Tunable Group Delay of the Optical Pulse Reflection From Fabry–Perot Cavity With the Insertion of Graphene Sheets
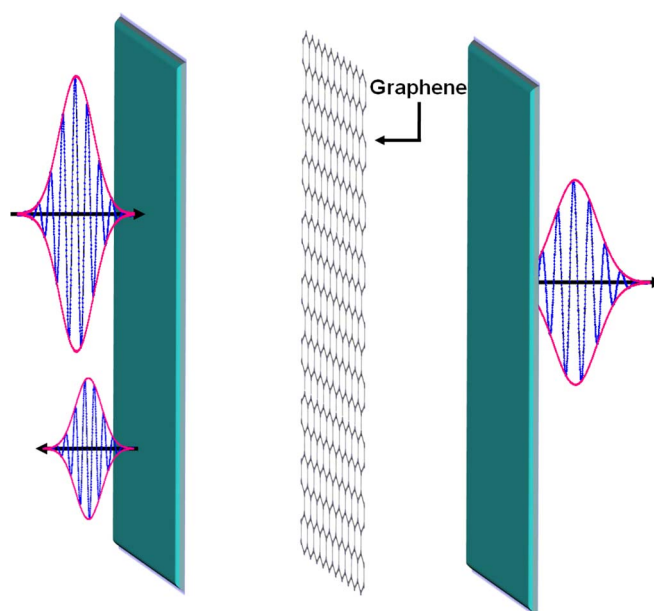
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Graphene

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Abstract: We have theoretically investigated the group delay of the TE-polarized beam reflected from a Fabry–Perot cavity with the insertion of the graphene sheets in the near-infrared band. It is shown that even a single-layer graphene allows for notable variation of group delay. Group delay can be enlarged negatively and can be switched from positive to negative, or vice versa. Importantly, the group delay depends on the Fermi energy of the graphene sheets, and thus, it can be actively controlled through electrical or chemical modification of the charge carrier density of the graphene. Furthermore, the influences of the position of graphene in the Fabry–Perot cavity, the mirror transmittance, and the number of graphene layers on group delay are clarified.

Index Terms: Group delay, graphene, Fabry–Perot cavity.

1. Introduction

The delay time for electromagnetic wave traversing through a medium is interesting and important [1], [2]. These delay techniques hold promise for uncovering new physical phenomena and for practical applications such as controllable optical delay lines, optical data storage, optical memories, and devices for quantum information [3]. It is well known that the delay time of the reflected and transmitted pulses can be realized and manipulated from subluminal to superluminal propagation by controlling the dispersive properties of medium. Therefore, the control of pulse propagation in different kinds of media has generated great interests for several decades [4], [5]. Various efforts have been devoted to the transmitted and reflected pulse from a medium [6]–[8]. Superluminal and subluminal pulses were theoretically and experimentally studied in different circumstances such as photonic crystal waveguides [9], left-handed medium [10], weakly absorbing dielectric slab [11], and graphene oxide solution [12]. In experiments, Longhi *et al.* first observed superluminal reflection of an optical pulse by using a double-Lorentzian fiber Bragg grating [13], and Painter *et al.* have reported superluminal light with a 1.4 microsecond signal advance in cavity optomechanical system [14], etc.
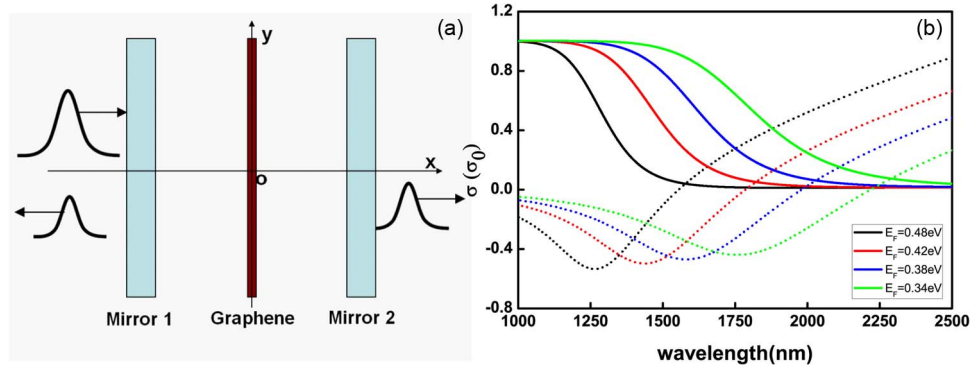
Fig. 1. (a) Schematic diagram of a Fabry–Perot cavity with the insertion of graphene sheets. (b) Dependence of the normalized optical conductivity of graphene sheet on the wavelength at different Fermi energies. The solid lines and the short dashed lines are the real and imaginary part of graphene conductivity, respectively, where $\sigma_0 = 6.08 \times 10^{-5}$ S/m, $T = 300$ K, and $\tau = 100$ fs.

Recently, graphene, a single-atom thick layer of covalently bonded carbon atoms has appeared as an alternative for tunable materials in optical systems due to its unusual properties [15]. It has shown great promise in many applications, such as optical modulator [16], ultrafast photodetector [17], surface plasmon polaritons [18], fiber laser [19], and nonlinear photonics [20]. Furthermore, one possible route to harnessing these properties for applications would be to incorporate graphene sheets in a composite material. Such as demonstrating the possibility to replace metals by grapheme [21] and depositing a graphene layer onto a complementary split-ring metamaterial using chemical vapor deposition [22], etc. More importantly, graphene shows a highly tunable carrier concentration under electrostatic gating, and this could provide an effective route to achieve electrical tunable devices for microwave photonics [23], plasmonic resonance [24] and bandgap [25].

In this paper, we will investigate theoretically the group delay of the optical pulse reflected from a Fabry–Perot cavity with the insertion of the graphene sheets, and we deduce that the superluminal properties of the reflected pulses may also be realized and tuned by using the tunable electrical conductivity properties of graphene. It is demonstrated that even a single-layer graphene (as thin as 0.34 nm) allows for greatly changing the properties of the optical pulse reflection and electrically tuning the value of the group delay. Moreover, we also point out that the reflected group delay depends on the position of graphene in the Fabry–Perot cavity, the mirror transmittance and the number of the layers of the graphene sheets.

2. Models and Methods

We consider the well-known case of an empty optical cavity, a single-layer graphene is placed inside the optical cavity, as shown schematically in Fig. 1(a), x direction is perpendicular to the mirror plane and y direction is parallel to the mirror. The scattering matrix of a partially silvered mirror is characterized by two pairs of reflectance and transmittance coefficients. Here, we assume the reflectance and transmittance coefficients of two mirrors are the same. In this case, the scattering matrix of mirror can be written as [26]

$$M_m = \frac{1}{t_m} \begin{bmatrix} -1 & -|r_m| \\ |r_m| & 1 \end{bmatrix} \qquad (1)$$

where r_m is the reflectance coefficient and t_m is the transmittance coefficient of the mirror, respectively. In the case of ignoring loss, we have $|r_m|^2 + |t_m|^2 = 1$. A light pulse normally incident along x direction, both sides of the mirror are vacuum. The graphene sheet is placed in the middle of the cavity ($x = 0$) with the surface conductivity σ. Within the random-phase
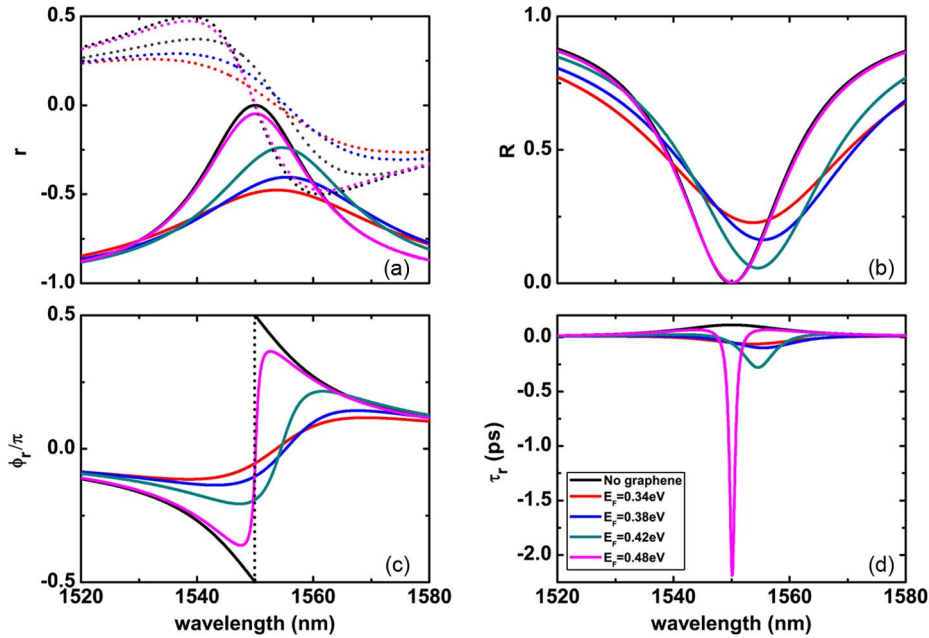
Fig. 2. (a) Reflection coefficient r, (b) reflectance R, (c) reflected phase ϕ_r, and (d) reflected group delay τ_r as functions of wavelength at different Fermi energy for TE-polarized light.

approximation, the graphene surface conductivity σ is the sum of the intraband σ_{intra} and the interband term σ_{inter} [24], [27]–[29], where

$$\sigma_{\text{intra}}(\omega) = \frac{ie^2 k_B T}{\pi \hbar^2 (\omega + i\tau^{-1})} \left[\frac{E_F}{k_B T} + 2\ln \left(e^{-\frac{E_F}{k_B T}} + 1 \right) \right] \tag{2}$$

$$\sigma_{\text{inter}}(\omega) = \frac{ie^2}{4\pi \hbar} \ln \left[\frac{2E_F - (\omega + i\tau^{-1})\hbar}{2E_F + (\omega + i\tau^{-1})\hbar} \right] \tag{3}$$

where ω is the frequency of incident light, e and $\hbar$ are universal constants related to the electron charge and reduced Planck's constants, respectively. E_F and τ are the Fermi energy (or chemical potential) and electron-phonon relaxation time, respectively. k_B is Boltzmann constant and T is a temperature in K. The Fermi energy E_F can be straightforwardly obtained from the carrier density (n_{2D}), in a graphene sheet, $E_F = \hbar \nu_F (\pi n_{2D})^{1/2}$, ν_F is the Fermi velocity of electrons. Here, the carrier density n_{2D} can be electrically controlled by an applied gate voltage. Thereby leading to a voltage-controlled Fermi energy E_F and hence the voltage-controlled surface conductivity σ. This could provide an effective route to achieving electrically controlled group delay in Fabry–Perot cavity with the insertion of the graphene sheets. Combining the conductivity of graphene, the transmission matrix for TE-polarized can be obtained [30]

$$M_g = \frac{1}{2} \begin{bmatrix} 1 + \eta_{TE} + \xi_{TE} & 1 - \eta_{TE} + \xi_{TE} \\ 1 - \eta_{TE} - \xi_{TE} & 1 + \eta_{TE} - \xi_{TE} \end{bmatrix} \tag{4}$$

where $\eta_{TE} = 1$, $\xi_{TE} = \sigma \mu_0 \omega$, then the full transfer matrix of the device can be expressed as

$$M = M_m M_f(x_g) M_g M_f(Lc - x_g) M_m \tag{5}$$

where

$$M_f(\triangle x) = \begin{bmatrix} e^{-ik\triangle x} & 0 \\ 0 & e^{ik\triangle x} \end{bmatrix} \tag{6}$$

where $k = \omega/c$, with the transfer matrix, the reflection and the transmission coefficients are defined as

$$r = \frac{M_{21}}{M_{11}}, \quad t = \frac{1}{M_{11}}. \tag{7}$$

We have assumed that the incident pulse is a Gaussian pulse. In the limit of the narrow spectral pulse [31], [32], the group delay of the reflected pulse can be written as [33]

$$\tau_r = \left[\frac{\partial \phi_r}{\partial \omega} \right]_{\omega = \omega_c} \tag{8}$$

where ω_c is the carrier frequency, ϕ_r is the phase of the reflection coefficients $r(\omega)$, namely, $r(\omega) = |r(\omega)| \exp(i\phi_r(\omega))$. With these relations, the group delay of the reflected beam is given by

$$\tau_r = \frac{1}{|r(\omega)|^2} \left[Re[r(\omega)] \frac{dIm[r(\omega)]}{d\omega} - Im[r(\omega)] \frac{dRe[r(\omega)]}{d\omega} \right]. \tag{9}$$

For a lossless system, the reflected group delay is equal to the transmitted group delay [11]. However, for the reflected group delay, it has no physical meaning at resonances $Re(kd) = m\pi$ ($m = 1, 2, 3, \ldots$), owing to the zero reflection [see Fig. 2(b)] and the undefined phases [see Fig. 2(c)]. If the monolayer graphene is inserted in the Fabry–Perot cavity, there is a large finite slope of the change of phase with a nonzero, albeit small, reflection. This large finite slope will lead to the large group delay. Wang *et al.* [11] theoretically showed that the negative group delay of TE- and TM-polarized reflected beams from the weakly absorbing dielectric slab can be very large near the resonance frequency. However, in these works, the group delay cannot be manipulated in a fixed configuration. The situation will be changed if we introduce the electrically doped graphene into the fixed configuration. In the next numerical calculation, in order to simplify we assume that the graphene sheet is suspended in the air and can be moved from left to right.

3. Theoretical Results and Discussions

We discuss the group delay near the communication wavelength. Fig. 1(b) is the calculated optical conductivity of graphene sheet at different Fermi energies, the solid lines and the short dash lines are the real part and the imaginary part of optical conductivity, respectively. It is indicated that the optical conductivity is dependent strongly on the Fermi energy. The imaginary part of the optical conductivity $Im(\sigma)$ exhibits some complex behaviors: It has a dip near the wavelength λ_{dip} for $\hbar\omega = 2E_F$, and at this dip, $Im(\sigma)$ can become negative. Moreover, λ_{dip} shifts to shorter wavelength with the increasing Fermi energy E_F. The real part of the optical conductivity $Re(\sigma)$ is large if $\lambda < \lambda_{dip}$, and it almost disappears if $\lambda > \lambda_{dip}$; the larger Fermi energy corresponding to a larger decline rate. The electrically tunable optical conductivity will provide us a method to manipulate the reflection coefficient, reflectivity, phase, and group delay, as shown in Fig. 2(a)–(d), respectively. Here, we assume the cavity length $Lc = 775$ nm and $t^2 = 0.045$. First, near the resonant frequency, $Re(kd) = m\, i\, (m = 1, 2, 3, \ldots)$, the imaginary part of the reflection coefficient $Im[r(\omega)]$ vanishes, but the non-vanishing real part of $Re[r(\omega)]$ makes the reflectance always set a tiny value [see Fig. 2(a)]. The reflectivity has been greatly enhanced by decreasing the Fermi energy [see Fig. 2(b)], which is induced by the increasing $Re(\sigma)$ near the $\lambda = 1550$ nm. Meanwhile, the wavelength for the minimum reflectivity shifts to larger wavelength. Significantly, we find that the group delay can be large negative near the $\lambda = 1550$ nm; here, $\tau_r \approx -2.2$ ps has been obtained for $E_F = 0.48$ eV, as shown in Fig. 2(d). However, decreasing Fermi energy E_F can decrease the peak value of the negative group delay. It should be noted that the group delay becomes -0.064 ps at $E_F = 0.34$ eV. In the absence of the graphene sheet, the reflected group delay immediately return to positive value, this is mainly due to the neglect of loss in the Fabry–Perot. This phenomenon is quite useful, and we can realize the manipulation of group
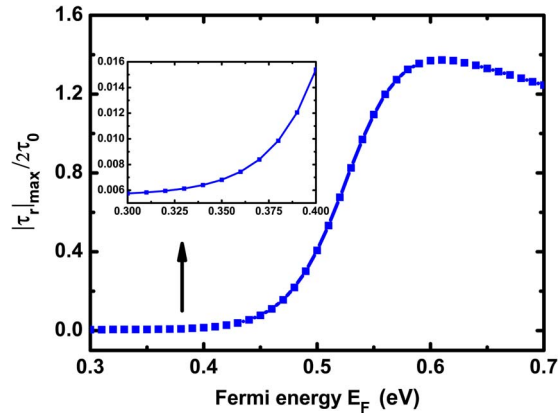
Fig. 3. The dependence of the maximum of absolute delay on the Fermi energy E_F, where $\tau_0 = 5$ ps, $T = 300$ K, and $\tau = 100$ fs.

delay easily by tuning the Fermi energy, chemical modification, or applied voltage. These novel behaviors of the group delay can be explained by relations of the reflection phase and the wavelength, as shown in Fig. 2(c); here, the slope of the reflection phase on the wavelength increases with increasing E_F, which leads to the increasing in the absolute value of the group delay. From Fig. 1(b), we can see that $Re(\sigma)$ is positive and it is decreasing with increasing Fermi energy E_F. We always have a negative $Re(r)$ near the resonance frequency and a change of $Im(r)$ from negative to positive across that frequency. The phase $\phi_r = tan^{-1}[Im(r)/Re(r)]$ is monotone increasing with the wavelength near the resonance wavelength and is limited in the range $[-\pi/2, \pi/2]$. Hence, the negative $d\phi_r/d\omega$ can be obtained, which leads to the negative group delay. In addition, increasing the Fermi energy E_F increases the slope of reflected phase and leads to the enhancement of the group delay. We have also plotted the dependence of the maximum of absolute delay on the Fermi energy E_F, as shown in Fig. 3. It is seen that The maximum of absolute delay increased rapidly with the Fermi energy while Fermi energy E_F is less than 0.6 eV. However, due to increased $Im(\sigma)$, The maximum of absolute delay decreased with the Fermi energy while $E_F > 0.6$ eV.

To demonstrate the validity of the reflected group delay that calculated in Fig. 2(d), we carried out a numerical simulation of pulse reflection through the Fabry–Perot cavity by means of the Fourier transform. Here, we consider the incident pulse is a Gaussian pulse and the electric field of the Gaussian pulse at the incident surface is expressed as $E_i(0, t') = A_0 exp(-t'^2/2\tau_0^2) \times exp(-i\omega_0 t')$. Here ω_0 is the center frequency and τ_0 is the temporal half-width of the Gaussian pulse. The Fourier spectrum of Gaussian pulse is given by $E_i(0, \omega) = (\tau_0 A_0/2\sqrt{\pi})exp[-\tau_0^2(\omega - \omega_0)^2/2]$. By calculating the dependence of the reflected pulse on frequency, we could qualitatively obtain the delay time of the pulse. We set the pulse temporal half-width $\tau_0 = 5$ ps. The numerical results of group delay of reflected pulse with different Fermi energies are shown in Fig. 4(a)–(d). Because the reflected pulse amplitude is very small compared to the incident pulse amplitude near the resonant frequency, all the pulse shapes have been normalized for better comparison. We can see that the delay times are negative and strongly influenced by the Fermi energy E_F. The delay times are -0.064 ps [Fig. 4(a)], -0.098 ps [Fig. 4(b)], -0.28 ps [Fig. 4(c)], -2.2 ps [Fig. 4(d)], for Fermi energy $E_F = 0.34$ eV, $E_F = 0.38$ eV, $E_F = 0.42$ eV, and $E_F = 0.48$ eV, respectively. These delay times are coinciding well with the results in Fig. 2(d).

We also observe that the positions of the graphene sheet can greatly affect the group delay of reflected pulse. This feature could provide an effective route for tuning the group delay of reflected pulse. Different positions of the graphene are calculated to study these effects. We change the position of the graphene in Fabry–Perot from $x = 0$ to $x = 0.4Lc$ as shown in Fig. 5. It can be seen that the reflectance, reflected phase and the reflected group delay vary according to the change of x. As the x increases, the peak wavelength of reflection coefficient shifts to
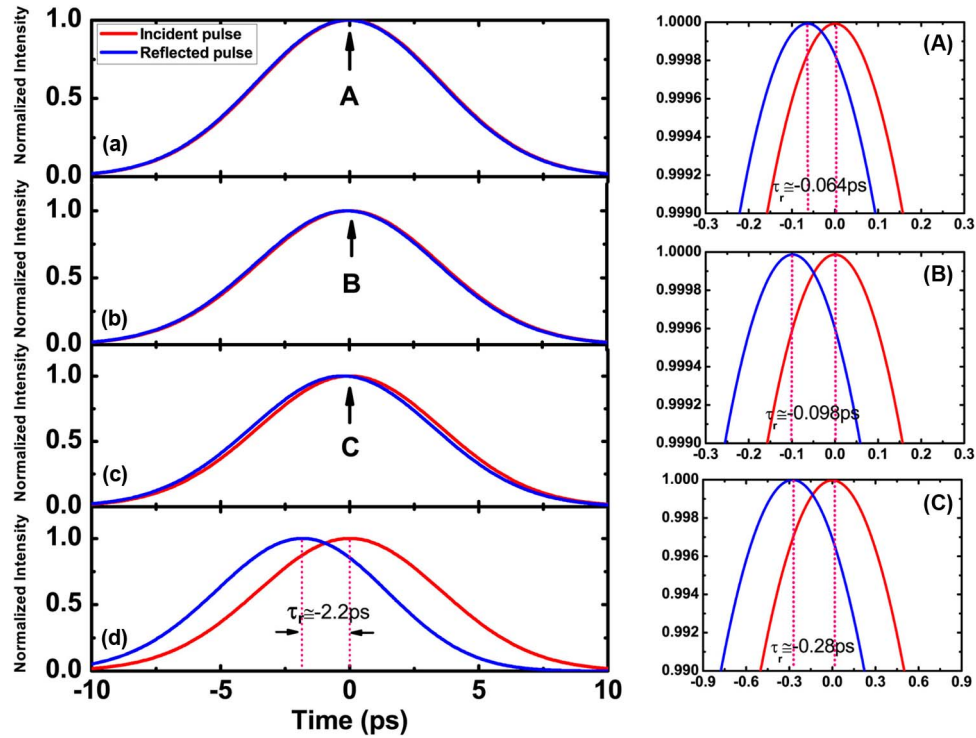
Fig. 4. Normalized intensity of an incident Gaussian pulse (red curve) and corresponding reflected pulse (blue curve) for TE-polarized with (a) $E_F = 0.34$ eV, (b) 0.38 eV, (c) 0.42 eV, and (d) 0.48 eV. Here, (a) $\omega_0 = 2\pi \times 193.05$ THz, (b) $2\pi \times 192.87$ THz, (c) $2\pi \times 192.98$ THz, and (d) $2\pi \times 193.54$ THz, respectively.

shorter wavelength and the reflectance dip get smaller and smaller, as shown in Fig. 5(a). Correspondingly, the peak wavelength of reflected group delay shifts to shorter wavelength, and the bandwidth becomes narrow and the delay dip gets more and more negative, as shown in Fig. 5(c). Therefore, Fabry–Perot cavity with graphene also has the potential to achieve tunable reflected group delay only by controlling the positions of the graphene in Fabry–Perot. It can be safely predicted that if a Fabry–Perot cavity is constructed, we are able to tune it by simply varying the position of graphene.

Besides the influence of the Fermi energy applied to graphene sheet and the positions of the graphene sheet on the group delay, the reflected phase and reflected group delay can also be engineered by changing the transmittance of mirror. The role of transmittance of mirror is clearly seen in (1) and (5). Fig. 6 is the contour plot of reflected phase and reflected group delay versus mirror transmittance t^2 and wavelength λ. Here, we only change the transmittance t^2, other parameters are the same as before. With increasing t^2, the resonant frequency is almost unchanged, this is mainly due to the fact that the resonance frequency and the transmittance in the expression have almost no relationship. However, the resonance strength of reflected phase becomes stronger with the increasing of t^2 [Fig. 6(a)], when t^2 increasing from near-zero to 0.3, the range of reflected phase is from near-zero to 0.8π [Fig. 6(a)]. As stated before, reflected phase can be engineered, thus the reflected group delay can be manipulated. The minimum value of the reflected group delay [Fig. 6(b)] varies from -0.05 ps to -0.22 ps by increasing t^2 from near-zero to 0.3. At the same time, the wavelength bandwidth for negative reflected group delay varies from 8 nm to 30 nm.

In addition to the sensitivity of the reflected group delay to the transmittance of mirror, we find that it is also sensitive to the number of layers of graphene sheets. For the few-layer graphene ($N < 6$), the individual graphene sheet can be considered as a noninteracting monolayer; the
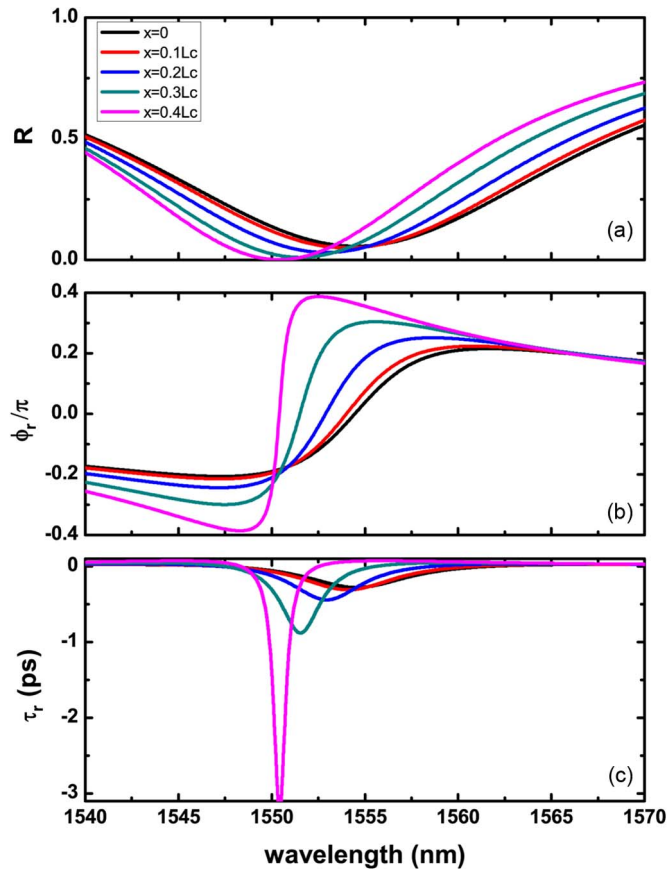
Fig. 5. (a) Reflectance R, (b) reflected phase ϕ_r, and (c) reflected group delay τ_r as functions of wavelength at different positions of graphene for TE-polarized light.
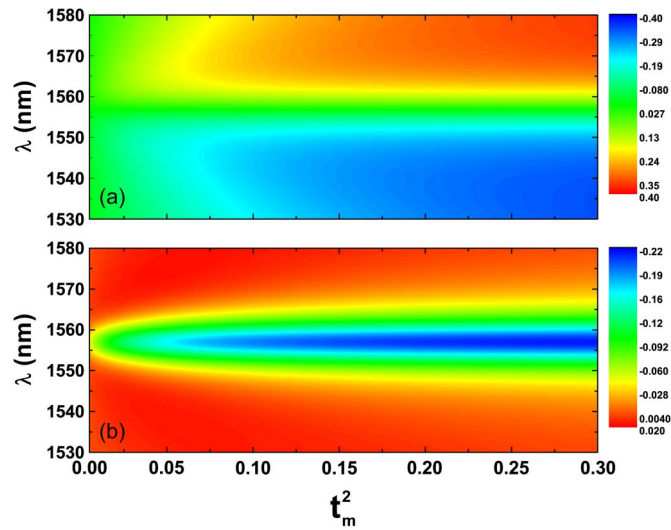

Fig. 6. Dependence of the normalized reflected phase (a) (ϕ_r/π) and (b) reflected group delay for TE-polarized on the wavelength and mirror transmittance. The calculations $E_F = 0.4$ eV, $T = 300$ K, and the unit of the group delay is ps.
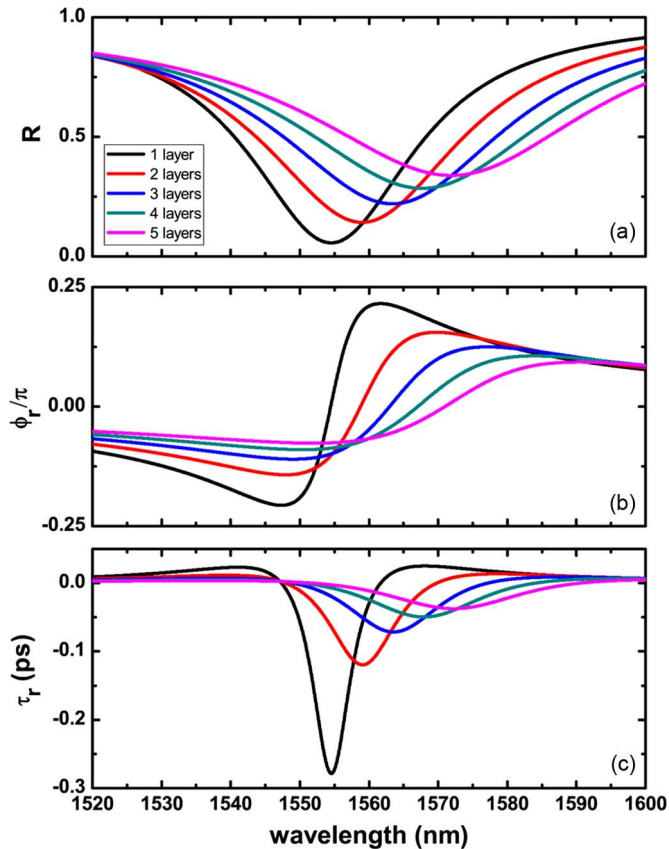
Fig. 7. (a) Reflectance R, (b) reflected phase ϕ_r and (c) reflected group delay τ_r as functions of wavelength at different numbers of layers, Here, $E_F = 0.42$ eV and $T = 300$ K.

optical conductivity of the few-layer graphene can be calculated as $N\sigma$ [34], [35], where N is the number of the layers. Then we can discuss the influence of the number of layers on the group delay, as shown in Fig. 7. It is apparent that increasing the number of layers of graphene can enhance the reflectivity due to the increased $Im(r)$ and $Re(r)$, as shown in Fig. 7(a). The slope of the phase of the reflected light beam is also decreasing [Fig. 7(b)]. The increasing of the number of the graphene sheets lead to the redshift of the delay peak, and the decreasing of the number of the graphene sheets bring about the blueshift of the delay peak. Moreover, both the reflectivity and reflected group delay get bigger and bigger with the increased number of the graphene sheets, which also implies that the reflected group delay can be engineered by manipulating the number of layers of graphene sheets. As a consequence, the number of the graphene sheets N plays an important role in determining the group delay.

4. Conclusion

In conclusion, we have proposed and theoretically demonstrated an electrically tunable reflected group delay composed of Fabry–Perot cavity and graphene sheet. A qualitative explanation has also been suggested based on transfer matrix method. With varied dc voltage, the reflected group delay of Fabry–Perot cavity could be dynamically tuned towards both higher and lower values. Besides, the positions of graphene, the mirror transmittance and the number of graphene layers also have significant impact on the reflected group delay. We believe that the controllable reflected group delay at near-infrared bands could find potential applications in optical delay devices and microelectronics. This would inspire the design of delay devices based on graphene.

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