IEEE Photonics Journal
An IEEE Photonics Society Publication

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Volume 7, Number 2, April 2015

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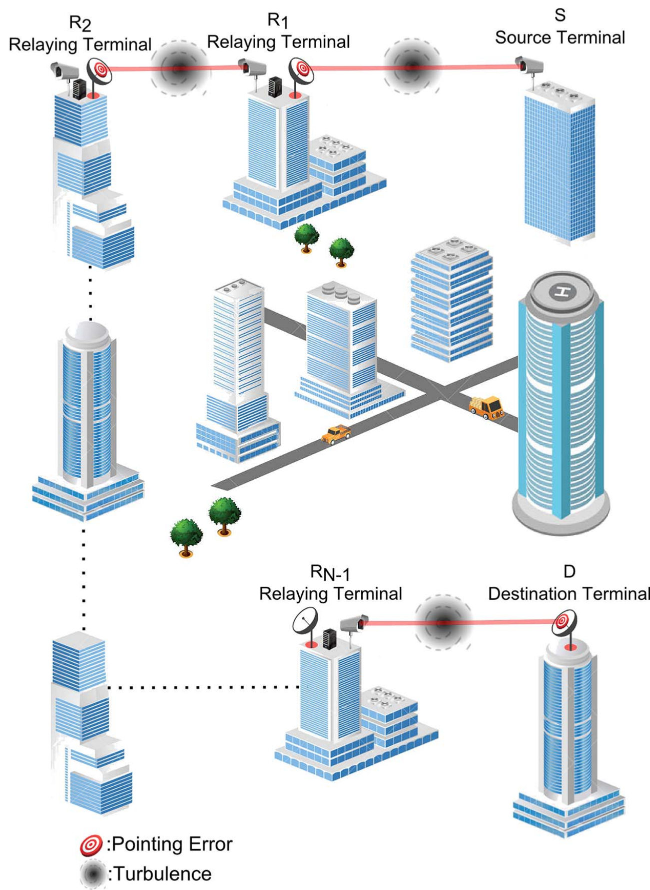

DOI: 10.1109/JPHOT.2015.2417867

On the Performance of Multihop Heterodyne FSO Systems With Pointing Errors

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DOI: 10.1109/JPHOT.2015.2417867

Manuscript received February 1, 2015; revised March 23, 2015; accepted March 23, 2015. Date of publication March 30, 2015; date of current version April 14, 2015. This work was supported in part by a grant from King Abdulaziz City of Science and Technology (KACST) under Project AT-34-145.
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Abstract: This paper reports the end-to-end performance analysis of a multihop free-space optical system with amplify-and-forward (AF) channel-state-information (CSI)-assisted or fixed-gain relays using heterodyne detection over Gamma–Gamma turbulence fading with pointing error impairments. In particular, we derive new closed-form results for the average bit error rate (BER) of a variety of binary modulation schemes and the ergodic capacity in terms of the Meijer's G function. We then offer new accurate asymptotic results for the average BER and the ergodic capacity at high SNR values in terms of simple elementary functions. For the capacity, novel asymptotic results at low and high average SNR regimes are also obtained via an alternative moments-based approach. All analytical results are verified via computer-based Monte-Carlo simulations.

Index Terms: Free-space optical (FSO) communication, multihop relaying, atmospheric turbulence, pointing errors, ergodic capacity.

1. Introduction

Multihop relaying, where several intermediate terminals relay the signal from the source terminal to the destination terminal, has gained significant research attention as an efficient technique mainly because it can expand the coverage of wireless networks with low power requirements while offering high data-rate at the end-to-end communication [1]. On the other hand, free-space optical (FSO) communication is a cost effective and wide bandwidth access technique operating at the unlicensed optical spectrum that allows higher capacity and higher data rates relative to the traditional radio frequency (RF) transmission [2], [3]. However, the atmospheric turbulence can degrade the system performance particularly over distances of 1 km or longer. Another major impairment over FSO links is pointing error as a result of the building sway [3], [4].

Recently, multihop relaying has been proposed to mitigate turbulence-induced fading and increase the reliability of the FSO link. In [1], the outage probability of a multihop FSO system with amplify-and-forward (AF) or decode-and-forward (DF) relays over strong turbulence fading channels is studied. Relay-assisted transmission over Log-normal turbulence-induced fading with path loss has been investigated in [5]. In [6], the end-to-end performance of the multihop

FSO system using channel-state-information (CSI)-assisted relays and fixed-gain relays over Gamma–Gamma turbulence under intensity modulation with direct detection (IM/DD) technique has been examined. The performance of the FSO multihop system using CSI-assisted and fixed-gain relays over Gamma–Gamma turbulence under heterodyne detection technique with pointing errors has been reported in [7]. However, the performance study carried out in [7] does not include the ergodic capacity analysis. The ergodic capacity, also known as average channel capacity, is an important performance metric of primary concern in the design of FSO systems. It gives an important benchmark on the data rate that can be supported by the communication link. Also, for the average BER, only a differential phase-shift keying (DPSK) modulation scheme is considered.

In this paper, we extend the work presented in [7] to study the average BER of a variety of binary modulation schemes, as well as the ergodic capacity of a multihop heterodyne FSO system with CSI-assisted or fixed-gain relays over Gamma–Gamma turbulence fading under pointing error impairments. Moreover, we present new asymptotic expressions at high SNR for the average BER and the ergodic capacity in terms of basic elementary functions by using the asymptotic expansion of the Meijer's G function. Then, for the case of the ergodic capacity, we use the moments to derive accurate asymptotic results in the low and high SNR regimes.

The rest of the paper is organized as follows. Section 2 presents the system and channel model of the multihop FSO system under consideration. The average BER, and the ergodic capacity results are derived in Section 3. The obtained analytical expressions are evaluated and interpreted in Section 4. Finally, Section 5 concludes this paper.

2. System and Channel Model

We consider the same multihop system model employed in [7], where the source terminal S communicates with the destination terminal D through $N-1$ intermediate terminals $R_1, R_2, \ldots, R_{N-1}$ which act as non-regenerative relays. The end-to-end SNR, when CSI-assisted (γ_{end}) relays are considered, can be derived as [8]

$$\gamma_{\text{end}} \triangleq \left(\sum_{i=1}^{N} \frac{1}{\gamma_i}\right)^{-1} \tag{1}$$

where γ_i is the instantaneous SNR for the ith hop. For a multihop FSO system equipped with fixed-gain relays, the overall SNR at the destination can then be written as [8]

$$\gamma'_{\text{end}} = \left(\sum_{i=1}^{N} \prod_{j=1}^{i} \frac{C_{j-1}}{\gamma_j}\right)^{-1} \tag{2}$$

where C_i is a positive constant $(C_0 = 1)$. The instantaneous SNR for the ith hop follows the Gamma–Gamma fading model including pointing error impairments under heterodyne detection with the PDF given by [9]

$$f_{\gamma_i}(\gamma) = \frac{\xi_i^2}{\gamma \Gamma(\alpha_i)\Gamma(\beta_i)} \mathrm{G}_{1,3}^{3,0}\left[\frac{\alpha_i \beta_i \xi_i^2 \gamma}{(1+\xi_i^2)\mu_i} \middle| \begin{matrix} \xi_i^2 + 1 \\ \xi_i^2, \alpha_i, \beta_i \end{matrix}\right] \tag{3}$$

where ξ_i denotes the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter) at the receiver [2], [9], which is given as $\xi_i = w_{z_{eq},i}/2\sigma_{s,i}$, where $\sigma_{s,i}^2$ is the jitter variance at the receiver, and $w_{z_{eq},i}$ is the equivalent beam radius at the receiver [4], [10] (for negligible pointing errors, $\xi \to \infty$), $\mathrm{G}_{\cdot,\cdot}^{\cdot,\cdot}(\cdot)$ is the Meijer's G function as defined in [11, Eq. (07.34.02.0001.01)], μ_i refers to the average SNR, and α_i and β_i are the fading/scintillation parameters related to the atmospheric turbulence conditions with small values of these two parameters pointing to severe fading conditions [12], [13]. More specifically, assuming a plane wave propagation with aperture averaging, these parameters can be determined from the Rytov variance as [14], [15]

$$\alpha_i = \left[\exp\left(\frac{0.49\sigma_{R,i}^2}{\left(1+0.18d_i^2+0.56\sigma_{R,i}^{\frac{12}{5}}\right)^{\frac{7}{6}}}\right)-1\right]^{-1} \tag{4}$$

$$\beta_i = \left[\exp\left(\frac{0.51\sigma_{R,i}^2\left(1+0.69\sigma_{R,i}^{\frac{12}{5}}\right)^{-5/6}}{\left(1+0.9d_i^2+0.62d_i^2\sigma_{R,i}^{\frac{12}{5}}\right)^{\frac{5}{6}}}\right)-1\right]^{-1} \tag{5}$$

where $\sigma_{R,i}^2 = 0.5C_n^2 k_w^{7/6} L_i^{11/6}$ denotes the Rytov variance, and $d_i^2 = k_w D_a^2/(4L_i)$, where D_a is the diameter of the receiver aperture, $k_w = 2\pi/\lambda_w$ is the optical wave number, λ_w is the wavelength, L_i is the propagation distance, and C_n^2 refers to the index of refraction structure parameter varying from 10^{-17} m$^{-2/3}$ for weak turbulence to 10^{-13} m$^{-2/3}$ for strong turbulence. It is noteworthy to mention that the equivalent SNRs in (1) and (2) are not easily tractable due to the difficulty in finding their statistics. However, upper bounds for the end-to-end SNRs γ_{end} and γ'_{end} can be derived by using the well-known inequality between harmonic and geometric mean of positive RVs as $\gamma_{\text{end}}\gamma_{\text{ub}} = (1/N)\prod_{i=1}^{N}\gamma_i^{1/N}$ and $\gamma'_{\text{end}}\gamma'_{\text{ub}} = (1/N)\prod_{i=1}^{N} C_i^{-((N-i)/N)}\gamma_i^{(N+1-i)/N}$ [7], respectively. Using [7, Eqs. (19) and (21)], the PDFs of γ_{ub} and γ'_{ub} can be determined in closed-form by[1]

$$f_{\gamma_{\text{ub}}}(\gamma) = \frac{N\gamma^{-1}\prod_{i=1}^{N}\xi_i^2}{\prod_{i=1}^{N}\Gamma(\alpha_i)\Gamma(\beta_i)} G_{N,3N}^{3N,0}\left[N^N\gamma^N\prod_{i=1}^{N}\frac{\alpha_i\beta_i\xi_i^2}{(1+\xi_i^2)\mu_i}\middle|\begin{matrix}\kappa_1\\ \kappa_2\end{matrix}\right] \tag{6}$$

where $\kappa_1 = 1+\xi_N^2,\ldots,1+\xi_1^2$ and $\kappa_2 = \xi_1^2,\alpha_1,\beta_1,\ldots,\xi_N^2,\alpha_N,\beta_N$, and

$$f_{\gamma'_{\text{ub}}}(\gamma) = \frac{N\gamma^{-1}\prod_{i=1}^{N}\xi_i^2(N+1-i)^{\alpha_i+\beta_i-2}}{(2\pi)^{\frac{N(N-1)}{2}}\prod_{i=1}^{N}\Gamma(\alpha_i)\Gamma(\beta_i)} \times G_{\nu,3\nu}^{3\nu,0}\left[N^N\gamma^N\prod_{i=1}^{N}C_i^{N-i}\left[\frac{\alpha_i\beta_i\xi_i^2}{\mu_i(1+\xi_i^2)(N+1-i)^2}\right]^{N+1-i}\middle|\begin{matrix}\mathcal{J}_1\\ \mathcal{J}_2\end{matrix}\right] \tag{7}$$

where $\mathcal{J}_1 = \Delta(1,1+\xi_N^2),\ldots,\Delta(N,1+\xi_1^2)$, $\mathcal{J}_2 = \Delta(N,\xi_1^2),\Delta(N,\alpha_1),\Delta(N,\beta_1),\ldots,\Delta(1,\xi_N^2),\Delta(1,\alpha_N)$, $\Delta(1,\beta_N)$, $\Delta(k,u) = u/k,(u+1)/k,\ldots,(u+k-1)/k$, $\nu = N(N+1)/2$, and μ_i is the average SNR of the ith hop.

3. Performance Metrics

3.1. Moments

Using the well-known inequality for positive RVs, upper bounds for the nth-order moment of γ_{ub} and γ'_{ub}, respectively, can be expressed as $E[\gamma_{\text{end}}^n]E[\gamma_{\text{ub}}^n]$ and $E[\gamma_{\text{end}}'^n]E[\gamma_{\text{ub}}'^n]$, where $E[\cdot]$

[1] We have corrected some typographical errors in [7, Eq. (21)], and as such, $(N+1-i)$ in $\mathcal{P}$ should be replaced by $(N+1-i)^{\alpha_i+\beta_i-2}$, and $(N+1-i)$ in $\mathcal{Q}$ should be replaced by $(N+1-i)^2$.

denotes the expectation operator. Since the RVs γ_i are independent, the above equations can be reformulated as

$$E\left[\gamma_{\text{end}}^n\right] \leq E\left[\gamma_{\text{ub}}^n\right] = \frac{1}{N^n} E\left[\prod_{i=1}^{N} \gamma_i^{\frac{n}{N}}\right] = \frac{1}{N^n} \prod_{i=1}^{N} E\left[\gamma_i^{\frac{n}{N}}\right] \tag{8}$$

$$E\left[\gamma'^n_{\text{end}}\right] \leq E\left[\gamma'^n_{\text{ub}}\right] = \mathcal{R}_N \prod_{i=1}^{N} E\left[\gamma_i^{\frac{(N+1-i)n}{N}}\right] \tag{9}$$

where $\mathcal{R}_N = (1/N)\prod_{i=1}^{N} C_i^{-((N-i)/N)}$. For Gamma–Gamma fading channels with pointing error impairments operating under heterodyne detection, the moments $E[\gamma_i^n]$ can be obtained using [11, Eq. (07.34.21.0009.01)] in a closed form as

$$E\left[\gamma_i^n\right] = \frac{\xi_i^2 \Gamma(n+\alpha_i)\Gamma(n+\beta_i)}{\Gamma(\alpha_i)\Gamma(\beta_i)\left(n+\xi_i^2\right)} \left[\frac{\left(1+\xi_i^2\right)\mu_i}{\alpha_i \beta_i \xi_i^2}\right]^n. \tag{10}$$

Using (8) and (9), we obtain the moments of γ_{ub} and $\gamma_{\text{ub}'}$ as

$$E\left[\gamma_{\text{ub}}^n\right] = \frac{1}{N^n} \prod_{i=1}^{N} \frac{\xi_i^2 \Gamma\left(\frac{n}{N}+\alpha_i\right)\Gamma\left(\frac{n}{N}+\beta_i\right)}{\Gamma(\alpha_i)\Gamma(\beta_i)\left(\frac{n}{N}+\xi_i^2\right)} \left[\frac{\left(1+\xi_i^2\right)\mu_i}{\alpha_i \beta_i \xi_i^2}\right]^{\frac{n}{N}} \tag{11}$$

$$E\left[\gamma'^n_{\text{ub}}\right] = \mathcal{R}_N^n \prod_{i=1}^{N} \frac{\xi_i^2 \Gamma\left(\frac{(N+1-i)n}{N}+\alpha_i\right)\Gamma\left(\frac{(N+1-i)n}{N}+\beta_i\right)}{\Gamma(\alpha_i)\Gamma(\beta_i)\left(\frac{(N+1-i)n}{N}+\xi_i^2\right)} \left[\frac{\left(1+\xi_i^2\right)\mu_i}{\alpha_i \beta_i \xi_i^2}\right]^{\frac{(N+1-i)n}{N}}. \tag{12}$$

The reason for including the moments is that they are useful in deriving closed-form expressions for the amount of fading performance metric [16], and asymptotic results for the ergodic capacity at high and low SNR ranges, as will be shown in the next section.

3.2. Average BER

The average BER for a variety of binary modulation schemes can be written as [17]

$$P_e = \frac{1}{2\Gamma(p)} \int_0^\infty \Gamma(p, q\gamma) f_{\gamma_{ub}}(\gamma) d\gamma \tag{13}$$

where $\Gamma(\cdot,\cdot)$ is the complementary incomplete Gamma function [11, Eq. (06.06.02.0001.01)], and the parameters p and q account for different binary modulation schemes [18, Tab. 1]. Using (6) and (7) together with (13), transforming $\Gamma(\cdot,\cdot)$ to the Meijer's G function [11, Eq. (06.06.26.0005.01)], and applying [19, Eq. (2.24.1.1)] in (13) leads to the following closed-form lower bounds for the average BER of the N-hop FSO system equipped with CSI-assisted and fixed gain relays:

$$P_{e,\gamma_{\text{ub}}} = \frac{N^{p-\frac{1}{2}} \prod_{i=1}^{N} \xi_i^2}{2\Gamma(p)(2\pi)^{\frac{N-1}{2}} \prod_{i=1}^{N} \Gamma(\alpha_i)\Gamma(\beta_i)} \mathrm{G}_{2N+1,3N+1}^{3N,N+1}\left[\left(\frac{N^2}{q}\right)^N \prod_{i=1}^{N} \frac{\alpha_i \beta_i \xi_i^2}{\left(1+\xi_i^2\right)\mu_i} \middle| \begin{matrix} \kappa_3 \\ \kappa_2, 0 \end{matrix}\right] \tag{14}$$

$$P_{e,\gamma'_{\text{ub}}} = \frac{N^{p-\frac{1}{2}} \prod_{i=1}^{N} \xi_i^2 (N+1-i)^{\alpha_i+\beta_i-2}}{2\Gamma(p)(2\pi)^{\frac{N^2-1}{2}} \prod_{i=1}^{N} \Gamma(\alpha_i)\Gamma(\beta_i)} \mathrm{G}_{(N+1)\left(\frac{N}{2}+1\right),3\nu+1}^{3\nu,N+1}$$
$$\times \left[\left(\frac{N^2}{q}\right)^N \prod_{i=1}^{N} C_i^{N-i} \left[\frac{\alpha_i \beta_i \xi_i^2}{\left(1+\xi_i^2\right)(N+1-i)^2 \mu_i}\right]^{N+1-i} \middle| \begin{matrix} \mathcal{J}_3 \\ \mathcal{J}_2, 0 \end{matrix}\right] \tag{15}$$

where $\kappa_3 = 1, \Delta(N, 1-p), \kappa_1$ and $\mathcal{J}_3 = 1, \Delta(N, 1-p), \mathcal{J}_1$. At high average SNR regimes, the average BER expressions in (14) and (15) can be approximated accurately in terms of elementary functions by using the Meijer's G function expansion given in (A.1) in the Appendix as

$$P_{e,\gamma_{\rm ub}} \underset{\mu_i \gg 1}{\approx} \frac{N^{p-\frac{1}{2}} \prod_{i=1}^{N} \xi_i^2}{2\Gamma(p)(2\pi)^{\frac{N-1}{2}} \prod_{i=1}^{N} \Gamma(\alpha_i)\Gamma(\beta_i)} \sum_{k=1}^{3N} \left[\left(\frac{q}{N^2}\right)^N \prod_{i=1}^{N} \frac{(1+\xi_i^2)\mu_i}{\alpha_i\beta_i\xi_i^2} \right]^{-\kappa_{2,k}} \times \frac{\prod_{l=1;l\neq k}^{3N} \Gamma(\kappa_{2,l}-\kappa_{2,k}) \prod_{l=1}^{N+1} \Gamma(1+\kappa_{2,k}-\kappa_{3,l})}{\Gamma(1+\kappa_{2,k}) \prod_{l=N+2}^{2N+1} \Gamma(\kappa_{3,l}-\kappa_{2,k})} \tag{16}$$

$$P_{e,\gamma'_{\rm ub}} \underset{\mu_i \gg 1}{\approx} \frac{N^{p-\frac{1}{2}} \prod_{i=1}^{N} \xi_i^2 (N+1-i)^{\alpha_i+\beta_i-2}}{2\Gamma(p)(2\pi)^{\frac{N^2-1}{2}} \prod_{i=1}^{N} \Gamma(\alpha_i)\Gamma(\beta_i)} \sum_{k=1}^{3\nu} \left[\left(\frac{q}{N^2}\right)^N \prod_{i=1}^{N} C_i^{i-N} \left[\frac{(1+\xi_i^2)(N+1-i)^2\mu_i}{\alpha_i\beta_i\xi_i^2}\right]^{N+1-i} \right]^{-\mathcal{J}_{2,k}} \times \frac{\prod_{l=1;l\neq k}^{3\nu} \Gamma(\mathcal{J}_{2,l}-\mathcal{J}_{2,k}) \prod_{l=1}^{N+1} \Gamma(1+\mathcal{J}_{2,k}-\mathcal{J}_{3,l})}{\Gamma(1+\mathcal{J}_{2,k}) \prod_{l=N+2}^{(N+1)\left(\frac{N}{2}+1\right)} \Gamma(\mathcal{J}_{3,l}-\mathcal{J}_{2,k})} \tag{17}$$

where $\kappa_{i,j}$ accounts for the jth term of κ_i, and $\mathcal{J}_{i,j}$ represents the jth term of $\mathcal{J}_i$. In addition, the average BER in (16) and (17) can be further expressed via only the dominant terms $d = \min(\alpha_i, \beta_i, \xi_i^2)$ and $d' = \min(\alpha_i/N, \beta_i/N, \xi_i^2/N)$, respectively. Furthermore, utilizing [20, Eq. (1)], the average BERs can be approximated as $P_{e,\gamma_{\rm ub}} \approx (G_c\mu)^{-G_d}$ and $P_{e,\gamma'_{\rm ub}} \approx (G'_c\mu)^{-G'_d}$, respectively. For the same average SNR per hop ($\mu_i = \mu$), the diversity orders of a multihop FSO system in operation under the heterodyne detection technique using CSI-assisted and fixed-gain relays can be obtained as $G_d = Nd$ and $G'_d = (N(N+1)d')/2$, respectively. Moreover, the coding gains can be easily derived as

$$G_c = \left[\left(\frac{q}{N^2}\right)^N \prod_{i=1}^{N} \frac{(1+\xi_i^2)}{\alpha_i\beta_i\xi_i^2} \right]^{\frac{1}{N}} \left[\frac{N^{p-\frac{1}{2}} \prod_{i=1}^{N} \xi_i^2}{2\Gamma(p)(2\pi)^{\frac{N-1}{2}} \prod_{i=1}^{N} \Gamma(\alpha_i)\Gamma(\beta_i)} \times \frac{\prod_{l=1;l\neq k}^{3N} \Gamma(\kappa_{2,l}-\kappa_{2,k}) \prod_{l=1}^{N+1} \Gamma(1+\kappa_{2,k}-\kappa_{3,l})}{\Gamma(1+\kappa_{2,k}) \prod_{l=N+2}^{2N+1} \Gamma(\kappa_{3,l}-\kappa_{2,k})} \right]^{\frac{-1}{Nd}} \tag{18}$$

$$G'_c = \left[\left(\frac{q}{N^2}\right)^N \prod_{i=1}^{N} C_i^{i-N} \left[\frac{(1+\xi_i^2)(N+1-i)^2}{\alpha_i\beta_i\xi_i^2}\right]^{N+1-i} \right]^{\frac{2}{N(N+1)}} \times \left[\frac{N^{p-\frac{1}{2}} \prod_{i=1}^{N} \xi_i^2 (N+1-i)^{\alpha_i+\beta_i-2}}{2\Gamma(p)(2\pi)^{\frac{N^2-1}{2}} \prod_{i=1}^{N} \Gamma(\alpha_i)\Gamma(\beta_i)} \frac{\prod_{l=1;l\neq k}^{3\nu} \Gamma(\mathcal{J}_{2,l}-\mathcal{J}_{2,k}) \prod_{l=1}^{N+1} \Gamma(1+\mathcal{J}_{2,k}-\mathcal{J}_{3,l})}{\Gamma(1+\mathcal{J}_{2,k}) \prod_{l=N+2}^{(N+1)\left(\frac{N}{2}+1\right)} \Gamma(\mathcal{J}_{3,l}-\mathcal{J}_{2,k})} \right]^{\frac{-2}{N(N+1)d'}}. \tag{19}$$

3.3. Ergodic Capacity

The ergodic capacity can be formulated as $\overline{C} = E[\log_2(1+\gamma)]$. Since $\gamma_{\rm end} \le \gamma_{\rm ub}$ and $\gamma'_{\rm end} \le \gamma'_{\rm ub}$, $\log_2(1+\gamma_{\rm end}) \le \log_2(1+\gamma_{\rm ub})$ and $\log_2(1+\gamma'_{\rm end}) \le \log_2(1+\gamma'_{\rm ub})$, and therefore, upper bounds for the ergodic capacity of both CSI-assisted and fixed-gain relays can be derived. Using the Meijer's G function representation of $\ln(1+\gamma)$ [11, Eq. (01.04.26.0003.01)] then integrating using [19, Eq. (2.24.1.1)], the ergodic capacity of an N-hop heterodyne FSO system employing CSI-assisted and fixed-gain relays can be upper bounded as

$$\overline{C}_{\gamma_{\rm ub}} = \frac{\prod_{i=1}^{N} \xi_i^2}{\ln(2)(2\pi)^{N-1} \prod_{i=1}^{N} \Gamma(\alpha_i)\Gamma(\beta_i)} G_{2N+1,4N+1}^{4N+1,N}\left[N^N \prod_{i=1}^{N} \frac{\alpha_i\beta_i\xi_i^2}{(1+\xi_i^2)\mu_i} \middle| \begin{matrix} \kappa_4 \\ \kappa_5 \end{matrix} \right] \tag{20}$$

$$\overline{C}_{\gamma'_{\rm ub}} = \frac{\prod_{i=1}^{N} \xi_i^2 (N+1-i)^{\alpha_i+\beta_i-2}}{\ln(2)(2\pi)^{(N-1)\left(\frac{N}{2}+1\right)} \prod_{i=1}^{N} \Gamma(\alpha_i)\Gamma(\beta_i)} \times G_{(N+1)\left(\frac{N}{2}+1\right),\vartheta}^{\vartheta,N}\left[N^N \prod_{i=1}^{N} C_i^{N-i} \left[\frac{\alpha_i\beta_i\xi_i^2}{(1+\xi_i^2)(N+1-i)^2\mu_i}\right]^{N+1-i} \middle| \begin{matrix} \mathcal{J}_4 \\ \mathcal{J}_5 \end{matrix} \right] \tag{21}$$

where $\kappa_4 = \Delta(N,0),1,\kappa_1$, $\kappa_5 = \kappa_2,\Delta(N,0),0$, $\vartheta = (N+1)((3N/2)+1)$, $\mathcal{J}_4 = \Delta(N,0),1,\mathcal{J}_1$, and $\mathcal{J}_5 = \mathcal{J}_2,\Delta(N,0),0$.

At high SNR, the ergodic capacity in (20) and (21) can be asymptotically approximated in terms of simple elementary functions as

$$\overline{C}_{\gamma_{\rm ub}} \underset{\mu_i \gg 1}{\approx} \frac{\prod_{i=1}^{N}\xi_i^2}{\ln(2)(2\pi)^{N-1}\prod_{i=1}^{N}\Gamma(\alpha_i)\Gamma(\beta_i)} \sum_{k=1}^{4N+1}\left[N^{-N}\prod_{i=1}^{N}\frac{(1+\xi_i^2)\mu_i}{\alpha_i\beta_i\xi_i^2}\right]^{-\kappa_{5,k}} \times \frac{\prod_{l=1;l\neq k}^{4N+1}\Gamma(\kappa_{5,l}-\kappa_{5,k})\prod_{l=1}^{N}\Gamma(1+\kappa_{5,k}-\kappa_{4,l})}{\prod_{l=N+1}^{2N+1}\Gamma(\kappa_{4,l}-\kappa_{5,k})} \tag{22}$$

$$\overline{C}_{\gamma'_{\rm ub}} \underset{\mu_i \gg 1}{\approx} \frac{\prod_{i=1}^{N}\xi_i^2(N+1-i)^{\alpha_i+\beta_i-2}}{\ln(2)(2\pi)^{(N-1)\left(\frac{N}{2}+1\right)}\prod_{i=1}^{N}\Gamma(\alpha_i)\Gamma(\beta_i)} \sum_{k=1}^{\vartheta}\left[N^{-N}\prod_{i=1}^{N}C_i^{i-N}\left[\frac{(1+\xi_i^2)(N+1-i)^2\mu_i}{\alpha_i\beta_i\xi_i^2}\right]^{N+1-i}\right]^{-\mathcal{J}_{5,k}} \times \frac{\prod_{l=1;l\neq k}^{\vartheta}\Gamma(\mathcal{J}_{5,l}-\mathcal{J}_{5,k})\prod_{l=1}^{N}\Gamma(1+\mathcal{J}_{5,k}-\mathcal{J}_{4,l})}{\prod_{l=N+1}^{(N+1)\left(\frac{N}{2}+1\right)}\Gamma(\mathcal{J}_{4,l}-\mathcal{J}_{5,k})}. \tag{23}$$

Alternatively, simple asymptotic expressions for the ergodic capacity in (20) and (21) in the high SNR region may also be obtained from the first derivative of the nth-order moment of $\gamma_{\rm ub}$ and $\gamma_{\rm ub'}$ [16, Eqs. (8) and (9)] as

$$\overline{C}_{\gamma_{\rm ub}} \approx \frac{\partial}{\partial n}E\left[\gamma_{\rm ub}^n\right]\Big|_{n=0} \underset{\mu_i \gg 1}{\approx} -\log(N)+\log\left(\prod_{i=1}^{N}\mu_i^{\frac{1}{N}}\right)+\frac{1}{N}\sum_{i=1}^{N}\left[\psi(\alpha_i)+\psi(\beta_i)-\log\left(\frac{\alpha_i\beta_i\xi_i^2}{1+\xi_i^2}\right)-\frac{1}{\xi_i^2}\right] \tag{24}$$

$$\overline{C}_{\gamma'_{\rm ub}} \approx \frac{\partial}{\partial n}E\left[\gamma_{\rm ub}^{\prime n}\right]\Big|_{n=0} \underset{\mu_i \gg 1}{\approx} \log(\mathcal{R}_N)+\log\left(\prod_{i=1}^{N}\mu_i^{\frac{N+1-i}{N}}\right) + \frac{1}{N}\sum_{i=1}^{N}(N+1-i)\left[\psi(\alpha_i)+\psi(\beta_i)-\log\left(\frac{\alpha_i\beta_i\xi_i^2}{1+\xi_i^2}\right)-\frac{1}{\xi_i^2}\right] \tag{25}$$

where $\psi(\cdot)$ is the psi (digamma) function [11, Eq. (06.14.27.0001.01)]. Furthermore, the ergodic capacity of a multihop FSO system using CSI-assisted and fixed-gain relays can be approximated by the first moment in the low SNR regime in terms of simple elementary functions by

$$\overline{C}_{\gamma_{\rm ub}} \approx E[\gamma_{\rm ub}] \underset{\mu_i \ll 1}{\approx} \frac{1}{N}\prod_{i=1}^{N}\frac{\xi_i^2\Gamma\left(\frac{1}{N}+\alpha_i\right)\Gamma\left(\frac{1}{N}+\beta_i\right)}{\Gamma(\alpha_i)\Gamma(\beta_i)\left(\frac{1}{N}+\xi_i^2\right)}\left[\frac{(1+\xi_i^2)\mu_i}{\alpha_i\beta_i\xi_i^2}\right]^{\frac{1}{N}} \tag{26}$$

$$\overline{C}_{\gamma'_{\rm ub}} \approx E[\gamma'_{\rm ub}] \underset{\mu_i \ll 1}{\approx} \mathcal{R}_N\prod_{i=1}^{N}\frac{\xi_i^2\Gamma\left(\frac{N+1-i}{N}+\alpha_i\right)\Gamma\left(\frac{N+1-i}{N}+\beta_i\right)}{\Gamma(\alpha_i)\Gamma(\beta_i)\left(\frac{N+1-i}{N}+\xi_i^2\right)}\left[\frac{(1+\xi_i^2)\mu_i}{\alpha_i\beta_i\xi_i^2}\right]^{\frac{(N+1-i)}{N}}. \tag{27}$$

4. Numerical Results and Discussion

In this section, we illustrate the performance of multihop heterodyne FSO systems under the effects of pointing error and atmospheric turbulence. Here, we consider the case that the average SNRs per hop for all hops are equal $\overline{\gamma}_i = \overline{\gamma}$. Weak ($\alpha = 2.902$ and $\beta = 2.51$), moderate ($\alpha = 2.296$ and $\beta = 1.822$), and strong ($\alpha = 2.064$ and $\beta = 1.342$) turbulence conditions are considered [15, Tab. I].

In Fig. 1, lower bounds for the average BER of BFSK ($p = 1/2$ and $q = 1/2$), BPSK ($p = 1/2$ and $q = 1$), and DPSK ($p = 1$ and $q = 1$) binary modulation schemes of a multihop FSO system ($N = 3$) using CSI-assisted relays along with their asymptotic results at high SNR are plotted as a function of the average SNR per hop under strong turbulence conditions. As expected, it can be shown that the average BER improves as the pointing error effect gets negligible ($\xi \to \infty$).

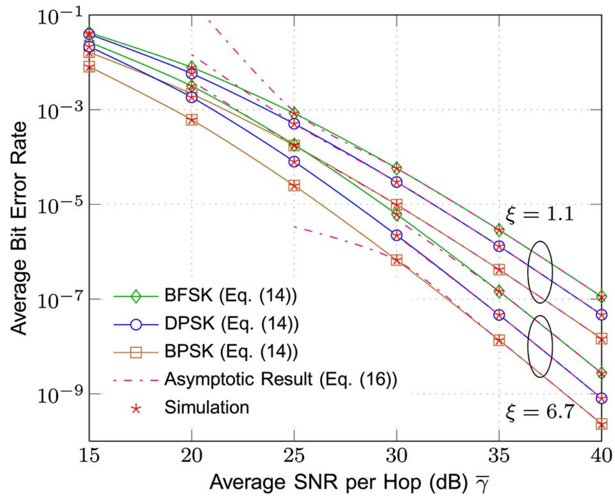

Fig. 1. Average BER of BFSK, BPSK, and DBSK binary modulation schemes for a multihop heterodyne FSO system using CSI-assisted relays under strong turbulence conditions for $N = 3$.

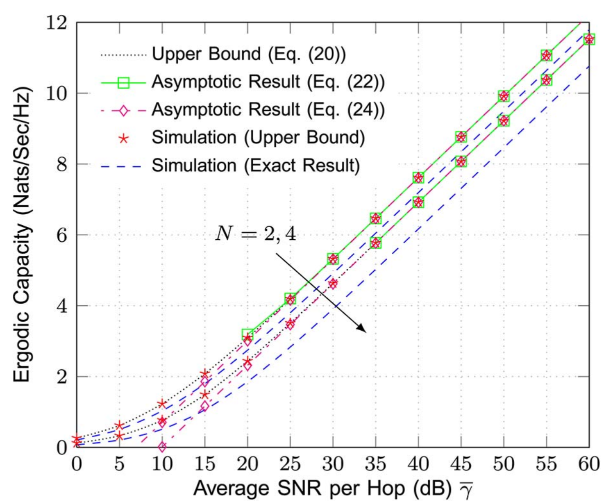

Fig. 2. Ergodic capacity for a multihop heterodyne FSO system using CSI-assisted relays under strong turbulence with strong pointing error ($\xi = 1.1$).

Monte-Carlo simulations are also provided, and a perfect match with the analytical results is observed. Additionally, it can be seen that at high SNR, the asymptotic results converge to the exact results proving the tightness and the accuracy of this asymptotic approximation. It can be also seen from this figure that BPSK performs better than the other modulation schemes. Moreover, DPSK and BFSK have the same performance at lower SNR, whereas as the SNR increases, DPSK outperforms BFSK.

Fig. 2 depicts the ergodic capacity of a multihop heterodyne FSO system using CSI-assisted relays for strong turbulence conditions with strong pointing error ($\xi = 1.1$) for $N = 2$ and $N = 4$. As illustrated in Fig. 2, the analytical results for the upper bound on the ergodic capacity obtained in (20) have been verified by means of computer simulations and a perfect agreement is observed. Moreover, the exact ergodic capacity results obtained via Monte-Carlo simulations based on (1) are also included to prove the tightness of the obtained bound. In addition, Fig. 2 indicates that the lower the values of N, the tighter the upper bounds are even at high average SNR. Also, we can see from Fig. 2 that the ergodic capacity degrades as the number of hop N

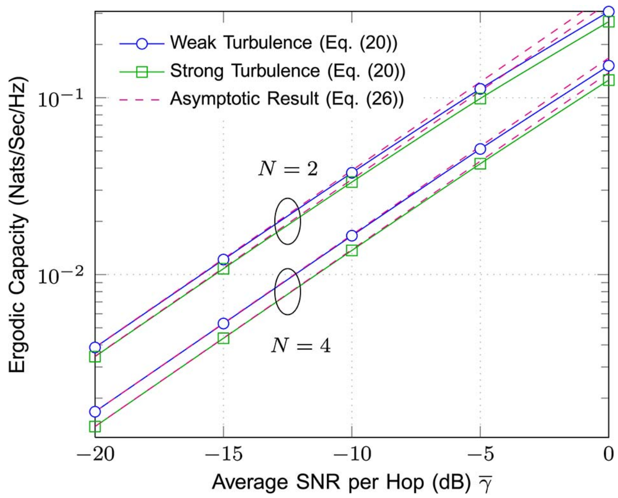

Fig. 3. Ergodic capacity for a multihop heterodyne FSO system using CSI-assisted relays under weak and strong turbulence conditions for strong pointing error ($\xi = 1.2$) along with the asymptotic results in the low SNR regime.

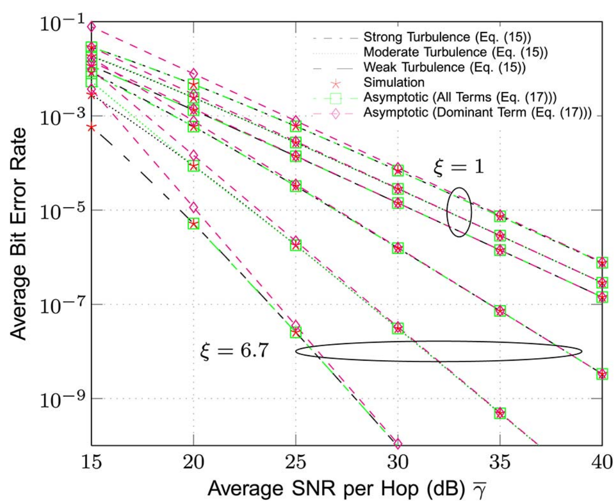

Fig. 4. Average BER of DPSK modulation scheme for a multihop FSO system using fixed-gain relays with heterodyne detection for $N = 3$ under weak, moderate, and strong turbulence conditions for varying effects of the pointing error along with the asymptotic results in the high SNR regime.

increases. Finally, the high accuracy of the asymptotic analysis via the Meijer's G function expansion in (22) or via the moments-based approach in (24) at high SNRs is clearly observed.

Fig. 3 presents tight asymptotic results for the upper bound on the ergodic capacity in the low SNR regime obtained in (26).

In Fig. 4, the average BER of DPSK modulation scheme for a 3-hop heterodyne FSO system equipped with fixed-gain relays is demonstrated with varying effects of the pointing error, $\xi = 1$ and 6.7, for strong, moderate, and weak turbulence conditions. Expectedly, as the pointing error gets severe ($\xi \rightarrow 0$) and/or as the atmospheric turbulence conditions get severe, the average BER increases (i.e., the higher the values of α and β, and/or ξ, the lower will be the average BER). It can be also seen that the asymptotic expression at high SNR obtained via the Meijer's G function expansion in (17) (utilizing all the terms in the summation) matches the exact results perfectly proving the tightness of this asymptotic approximation. Moreover, the asymptotic result based on the appropriate dominant term converges to the exact result though relatively slower.

5. Conclusion

In this paper, we have derived closed-form bounds for the average BER for a variety of binary modulation schemes, and the ergodic capacity of a multihop heterodyne FSO system using AF CSI-assisted or fixed-gain relays over Gamma–Gamma fading channels with pointing errors. Furthermore, new asymptotic results were presented for the average BER, as well as the ergodic capacity at high SNR. Moreover, using the moments-based approach, new accurate asymptotic results for the ergodic capacity are obtained in the low and high SNR regimes. Overall, the performance degrades as the pointing error effect and/or the atmospheric turbulence conditions become severe and with an increase in the number of hops.

Appendix
Asymptotic Expansion of the Meijer's G Function

The asymptotic expansion of the Meijer's G function can be written using [21, Eq.(1.4.13)] as

$$\lim_{x\to+\infty} G_{p,q}^{m,n}\left(x \middle| \begin{matrix} a_1,\ldots,a_n,\ldots,a_p \\ b_1,\ldots,b_m,\ldots,b_q \end{matrix}\right) = \sum_{k=1}^{n} x^{a_k-1} \times \frac{\prod_{l=1;=;l\neq k}^{n}\Gamma(a_k-a_l)\prod_{l=1}^{m}\Gamma(1+b_l-a_k)}{\prod_{l=n+1}^{p}\Gamma(1+a_l-a_k)\prod_{l=m+1}^{q}\Gamma(a_k-b_l)} \tag{A.1}$$

with $a_k - a_l \neq 0, \pm1, \pm2, \ldots; (k, l = 1, \ldots, n; k \neq l)$ and $a_k - b_l \neq 1, 2, 3, \ldots; (k = 1, \ldots, n; l = 1, \ldots, m)$.

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