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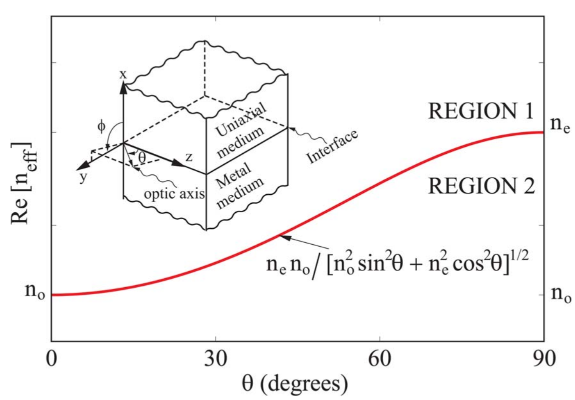

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Leaky Surface Plasmon Polariton Modes at an Interface Between Metal and Uniaxially Anisotropic Materials

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Abstract: By solving analytically derived characteristic equations, leaky surface waves are found to exist at an interface between a metal material and a uniaxially anisotropic dielectric material with its optic axis falling in the plane of the interface. Both the assumed lossless metal material with negative relative permittivity and the real metal such as silver having an imaginary part in its permittivity are considered to reveal the difference in the solved leakage loss behavior of the leaky modes. Analytical solutions are also confirmed by the finite-element eigenmode analysis.

Index Terms: Anisotropic optical materials, leaky modes, liquid crystals, surface plasmons, surface waves.

1. Introduction

The phenomenon of the surface plasmon polariton (SPP) at an interface between metal and isotropic dielectric material has been well known and exploited to achieve different applications [1], [2]. In this paper, we consider the SPP solution of the more complicated situation that the isotropic dielectric is replaced by a uniaxially anisotropic dielectric material and a new leaky mode solution will be presented. The characteristic equation for this solution is basically that derived by Dyakonov in 1988 when predicting the existence of a new type of surface wave at an interface between an isotropic dielectric material and a uniaxial-birefringent dielectric material with its optic axis falling in the plane of the interface [3], which was called the Dyakonov wave [4]–[6]. Related phenomena involving different materials have been continuously investigated [5]. A more recent related work was that Li *et al.* numerically studied surface plasmon polaritons (SPPs) at the interface between a metal and a uniaxial crystal [7]. In these studies, only surface modes with the field amplitudes decaying away from the interface in the pure exponential manner were mostly concerned, i.e., the modes are of no loss, and such solutions might exist only in some angular regime of the optic axis. An earlier work by Avrutsky [8] presented an algorithm for solving guided modes in a multilayer uniaxial structure and gave several numerical examples including a single interface case between gold and the uniaxial lithium-niobate material at infrared wavelength. In this paper, we first provide possible leaky-mode solutions in addition to the pure guided modes based on solving suitable

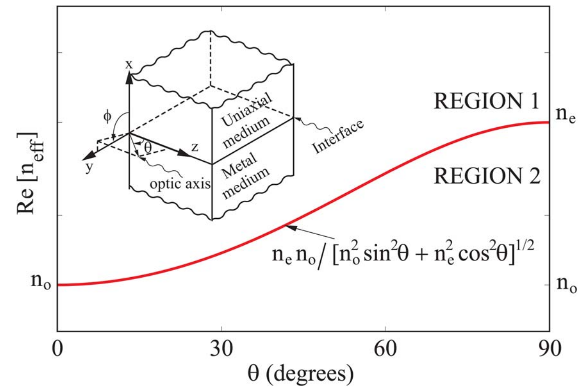

Fig. 1. The range of the allowable Re[n_{eff}] for the surface modes guided by the interface between a metal material and a uniaxially anisotropic dielectric material with $n_e > n_o$, as depicted in the inset.

characteristic equations, and then consider the practical situation that the metal is lossy. These analytical solutions are shown to excellently agree with a finite-element (FE) eigenmode analysis.

Recent researches related to SPPs involving anisotropic dielectrics include the following studies. Nagaraj and Krokhin investigated long-range SPPs in dielectric-metal-dielectric structure with anisotropic substrates [9], but only transverse magnetic (TM) waves for the case of the optic axis perpendicular to the plane of the interface were considered. Liscidini and Sipe conducted a theoretical study of SPPs in a configuration in which a metal layer is on top of an anisotropic dielectric [10] and discovered the existence of quasiguided SPPs with numerical demonstration of the coupling of a TM-polarized incident beam to such SPP in the Kretschmann configuration at surface plasmon resonance. The leaky-mode solutions presented in this paper are related to the aforementioned quasiguided SPPs. We nevertheless can provide the complete mode-field characteristics and dispersion curves for such modes using standard waveguide mode analysis. The analytical modal analysis is described in Section 2, numerical results are presented in Section 3, and the conclusion is given in Section 4.

2. The Analytical Modal Analysis

The inset in Fig. 1 shows the structure and coordinate systems for this study: an interface (the $x = 0$ plane) between the metal material ($x < 0$) and the uniaxially anisotropic material ($x > 0$). We consider surface wave modes propagating in the z direction and guided by this interface. As shown in this inset, the optic-axis orientation is described by the spherical-coordinate angles ϕ and θ. The index of refraction of the metal material is denoted as n_m and the ordinary and extraordinary indices of refraction of the uniaxial material are denoted as n_o and n_e, respectively. The corresponding relative permittivities are $\epsilon_m = n_m^2$, $\epsilon_o = n_o^2$, and $\epsilon_e = n_e^2$. The situation that $\phi = 90^\circ$ is considered here, and then the elements of the relative permittivity tensor for the $x > 0$ region are $\epsilon_{xx} = n_{xx}^2 = n_o^2$, $\epsilon_{xy} = n_{xy}^2 = 0$, $\epsilon_{xz} = n_{xz}^2 = 0$, $\epsilon_{yx} = n_{yx}^2 = 0$, $\epsilon_{yy} = n_{yy}^2 = n_o^2 + (n_e^2 - n_o^2)\sin^2\theta$, $\epsilon_{yz} = n_{yz}^2 = (n_e^2 - n_o^2)\sin\theta\cos\theta$, $\epsilon_{zx} = n_{xz}^2 = 0$, $\epsilon_{zy} = n_{zy}^2 = (n_e^2 - n_o^2)\sin\theta\cos\theta$, and $\epsilon_{zz} = n_{zz}^2 = n_o^2 + (n_e^2 - n_o^2)\cos^2\theta$.

In the following analysis, we assume that ϵ_m is complex with lossy imaginary part, and ϵ_o and ϵ_e are real and positive. Note that in [7], it was assumed ϵ_m is real and negative and $|\epsilon_m| > \epsilon_e > \epsilon_o$. In the metal material, we write the trial solutions as $E_{ym}(x)$ and $H_{ym}(x)$. The transverse electric (TE) mode and the TM mode trial solutions are assumed to be $E_{ym}(x) = A_{\mathrm{TE}}\exp(\kappa_m x)$ and $H_{ym}(x) = A_{\mathrm{TM}}\exp(\kappa_m x)$, respectively, where A_{TE} and A_{TM} are arbitrary constants. In the anisotropic material region, the trial solutions are expressed as $E_y(x) = E_{yo} + E_{ye}$, where the subscripts o and e denote the ordinary and the extraordinary components, respectively. We take $E_y(x) = A_o\exp[-\kappa_o x] + A_e\exp[-\kappa_e x]$ for guided-mode solutions with $n_e > n_o$, and $E_y(x) = A_o\exp[-\kappa_o x] + A_e\exp[-j\kappa_e x]$ for leaky-mode solutions, where A_o and A_e are arbitrary constants. Note that the inclusion of j in the second term of $E_y(x)$ for leaky modes would assure the complex κ_e can be obtained from solving the derived characteristic equation, as was similarly treated in [11].

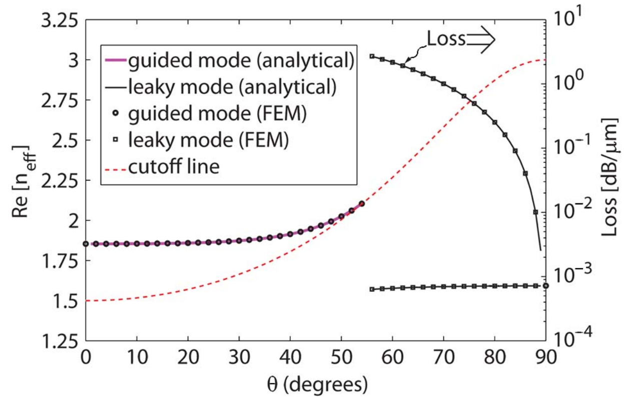

Fig. 2. Re$[n_{eff}]$ and loss in dB/μm versus θ results from both analytical analysis and FE analysis for the case with $\epsilon_o = 2.25$, $\epsilon_e = 9$, $\epsilon_m = -20$, $\lambda = 0.694\,\mu$m, and $\phi = 90°$ (the optic axis in the $y-z$ plane).

In the metal material region, the wave vector components satisfy $\kappa_m^2 = (\beta^2 - k_0^2\epsilon_m)$, and in the anisotropic material region, the corresponding relations are $\kappa_o^2 = (\beta^2 - k_0^2 n_o^2)$ and $\kappa_e^2 = \pm(\beta^2\epsilon_{zz}/n_o^2 - k_0^2\epsilon_e)$ for the ordinary and extraordinary components, respectively, where k_0 is the free-space wavenumber, $k_0^2 = \omega^2\epsilon_0\mu_0$, and β is the modal propagation constant along the z direction. Note that we have $\pm$ in κ_e^2 with the upper and lower signs corresponding to the guided and leaky modes, respectively. In the corresponding (2) in [3], only the plus sign was given and thus only guided modes could be obtained.

By fulfilling the continuity conditions of the four field components, $E_y(x)$, $E_z(x)$, $H_y(x)$, and $H_z(x)$, at the interface, we can respectively obtain the characteristic equations for

i) *Guided surface modes (the Dyakonov surface wave)*:

$$\kappa_o(\kappa_m + \kappa_o)\left(\epsilon_m\kappa_o^2 + n_o^2\kappa_m\kappa_e\right)\cos^2\theta = k_0^2 n_o^2(\kappa_e + \kappa_o)\left(\epsilon_m\kappa_o + n_o^2\kappa_m\right)\sin^2\theta \quad (1)$$

ii) *Leaky surface modes*:

$$\kappa_o(\kappa_m + \kappa_o)\left(\epsilon_m\kappa_o^2 + jn_o^2\kappa_m\kappa_e\right)\cos^2\theta = k_0^2 n_o^2(j\kappa_e + \kappa_o)\left(\epsilon_m\kappa_o + n_o^2\kappa_m\right)\sin^2\theta. \quad (2)$$

As in the modal analysis of planar waveguides involving uniaxially anisotropic materials [12], considering the extraordinary wave, the range of the modal effective index, defined as $n_{eff} = \beta/k_0$, for the guided modes can be determined by a θ-dependent cutoff line which corresponds to $\kappa_e = 0$ and defined by the formula, $n_{\text{cutoff}} = n_e n_o/[n_o^2\sin^2\theta + n_e^2\cos^2\theta]^{1/2}$, as shown by the red curve in Fig. 1. The dispersion curves of the guided modes would fall above this curve, i.e., region 1 in Fig. 1, and those of the leaky modes would appear below this curve, i.e., region 2.

In (1), κ_e is one unknown, and if the corresponding unknown in (2) is considered as $j\kappa_e$, then (1) and (2) seem to be identical. The appearance of j in (2) is due to $\pm$ in κ_e^2. When the lossy metal is taken into account, the solution of κ_e in (1) would typically be a complex number with Re$[\kappa_e] \gg$ Im$[\kappa_e]$. Usually the solution of κ_e in (2) is also with Re$[\kappa_e] \gg$ Im$[\kappa_e]$. Therefore, the form of (2) gives an advantage of searching for the complex solution in practical numerical implementation.

3. Numerical Results

We first consider the case analyzed in [7] to demonstrate the leaky surface-mode solution. The material parameters are $\epsilon_o = 2.25$, $\epsilon_e = 9$, and $\epsilon_m = -20$. The wavelength is taken to be $\lambda = 0.694\ \mu$m. By following a similar solution procedure described in [13], the complex solutions of the characteristic equations, (1) and (2), are determined. The real part of n_{eff} and the loss in dB/μm versus θ of the analytical solution are shown as the continuous curves in Fig. 2. For $\theta <\ \sim 55°$, it is the pure guided mode as was shown in [7], which is a lossless wave, and no corresponding loss curve appears in Fig. 2. And it is seen that the Re$[n_{eff}]$ curve ends at the cutoff line (the red dashed curve). Please note that in [7] $\lambda = 1\ \mu$m was used. Our analysis based on $\lambda = 1\ \mu$m shows good

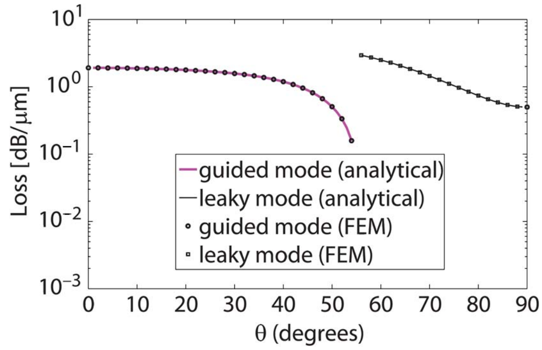

Fig. 3. The loss versus θ results from both analytical analysis and FE analysis for the case with $\epsilon_o = 2.25$, $\epsilon_e = 9$, $\epsilon_m = -20 - 1.2632j$, $\lambda = 0.694$ μm, and $\phi = 90°$ (the optic axis in the $y-z$ plane).

agreement with [7], and in fact, we have found analytical solutions show very little difference between $\lambda = 0.694$ μm and $\lambda = 1$ μm for $\theta < \sim 55°$. We obtain the leaky mode for $\theta > \sim 55°$, for which the loss in dB/μm, which is calculated from Im[n_{eff}] decreases to small values as θ gets toward 90°, as seen in Fig. 2. The corresponding FE analysis results are shown as circle-dots and square-dots in Fig. 2 and excellent agreement with the analytical solution is seen. This FE eigenmode solver [14] was formulated using three field components with suitable perfectly matched layers (PMLs), which has successfully been applied to solve guided and leaky modes on planar optical waveguides with the core and cladding composed of uniaxially anisotropic materials, such as those analyzed in [15].

Next, we consider a real-metal case. From [16] we find that for silver, ϵ_m is about $-20 - 1.2632j$ at $\lambda = 0.694$ μm. These parameters are used together with $\epsilon_o = 2.25$ and $\epsilon_e = 9$. From solving (1) and (2), the analytical loss in dB/μm versus θ results are shown in Fig. 3 as continuous curves for both the guided and the leaky SPP modes. Again, the corresponding FE analysis results are shown in Fig. 3 with excellent agreement with the analytical ones. Here, we do not show the corresponding Re[n_{eff}] versus θ results since they would appear almost indistinguishable from Fig. 2, indicating that the addition of the imaginary part, $-1.2632j$ to $\epsilon_m = -20$ does not give noticeable change in Re[n_{eff}]. This imaginary part of course contributes to the guided-mode losses, as seen in Fig. 3, and the metal-induced loss adds to the leakage loss of the leaky mode for $\theta > \sim 55°$ such that the modal loss function changes from that in Fig. 2 to that in Fig. 3. It is also observed that the modal losses are on the similar orders of magnitude for both guided and leaky modes.

We show in Fig. 4(a) and (b), respectively, the $\eta_0 H_y$ profiles versus the *x* position, where η_0 is the free-space impedance, for $\theta = 75°$ and 80°, as obtained from the FE analysis, and the corresponding E_y profiles in Fig. 4(c) and (d), respectively. Due to the anisotropic substrate, the six field components are all non-zero, forming a hybrid mode, although Fig. 4 shows the mode is more like the TM mode since E_y is relatively small. In the FE analysis, the 5 μm $\leq x \leq$ 10 μm region is the PML region for absorbing the leaky fields.

Finally, a more practical structure is considered, i.e., the nematic liquid crystal (5CB) on top of silver at $\lambda = 0.644$ μm. At this wavelength, the relative permittivities for this liquid crystal are $\epsilon_o = (1.5292)^2$ and $\epsilon_e = (1.7072)^2$ [17] and that for silver is $\epsilon_m = (0.13763 - 4.0790j)^2$ [16]. The Re[n_{eff}] and loss in dB/μm versus θ results are shown in Fig. 5 along with the cutoff line (the red dashed curve). Now the guided mode exists when $\theta < \sim 61°$ and the leaky mode appears when $\theta > \sim 64°$.

We give a remark on the appearance of dispersion-curve discontinuity near the cutoff angle in Figs. 2, 3, and 5. In each figure, for the guided-mode part, we have searched for the solutions starting from $\theta = 0°$ to the cutoff angle, while for leaky-mode part, we have done it starting from $\theta = 90°$ toward the cutoff angle. We found it becomes more difficult to identify the leaky mode near the cutoff angle and the solution searching stopped at our best trial. The behavior near the cutoff angle could be worth being further investigated.

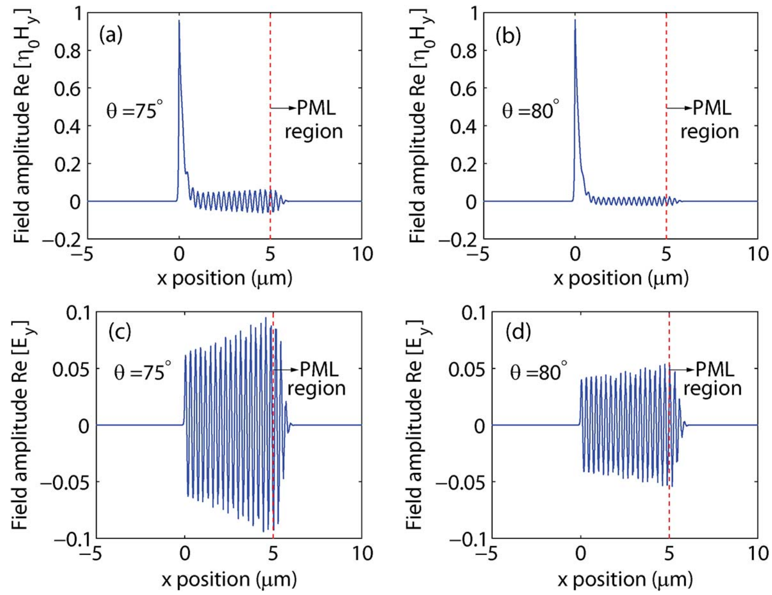

Fig. 4. η_0 H_y versus x profiles for $\theta =$ (a) 75° and (b) 80°, and E_y versus x profiles for $\theta =$ (c) 75° and (d) 80°, for the case of Fig. 3 obtained from the FE analysis.

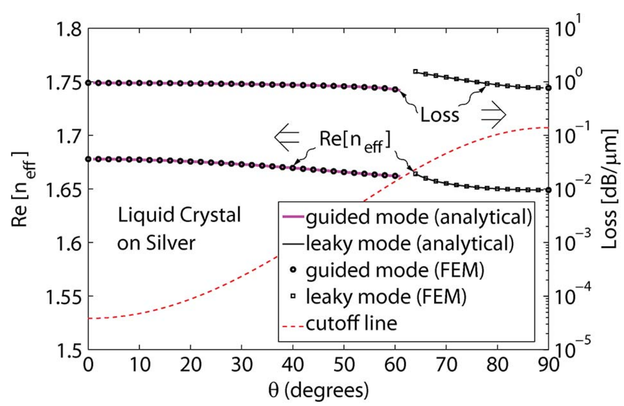

Fig. 5. Re[n_{eff}] and loss in dB/μm versus θ results from both analytical analysis and FE analysis for the case with the 5CB liquid crystal ($\epsilon_o = (1.5292)^2$ and $\epsilon_e = (1.7072)^2$) and silver ($\epsilon_m = (0.13763 - 4.0790j)^2$) at $\lambda = 0.644$ μm for $\phi = 90°$ (the optic axis in the $y-z$ plane).

4. Conclusion

In conclusion, we have demonstrated that for an interface between a lossy metal material and a uniaxially anisotropic dielectric material with the optic axis falling in the interface plane, the leaky SPP mode propagating along the interface can exist in addition to the known guided SPP mode. Analytical solutions for both the guided and leaky modes are confirmed by the FE eigenmode analysis. The FE eigenmode analysis would be more conveniently applied to solving multilayer structures having more than one interface.

References

[1] H. Raether, *Surface Plasmons on Smooth and Rough Surfaces and on Gratings*. Berlin, Germany: Springer-Verlag, 1988.
[2] S. A. Maier, *Plasmonics: Fundamentals and Applications*. New York, NY, USA: Springer-Verlag, 2007.

[3] M. I. Dyakonov, “New type of electromagnetic wave propagating at an interface,” *Sov. Phys. JETP*, vol. 67, no. 4, pp. 714–716, Apr. 1988.
[4] D. Artigas and L. Torner, “Dyakonov surface waves in photonic metamaterials,” *Phys. Rev. Lett.*, vol. 94, no. 1, pp. 013901-1–013901-4, Jan. 2005.
[5] O. Takayama, L.-C. Crasovan, S. K. Johansen, D. Mihalache, D. Artigas, and L. Torner, “Dyakonov surface waves: A review,” *Electromagnetics*, vol. 28, no. 3, pp. 126–145, Mar. 2008.
[6] J. A. Polo, Jr. and A. Lakhtakia, “Surface electromagnetic waves: A review,” *Laser Photon. Rev.*, vol. 5, no. 2, pp. 234–246, Mar. 2011.
[7] R. Li, C. Cheng, F. F. Ren, J. Chen, Y. X. Fan, J. Ding, and H. T. Wang, “Hybridized surface plasmon polaritons at an interface between a metal and a uniaxial crystal,” *Appl. Phys. Lett.*, vol. 92, no. 14, pp. 141115-1–141115-3, Apr. 2008.
[8] I. Avrutsky, “Guided modes in a uniaxial multilayer,” *J. Opt. Soc. Amer. A, Opt. Image Sci.*, vol. 20, no. 3, pp. 548–556, Mar. 2003.
[9] Nagaraj and A. A. Krokhin, “Long-range surface plasmons in dielectric–meta–dielectric structure with highly anisotropic substrates,” *Phys. Rev. B, Condens. Matter*, vol. 81, no. 8, pp. 085426-1–085426-9, Feb. 2010.
[10] M. Liscidini and J. E. Sipe, “Quasiguided surface plasmon excitations in anisotropic materials,” *Phys. Rev. B, Condens. Matter*, vol. 81, no. 11, pp. 115335-1–115335-9, Mar. 2010.
[11] J. Hu and C. R. Menyuk, “Understanding leaky modes: Slab waveguide revisited,” *Adv. Opt. Photon.*, vol. 1, no. 1, pp. 58–106, Jan. 2009.
[12] A. Knoesen, T. K. Gaylord, and M. G. Moharam, “Hybrid guided modes in uniaxial dielectric planar wave-guides,” *J. Lightwave Technol.*, vol. 6, no. 6, pp. 1083–1104, Jun. 1988.
[13] R. D. Kekatpure, A. C. Hryciw, E. S. Barnard, and M. L. Brongersma, “Solving dielectric and plasmonic waveguide dispersion relations on a pocket calculator,” *Opt. Exp.*, vol. 17, no. 26, pp. 24 112–24 129, Dec. 2009.
[14] H. H. Liu and H. C. Chang, “Calculation of leaky surface modes in planar structures involving uniaxially anisotropic interfaces,” in *Proc. 6th IEEE/ICAIT*, Hsinchu, Taiwan, 2013, pp. 5–6.
[15] D. Marcuse and I. P. Kaminow, “Modes of a symmetric slab optical waveguide in birefringent media—Part II: Slab with coplanar optical axis,” *IEEE J. Quantum Electron.*, vol. QE-15, no. 2, pp. 92–101, Feb. 1979.
[16] E. D. Palik, *Handbook of Optical Constants of Solids I*. New York, NY, USA: Academic Press, 1998, p. 356.
[17] P. Yeh and C. Gu, *Optics of Liquid Crystal Displays*. New York, NY, USA: Wiley, 1999, p. 61.