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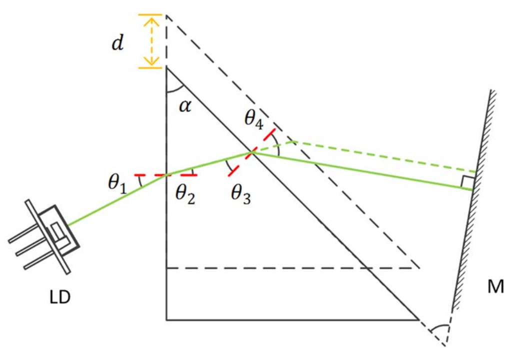

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Refractive Index Measurement With High Precision by a Laser Diode Self-Mixing Interferometer

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Abstract: A new method based on the laser diode self-mixing interference effect for refractive index measurement is demonstrated. It employs a simple translation method to measure the optical phase shift as a function of the moving distance of the sample. The refractive index is determined by analyzing a fringe number of self-mixing signals with respect to the moving distance and the incidence angle, with an experimental accuracy of 0.004. Interestingly, the setting error of the proposed system can be effectively decreased by modifying the incidence angle. This method also shows the advantage of a large measurable range of the refractive index.

Index Terms: Self-mixing interference (SMI), refractive index.

1. Introduction

Refractive index n is an essential optical property of materials, and its accurate value is always of great importance in many branches of physics and chemistry. There are numerous methods to measure the refractive index, including the critical angle method [1], the minimum-deviation method [2], the spectroscopic ellipsometric method [3], and the interferometric method [4], which includes the Michelson interferometer method [5] and the Mach-Zehnder interferometer method [6]. Although high precision of refractive index can be achieved through above methods, the uses of these methods are still trapped in the complexity of the operation and the limitation of the measurable range.

Over the past several decades, as a new laser technique called, self-mixing interference (SMI), which is based on the interaction of cavity field with the field backscatter from the remote target, has increasingly garnered intense attention. The SMI has advantages of low dependence on the coherence length of the laser and the use of a single-mode or multimode laser as the light source [7]. Therefore, the applications of the SMI have been popularized in many fields, including metrology [8], laser parameters [9], terahertz imaging [10], velocimeter [11], and biomedical signals sensing [12]. Recently, Fathi and Donati [13], [14] presented a method to simultaneously

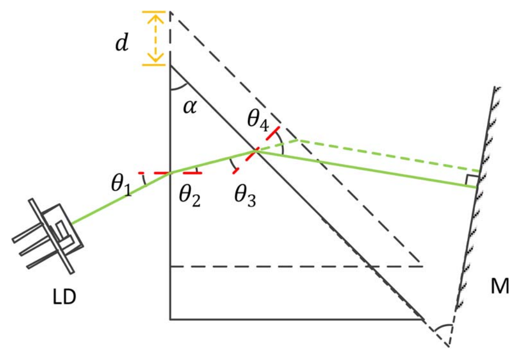

Fig. 1. Schematic of the measuring optical path.

determine thickness and refractive index using SMI. They introduce a transparent slab and make the slab rotate to change the optical path. As a consequence, they got an accuracy that $\Delta n = 0.02$, $d = 1\%$ in the ideal case, without considering the setting error of the slab.

In this paper, we introduce a new method based on SMI for measuring the refractive index. Namely, the refractive index is obtained by analyzing the optical phase shift by means of translation of the transparent object. With optimal incidence angle, Δn caused by setting error is extremely small and can be ignored as compared with that caused by fringe counting. Therefore, the precision of this system is mainly determined by precision of fringe counting. Moreover, the measurement range of refractive index can be expanded, especially suitable for measuring refractive index of liquid.

2. Method for Refractive Index Measurement

2.1 Theory of the SMI

The SMI effect has been studied deeply and described by [15]–[18]. When the light partially re-injected back into the laser by the external reflector, both the gain and the frequency of the laser will be affected; hence, the modulated output power of the laser is related to variation of external optical distance. Based on an analytical steady-state solution, the emitted power P is usually expressed as

$$P(\phi) = P_0[1 + mF(\phi)] \tag{1}$$

which is amplitude modulated by a periodic interferometric function $F(\phi)$ of which the period is phase shift of 2π. In (1), P_0 is the laser power without optical feedback and m is the modulation index; ϕ is the optical phase shift of the external path with feedback, given by $\phi = 2kL = 4\pi L/\lambda$ with k being the wave vector, λ being the wavelength, and L being the variation of optical distance from the LD to the reflector. It is recognized that the phase difference is caused by the external optical path difference. Each fringe of SMI signal corresponds to phase shift of 2π (or displacement shift of $\lambda/2$). Therefore, the relationship between the optical phase shift $(\Delta\phi)$ and the fringe numbers can be given as

$$|\Delta\phi| = 2\pi(N + N') \tag{2}$$

where N is the number of integer fringes, and N' is the number of decimal fringes often observed when the phase difference is bigger than $2\pi N$ and smaller than $2\pi(N + 1)$.

2.2 Theory of the Method for Refractive Index Measurement

In our design (see Fig. 1), the optical phase shift $(\Delta\phi)$ is caused by the movement of the target, i.e., a transparent solid object that sets in the optical path. The emitted light from LD will go through the object, which has a certain included angle α between front and rear surface. The refractive index n of the object is unknown. The external reflecting mirror (M) is employed to make

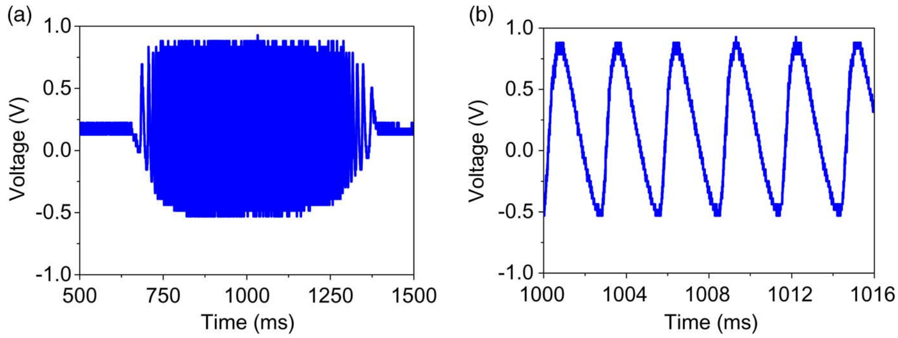

Fig. 2. (a) Experimental results of measurement and (b) typical SMI signals.

the light backtrack and re-enter the laser cavity. The optical phase shift that caused by moving target can be written as

$$\Delta\phi = 2kd[\sin(\theta_1) - \sin(\alpha - \theta_4)] \tag{3}$$

where d is the displacement of the object. The relationship of N' entire angles in optical path can be described by

$$\sin\theta_1 = n\sin\theta_2 \tag{4}$$

$$n\sin\theta_3 = \sin\theta_4 \tag{5}$$

$$\alpha = \theta_2 + \theta_3 \tag{6}$$

where $\theta_{1,2}$ is the incident and refractive angle of front surface, respectively, and $\theta_{3,4}$ is the incident and refractive angle of rear surface, respectively.

Hence, combining with equations (2) and (3) and substituting the angles' relationship (4) and (5), the refractive index n can be expressed as follows:

$$n = \sqrt{\left[\frac{\lambda}{2d\sin\alpha}(N + N') + \cos\theta_4\right]^2 + \sin^2\theta_4}. \tag{7}$$

Considering a certain situation ($\alpha = 45^\circ$), equation (7) can be rewritten as

$$n = \sqrt{\left[\frac{\sqrt{2}\lambda}{2d}(N + N') + \cos\theta_4\right]^2 + \sin^2\theta_4}. \tag{8}$$

Therefore, according to (8), the refractive index n can be solved though the parameters θ_4 and $(N + N')$ that can be obtained from signal processing on SMI waveforms.

3. Experiment and Discussion

In the experiment, we use an LD (FU654AD5_C9N) with nominal wavelength of 654 nm and output power of 5 mW as the light source, driven by a constant current supply. We use a prism that is made of K9 glass ($n_d = 1.51630$, $v_d = 64.06$) and has an included angle of 45° as the sample. It is placed on a translation stage between laser and reflector, at a distance of 12 cm from laser. By adjusting the focus lens packaged in the LD, the laser beam passes through prism and focuses onto the reflector. Parts of the laser beam return back to the laser cavity. A power-monitor PD packaged in the LD detects the change of laser power and transforms light power into current, which will be amplified by a transimpedance amplifier. Finally, the SMI signals are acquired by a computer via a DAQ card (ISDS205A). Experimental results of measurement with translating prism are shown in Fig. 2(a), and the SMI waveform after zooming in on Fig. 2(a) is shown in Fig. 2(b). The recording fringe numbers for eight groups under different

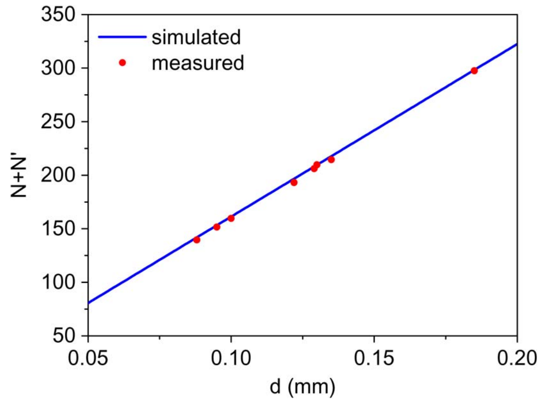

Fig. 3. Fringe numbers $(N + N')$ as a function of displacement d.

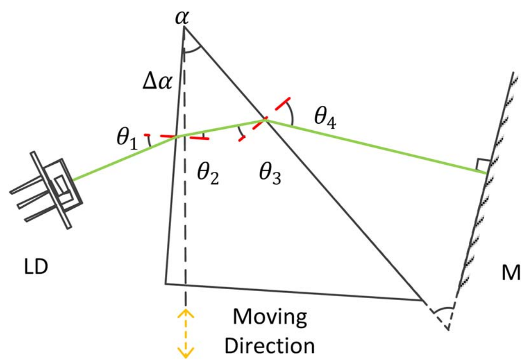

Fig. 4. Schematic of the measuring optical path with setting error.

translated distance are shown in Fig. 3. The several red dots show the measured results by calculating fringe numbers, which contain more than 100 fringe numbers, and the blue line shows the simulated results about fringe numbers $(N + N')$ and displacement d. The experimental result of refractive index n is 1.510, of which the standard deviation is $\delta n = 0.004$. The standard deviation is mainly determined by error of fringe counting. According to theory of the SMI, each fringe of SMI signal corresponds to displacement shift of $\lambda/2$. For improvement of accuracy, we can obtain double precision, i.e., $\lambda/4$ by segmenting self-mixing signal at crest and trough of wave, and higher precision by phase unwrapping.

However, the incidence surface of object is unable to be strictly parallel to the moving direction actually, as shown in Fig. 4. Hence, we introduce a variable $\Delta\alpha$ for modifying the expression of $\Delta\phi$, and (3) can be rewritten as

$$\Delta\phi = 2kd[\sin(\theta_1 - \Delta\alpha) - \sin(\alpha - \Delta\alpha - \theta_4)]. \tag{9}$$

Consequently, the setting error will give rise to the deviation of optical phase shift, which also causes the change of fringe numbers. In order to evaluate the effect of setting error $\Delta\alpha$ on fringe numbers $(N + N')$, (9) is substituted into (2), and the fringe numbers can be expressed as

$$(N + N') = \frac{2d}{\lambda}[\sin(\theta_1 - \Delta\alpha) - \sin(\alpha - \Delta\alpha - \theta_4)]. \tag{10}$$

Without loss of generality, we only consider the situation that the uncertainty of setting error $\Delta\alpha$ is kept within $\pm 1°$. Then, the simulated results $\Delta n = n_{\text{with error}} - n_{\text{without error}}$ with $d = 0.1$ mm,

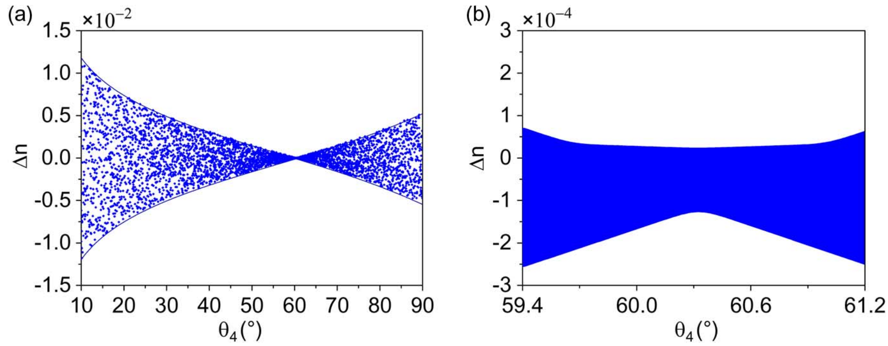

Fig. 5. (a) Error of refractive index Δn for $\Delta\alpha$ within $\pm 1^\circ$ as a function of the refraction angle θ_4 and (b) the details after zooming in for a range of angle from $\theta_4 = 59.4^\circ$ to 61.2°.

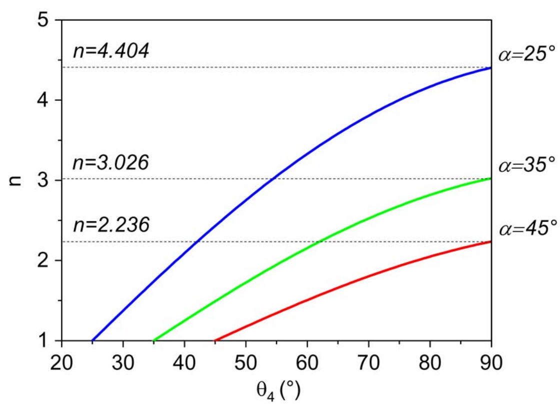

Fig. 6. Refractive index n as a function of the angle θ_4 for $\alpha = 45^\circ,\ 35^\circ,$ and 25°.

$\lambda = 654$ nm, $n = 1.5163$, and $\alpha = 45^\circ$ are shown in Fig. 5(a). Obviously, the error of refractive index is much affected by the refraction angle θ_4, i.e., the incidence angle θ_1. There is a specific angle $\theta_4 = 60.12^\circ$, which can make refractive index error Δn to be greatly reduced to less than 3×10^4, as shown in Fig. 5(b). Compared with Δn caused by fringe counting, Δn caused by setting error is extremely small, which can be ignored.

From above, we can clearly know that the optimal refraction angle θ_4 is closely related to the included angle α and the refractive index n. Using Taylor expansion of the trigonometric function, (10) turns to be as follows:

$$(N + N') \approx \frac{2d}{\lambda}\{\sin\theta_1 - \sin(\alpha - \theta_4)\} - \frac{2d}{\lambda}\{\cos\theta_1 - \cos(\alpha - \theta_4)\}\Delta\alpha$$
$$\approx \{N + N'\}_{\text{idea}} - \{N + N'\}_{\text{error}}. \tag{11}$$

Apparently, $\{N + N'\}_{\text{error}}$ should be equal to zero with the purpose of minimizing the influence of $\Delta\alpha$, so $\{\cos\theta_1 - \cos(\alpha - \theta_4)\} = 0$. Therefore, the angle θ_4 can be determined by solving this equation. Through substituting the angles' relationship (4) and (5), the solution of $\{\cos\theta_1 - \cos(\alpha - \theta_4)\} = 0$ is that

$$\sin^2\theta_4 + \left(\frac{2\sin\theta_4}{\tan\alpha} - \cos\theta_4\right)^2 = n^2. \tag{12}$$

There is an optimal angle θ_4 for every α and n, the results are shown in Fig. 6. The maximum measurable refractive index n are 2.236, 3.026, 4.404 for $\alpha = 45^\circ,\ 35^\circ,\ 25^\circ$, respectively.

Considering expansion of refractive index range and convenience of experiment, the smaller included angle α is preferred.

4. Conclusion

We have presented an SMI-based method of refractive index measurement that is applicable to a wide range of refractive index. By analyzing of derivative phase difference and using a simple fringe counting method, we have been able to determine refractive index and have achieved a relative repeatability estimated in 0.004. The setting error of this system is significantly decreased and can be ignored by selecting optimal incidence angle. This method is very easy to implement in the laboratory and requires only a few simple components and is especially suitable for measuring the refractive index of liquid.

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