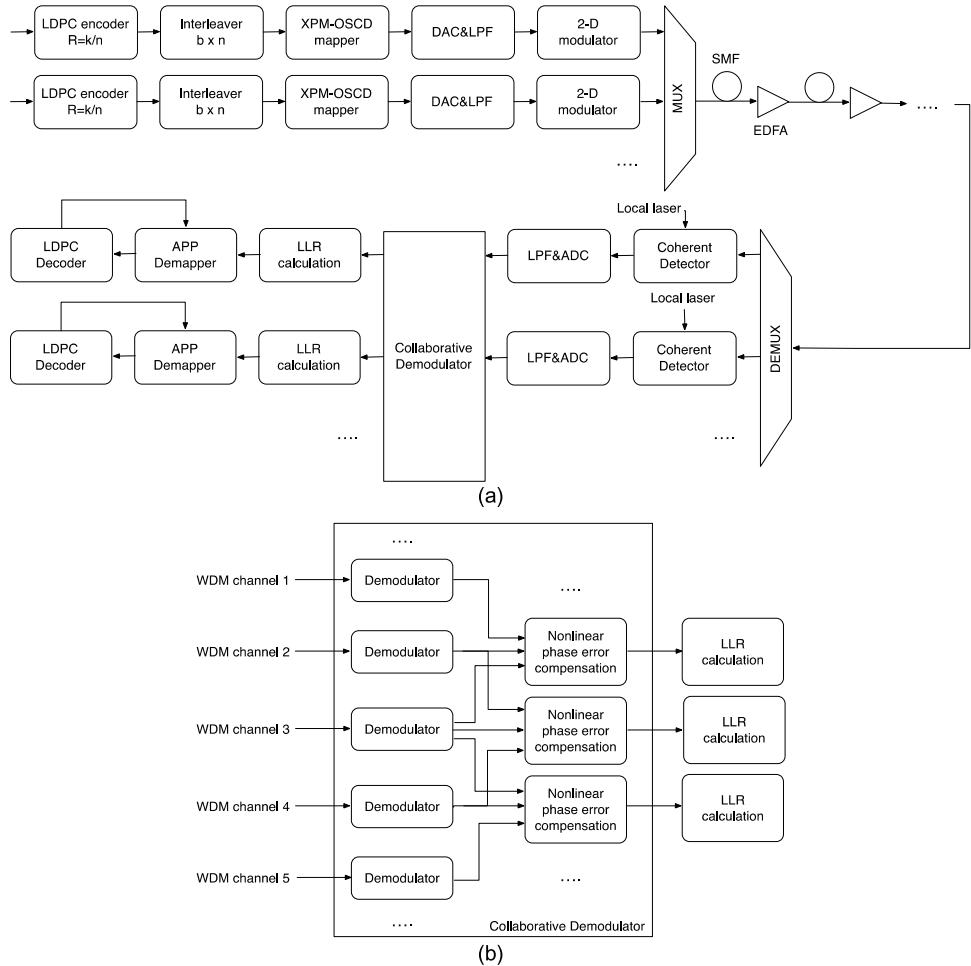


Fig. 3. (a) Collaborative LDPC coded WDM transmission scheme. (Only single polarization is shown to facilitate the explanations.) (b) Details of the collaborative demodulation (CDM) scheme.

between the a posteriori probability (APP) demapper and LDPC decoders. This system can straightforwardly be generalized to the multi-dimensional case. Note that at the receiver side, we have implemented the Monte Carlo integration approach, introduced in [19], in order to estimate the log-likelihood function of transmitted sequence  $\mathbf{a}$ :

$$I(\mathbf{a}) = \log E_{\Phi_{XPM}} \{ \exp[I(\mathbf{a}, \Phi_{XPM})] \} \quad (6)$$

where  $I(\mathbf{a}, \Phi_{XPM})$  is the log-likelihood function for the transmitted symbols in the presence of XPM, and  $E_{\Phi_{XPM}}$  is the expectation average over the nonlinear phase introduced by XPM. Based on (6), the log-likelihood can be calculated after the carrier phase estimation (CPE).

In order to get accurate phase error estimation, we propose a collaborative demodulation (CDM) scheme, with details shown in Fig. 3(b). The received signal is first passed to the demodulator, whose output is one of the constellation points with the minimum distance to the received symbol. Then, this output will be sent to the nearby channels. With all three outputs available, including one from target channel itself, the phase estimator will perform the propagation using the channel model introduced in Section 2 and generate an estimation of the phase error, which will be used as  $\Phi_{XPM}$  in (6). This process can be considered as a digital back propagation on a XPM channel model of low complexity. We first estimate the XPM introduced the nonlinear phase noise by employing the channel model introduced in Section 2, followed by the Monte Carlo integration approach to

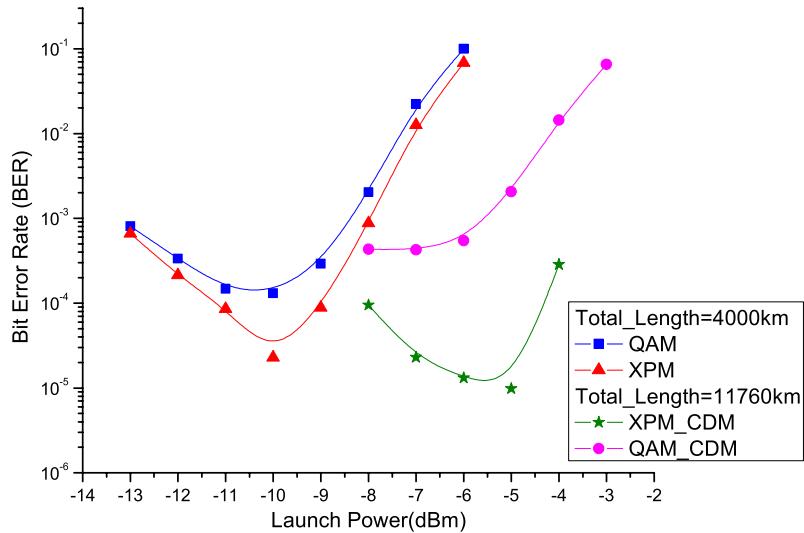


Fig. 4. Uncoded BERs versus launch power for 8-ary signal constellations.

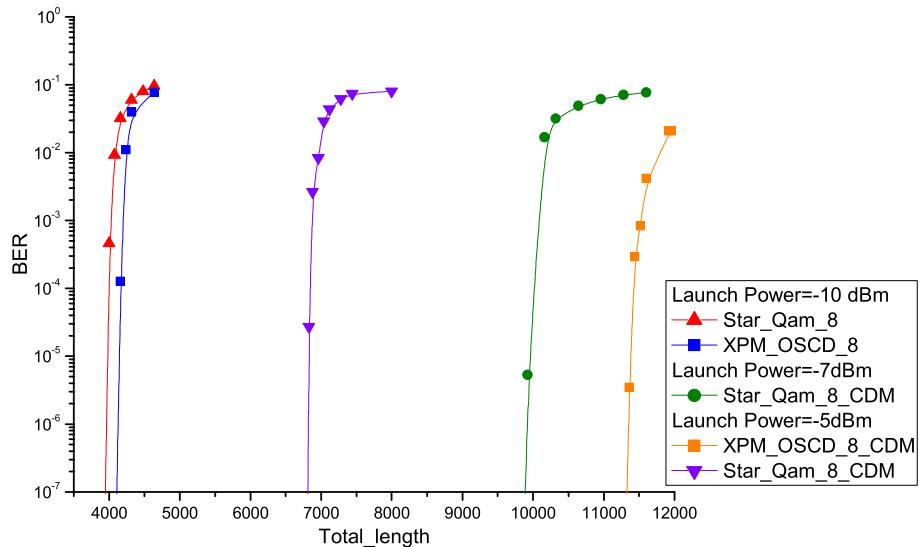


Fig. 5. LDPC-coded BERs versus total transmission distance for 8-ary signal constellations.

estimate the log-likelihood function of transmitted sequence. The estimation of XPM introduced nonlinear phase noise is performed by our proposed collaboration demodulation scheme, which takes the nonlinear phase noise from neighboring WDM channels into account. The demodulator also sends the original received symbols to LLR calculation block for the target channel.

## 5. Numerical Results

The simulation results for LDPC coded WDM transmission based on XPM-OSCD constellations are summarized in Figs. 4 and 5. The scheme is based on 3-channel DWDM. The symbol rate is set to  $R_s = 32.25$  GS/s and the quasi-cyclic LDPC (16935, 13550) code of rate 0.8 is used. All the results are obtained for three outer iterations and 20 inner (LDPC decoder) iterations. For 8-ary OSCD-XPM simulations, the nearby channels all use 8-QAM as modulation format. With PDM and

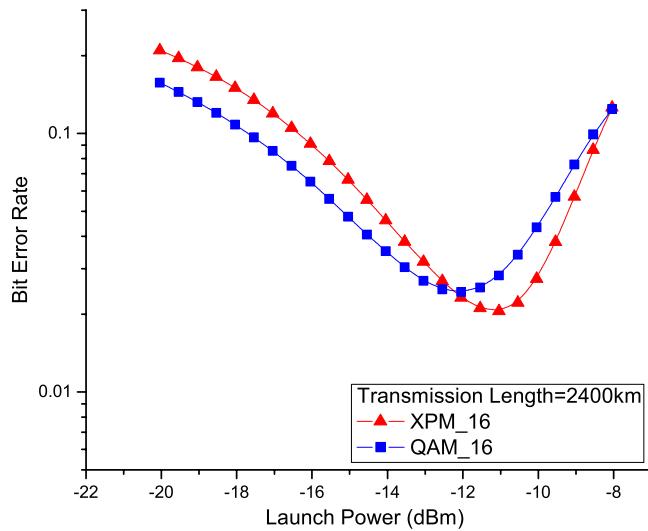


Fig. 6. Uncoded BERs versus launch power for 16-ary constellations.

when two orthogonal subcarriers are used, each WDM channel is compatible with 400 Gb/s per WDM channel transmission. In simulations, we first fix the transmission length, which is chosen as the largest error-free transmission length possible at coded case, while varying the launch power in order to determine the optimum launch power (in minimizing the BER sense). Fig. 4 clearly indicates that the optimal launch power for 8-ary XPM-OSCD is about  $-10$  dBm. It can also be noticed that the XPM tolerance of 8-XPM-OSCD is better. Meanwhile, the gap between QAM and XPM-OSCD has become larger when applying the CDM technique, compared to the case when CDM is not used. On the other hand, when the XPM effect is too strong or not compensated for at the receiver side, the improvement brought by XPM-OSCD is smaller. We can also see the optimal launch power for the system with collaborative demodulation scheme is about  $-5$  dBm.

In Fig. 5, we show the result for 8-ary XPM-OSCD constellations as well as 8-QAM provided for comparison purpose. Additionally, the BER performance versus total transmission distance with or without collaborative demodulation is also shown in the figure. The “CDM” notation is used to denote the WDM system equipped with collaborative demodulation scheme, as indicated above. It is clear to see that the XPM-OSCD constellations perform better than 8-QAM about 240 km without collaborative demodulation scheme. For the system with the collaborative demodulation scheme, the XPM-OSCD outperforms 8-QAM for about 1500 km with each systems operating at their optimal power. If operating at the same power ( $-5$  dBm), the XPM-OSCD outperforms 8-QAM for about 3000 km.

In Figs. 6 and 7, we show the corresponding results for 16-ary XPM-OSCD constellations. Note that the optimal power for the WDM system based on 16-ary constellations is about  $-11$  dBm. It is clear to see that the XPM constellation outperforms 16-QAM more than 600 km when the collaborative demodulation scheme is not used. It is evident that our proposed collaborative demodulation scheme can further extend the transmission length by even 2000 km. When the CDM scheme is also applied for 16-QAM, at this scenario, XPM constellation still performs better than QAM for about 600 km.

## 6. Conclusion

In conclusion, we have proposed the signal constellation design algorithm suitable for XPM dominated WDM channels. Additionally, the collaborative demodulation scheme has been proposed, compensating for the nonlinear phase noise due to XPM. The proposed signal constellation design algorithm employs the log-likelihood function as the optimization criterion. The Monte Carlo simulation shows that both 8-ary and 16-ary constellations obtained by XPM-OSCD algorithm outperform

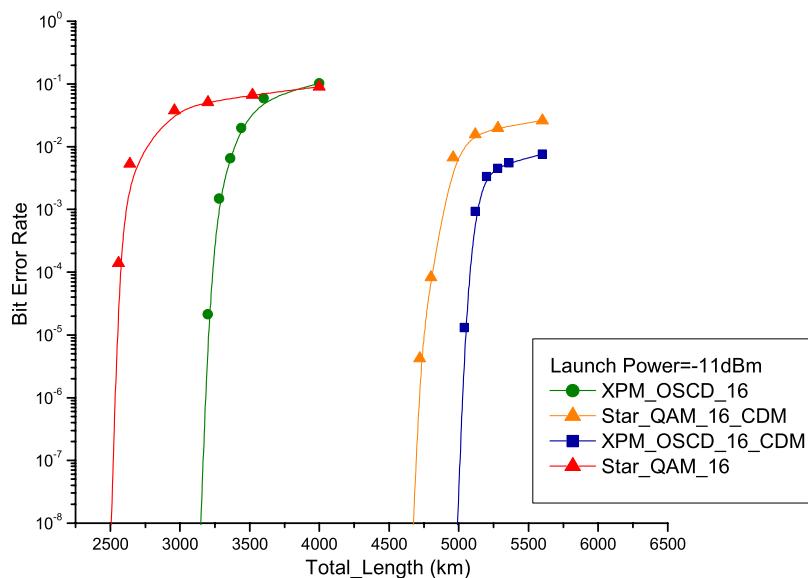


Fig. 7. LDPC-coded BERs versus total transmission distance for 16-ary constellations.

corresponding QAM constellations when a non-collaborative demodulation scheme is used. On the other hand, when the proposed collaborative demodulation scheme has been used, the transmission length can be extended significantly.

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