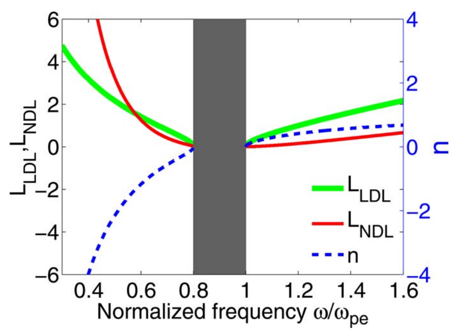
IEEE Photonics Journal
An IEEE Photonics Society Publication

Open Access

Collapse of Optical Wave Controlled by Anomalous Nonlinear Diffraction in Metamaterials

Volume 7, Number 2, April 2015

Jinggui Zhang
Ying Li
Lifu Zhang
Yuanjiang Xiang
Xiongjie Kang
Shuangchun Wen



DOI: 10.1109/JPHOT.2015.2413592

IEEE photonics SOCIETY

IEEE

Collapse of Optical Wave Controlled by Anomalous Nonlinear Diffraction in Metamaterials

Jinggui Zhang,[1,2] Ying Li,[1] Lifu Zhang,[1] Yuanjiang Xiang,[1] Xiongjie Kang,[4] and Shuangchun Wen[1,3]

[1]Key Laboratory of Optoelectronic Devices and Systems of the Ministry of Education and Guangdong Province, College of Optoelectronic Engineering, Shenzhen University, Shenzhen 518060, China
[2]Department of Information Science and Engineering, Hunan First Normal University, Changsha 410205, China
[3]Key Laboratory of Micro-/Nano-Optoelectronic Devices of the Ministry of Education, School of Physics and Electronics, Hunan University, Changsha 410082, China
[4]Department of Information Engineering, Shaoyang University, Shaoyang 422000, China

DOI: 10.1109/JPHOT.2015.2413592

Manuscript received January 23, 2015; revised March 8, 2015; accepted March 11, 2015. Date of publication March 24, 2015; date of current version April 9, 2015. This work was supported in part by the Research Foundation of Education Bureau of Hunan Province under Grant 14B038, by the Natural Science Foundation of Hunan Province under Grant 2015JJ2036, by the Natural Science Foundation of Shenzhen University under Grant 201449 and Grant 201452, and by the Program of Fundamental Research of the Shenzhen Science and Technology Plan under Grant JCYJ20140828163634005 and Grant JCYJ20140828163633996. Corresponding author: L. Zhang (e-mail: zhanglifu68@hotmail.com).

Abstract: In this paper, we put forward a novel method for the management of optical collapse through controlling anomalous nonlinear diffraction (NLD) originating from the vectoral effect in metamaterials with simultaneous cubic electric and magnetic nonlinearity, particularly focusing on the unusual behaviors different from those in ordinary positive-index materials. First, NLD in metamaterials can be positive or negative, which makes it possible for us to offer greatly enhanced design freedom to manipulate the behavior of optical collapse at will. Then, it is further demonstrated that the negative NLD can arrest the catastrophic self-focusing of optical wave; however, the positive NLD will lead to the acceleration of its occurrence, as opposed to the case in ordinary positive-index materials. Our analysis is performed by directly numerically solving the nonlinear Schrödinger equation, as well as by using the modulation theory, both showing consistent results. These findings show that NLD in metamaterials can provide a more powerful tool in controlling the collapse of optical wave when compared with the corresponding case in ordinary positive-index materials.

Index Terms: Nonlinear optics, metamaterials, self-focusing.

1. Introduction

Nonlinear wave collapse is an intrinsic feature in many areas of physics, including optics [1], [2]; plasma physics [3]; hydrodynamics [4]; and Bose–Einstein condensates [5], [6]. In optics, propagation of a laser beam through a transparent medium is governed by the two-dimensional NLSE, and wave collapse occurs when nonlinear focusing due to the intensity-dependent refractive index overcomes linear diffraction [1], [2], [7]. Over the past several years, a great deal of physical mechanisms for preventing the optical collapse are gradually unfolded, such as multiphoton absorption, plasma formation, temporal dispersion, *et al.* [1], [2], [7]–[16]. In particular, when close to the point of collapse, nonlinear diffraction resulting from vectoral effect as another important physical mechanism must be considered to capture properly the optical propagation dynamics during the course of arresting optical collapse [16]–[22]. Up to now, its roles in controlling the nonlinear propagation of optical wave have been well understood in ordinary positive-index materials [16]–[22]. Specifically speaking, Chi *et al.* has taken into account effect of nonlinear diffraction in the deriving the vector model of self-focusing, and found that nonlinear diffraction can lead to the occurrence of noncatastrophic self-focusing [15]. Prior to this, nonlinear diffraction was also included in calculations of nonlinear coefficients in the NLSE by Boardman *et al.* and Shivarova *et al.* [19], [20]. Their studies of self-phase modulation in optical fibers show that the nonzero divE-term with E being electric field vector influences temporal soliton formation. Meanwhile, Boardman *et al.* also discussed that the effects of nonlinear diffraction on spatial solitary waves in cubic, as well as quadratic, nonlinear optical waveguides [21], [22]. In particular, it is further identified as being the main contributor to the stabilization of the beam giving an effect which even prevails over those of nonparaxiality and longitudinal field component in ordinary positive-index materials [16]–[22].

Recent progress in nanofabrication has led to design of new metamaterials with highly unusual optical properties unattainable in naturally occurring materials, as well as a variety of unprecedented applications [23]–[26]. The existence of such media was demonstrated experimentally first in the microwave, and then in the optical ranges. Very recently, quick advances in the fabrication of metamaterial, ranging from linearity to nonlinearity have stimulated a great deal of research including the revaluation and characterization of the classical nonlinear optical processes [28]. Since 2005, some dynamical models for describing ultrashort pulse propagation in metamaterials with a Kerr polarization and/or a Kerr magnetization have been established and several intriguing and counterintuitive nonlinear phenomena associated with optical solitons as well as modulation instability (MI) [28]–[45]. Meanwhile, the theoretical study of self-focusing phenomena in Kerr nonlinear metamaterials recently become a new and exciting field of research that has been the subject of intense investigations [28]–[45]. It is found that the negative index will lead to the occurrence of optical collapse in the self-defocusing metamaterial while it only occurs in the focusing regime in ordinary positive-index materials [38]. Afterwards, we also showed that it is the divergent rather than convergent incident beams which are self-focused more quickly in metamaterials, in sharp contrast with the propagation property of beams in conventional Kerr media, in which a convergent incident beam self-focuses more quickly than a divergent one [40]. Very recently, we further demonstrated that the anomalous self-steepening effect in metamaterial s can be tailored through structure design, showing significant potential to control the collapse of ultrashort pulse [42]. In addition, we disclosed the anomalous behavior of induced focusing in metamaterials through designing probe-pump configuration where the coupled waves may simultaneously experience negative refractive index or positive refractive index, or one experiences positive refractive index and another experiences negative refractive index [43]. All the studies suggest that Kerr-type metamaterials may be a new but important candidate for application in control of light beam propagation due to rich and unusual linear and nonlinear electromagnetic properties. However, up to now, all foregoing researches on optical self-focusing in metamaterials have only been confined to the scalar approximation. In fact, this condition have previously demonstrated to be broken down when the contribution of the nonlinear polarization to the displacement vector are considered in ordinary positive-index materials, thus leading to a significant influence on the soliton dynamics, as the beams become narrow [16]–[22]. Meanwhile, a metamaterial may exhibit additional nonlinear magnetization, in which case, the common assumption that $\nabla \cdot \mathbf{H} = 0$, where H is the magnetic field vector, also breaks down, leading once again to nonlinear diffraction term [44], [45]. Therefore, motivated by the foregoing researches, we can predict that the NLD-induced novel phenomena of optical collapse may also arise in metamaterials with simultaneous cubic electric and magnetic nonlinearity, just as the case in ordinary positive-index materials. However, what is the difference between them and how do the properties of optical collapse change in metamaterials? We will reply them in this paper. Therefore, based on our pioneer research we further put forward the novel method for the management of optical wave collapse through controlling nonlinear diffraction originating from the vectoral effect in metamaterials with simultaneous cubic electric and magnetic nonlinearity. Here, using both the direct simulations as well as the modulation theory applied to the derived physical system that describe beam propagation in nonlinear metamaterials, we try to disclose the unusual behaviors of optical collapse associating with the unique and engineerable electromagnetic properties of such materials and further elucidate the underlying physical mechanisms.

The rest of this paper has been split into several parts. In Section 2, we will derive the theoretical model that describe optical waves propagating including the vectoral nonparaxial effect in metamaterials with simultaneous cubic electric and magnetic nonlinearity. In Section 3, first we will discuss the anomalous linear and nonlinear properties in metamaterials. Then by utilizing the modulation theory we will arrive at an explicit, although approximate, analytical solution for NLSE. Further based on the numerical simulation and the approximate, analytical solution, we will give a detailed discussion of our main observation involving the collapse of optical wave controlled by nonlinear diffraction. Finally, a brief conclusion will be given in Section 4.

2. Theoretical Model

In this section, taking into full consideration the unique characteristics of such materials, we will derive the theoretical model including the vectoral nonparaxial effect in metamaterials mainly by following the classical steps for deducing NLSEs in ordinary dielectrics. Therefore, we assume that the linearly polarized electromagnetic plane wave propagates in metamaterials having simultaneous nonlinear electric polarization and nonlinear magnetization, in which there are no free charges and no free currents flow. Based on our previous works, we easily obtain the following coupled wave equations from the Maxwell's equations [39], [44]:

$$\left(\partial_z^2+\nabla_\perp^2\right)\mathbf{E}-\nabla(\nabla\cdot\mathbf{E})-\mu\varepsilon\partial_t^2\mathbf{E}=\mu\partial_t^2 P_{\mathrm{NL}}+\partial_t(\nabla\times M_{\mathrm{NL}}) \tag{1a}$$

$$\left(\partial_z^2+\nabla_\perp^2\right)\mathbf{H}-\nabla(\nabla\cdot\mathbf{H})-\mu\varepsilon\partial_t^2\mathbf{H}=\varepsilon\partial_t^2 M_{\mathrm{NL}}-\partial_t(\nabla\times \mathbf{P}_{\mathrm{NL}}) \tag{1b}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields, respectively, and $\mathbf{P}_{\mathrm{NL}}$ and M_{NL} are the nonlinear polarization and magnetization, respectively, The electric and magnetic fields here are assumed to propagate along the z direction, and so $\nabla_\perp^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is transverse Laplace operator. $\varepsilon=\varepsilon_0\varepsilon_r$ and $\mu=\mu_0\mu_r$ are the electric permittivity and magnetic permeability in medium, respectively. Here, ε_0 and ε_r are the electric permittivity in vacuum and relative electric permittivity of media, respectively, and μ_r and μ_0 are magnetic permeability in vacuum and relative magnetic permeability of media. Both $\nabla(\nabla\cdot\mathbf{E})$ and $\nabla(\nabla\cdot\mathbf{H})$ account for the transverse inhomogeneity of the medium polarization and magnetization. This possibility is previously eliminated in metamaterials [31]–[39], by the assumption that $\nabla\cdot\mathbf{E}=0$ and $\nabla\cdot\mathbf{H}=0$. In fact, in ordinary positive-index materials this condition has demonstrated to be broken down when the contribution of the nonlinear polarization to the displacement vector are considered [16]–[22], thus leading to a significant influence on the self-focusing. Meanwhile, a metamaterial may exhibit additional nonlinear magnetization, in which case, the common assumption that $\nabla\cdot\mathbf{H}=0$ also breaks down, leading once again to nonlinear diffraction [44], [45]. Therefore, a reexamination of optical wave collapse in metamaterials is necessary due to such unusual electromagnetic characteristics of metamaterials.

To further process (1a) and (1b), we need to make an approximate assumptions as following: since the longitudinal field components for both electric and magnetic field are an order of magnitude smaller than the corresponding transverse component, and therefore, we neglect the longitudinal field components for simplicity, as does Boardman *et al.* In addition, we restrict our attention to only linearly x-(y-)polarized electric (magnetic) field. Therefore, when we introduce an envelope and carrier form for field in usual way, the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ can be written in the form $\mathbf{E}=\hat{x}E\exp[i(\beta_0 z-\omega_0 t)]+c.c$, and $\mathbf{H}=\hat{y}H\exp[i(\beta_0 z-\omega_0 t)]+c.c$, respectively. And the nonlinear polarization $\mathbf{P}_{\mathrm{NL}}$ and nonlinear magnetization $\mathbf{M}_{\mathrm{NL}}$ can also be expressed in a similar way by writing $\mathbf{P}_{\mathrm{NL}} = \hat{x} P_{\mathrm{NL}} \exp[i(\beta_0 z - \omega_0 t)] + c.c$ and $\mathbf{M}_{\mathrm{NL}} = \hat{y} M_{\mathrm{NL}} \times \exp[i(\beta_0 z - \omega_0 t)] + c.c$, where $\beta_0 = n\omega_0/c$ is the wave number with n being the refractive index of the material and c being light velocity in vacuum, ω_0 is the carrier frequency of the electromagnetic pulse, and c.c. denotes the complex conjugate. Under these assumptions, from (1a) and (1b) we easily obtain the following scalar coupled equations that describe the evolution of the envelop amplitudes of electric and magnetic fields

$$\left(\partial_z^2 + 2i\beta_0\partial z + \nabla_\perp^2\right)E + \partial_x^2(P_{\mathrm{NL}})/(\varepsilon_0\varepsilon_r) = -\omega_0^2\mu P_{\mathrm{NL}} - \omega_0\beta_0 M_{\mathrm{NL}} \tag{2a}$$

$$\left(\partial_z^2 + 2i\beta_0\partial z + \nabla_\perp^2\right)H + \partial_y^2\left(M_{\mathrm{NL}}/(\mu_0\mu_r)\right) = -\omega_0^2\varepsilon M_{\mathrm{NL}} - \omega_0\beta_0 P_{\mathrm{NL}}. \tag{2b}$$

In deriving the above coupled equations, we also employ the approximate relation $\partial P_{\mathrm{NL}}/\partial z \approx 0$ and $\partial M_{\mathrm{NL}}/\partial z \approx 0$ that have been used in [39] and [44]. Obviously, the system above is suitable for any form of nonlinear polarization and magnetization because we have not exerted any limitations on them up to now. In the following analysis, we consider a nonlinear metamaterial created by arrays of wires and split-ring resonators embedded into a nonlinear Kerr dielectric [27]. This metamaterial has a Kerr-type nonlinear polarization and a comparatively complicated form of nonlinear magnetization. For a relatively small magnetic field intensity, however, the nonlinear magnetization can also be taken as of the Kerr type [29], [30]. This assumption has been justified numerically in [29]. We thus assume that the nonlinear polarization PNL and the nonlinear magnetization M_{NL} take the forms $P_{\mathrm{NL}} = \varepsilon_0\chi_e^{(3)}|E|^2E$, $M_{\mathrm{NL}} = \mu_0\chi_m^{(3)}|H|^2H$, where $\chi_e^{(3)}$ and $\chi_m^{(3)}$ are the third-order electric and magnetic susceptibility, respectively, and therefore, the above coupled (2a) and (2b) can be transformed into the following form:

$$i\frac{\partial E}{\partial z} + \frac{1}{2\beta_0}\nabla_\perp^2 E + \frac{\omega_0^2\mu\varepsilon_0\chi_e^{(3)}|E|^2E}{2\beta_0} + \frac{\omega_0\mu_0\chi_m^{(3)}|\mathrm{H}|^2H}{2} + \frac{1}{2\beta_0\varepsilon_0\varepsilon_r}\frac{\partial^2\varepsilon_0\chi_e^{(3)}|E|^2E}{\partial x^2} + \frac{1}{2\beta_0}\frac{\partial^2 E}{\partial z^2} = 0 \tag{3a}$$

$$i\frac{\partial H}{\partial z} + \frac{1}{2\beta_0}\nabla_\perp^2 H + \frac{\omega_0^2\varepsilon\mu_0\chi_m^{(3)}|H|^2H}{2\beta_0} + \frac{\omega_0\varepsilon_0\chi_e^{(3)}|E|^2E}{2} + \frac{1}{2\beta_0\mu_0\mu_r}\frac{\partial^2\mu_0\chi_m^{(3)}|H|^2H}{\partial y^2} + \frac{1}{2\beta_0}\frac{\partial^2 H}{\partial z^2} = 0. \tag{3b}$$

Obviously, if we only consider one-dimensional case, equations (3a) and (3b) are reduced to the propagation equation obtained by Boardman *et al.* [44] in metamaterials. If there is no the last two terms in the left side of (3a) and (3b), the system is identical to our result obtained in the case where there is no temporal term in metamaterials [39], [41]. In addition, one notes that when $M_{\mathrm{NL}} = 0$ and $\mu_r = 1$, the propagation equation for electric field, i.e., (3a), is identical to the conventional theoretical model in ordinary dielectrics if the same form of nonlinear polarization is assumed. Obviously, the equations for electric field [see (3a)] magnetic field [see (3b)] exhibit an evident symmetry. In fact we can formally obtain (3b) from (3a) and *vice versa* with the formal substitutions $\varepsilon_0 \to \mu_0, E \to H, x \to y$. Therefore, following the procedure [41], we can incorporate the coupled system (3) into a single equation for electric field as following by using the relation between magnetic and electric field $H \approx E/\eta$, where η is the impedance of metamaterial:

$$i\frac{\partial E}{\partial z} + \frac{1}{2\beta_0}\nabla_\perp^2 E + \frac{\eta\omega_0\varepsilon_0}{2}\left(\chi_e^{(3)} + \frac{\mu_0\chi_m^{(3)}}{\eta^4\varepsilon_0}\right)|E|^2E + \frac{\chi_e^{(3)}}{2\beta_0\varepsilon_r}\frac{\partial^2|E|^2E}{\partial x^2} + \frac{1}{2\beta_0}\frac{\partial^2 E}{\partial z^2} = 0. \tag{4}$$

For convenience of computation, we define four characteristic lengths for linear diffraction L_{LDL}, nonlinear diffraction L_{NDL}, nonparaxiality L_{NPL}, nonlinear electric field L_{ENL}, and magnetic field L_{HNL}, respectively, as follows:

$$L_{\mathrm{LDL}} = \left|2\beta_0 w_0^2\right|, \quad L_{\mathrm{NDL}} = \frac{2w_0^2|\beta_0\varepsilon_r|}{|E_0|^2|\chi_e^{(3)}|}, \quad L_{\mathrm{NPL}} = \left|\frac{1}{2\beta_0}\right|, \quad L_{\mathrm{ENL}} = \frac{2}{\omega_0\varepsilon_0\eta\left|\chi_e^{(3)}\right||E_0|^2}, \quad L_{\mathrm{HNL}} = \frac{2\eta}{\omega_0\mu_0\left|\chi_m^{(3)}\right||H_0|^2} \tag{5}$$

where w_0 is the initial input beam radius (1/e), while E_0 and H_0 are the initial amplitudes of electric and magnetic envelop, respectively. Now, we normalize (5) using the transformations: $Z = z/L_{LDF}$, $X = x/w_0$, $Y = y/w_0$, $A = E/E_0$ and obtain the following normalized equation:

$$i\frac{\partial A}{\partial Z} + \mathrm{sgn}(n)\nabla_\perp^2 A + p|A|^2 A + q\frac{\partial^2 |A|^2 A}{\partial X^2} + m\frac{\partial^2 A}{\partial Z^2} = 0 \qquad (6)$$

where the second and third terms in the left-hand side of (6) account for the diffraction and self-focusing effect, respectively, and the last two terms account for the nonlinear diffraction and nonparaxial effect, respectively. Here, $\mathrm{sgn}(n) = \pm 1$ corresponds to positive-index and negative-index material, respectively. The dimensionless Kerr nonlinear coefficient p, nonlinear diffraction coefficient q and nonparaxial coefficient m in (6) are given by

$$p = \mathrm{sgn}\left(\chi_e^{(3)}\right)\frac{L_{\mathrm{LDL}}}{L_{\mathrm{ENL}}} + \mathrm{sgn}\left(\chi_m^{(3)}\right)\frac{L_{\mathrm{LDL}}}{L_{\mathrm{HNL}}}, \quad q = \mathrm{sgn}\left(\chi_e^{(3)}\right)\frac{L_{\mathrm{LDL}}}{L_{\mathrm{NDL}}}, \quad m = \mathrm{sgn}(n)\frac{L_{\mathrm{NPL}}}{L_{\mathrm{LDL}}} \qquad (7)$$

where $\mathrm{sgn}(\chi_e^{(3)}) = \pm 1$ stands for focusing and defocusing electric nonlinearity, respectively, and $\mathrm{sgn}(\chi_m^{(3)}) = \pm 1$ stands for focusing and defocusing magnetic nonlinearity, respectively.

The system obtained is suitable for any form of practical metamaterials because we have not exerted any limitations on the permittivity and magnetic permeability of such materials during the deriving process. As an example, we consider a practical metamaterial described by the well-known Drude model [32]–[46]: The relative permittivity $\varepsilon_r(\omega) = 1 - \omega_{pe}^2/(\omega^2 + i\omega\gamma_e)$ and the relative magnetic permeability $\mu_r(\omega) = 1 - \omega_{pm}^2/(\omega^2 + i\omega\gamma_m)$, where ω_{pe} and ω_{pm} are the respective electric and magnetic plasma frequencies, γ_e and γ_m are the respective electric and magnetic losses. It should be pointed out that the loss of metamaterials is an issue one must deal with in any bulk medium and at any frequency. However, in the Gigahertz range, it is possible to build a metamaterial with small (and even negligible) imaginary parts of $\varepsilon_r(\omega)$ and $\mu_r(\omega)$, but for high frequencies, the former is not true. Nevertheless, many strategies for reducing losses at high frequencies have been proposed, such as improving fabrication techniques [47], [48] or introducing materials with optical gain [26], [49]. Even when dissipation is considered, following the procedure introduced in [50] and [51], we can set the complex wave number $\beta(\omega_0) = \beta_0 + i\alpha$ where β_0 is the real wave number of the plane wave modulated by slowly varying amplitude, and α is the imaginary part. Therefore, the corresponding loss term $i\alpha E$ can be added to the left-hand side of (4) [50], [51]. In this case, one notes that our physical model is formally identical to the corresponding case with considering loss in ordinary positive-index materials [1], [2], [7]. According to the theory of conventional nonlinear optics, we can conclude that the loss resulting from the electric and magnetic dissipations in metamaterials only are to reduce the value of optical intensity or delay the occurrence of the catastrophic self-focusing, while the main role of nonlinear diffraction in optical collapse is not strongly affected by losses. Similarly, Chen *et al.* recently showed that the loss in metamaterials does not fundamentally alter the dynamical behavior of coupled optical solitons except that it is to reduce the intensity along the propagation distance [52]. Thus, to more explicitly illustrate the dynamics of optical wave controlled by anomalous nonlinear diffraction, we will neglect the electric and magnetic losses in the following analysis. To further disclose the electromagnetic properties of metamaterials governed by the Drude model, in Fig. 1, we plot the variations of the refractive index n: the five characteristic lengths with normalized frequency ω/ω_{pe}. Here, L_{LDL}, L_{NDL}, L_{NPL} L_{ENL}, and L_{HNL} are calculated in units of $\omega_{pe}w_0^2/c$,$2w_0^2\omega_{pe}/c|\chi_e^{(3)}||E_0|^2$, $c/2\omega_{pe}$, $2c/\omega_{pe}|\chi_e^{(3)}||E_0|^2$, and $2c/\ \omega_{pe}|\chi_m^{(3)}||H_0|^2$, respectively. We clearly see that in Drude model the electromagnetic metamaterials can be divided into three regions: 1) the negative-index region ($n < 0$), 2) the stopband region (gray region), and 3) the positive-index region ($n > 0$). Therefore, in order to limit optical wave propagating in the negative-index region, we need to tune the plasma frequency ω_{pm}/ω_{pe} bigger than the optical frequency ω_0/ω_{pe}. In addition, one notes that L_{HNL} will become very long while L_{ENL} will become very short in the positive-index region near the band edge of materials. However, in the

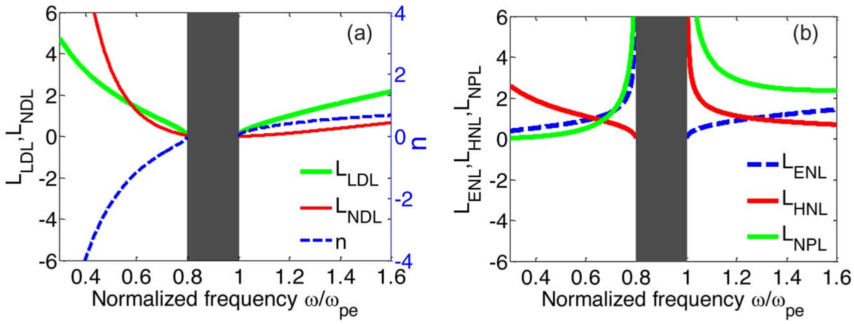


Fig. 1. Variations of the refractive index n: the five characteristic lengths versus normalized frequency ω/ω_{pe} for $\omega_{pm}/\omega_{pe} = 0.8$.

negative-index region the opposite situation will occur, i.e., L_{HNL} will become very short while L_{ENL} will become very long. In both positive-index and negative-index regions, both L_{DF} and L_{DS} tends to become very short while L_{NPL} tends to become very long. Most interestingly, the five characteristic lengths for describing the optical propagation are dependent on not only the parameters of optical wave itself, but also the electric and magnetic plasma frequency relating to the size of split-ring resonators and wires for metamaterials, as shown in Fig. 1. Therefore, the value of the parameters in (6) can theoretically cover over a wide range including those that are used in the next section by adjusting the plasma frequencies of materials or the input wave parameters. Ultimately, we want to stress that the qualitative behavior of optical collapse we describe also occurs for the other dispersive-model or structure parameters of metamaterials but does not depend sensitively on the values governed by the specific Drude model.

3. Results and Discussions

3.1. Anomalous Electromagnetic Properties of Metamaterials

Now, let us concentrate on (6) again; apparently, it is formally the same as that for optical nonlinear propagation in ordinary positive-index materials. However, when compared to the counterpart for the latter case, we clearly see that, both linear and nonlinear parameters in metamaterials exhibit some noticeable difference. First, the sign of both diffraction and nonparaxial terms become negative while they are always positive in ordinary positive-index materials. Therefore, self-focusing of optical beam will occur for $\mathrm{sgn}(p) = -1$ since $n < 0$, contrary to that in ordinary positive-index materials, in which it only occurs in the focusing regime. Second, as is well known, the sign of nonlinearity is only decided by the electric susceptibility $\chi_e^{(3)}$ in conventional materials. However, as (6) shows, the sign of the nonlinearity parameter p is determined by the combined effect of electric susceptibility $\chi_e^{(3)}$ and magnetic susceptibility $\chi_m^{(3)}$. Therefore, we will define $\mathrm{sgn}(p) = \pm 1$ as the focusing and defocusing nonlinearity for metamaterials, respectively, as does in conventional materials. Further analysis shows that, for $\chi_e^{(3)}$ and $\chi_m^{(3)}$ having different signs, the sign of p can be engineered by varying the relative magnitude between the electric nonlinearity and magnetic nonlinearity otherwise inaccessible for conventional material. Namely, the parameter p can be positive or negative, and its sign is dependent on their relative magnitude of electric and magnetic nonlinearity. Concretely speaking, for $\mathrm{sgn}(\chi_e^{(3)}) = 1$ and $\mathrm{sgn}(\chi_m^{(3)}) = -1$, if $L_{ENL}/L_{HNL} > 1$, metamaterial will exhibit defocusing nonlinear characteristics ($p < 0$) while it will exhibit focusing nonlinear characteristics ($p > 0$) if $L_{ENL}/L_{HNL} < 1$, In contrast, for $\mathrm{sgn}(\chi_e^{(3)}) = -1$ and $\mathrm{sgn}(\chi_m^{(3)}) = 1$, the opposite situation will occur. Namely, metamaterial will exhibit focusing nonlinear characteristics ($p > 0$) if $L_{ENL}/L_{HNL} > 1$ while it will exhibit defocusing nonlinear characteristics ($p < 0$) if $L_{ENL}/L_{HNL} < 1$. In fact, we have demonstrated that the nonlinear magnetization makes the sign of effective nonlinear effect switchable due to the combined action of electric and magnetic nonlinearity, exerting a significant influence on the propagation of electromagnetic pulses [41]. Finally, and most importantly, another noticeable anomaly in (6) is the normalized nonlinear diffraction parameter q. One notes that its sign only depends on the combined effect of electric susceptibility $\chi_e^{(3)}$, as shown in (6). Meanwhile, it is interesting, now, that a sufficient condition to make p < 0 (or > 0) is to make $\mathrm{sgn}(\chi_e^{(3)}) < 0$ (or > 0) and $\mathrm{sgn}(\chi_m^{(3)}) < 0$ (or > 0) but this is not a necessary condition, and it is possible to have either $\mathrm{sgn}(\chi_e^{(3)}) > 0$ or $\mathrm{sgn}(\chi_m^{(3)}) > 0$, depending on their relative magnitude, but not simultaneously. Therefore, when $\mathrm{sgn}(\chi_e^{(3)}) = 1$ and $\mathrm{sgn}(\chi_m^{(3)}) = -1$, NLD is always positive while p can be positive (or negative) if $L_{\mathrm{ENL}}/L_{\mathrm{HNL}} < 1$ (or $L_{\mathrm{ENL}}/L_{\mathrm{HNL}} > 1$). When $\mathrm{sgn}(\chi_e^{(3)}) = -1$ and $\mathrm{sgn}(\chi_m^{(3)}) = 1$, NLD is always negative, while p still can be positive (or negative) if $L_{\mathrm{ENL}}/L_{\mathrm{HNL}} > 1$ (or $L_{\mathrm{ENL}}/L_{\mathrm{HNL}} < 1$). As a consequence, the coefficient of nonlinear diffraction can, in principle, be positive or negative, independent on the sign of the nonlinear parameter p, while both of them show the same sign in ordinary positive-index materials with electronic nonlinearity. As demonstrated previously, the possibility of controlling the group-velocity dispersion in metamaterials can be exploited to obtain dispersion-free propagation in spectral regions otherwise inaccessible using ordinary positive-index materials [46]. Meanwhile, the controllable SS of role in both solitons and MI has been well known in metamaterials [35]–[42]. Similarly, by controlling the anomalous nonlinear diffraction in metamaterials we believe that we can obtain some novel properties of the optical collapse unseen in ordinary positive-index materials.

3.2 Role of Anomalous Nonlinear Diffraction in Controlling Optical Collapse

In this section, we will in detail discuss the role of anomalous nonlinear diffraction in the controlling the optical collapse both analytically and numerically. As is well known, although the numerical solution is welcomed for accurately describing the evolution of wave in dispersive nonlinear materials, some qualitative physical insight can be more easily gained if the NLSE can be solved analytically. However, it is very difficult to obtain the accurate analytical solution to (6). Here we will borrow an approach derived from modulation theory to directionally describe the unusual behavior of optical self-focusing in metamaterials. Following the standard procedure of modulation theory [8]–[10], [18], near the focal point, the solution of (6) will have the form

$$A(Z,r) \sim \frac{1}{L(Z)} R\left(\frac{r}{L}\right) \exp\left(i\frac{r}{L} + i\frac{L_Z r^2}{4L}\right) \tag{8}$$

where $r^2 = X^2 + Y^2$ and $R(r) > 0$, the radial profile of Townes soliton satisfies $\Delta_\perp R - R + R^3 = 0$, and $\int R^2 r dr = N_c$. According the definition of modulation theory, self-focusing dynamics is described by the modulation variable L, which is proportional to beam-width and also to 1/(on-axis amplitude). In particular, $L \to 0$ and $L \to \infty$ correspond to blowup and to complete defocusing, respectively. By averaging over the transverse coordinates and skipping the straightforward details of the calculations, we present the following differential equations to govern the evolution of the modulation function L

$$\frac{\partial^2 L}{\partial Z^2} = -\frac{\beta}{L^3}, \quad \frac{\partial \beta}{\partial Z} = \frac{(m + 23q/3)N_c}{2M} \frac{d}{dZ}\left(\frac{1}{L^2}\right). \tag{9}$$

Inspection of the derivation of (9) shows that the terms with m and q correspond to nonparaxial and vectorial effects, respectively. The reduced system (9) clearly shows that nonparaxiality and vectorial effects have the same qualitative effect on the self-focusing dynamics of a single filament if mq > 0. On the other hand, their contributions to optical evolution tend to counteract with each other if mq < 0. In addition, the reduced system shows that vectorial effects dominate over nonparaxiality in metamaterials since both the parameters m and q generally remain the

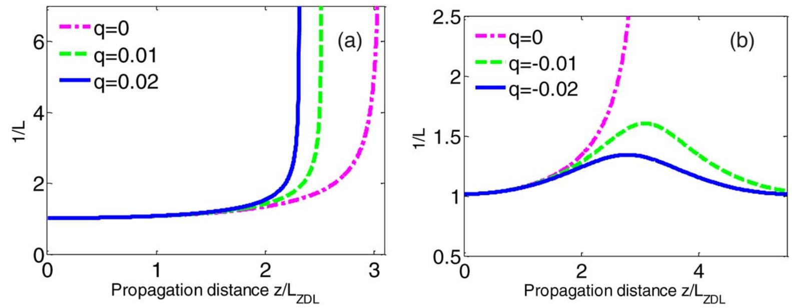


Fig. 2. Analytical prediction describes 1/L as the function of normalized propagation distance z for different NLD coefficients. (a) Positive value. (b) Negative value.

same order of magnitude [16]–[22]. Similarly, some pioneer studies have verified that the effect of nonparaxiality plays a small role in controlling the optical collapse, when compared with that of vectorial effect [16]–[22]. Therefore, in the following analysis, we only focus on the effect of nonlinear diffraction on the dynamical behavior of optical wave by neglecting the effect of nonparaxiality for simplicity. To directly disclose the effect of NLD, we solve (9) numerically by using Runge–Kutta method. In our simulations, it is assumed the initial condition is $\beta(Z=0)=0.1$, $dL/dZ|_{Z=0}=0$, and $L(Z=0)=0.99$. The qualitative picture predicted by (9) can be observed in Fig. 2, where we plot the parameter 1/L proportional to on-axis amplitude as a function of normalized propagation distance Z for different nonlinear diffraction coefficients, (a) positive value; (b) negative value. In the absence of NLD effect, as shown in Fig. 2, catastrophic collapse occurs at about $Z=3$, as one expected. When comparing Fig. 2(a) and (b), it is observed that the NLD effect is important in changing the dynamical behavior of optical collapse, depending on its sign. For positive value of nonlinear diffraction, as is well known, some pioneer authors reported that it can arrest the collapse of optical wave in ordinary positive-index materials [16]–[18]. However, in metamaterials, the situation becomes qualitatively different, as can be seen from Fig. 2(a). We clearly observe that peak intensity of optical wave will monotonically increase with increasing propagation until collapse occurs, regardless of whether the nonlinear diffraction is considered. In particular, the increasing its value will decrease the distance of collapse, meaning that the positive NLD will promote the occurrence of collapse. On the other hand, in Fig. 2(b), it can be seen that the peak intensity initially increases gradually up to a certain distance but then begins to decrease when the negative NLD is considered. Obviously, the larger the value of the nonlinear diffraction, the smaller the parameter 1/L. These results clearly demonstrate that the collapse of optical wave propagating in metamaterials can be arrested by negative NLD rather than positive case.

The modulation theory allows one to capture the main physical effects and predict the behavior of the optical propagating with much shorter computational times required for the simulations. However, in our deriving (9), the modulation theory only enables us to present an approximate analytical solution through the certain assumption that optical wave is close to a modulated Townes profile, which is cylindrically symmetric during the process of the whole self-focusing [8]–[10], [18]. Therefore, to verify our approximate analytical prediction above resulting from the modulation theory, we will directly solve (6) by using the standard split-step Fourier method. In our simulations, it is assumed the initial envelope of the electric field of the light beam retains Gaussian-shaped distribution along spatial direction and is written as $A(Z=0,X,Y)=2.2\exp(-X^2/2-Y^2/2)$, which is above the critical power for blowup of input Gaussian beams. The typical numerical results are summarized in Fig. 3(a), where we plot peak intensity as the function of propagation distance for positive m and the corresponding case for negative m is also illustrated in Fig. 3(b). It is evident that both numerical method and modulation theory yield nearly identical qualitative predictions regarding the possibility to control the optical collapse by means of the vectoral effect-induced nonlinear diffraction. Namely, negative

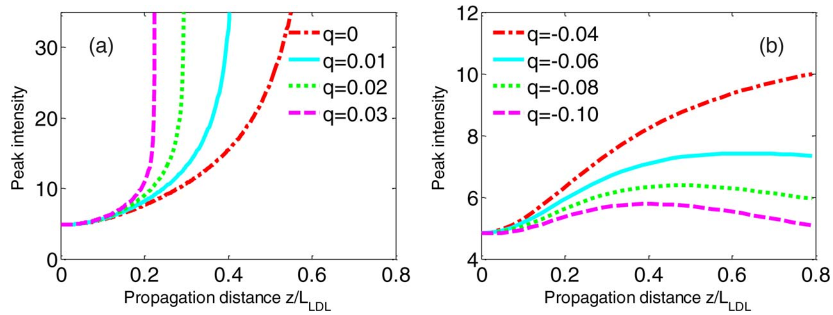


Fig. 3. Numerical prediction describes the normalized peak intensity as a function of normalized propagation distance z for different NLD coefficients. (a) Positive value. (b) Negative value.

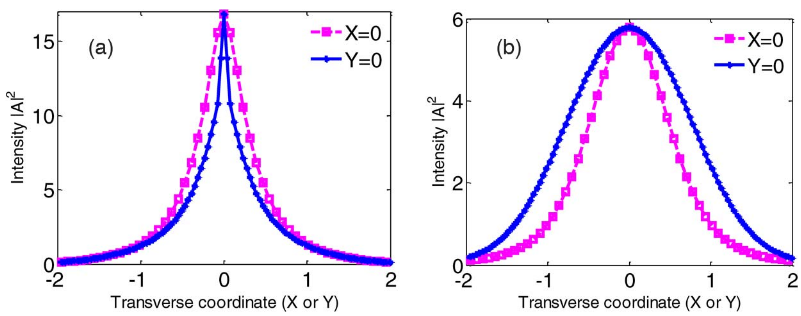


Fig. 4. Transverse normalized intensity distribution of on-axis spatial profile. The blue and magenta curves denote on-axis x profile and on-axis y profile, respectively. (a) $q = 0.03$. (b) $q = -0.10$.

NLD is found to be cable to prevent the catastrophic self-focusing of optical wave; however, the positive NLD will lead to the acceleration of collapse, just as opposed to the case in ordinary positive-index materials [16]–[18]. Obviously, large differences between the results obtained by direct simulations as well as investigations based on the modulation theory are observed. One of the reasons for the different results produced by the analytical solution and the numerical prediction is due to the fact that the modulation theory assumes that the spatial-beam profile always remains symmetric Townes profile during the process of optical evolution, while the direct simulations allow for a change of the beam shape along propagation. In fact, in our simulations, the shape of optical wave is found to be deviated from a perfect Gaussian, please see Fig. 4 below. In addition, another reason for large difference is that the parameter 1/L of the analytical solution is only proportional to on-axis amplitude but not the corresponding accurate value. Finally, it is worthwhile to stress metamaterial is artificial materials with dispersive permeability in addition to dispersive permittivity; therefore, the corresponding parameters of nonlinear diffraction are associated with the structural parameters of materials, i.e., the magnetic and electric plasma frequencies, just as in the case of anomalous SS [38]–[42]. As a consequence, we can similarly manipulate self-focusing of optical wave at our will by engineering nonlinear diffraction effect through choosing the structural parameters of materials [38]–[45].

Finally, another interesting feature is that for an initial transverse symmetric shape the beam will become transverse asymmetrical after some propagation distance due to the effect of nonlinear diffraction. Obviously, this feature can be directly observed from (6), where the last terms show an anisotropic-like property. In fact, the similar phenomena have previously been reported in ordinary positive-index materials [16]. However, due to the anomalous electromagnetic properties in metamaterials, it is nature to expect that such features will exhibit strikingly different behaviors. Fig. 4(a) and (b) give the normalized transverse intensity distribution at $Z = 0.22$ for the case of positive NLD and $Z = 0.4$ for the negative case, respectively. The results clearly show that for the initial symmetric beam linearly polarized along the X axis, its beam width along the

X axis is smaller than its beam width along the Y axis for the case of positive NLD. The observation suggests that the effect of positive NLD on optical wave dynamics is just opposite as the corresponding case in ordinary positive-index materials where the stronger focusing effect occurs along the Y axis [16]. On the other hand, for the negative case, the opposite situation will occur, namely, the self-focusing of beam along the X axis will become more pronounced than that along the Y axis. From a simple physical standpoint, we understand the interesting phenomena of optical self-focusing as following. Equation (6) is equivalent to the equation that describes the static electric field distribution generated by an equivalent normalized charge density source $\partial^2|A|^2A/\partial X^2$, which does not hold the symmetry of the optical field. Therefore, the NLD-induced refractive index change will differ along both X and Y directions. Specifically speaking, for the case of positive NLD, the induced refractive-index change curve is narrower along the X direction when compared with that along the Y direction so that focusing is stronger along the x direction than along the y direction. However, for the negative case, the induced refractive index change curve becomes wider along the Y direction, which explains the stronger focusing effect along the X direction.

4. Conclusion

In conclusion, we derive a general NLSE including the vectoral nonparaxial effect in metamaterials with simultaneous cubic electric and magnetic nonlinearity. In particular, the possibility of controlling the collapse dynamics by means of nonlinear diffraction resulting from the vectoral effect is demonstrated. We find nonlinear diffraction can provide a more powerful tool in manipulating the collapse of optical wave, when compared with the corresponding case in conventional materials. First, its value in principle can be positive or negative in metamaterials, which makes it possible for us to offer greatly enhanced design freedom to manipulate the behavior of optical collapse at will. Then, we further demonstrate that the acceleration and the arrest of the collapse may be achieved via negative and positive nonlinear diffraction, respectively. Direct simulations, as well as investigations based on the modulation theory, have been carried out, yielding consistent results. Our analysis suggests that the ability to control the catastrophic self-focusing of optical beams in metamaterials may be important for possible practical applications in many areas, including nonlinear optics and soliton dynamics.

References

[1] A. Couairon and A. Mysyrowicz, “Femtosecond filamentation in transparent media,” *Phys. Rep.*, vol. 441, no. 2–4, pp. 47–189, Mar. 2007.
[2] L. Bergé, “Wave collapse in physics: Principles and applications to light and plasma waves,” *Phys. Rep.*, vol. 303, no. 5/6, pp. 259–370, Sep. 1998.
[3] P. A. Robinson, “Nonlinear wave collapse and strong turbulence,” *Rev. Mod. Phys.*, vol. 69, no. 2, pp. 507–574, Apr. 1997.
[4] B. W. Zeff, B. Kleber, J. Fineberg, and D. P. Lathrop, “Singularity dynamics in curvature collapse and jet eruption on a fluid surface,” *Nature*, vol. 403, no. 6768, pp. 401–404, Jan. 2000.
[5] E. A. Donley *et al.*, “Dynamics of collapsing and exploding Bose–Einstein condensates,” *Nature*, vol. 412, no. 6844, pp. 295–299, Jul. 2001.
[6] M. Greiner, O. Mandel, T. W. Hänsch, and I. Bloch, “Collapse and revival of the matter wave field of a Bose–Einstein condensate,” *Nature*, vol. 419, no. 6902, pp. 51–54, Sep. 2002.
[7] R. W. Boyd, S. G. Lukishova, and Y. R. Shen, *Self-focusing: Past and Present*. New York, NY, USA: Springer-Verlag, 2009.
[8] G. Fibich and G. C. Papanicolaou, “Self-focusing in the presence of small time dispersion and nonparaxiality,” *Opt. Lett.*, vol. 22, no. 18, pp. 1379–1381, Sep. 1997.
[9] G. Fibich and B. Ilan, “Optical light bullets in a pure Kerr medium,” *Opt. Lett.*, vol. 29, no. 8, pp. 887–889, Apr. 2004.
[10] G. Fibich, “Small beam nonparaxiality arrests self-focusing of optical beams,” *Phys. Rev. Lett.*, vol. 76, no. 23, pp. 4356–4369, Jun. 1996.
[11] M. D. Feit and A. J. Fleck, “Beam nonparaxiality, filament formation, and beam breakup in the self-focusing of optical beams,” *J. Opt. Soc. Amer. B*, vol. 5, no. 3, pp. 633–640, Mar. 1988.
[12] B. A. Malomed, D. Mihalache, F. Wise, and L. Torner, “Spatiotemporal optical solitons,” *J. Opt. B: Quantum Semiclassical Opt.*, vol. 7, no. 5, pp. R53–R72, Apr. 2005.
[13] S. Gatz and J. Herrmann, “Propagation of optical beams and the properties of two-dimensional spatial solitons in media with a local saturable nonlinear refractive index,” *J. Opt. Soc. Amer. B*, vol. 14, no. 7, pp. 1795–1806, Jul. 1997.

[14] O. Bang, W. Krolikowski, J. Wyller, and J. J. Rasmussen, "Collapse arrest and soliton stabilization in nonlocal nonlinear media," *Phys. Rev. E*, vol. 66, no. 4, Oct. 2002, Art. ID. 046619.
[15] Y. Linzon, K. A. Rutkowska, B. A. Malomed, and R. Morandotti, "Magneto-optical control of light collapse in bulk Kerr media," *Phys. Rev. Lett.*, vol. 103, no. 5, Jul. 2009, Art. ID. 053902.
[16] S. Chi and Q. Guo, "Vector theory of self-focusing of an optical beam in Kerr media," *Opt. Lett.*, vol. 20, no. 15, pp. 1598–1600, Aug. 1995.
[17] C. S. Milsted, Jr. and C. D. Cantrell, "Vector effects in self-focusing," *Phy. Rev. A*, vol. 53, no. 5, p. 3536, May 1996.
[18] G. Fibich and B. Ilan, "Vectorial and random effects in self-focusing and in multiple filamentation," *Physi. D: Nonlinear Phenom.*, vol. 157, no. 1, pp. 112–146, Sep. 2001.
[19] A. D. Boardman, T. Popov, A. Shivarova, S. Tanev, and D. Zyapkov, "Nonlinear dispersion coefficients of a cylindrical optical waveguide," *J. Modern Opt.*, vol. 39, no. 5, pp. 1083–1096, Aug. 1992.
[20] A. Shivarova and S. Tanev, "Effective area of the LP01 mode in non-linear optical fibres," *Pure Appl. Opt.: J. Eur. Opt. Soc. Part A*, vol. 3, no. 5, p. 725, Sep. 1994.
[21] A. D. Boardman, K. Marinov, D. I. Pushkarov, and A. Shivarova, "Wave-beam coupling in quadratic nonlinear optical waveguides: Effects of nonlinearly induced diffraction," *Phy. Rev. E*, vol. 62, no. 2, p. 2871, Aug. 2000.
[22] A. D. Boardman, K. Marinov, D. I. Pushkarov, and A. Shivarova, "Influence of nonlinearly induced diffraction on spatial solitary waves," *Opt. Quantum Electron.*, vol. 32, no. 1, pp. 49–62, Jan. 2000.
[23] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Phys. Rev. Lett.*, vol. 84, no. 18, pp. 4184–4187, May 2000.
[24] J. B. Pendry, D. Shurig, and S. R. Smith, "Controlling electromagnetic fields," *Science*, vol. 312, no. 5781, pp. 1780–1782, Jun. 2006.
[25] J. B. Pendry, "Negative refraction makes a perfect lens," *Phys. Rev. Lett.*, vol. 85, no. 18, pp. 3966–3969, Oct. 2000.
[26] V. M. Shalaev, "Optical negative-index metamaterials," *Nat. Photon.*, vol. 1, no. 1, pp. 41–48, Jan. 2007.
[27] A. Zharov, I. V. Shadrivov, and Y. S. Kivshar, "Nonlinear properties of left-handed metamaterials," *Phys. Rev. Lett.*, vol. 91, no. 3, Jul. 2003, Art. ID. 037401.
[28] M. Lapine, I. V. Shadrivov, and Y. S. Kivshar, "Colloquium: Nonlinear metamaterials," *Rev. Mod. Phys.*, vol. 86, no. 3, pp. 1093–1123, Jul./Sep. 2014.
[29] N. Lazarides and G. P. Tsironis, "Coupled nonlinear Schrödinger field equations for electromagnetic wave propagation in nonlinear left-handed materials," *Phys. Rev. E.*, vol. 71, no. 3, Mar. 2005, Art. ID. 036614.
[30] N. L. Tsitsas, N. Rompotis, I. Kourakis, P. G. Kevrekidis, and D. J. Frantzeskakis, "Higher-order effects and ultrashort solitons in left-handed metamaterials," *Phys. Rev. E*, vol. 79, no. 3, Mar. 2009, Art. ID. 037601.
[31] I. Kourakis and P. K. Shukla, "Nonlinear propagation of electromagnetic waves in negative-refraction-index composite materials," *Phys. Rev. E*, vol. 72, no. 1, Jul. 2005, Art. ID. 016626.
[32] M. Scalora *et al.*, "Generalized nonlinear Schrödinger equation for dispersive susceptibility and permeability: Application to negative index materials," *Phys. Rev. Lett.*, vol. 95, no. 1, Jun. 2005, Art. ID. 013902.
[33] P. Kinsler, "Unidirectional optical pulse propagation equation for materials with both electric and magnetic responses," *Phys. Rev. A*, vol. 81, no. 2, Feb. 2010, Art. ID. 023808.
[34] P. Kinsler, "Optical pulse propagation with minimal approximations," *Phys. Rev. A*, vol. 81, no. 1, Feb. 2010, Art. ID. 013819.
[35] A. Kumar and A. K. Mishra, "Anomalous self-steepening, temporal pulse splitting and ring formation in a left-handed metamaterial with cubic nonlinearity," *J. Opt. Soc. Amer. B*, vol. 29, no. 6, pp. 1330–1337, Jun. 2012.
[36] A. K. Sarma and M. Saha, "Modulational instability of coupled nonlinear field equations for pulse propagation in a negative index material embedded into a Kerr medium," *J. Opt. Soc. Amer. B*, vol. 28, no. 4, pp. 944–948, Apr. 2011.
[37] A. Joseph and K. Porsezian, "Stability criterion for Gaussian pulse propagation through negative index materials," *Phys. Rev. A*, vol. 81, no. 2, Feb. 2010, Art. ID. 023805.
[38] S. Wen *et al.*, "Modulation instability in nonlinear negative-index material," *Phys. Rev. E*, vol. 73, no. 3, Mar. 2006, Art. ID. 036617.
[39] S. Wen *et al.*, "Theoretical models for ultrashort electromagnetic pulse propagation in nonlinear metamaterials," *Phys. Rev. A*, vol. 75, no. 3, Mar. 2007, Art. ID. 033815.
[40] Y. Hu, S. Wen, Y. Wang, and D. Fan. "Propagation of Gaussian beams in negative-index metamaterials with cubic nonlinearity," *Opt. Commun.*, vol. 281, no. 9, pp. 2663–2669, May 2008.
[41] J. Zhang, S. Wen, Y. Xiang, Y. Wang, and H. Luo, "Spatiotemporal electromagnetic soliton and spatial formation in nonlinear metamaterials," *Phys. Rev. A*, vol. 81, no. 2, Feb. 2010, Art. ID. 023829.
[42] J. Zhang, Y. Xiang, S. Wen, and Y. Li, "Controlling self-focusing of ultrashort pulses with anomalous self-steepening in nonlinear negative-index materials," *J. Opt. Soc. Amer. B*, vol. 31, no. 1, pp. 45–52, Jan. 2014.
[43] J. Zhang, Y. Xiang, L. Zhang, Y. Li, and Z. Luo, "Induced focusing of optical wave from cross-phase modulation in nonlinear metamaterials," *IEEE J. Quantum Electron.*, vol. 50, no. 10, pp. 823–830, Aug. 2014.
[44] A. D. Boardman, R. C. Mitchell-Thomas, N. J. King, and Y. G. Rapoport, "Bright spatial solitons in controlled negative phase metamaterials," *Opt. Commun.*, vol. 283, no. 8, pp. 1585–1597, Apr. 2010.
[45] A. D. Boardman, N. King, R. C. Mitchell-Thomas, V. N. Malnev, and Y. G. Rapoport, "Gain control and diffraction-managed solitons in metamaterials," *Metamaterials*, vol. 2, no. 2, pp. 145–154, Sep. 2008.
[46] G. D'Aguanno *et al.*, "Dispersion-free pulse propagation in a negative-index material," *Opt. Lett.*, vol. 30, no. 15, pp. 1998–2000, Aug. 2005.
[47] G. Dolling, C. Enkrich, M. Wegener, C. M. Soukoulis, and S. Linden, "Low-loss negative-index metamaterial at telecommunication wavelengths," *Opt. Lett.*, vol. 31, no. 12, pp. 1800–1802, Jun. 2006.
[48] V. M. Shalaev *et al.*, "Negative index of refraction in optical metamaterials," *Opt. Lett.*, vol. 30, no. 24, pp. 3356–3358, Dec. 2005.
[49] A. K. Popov and V. M. Shalaev, "Compensating losses in negative-index metamaterials by optical parametric amplification," *Opt. Lett.*, vol. 31, no. 14, pp. 2169–2171, Jul. 2006.

[50] D. A. Mártin and M. Hoyuelos, “Cavity equations for a positive-or negative-refraction-index material with electric and magnetic nonlinearities,” *Phy. Rev. E*, vol. 80, no. 5, Nov. 2009, Art. ID. 056601.
[51] Y. Xiang, X. Dai, S. Wen, and J. Guo, “Nonlinear absorption due to linear loss and magnetic permeability in metamaterials,” *Phys. Rev. E*, vol. 85, no. 6, Jun. 2012, Art. ID. 066604.
[52] P. Y. P. Chen and B. A. Malomed, “Single- and multi-peak solitons in two-component models of metamaterials and photonic crystals,” *Opt. Commun.*, vol. 283, no. 8, pp. 1598–1606, Apr. 2010.