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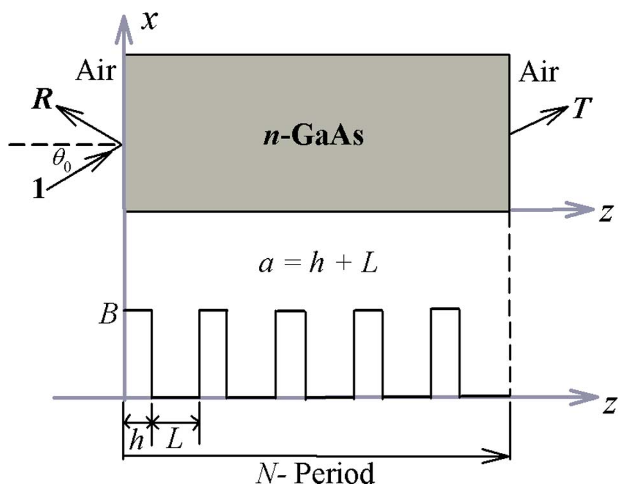
Chao-Chin Wang
Wen-Kai Kuo
Chien-Jang Wu, Member, IEEE



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Analysis of Effective Plasma Frequency in a Magnetized Extrinsic Photonic Crystal

Tzu-Chyang King,[1] Chao-Chin Wang,[1] Wen-Kai Kuo,[2] and Chien-Jang Wu,[3] *Member, IEEE*

[1]Department of Applied Physics, National Pingtung University of Education, Pingtung 900, Taiwan
[2]Department of Electro-Optical Engineering, National Formosa University, Yunlin 632, Taiwan
[3]Institute of Electro-Optical Science and Technology, National Taiwan Normal University, Taipei 116, Taiwan

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Abstract: The effective plasma frequency (EPF) in an extrinsic photonic crystal is theoretically analyzed. The extrinsic PC is a single homogeneous doped semiconductor (*n*-GaAs) that is influenced by an externally and periodically applied magnetic field. Based on the calculated photonic band structure, we investigate the magnetic-field dependence of EPF. The results reveal that the EPF will be smaller in the absence of the magnetic field and lowered down when the magnetic field increases. The EPF is shown to be a decreasing function of the filling factor of the magnetized region. Additionally, investigation of the first passband and band gap is also given. The study illustrates that such an extrinsic PC possesses tunable optical properties that are of technical use in semiconductor photonic applications.

Index Terms: Effective plasma frequency, extrinsic photonic crystal, doped semiconductor, transfer matrix method.

1. Introduction

The concept of effective plasma frequency (EPF) $\omega_{p,eff}$ was introduced in a metal-dielectric photonic crystal (MDPC) by Xu *et al.* [1]. It is defined as the lowest frequency above which the electromagnetic wave can propagate through the MDPC. Thus, the EPF can be regarded as the fundamental quantity that characterizes the properties of wave propagation in the MDPC. In [1], it has been shown that the EPF of MDPC is less than ω_p, the plasma frequency of a bulk metal which usually appears in the dielectric function given by

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - j\gamma\omega}, \tag{1}$$

according to the Drude model [2]. Here, γ is the damping frequency, and the plasma frequency ω_p is given by

$$\omega_p = \left(\frac{Ne^2}{m\varepsilon_0}\right)^{1/2}, \tag{2}$$

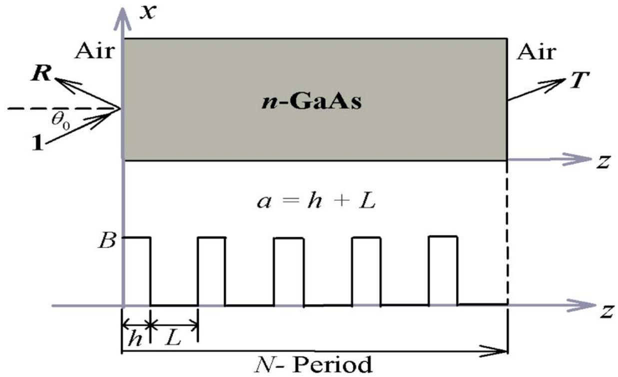

Fig. 1. The structures of n-GaAs-based extrinsic PC, where the homogeneous n-GaAs (top) is influenced by a spatially periodic magnetic field (bottom). The magnetic field, $\mathbf{B} = \mathbf{x}B$, is applied with a spatial period of $a = h + L$. The corresponding thicknesses and permittivities of this extrinsic PC are denoted by h, L, and ε_h, ε_L, respectively. ε_h is for the magnetized region whereas ε_L is for the one without magnetic field. Here, the angle of incident is denoted by θ_0. R and T are reflectance and transmittance, respectively.

where e is the electronic charge, m is the mass of free electron, N is the electron density, and ε_0 is the permittivity of vacuum. The analytical expression for $\omega_{p,eff}$ in an MDPC, which can be derived based on the effective medium concept, is now available [3]. The related studies of EPF on different PCs made of plasma or superconductor have been reported recently [4], [5].

The aforementioned studies focus mainly on conventional PCs which are made of two or more materials stacked in a spatially periodic manner. In addition to conventional PCs, there exists another type of PC, extrinsic PCs [6]. Unlike the usually conventional PCs, an extrinsic PC is made of a single homogeneous medium that is exerted by a spatially periodic external agent like electric field or magnetic field. With the applied field, the permittivity or permeability of the medium can become a periodic function of space, and thus, the homogeneous medium behaves effectively like a PC. As illustrated in [6], an extrinsic PC can be obtained in a doped semiconductor under a spatially periodic magnetic field. This extrinsic PC can be used to design a tunable photonic device since its permittivity or permeability is varied by the external agent. A tunable PC is of technical use in the photonic applications because it can be used to design as a tunable optical reflector [7], [8].

In this work, based on the consideration of an extrinsic PC made of a doped semiconductor, n-GaAs, we would like to investigate the related properties of EPF. We analyze the EPF from the calculated photonic band structure (PBS) and transmittance spectrum. The EPF will be studied as a function of the magnitude of external magnetic field and the filling factor of the magnetized region. In addition, the related studies on the first photonic passband and band gap will also be given.

2. Basic Equations

As shown on the top in Fig. 1, where a single bulk and homogeneous doped semiconductor, n-GaAs, occupies the space, $0 < z < Na$. To form an extrinsic PC, an external magnetic field is applied to the medium, as illustrated on the bottom in Fig. 1. The applied magnetic field $\mathbf{B} = \mathbf{x}B$ is spatially periodic with a period $a = h + L$, where h denotes the region influenced by the field, whereas L is the region in the absence of the field. The extrinsic PC is finite and has a number of periods, N. In the region where there is no magnetic field, the dielectric function of n-GaAs can, based on the plasma model, be expressed as [6]

$$\varepsilon_L(\omega) = \varepsilon_S\left(1 - \frac{\omega_p^2}{\omega^2}\right), \tag{3}$$

where ε_S is the static dielectric constant, and ω_p is the same as Eq. (2) with the mass being replaced by the effective mass of electrons m^*. In general, the dielectric constant of a semiconductor is contributed by the phonons and carriers (electrons and holes) [9]. In expressing Eq. (3), we have limited the frequency to be well below the phonon resonance frequency. In the presence of applied magnetic field, dielectric function of *n*-GaAs is modified and takes the form [10]

$$\varepsilon_h(\omega, B) = \varepsilon_S\left[1 - \frac{\omega_p^2\left(\omega^2 - \omega_p^2\right)}{\omega^2\left(\omega^2 - \omega_c^2 - \omega_p^2\right)}\right], \tag{4}$$

where the magnetic field dependence is related by the cyclotron frequency, $\omega_c = eB/m^*$. It is clear that Eq. (4) will reduce to Eq. (3) when $B = 0$. It should be mentioned that the expression of Eq. (4) is valid for the configuration of *H* polarization, i.e., the electric field must be perpendicular to the external magnetic field. In other words, the oblique incidence in Fig. 1 belongs to *TE* wave. In addition, the considered Fig. 1 is the Voigt configuration. As for *E* polarization or TM wave, the dielectric constant will not be influenced by the externally applied magnetic field [6], [10].

With the dielectric functions described in Eqs. (3) and (4), the equivalent refractive index profile for this extrinsic PC can be written as

$$n(z) = \begin{cases} n_h = \sqrt{\varepsilon_h(\omega, B)}, & ma < z < ma + h, \\ n_L = \sqrt{\varepsilon_L(\omega)}, & ma + h < z < (m+1)a, \end{cases} \qquad m = 0, 1, 2, \ldots N. \tag{5}$$

Then, the EPF can be read out from the PBS which can be calculated within the framework of the Kronig-Penney (KP) model in solids. The central equation determining the PBS is expressed as [2]

$$\cos(Ka) = \cos(k_{h,z}h)\cos(k_{L,z}L) - \frac{1}{2}\left(\frac{k_{L,z}}{k_{h,z}} + \frac{k_{h,z}}{k_{L,z}}\right)\sin(k_{h,z}h)\sin(k_{L,z}L), \tag{6}$$

for TE wave. Here, K is the Bloch wave number, and

$$k_{h,z} = \frac{\omega n_h}{c}\cos\theta_h, \quad k_{L,z} = \frac{\omega n_L}{c}\cos\theta_L. \tag{7}$$

In addition, we can also obtain the EPF from the transmittance spectrum calculated by making use of the transfer matrix method (TMM) [2].

3. Numerical Results and Discussion

In Fig. 2, we plot the calculated PBS at $B = 0$ (left) and $B \neq 0$ (center and right). Here, normal incidence is considered, and material parameters used are $\omega_p = 0.707 \times 10^{12}$ Hz, $m^* = 0.066m$, $m = 9.1 \times 10^{-31}$ kg, and $\varepsilon_s = 12.9$. In addition, the thicknesses are $h = 0.7a$, $L = 0.3a$, where $a = 0.8$ mm [6]. It can be seen that, at $B = 0$, the whole structure is a homogeneous medium like a metal with a plasma frequency of $\omega_p = 0.707$ THz. This agrees with Eq. (3). At frequencies above ω_p, waves can propagate through the medium, and band structure is a single continuous passband, as illustrated in the left panel in Fig. 2. In the presence of applied magnetic field, two PBSs are shown in the center and right panels. In this case, in addition to the low-frequency gap $(0 < \omega < \omega_{p,eff})$, a first photonic band gap $(\omega_L < \omega < \omega_H)$ is opened up because an extrinsic PC is established. The presence of applied magnetic field causes $\omega_{p,eff}$ (indicated by the arrow) to be lowered down to 0.491 THz and 0.394 THz at $B = 0.264$ and 0.528 T, respectively. The EPF decreases as B increases. The first photonic gap size, $\Delta = \omega_H - \omega_L$, is enhanced when the magnetic field increases. In addition, the PBS is red-shifted as B increases. The field-dependent $\omega_{p,eff}$, ω_L, and ω_H are depicted in Fig. 3, where shaded areas represent the forbidden bands. It is seen that these three frequencies are a decreasing function of B. The decreasing trend is more pronounced for both $\omega_{p,eff}$ and ω_L. The variation in ω_H is, however, a weak function of B.

Next, we try to investigate the size effect in the PBS as well as the EPF. Let us now fix the magnetic field at $B = 0.264$ T and let the ratio of h/L be 7/3. The three characteristic frequencies

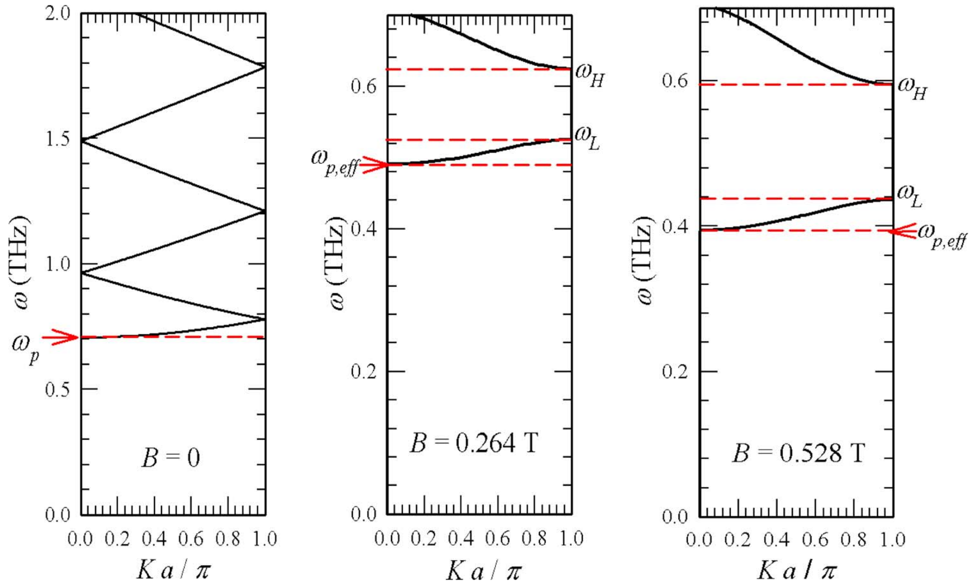

Fig. 2. The calculated PBSs for the extrinsic PC at three different static magnetic fields, $B = 0$, 0.264, and 0.528 T, respectively. The corresponding effective plasma frequencies are denoted by the arrows.

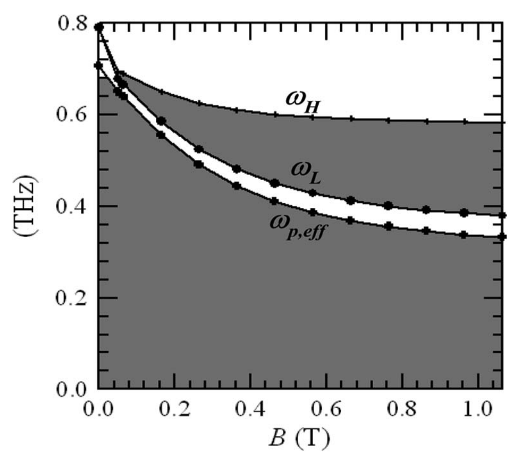

Fig. 3. The magnetic field dependence for $\omega_{p,eff}$, ω_L, and ω_H, respectively. All these three frequencies are decreasing functions of magnetic field. It can be seen that ω_H and ω_L coincide as B approaches zero, consistent with Fig. 2 at $B = 0$.

$\omega_{p,eff}$, ω_L, and ω_H versus the lattice constant a are plotted in Fig. 4. It is seen that PBS for an extrinsic PC is indeed affected by the size of the medium (or the period). First, all frequencies decrease when a increases. The decreasing trend in ω_L and ω_H is more pronounced than $\omega_{p,eff}$. Second, the first passband, $\omega_{p,eff} < \omega < \omega_L$, is sensitive to a and shrinks as function of a. However, there still exists a quite small gap at $a > 1.4$ mm. This negligibly small gap can be regarded as narrowband state.

Let us finally investigate how the applied-field region affects the PBS. To do this, we define the filling factor $\rho = h/a$ as the fraction of applied-field region for each period. Keeping $B = 0.264$ T and $a = 0.8$ mm, the calculated results for $\omega_{p,eff}$, ω_L, and ω_H are shown in Fig. 5. It is of interest to see that the EPF will decrease with the increase in ρ. The significant lowering in EPF at a large ρ reveals that the extrinsic PC exhibits a dielectric-like behavior because it is known that an all-dielectric PC

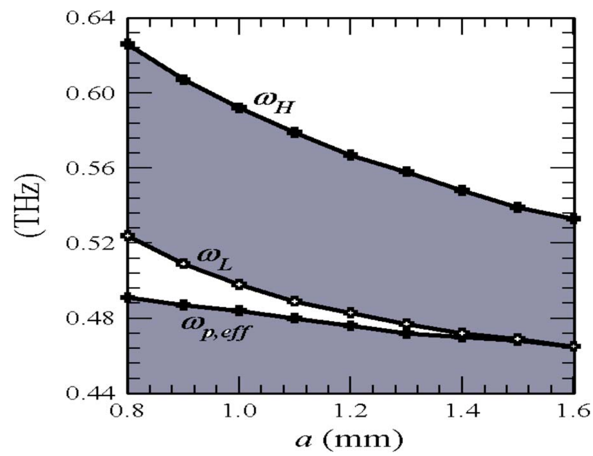

Fig. 4. The lattice-constant dependence for $\omega_{p,eff}$, ω_L, and ω_H, respectively. The applied magnetic field is $B = 0.264$ T and the ratio of h/L is fixed at 7/3. All these three frequencies are decreasing functions of magnetic field.

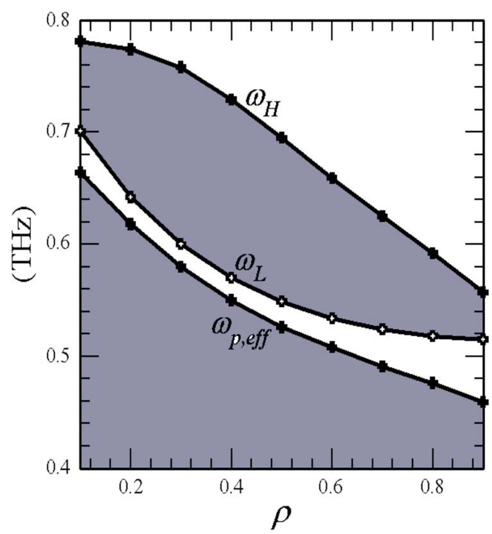

Fig. 5. The filling-factor dependence for $\omega_{p,eff}$, ω_L, and ω_H, respectively. The applied magnetic field is $B = 0.264$ T and $a = 0.8$ mm. All these three frequencies are decreasing functions of magnetic field.

has a zero EPF [2]. Conversely, by reducing ρ, the extrinsic is more like an MDPC because of a high EPF.

4. Conclusion

The magnetic-field dependence of EPF for an extrinsic photonic crystal made of a doped semiconductor has been numerically investigated based on the calculated PBS. In the absence of the applied magnetic field, the entire medium is homogeneous, and its wave properties can be characterized by the plasma frequency like a metal. When the applied magnetic field is present, the EPF is substantially decreased compared to the case of without field. With the increasing of the magnetic field, the EPF is shown to be lowered down. The decreasing in EPF leads to dielectric-like behavior for the extrinsic PC. The PBS and EPF are also shown to be dependent on the size of the medium and the filling factor of the applied-field region. The first three characteristic frequencies are all decreasing functions of medium size and filling factor as well.

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