IEEE Photonics Journal
An IEEE Photonics Society Publication

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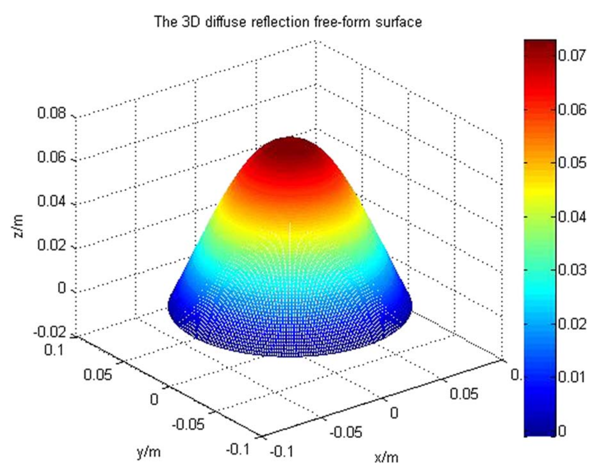
Volume 7, Number 3, June 2015

Zhen-min Zhu
Hui Liu
Shi-ming Chen



DOI: 10.1109/JPHOT.2015.2424402
1943-0655

The Design of Diffuse Reflective Free-Form Surface for Indirect Illumination With High Efficiency and Uniformity

Zhen-min Zhu, Hui Liu, and Shi-ming Chen

School of Electrical and Electronic Engineering, East China Jiaotong University, Nanchang 330013, China

DOI: 10.1109/JPHOT.2015.2424402

Manuscript received March 23, 2015; revised April 12, 2015; accepted April 13, 2015. Date of publication April 20, 2015; date of current version April 28, 2015. This work was supported in part by the National Natural Science Foundation of China under Grant 51305137 and in part by the Foundation of Jiangxi Educational Committee under Grant GJJ14388. Corresponding author: Z. Zhu (e-mail: zhuzhenmin1984@163.com).

Abstract: Indirect illumination with light-emitting diodes (LEDs) and a diffuse reflector is widely applied in various fields that require uniform illumination, whereas the high uniformity obtained is always at the cost of sacrificing the light efficiency. Hence, in this paper, the energy conservation is implemented to greatly compensate for an inefficient design. A mathematical model of a free-form reflective surface based on the Lambertian characteristic of LED, ideal diffuse surface, and energy conservation is established. The diffuse free-form surface is constructed by solving a series of equations numerically. Furthermore, several array configurations are used to illustrate the advantages of the free-form surface. Finally, not only in the far-field simulation but in the near-field simulation as well, the results show better uniformity and higher efficiency compared with traditional design methods (direct illumination and diffuse illumination with a hemispherical inner surface).

Index Terms: Optics, advanced optics design.

1. Introduction

The rapid development of light-emitting diodes (LEDs) over the past few years has surpassed the characteristics of incandescent lamps in luminous efficiency, durability, reliability, safety, and power requirements [1]. Regrettably, LED's usually cannot be directly applied in general lighting systems with perfect illumination effect and high efficiency because of their Lambertian irradiation distribution. Considering the Lambertian characteristic of LED, an appropriate optical design is necessary to meet the need of illumination systems.

Therefore, plenty of works have been done in recent years. The direct illumination and indirect illumination are main parts of the research. Moreno [2], [3] presented a direct illumination method to achieve uniform distribution with different configurations of LED arrays. Ding [4], [5] proposed a method combined refractive equation with energy conservation, a set of first order partial differential equations were established. For example, Luo [6] developed a feedback modification method to design a refractive lens for a non-negligible size of LED source with variable separation mapping, the uniformity in target plane becomes saturated at 81% when the model was modified nine times. Many optimization methods have been proposed [7]–[17], these literature are all focused on the design of free-form lens for uniform illumination. In addition to the direct illumination method, indirect illumination could also be applied to the design of secondary optics. In order to overcome the discomfort glare effect and the over-exposure problem, Tsuei [18]–[20] developed a method for making the illumination more uniform in an indoor lighting environment and weakening the glare effect by using diffuse reflector. Zhu [21], [22] presented a design method of a diffuse reflection free-form surface by solving a series of nonlinear algebraic equations which are coming from a mathematical model. In this way, the indirect design methods mentioned above could guarantee a certain degree of uniformity of the target scene. Comparing with LEDs based on conventional parabolic or ellipsoidal reflectors, our method has two advantages. On the one hand, in order to satisfy the various applications, the space structure of the free-form surface reflector we designed is different once its parameters change, while the conventional parabolic or ellipsoidal reflectors can not do this freely. On the other hand, in our process of designing, the energy conservation is implemented to greatly compensate for the efficiency but the uniformity and efficiency of the two traditional reflectors are unchanging. In summary, the uniformity and efficiency of the target plane still could be further improved by designing the shape of the diffuse surface in other ways.

Therefore, in this paper, energy conservation is implemented to greatly compensate for an inefficient design. The energy emitted from a LED is redistributed by the designed free-form inner surface and then a uniform illumination area could be acquired on the target plane. Firstly, considering the characteristics of Lambertian sources, the irradiate distribution function over the free-form inner surface to a LED is obtained. Secondly, the desired value uniformity over the target plane easily could be calculated by energy conservation law and then use bidirectional scattering distribution function of the diffuse free-form inner surface, a mathematical simulated algorithm is developed to represent the irradiate of the illuminated plane. Thirdly, according to the desired value over the target plane calculated by the conservation law, a set of nonlinear algebraic equations were established. Finally, the results by this design turn out to be better uniformity and higher efficiency than traditional design methods (i.e., direct illumination and diffuse illumination with hemispherical inner surface). The simulation results illustrate that, using an LED as the source, not only the illumination uniformity but also the lighting efficiency, our design both have a good lighting performance and with illumination uniformity near to 86%. In addition, in the far-filed illumination area, using a multiple-ring array of LED could carry out the same conclusions and with the lighting efficiency near to 45%, which has an advantage to the diffuse illumination with hemispherical inner surface whose efficiency is 34% and the direct illumination whose efficiency is 30%. In this paper, the secondary optical free-form surface could be applied in various fields that require uniform illumination.

2. Optical Model From a Single LED

Ideally, one LED source is a Lambertian emitter, which means the irradiate distribution is also a cosine function of the viewing angle [21]–[23]. In practice, this dependence turns out to be a power law that primarily depends on encapsulates and semiconductor region shapes. A practical approximation for the irradiate distribution is shown as

$$E_p(D,\varphi) = \frac{I_{\text{LED}}(\cos^m \varphi)}{D^2} \tag{1}$$

where D is the distance between the LED and a point p at the illuminated plane, and φ is the viewing angle of light source. $E_p(D,\varphi)$ is the irradiate distribution at a distance D and viewing angle φ. I_{LED} is the radiant intensity (w/sr) of the LED.

The value of m depends on the relative position of the LED emitting region from the curvature center of the spherical encapsulation. If the chip position coincides with the curvature center, the number m $\approx$ 1, and the source is nearly a perfect Lambertian. The number m is given by the angle $\theta_{1/2}$(a value typically provided by the manufacturer, which is defined as the view angle when distance is half of the value at 0°)

$$m = \frac{-\ln 2}{\ln(\cos\theta_{1/2})}. \tag{2}$$

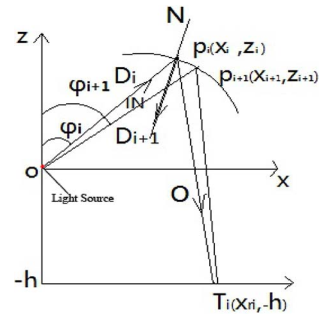


Fig. 1. Schematic diagram of the diffuse reflection free-form surface solution.

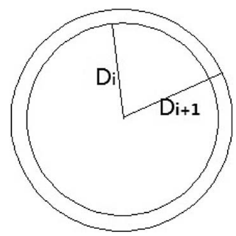


Fig. 2. Circle model.

In our study, the free-form diffuse reflection inner surface is supposed to have rotational symmetry. Accordingly, the two-dimensional mode of x–z plane was taken for instance. As illustrated in Fig. 1, the points on free-form inner surface P and target plane T can be further expressed as $p_i(x_i, z_i)$ and $T_i(x_{ri}, -h)$ in 2-D plane of x–z, respectively. If we take ***N***, ***I***, and ***O*** as the unit vectors of normal, incident and refractive vectors respectively, they can be expressed as

$$\begin{cases} \cos\varphi_i' = \dfrac{\overrightarrow{\mathrm{O}} \bullet \overrightarrow{\mathrm{N}}}{|\overrightarrow{\mathrm{O}}||\overrightarrow{\mathrm{N}}|} \\ \overrightarrow{\mathrm{O}} = (x_{ri} - x_i, -h - z_i) \\ \overrightarrow{\mathrm{N}} = (-dz, dx) \end{cases}$$

where ***O*** is the vector that represents the emergent ray from point p_i to T_i reflected by the free-form surface. ***N*** is the normal vector of the free-form surface at point p_i, dx, and dz represent the differential of x and z, respectively. φ_i' is the angle between ***O*** and ***N***.

Fig. 2 shows that a circle with the output view by φ_i and φ_{i+1} can be built while φ_i and φ_{i+1} are close enough. Then the total flux with a circle is given by

$$\phi_c = \mathrm{E}_p(D_i, \varphi_i) \bullet s_i \tag{4}$$

$$s_i = 2\pi D_i \sin\varphi_i D_i(\varphi_{i+1} - \varphi_i) \tag{5}$$

where ϕ_c is the received irradiation of the ith circle including by free-form inner surface. s_i represents the area on the ith circle.

In this paper, the free-form inner surface is defined as a Lambertian surface. It is also called an ideal diffuse reflection surface with the cosine characteristic expressed as

$$I\varphi = I_0 \cos\varphi \tag{6}$$

where $I\varphi$ is the emergent light intensity in a certain emitting angle φ. I_0 is the maximum radiant intensity in vertical direction of the free-form inner surface $(\varphi = 0^\circ)$. The reflected light emitted from this free-form inner surface is scattered equally at random direction. The free-form surface's inner surface approximately seems as a Lambertian surface which is defined as a surface has invariant irradiance at random direction. It is also called a perfect diffuse reflection surface with hemisphere reflectance $\rho = 1$, which reflect light entirely. Hemisphere reflectance is the ratio of the total amount of light radiation reflected by a surface to the total amount of light radiation incident on the surface, but the free-form surface reflectance only represents the reflectance of light radiation. BRDF [22], [24] is better than hemisphere reflectance to describe the uniformity of light at random direction in the space. In practice, typical diffuse reflection surface often has values $\rho < 1$. Bidirectional scattering distribution function (BRDF) is used to describe the uniformity of light at random direction in space. It can be given by

$$\text{BRDF} = \rho/\pi. \tag{7}$$

At this time, the irradiation of the i-th circle reflected by free-form inner surface can be given as

$$\phi_i' = \phi_c \bullet \cos\varphi_i' \bullet \frac{\rho}{\pi} \tag{8}$$

where ϕ_c is a part of the received irradiation of the free-form inner surface and φ_i' represents the irradiation received in target plane reflected by the free-form inner surface. Considering the practical situation, we make the reflectance of the diffuse free-form inner surface $\rho = 0.9$ [22]

$$\phi_c' = \frac{1}{u}\sum_{i=1}^{u}\phi_i' = \frac{1}{u}\left(\phi_1' + \phi_2' + \phi_3' \ldots + \phi_u'\right) = \sum_{i=1}^{u} E_i \bullet S_i \tag{9}$$

$$S_i = \pi x_{ri}^2 \tag{10}$$

where ϕ_c represents the average of the values from ϕ_1' to ϕ_u' $(u = 81)$. E_i is the irradiation of the detector plane at point T_i. S_i represents the area of a circle with radius is x_{ri}. Well, the irradiation coming from the point T_i can be expressed as

$$\pi\sum_{i=1}^{u} E_i x_{ri}^2 = \frac{1}{u}\sum_{i=1}^{u}\frac{-2I_o(\cos^m\varphi_i)\rho[(x_{ri} - x_i)dz + (h + z_i)dx]\sin\varphi_i(\varphi_{i+1} - \varphi_i)}{\sqrt{(x_{ri} - x_i)^2 + (h + z_i)^2} \bullet \sqrt{(dz)^2 + (dx)^2}} \tag{11}$$

where the relationship could be obtained between x_i and z_i through calculating the value of E_i $(E_1, E_2, \ldots, E_u)$. Finally, the irradiation of the detector plane can be expressed as

$$E_t = \frac{1}{u}\sum_{i=1}^{u} E_i \tag{12}$$

3. Energy Conservation

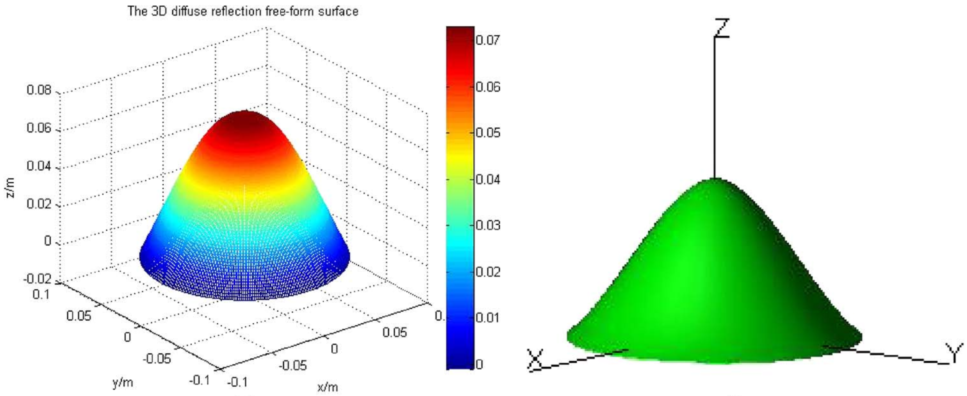
According to the light transmission of energy conservation, the output of source is equal to the flux incident in the target plane. Assuming the half viewing angle of the source is φ_N

$$\int_0^{2\pi} d\theta \int_0^{\varphi_N} I(I(\varphi))\sin\varphi d\varphi = \int E_t \bullet dA \tag{13}$$

where E_t is the irradiation at point T_i, A represents the area in the target plane. $I(I(\varphi))$ is LED emitting intensity in the direction of $I(\varphi)$. It can be simply expressed

$$\text{E}_t = \frac{-2\text{I}_o(\cos^{m+1}\varphi_N - 1)}{(m+1) \bullet x_r^2}. \tag{14}$$

Equations (11) and (12) indicate the relationship between θ, φ, x, and z, and its exact form depends on the topological mapping from the source to the target plane. When target plane T is fixed, E_t can be gotten through (14).



(a) (b)

Fig. 3. Three-dimensional diffuse reflection free-form surface.

In order to establish sufficient number of equations for solving the free-form surface in two-dimensional mode, the same amounts of sample points are chosen on the target plane. According to the goal of uniform illumination, the reflected irradiation expressions of these sample points given by (11) should be assigned to a same constant value of E_t expressed by (14). Meanwhile, Euler numerical method is employed to replace the first derivatives *dx*, *dz* into their difference scheme for advancing the calculation speed. In this way, a set of nonlinear algebraic equations which take $p_i(x_{ri}, 0, z_i)$ as unknowns are established by

$$E_t = \frac{-2I_o(\cos^{m+1}\varphi_N - 1)}{(m+1)\bullet x_r^2} = \frac{1}{u}\sum_{i=1}^{u} E_i \qquad (i = 1, 2, 3, \cdots, u). \tag{15}$$

According to the size of the illumination system in our study, the abscissa value range of the sample points on the free-form surface and its step-length are determined. Then, the unknowns of the equations given by (15) only leave the ordinate of the points p_i $(i = 1, 2, \ldots, u)$.

Using MATLAB software to solve the nonlinear algebraic equations in numerical way, a series of discrete points that used to describe the outline of free-form inner surface in two-dimensional mode—$(x_1, 0, z_1), (x_2, 0, z_2), (x_3, 0, z_3), \ldots, (x_u, 0, z_u)$ could be derived. Next, the least squares method is added into the coordinate data of these discrete points to fit a smooth curve. Finally, by revolving the smooth curve around *Z*-axis to create a surface in three-dimensional mode, the design of the desired free-form surface is completed. Fig. 3 shows the diffuse reflection free-form surface in solid form. The Projected length of the free-form surface on the *x*-axis and *y*-axis both are 160 mm, its height is 80 mm, and the unit of axis is meter (m) in Fig. 3.

4. Simulation the Illumination Uniformity Over the Illuminated Plane

4.1. The Near-Field Situation

Once the free-form surface was obtained, TracePro is used to simulate and verify the results of the diffuse free-form surface illumination performance. Here, a single LED located at the origin can be treated as the source for the simulation and the LED has an analogous Lambertian radiation distribution, with the die size of 1×1 mm^2 and the power of the single Led is 1 w. Compared to the size of the free-form surfaces, the Led can be regarded as a point light source. The range of near-field distant defined was from 100 mm to 800 mm [2]. Fig. 4 shows the simulated results of 1 million light rays tracing with $Z = 400$. Meanwhile, the simulation results at different distances and radius further show the importance of the near-field approach in Table 1. It shows efficiency and uniformity at target plane under different conditions.

In order to observe the trends of uniformity and efficiency with three different methods clearly, we selected the radius $R = 150$ mm of the target surface and three different distances $Z = 400$,

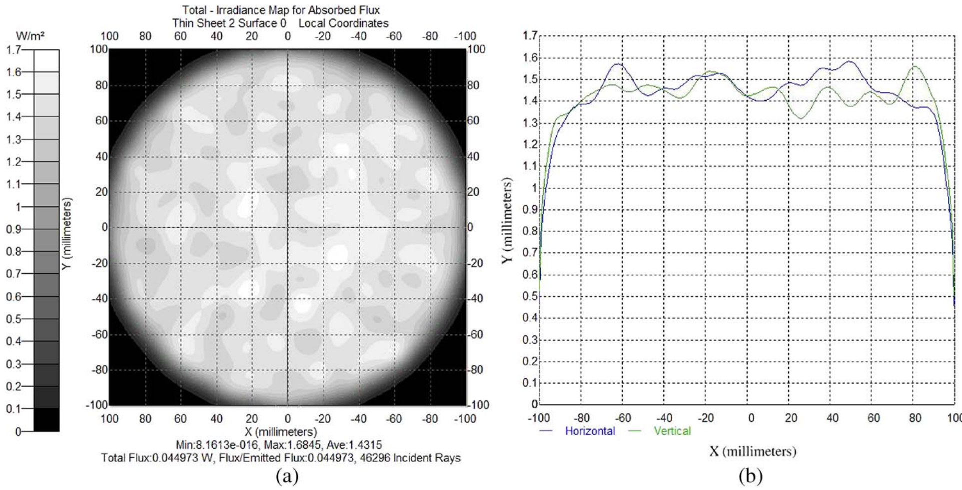


Fig. 4. Simulation results of the irradiation uniformity over the target plane at the distance $Z =$ 400 mm. (a) Irradiation distribution of the illuminated scene which is a circular area with the radius $R = 100$ mm. (b) Uniformity across the center of the target plane along x axis and y axis.

TABLE 1

Efficiency and uniformity of near field distance

A single LED	Efficiency			Uniformity		
	Diffuse free-form	Diffuse hemisphere	direct	**Diffuse free-form**	Diffuse hemisphere	direct
Z=150, R=40	**4.2%**	3.3%	3.3%	**84.3%**	81.5%	80.5%
Z=200, R=40	**2.6%**	2%	1.9%	**82.6%**	81.2%	77.4%
Z=400, R=100	**4.5%**	3.6%	3%	**85%**	80.1%	79.6%
Z=400, R=150	**9.5%**	7.7%	6.2%	**84.9%**	81.5%	80.7%
Z=400, R=200	**15.7%**	12.6%	10.2%	**80.7%**	77.8%	76%
Z=600, R=150	**4.6%**	3.7%	3%	**85.7%**	82.2%	79.4%
Z=800, R=150	**2.7%**	2%	1.7%	**84.2%**	79.4%	76.7%
Z=800, R=200	**4.7%**	3.7%	3%	**85.2%**	82.1%	79.6%
Z=1000, R=400	**11.1%**	8.9%	7.0%	**83.3%**	82.8%	79.8%

600 and 800, respectively. A brief comparison with other LED illumination systems [2], [3], [22] is necessary to demonstrate the correctness and effectiveness of our study shown in Fig. 5.

It seemed that the uniformity has been improved significantly to 85.7% by using diffuse reflection free-form surface compared with the other two lighting patterns at the distance $H = 600$ and the luminous efficiency was higher than the others at the same time. According to the above comparison and analysis, indirect lighting with designed free-form diffuse surface is considered to be a feasible method for improving the illumination uniformity in near-field application and the efficiency also has a slight raise as well.

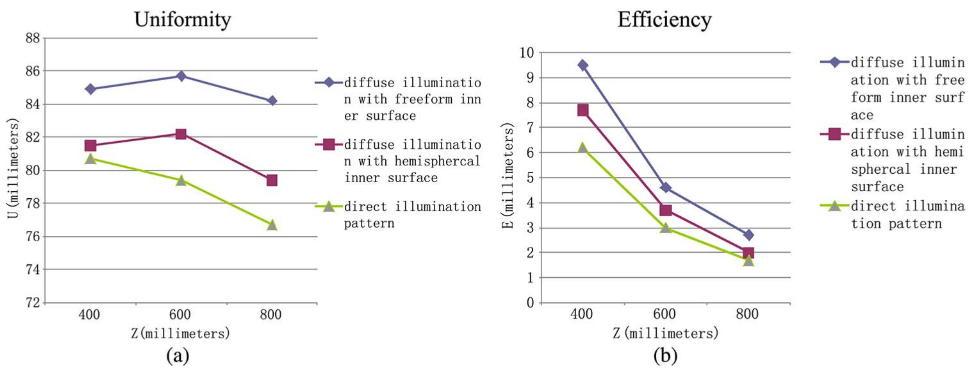


Fig. 5. Simulation results with different illumination schemes and at different source–target distances Z with the same radius of the target plane $R = 100$. (a) Irradiation uniformity over the target plane. (b) Luminous efficiency of the illumination system.

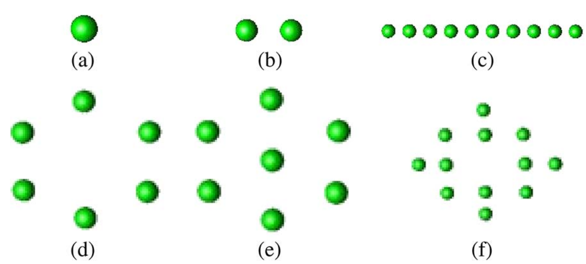


Fig. 6. (a) Single LED. (b) Two-LED array. (c) Linear LED array. (d) Ring array of LED. (e) Ring array with one LED in the center. (f) Multiple-ring array of LED.

4.2. The Far-Field Situation

In far-field illumination system, several array configurations [2], [3] were used to validate the correctness and advantage of this method. As Fig. 6 shows different configurations of LED arrays: a single LED, two-LED array, a ring array of LED, a ring array with one LED in the center, linear LED array and a multiple-ring array of LED. The coordinate potion of the LED in the 2-D (x, y) and the unit of axis x and axis y were both in millimeters:

(a) Single LED. (0, 0);
(b) Two-LED array. (−6.5, 0), (6.5, 0);
(c) Ring array of LED. (−5, 5), (−5, −5), (0, 7), (0, −7), (5, 5), (5, −5);
(d) Ring array with one LED in the center. (−50, 50), (−50, −50), (0, 70), (0, 0), (0, −70), (50, 50), (50, −50);
(e) Linear LED array. (−20, 0), (−16, 0), (−12, 0), (−8, 0), (−4, 0), (4, 0), (8, 0), (12, 0), (16, 0), (20, 0);
(f) Multiple-ring array of LED. (−70, 0), (−50, 50), (−50, 0), (−50, −50), (0, 70), (0, 50), (0, −50), (0, −70), (50, 50), (50, 0), (50, −50), (70, 0).

Next, the efficiency and uniformity results of different arrays could be easily shown in Table 2. Although in different arrays, it can be clearly seen that our design has significantly improved the efficiency of the luminous, what is more, the efficiency increased about 10% in the multiple-ring array.

Then, we performed a simple demonstrative simulation with a ring array of LED, The radius of the target plane $R = 1500$. In order to display the advantage of our method with a ring array of LED is more intuitive, Fig. 7 shows the uniformity and efficiency at target surface under different situations $Z = 2000, 3000, 3500,$ and 4000, respectively.

As we all know, the longer distance from light source to target plane, the greater energy loss. It leads to the uniformity over the target plane looks the same inevitably when the source–target

TABLE 2

Efficiency and uniformity

Array configuration of LED		efficiency			Uniformity		
		Diffuse free-form surface	Diffuse hemisphere	direct	**Diffuse free-form surface**	Diffuse hemisphere	direct
A single LED	Z=2500 R=1000	**13.4%**	10.4%	8.3%	**78.7%**	78.1%	75%
Two-LED array	Z=2500 R=1000	**13.5%**	10.7%	8.3%	**78%**	77.3%	77%
Ring array of LED	Z=3000 R=2000	**27.3%**	19.3%	18.4%	**67.7%**	67.1%	64.5%
A ring array with one LED in the center	Z=3000 R=1500	**18.3%**	12.9%	12%	**74%**	73.8%	73%
Linear LED Array	Z=2500 R=2000	**37.3%**	29.3%	23%	**56.7%**	55.6%	56%
A multiple-ring array of LED	Z=3000 R=3000	**44.6%**	34%	30%	**48%**	48%	46%

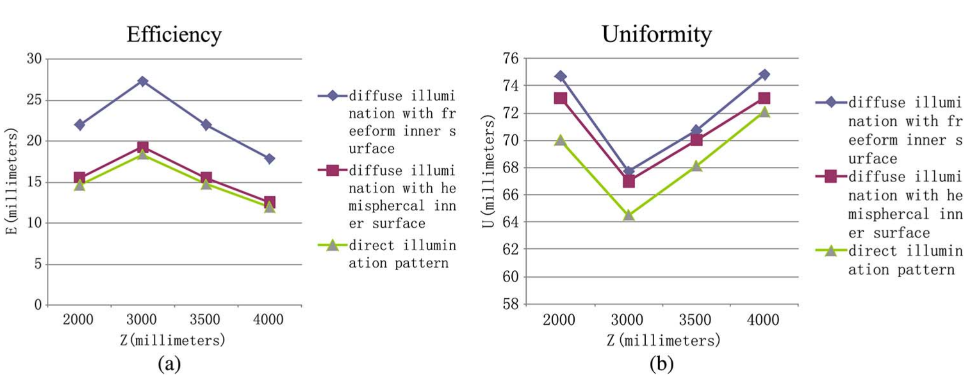


Fig. 7. Simulation results with different illumination schemes and at different source–target distances. (a) Luminous efficiency of the illumination system. (b) Irradiation uniformity over the target plane.

distances increasing in far-field illumination system by three different methods, but the efficiency of the target plane simulated by our method is greatly been improved comparing with the other two methods. At four different source–target distances, the efficiency and uniformity results over the target plane with three different illumination schemes are shown in Fig. 7, respectively. It is clearly seen that: first, with the distance of Z increasing, the uniformity of the target plane appears alike, but luminous efficiency improved significantly by using diffuse reflection free-form surface. Second, a better result occurs at the distant $Z = 3000$. In summary, the uniformity with different methods seemed approach the same, but the luminous efficiency was been significantly improved with our design.

5. Conclusion

In this paper, a design of diffuse reflection free-form surface containing the energy conservation is used to improve the luminous efficiency. According to the comparison and analysis discussed above, in the near-field applications, indirect lighting with the free-form diffuse surface is considered to be a feasible method for improving the illumination uniformity apparently and the efficiency also has a slight raise as well. In the far-field applications, with the distance of source–target is increasing, the uniformity of the target plane appears alike, but the lighting efficiency is been improved significantly by our method. What's more, similar conclusions (just as a two-LED array, a ring with one middle LED, a linear array of LED, a square array of LED, and a triangular array of LED) are obtained as well. For diffuse indirect illumination, the significantly increasing of the efficiency and uniformity can be effectively broadened the outlook of indirect illumination in the practical application.

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