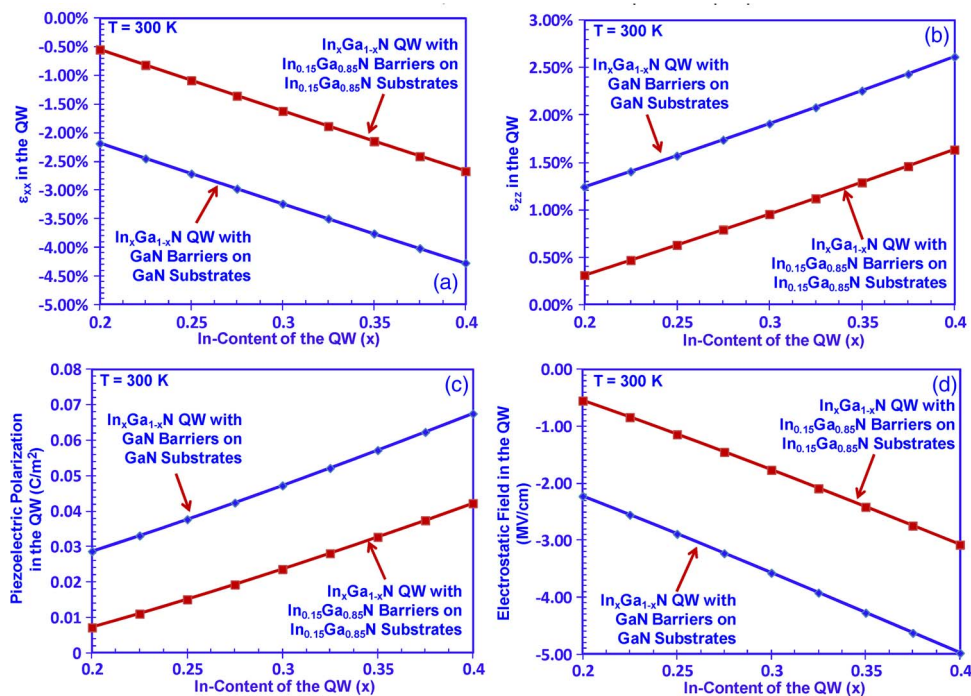


# Optical Gain and Laser Characteristics of InGaN Quantum Wells on Ternary InGaN Substrates

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# Optical Gain and Laser Characteristics of InGaN Quantum Wells on Ternary InGaN Substrates

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**Abstract:** The optical gain and threshold characteristics of InGaN quantum wells (QWs) on ternary InGaN substrate emitting in green and yellow spectral regimes are analyzed. By employing the ternary substrates, the material gains were found as  $\sim 3$ – $5$  times higher than that of conventional method with reduced wavelength shift. The threshold carrier density is reduced by  $\sim 15\%$ – $50\%$  from the use of ternary substrate method for green- and yellow-emitting lasers.

**Index Terms:** III-Nitride, InGaN quantum wells (QWs), ternary InGaN substrate, optical gain, threshold current, laser diodes.

## 1. Introduction

Significant advances in the III-Nitride materials have led to applications for lasers and light-emitting diodes (LEDs) [1]–[15], thermoelectricity [16], [17], and solar cells [18]. The use of conventional GaN substrate leads to large lattice mismatch strain between the substrate and quantum well (QW). The QW large strain leads to a large internal field in the QW, which reduces the optical matrix element from the charge separation effect [19]–[22]. The large strain also results in increased misfit dislocation density in the high In-content InGaN QWs. To address the charge separation, approaches based on semi/nonpolar InGaN QWs [23]–[27] and InGaN QWs with large overlap design [28]–[36] had been used.

Previous works had reported the growths of InGaN templates and substrates by metalorganic vapor phase epitaxy [37], [38] and hydride vapor phase epitaxy [39]. Recent works [40], [41] had shown the feasibility for accessing the green up to red spectral regimes by using ternary substrate method. Our recent work [42] revealed that the use of ternary InGaN substrates resulted in  $\sim 2$ – $3$  times increase in the radiative recombination rates for green- and red-emitting InGaN QWs attributing to the reduced charge separation effect. Recent works [43] had also reported an electrically injected semipolar laser grown on an intentionally stress-relaxed  $n$ - $\text{In}_{0.09}\text{Ga}_{0.91}\text{N}$  template, which shows the feasibility of the ternary template for lasers. These initial works have shown the potential of this method for achieving high-performance visible LEDs and lasers. However, comprehensive studies on the optical gain and threshold characteristics of InGaN QWs on ternary InGaN substrate or template are still lacking.

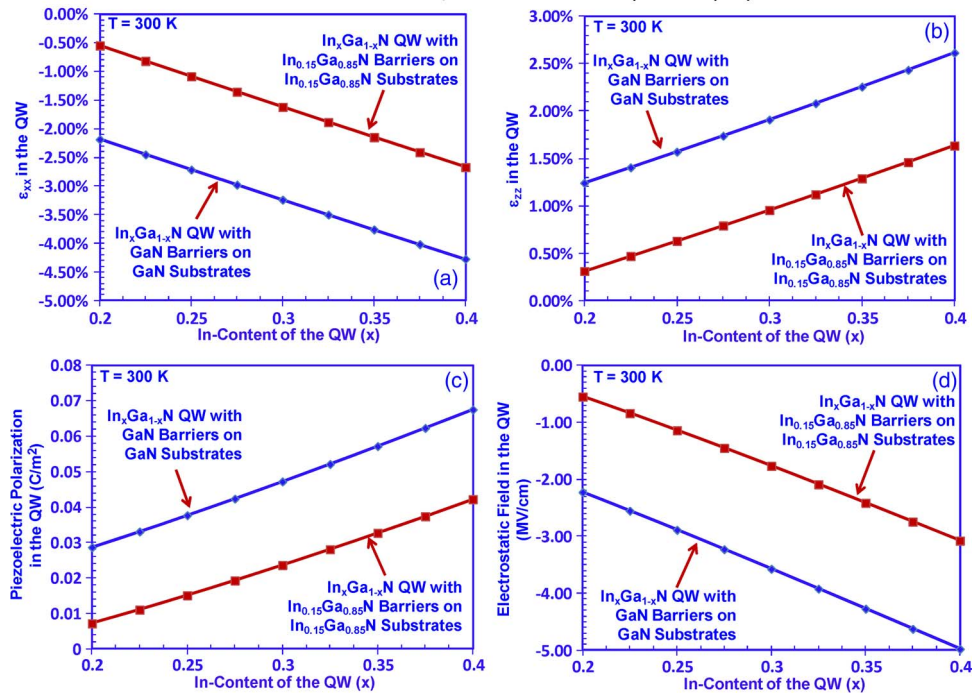


Fig. 1. (a) The strain parameter at x-direction, (b) the strain parameter at z-direction, (c) the piezoelectric polarization field, and (d) the electrostatic field in the  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs as a function of In-content of the QW, for both  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs with  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  barriers on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate and conventional  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs with GaN barriers on GaN substrate.

Here, we present a comprehensive study on the optical gain and threshold characteristics of InGaN QWs on ternary InGaN substrates for green- and yellow-emitting lasers. These results are also compared with those of the conventional InGaN QWs/GaN substrate approach. The band structures and wave functions in this paper were calculated by using self-consistent six-band  $\mathbf{k} \cdot \mathbf{p}$  formalism for wurtzite semiconductor [44]–[47] taking into account the valence band mixing, strain effect, polarization fields, and carrier screening effect, with the band parameters obtained from [47]–[49].

## 2. Strain and Polarization Field for InGaN QW Lasers With Ternary Substrates

Fig. 1(a) and (b) shows the in-plane ( $\epsilon_{xx}$ ) and cross-plane ( $\epsilon_{zz}$ ) strains in the active region layers as a function of In-contents (x) in the  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QW. The comparisons of the strains in the  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate and conventional GaN substrate for  $x = 0.2$  up to  $x = 0.4$  were carried out. Attributing to the smaller lattice mismatch, both the  $\epsilon_{xx}$  and  $\epsilon_{zz}$  are reduced by  $\sim 37.5\%$ – $75\%$  in the QWs from the use of the ternary substrates, which, in turn, reduces the piezoelectric polarization fields [see Fig. 1(c)] and electrostatic fields [see Fig. 1(d)] in the QWs. The internal fields in the QWs are reduced by  $\sim 40\%$  up to  $75\%$ , which, in turn, leads to suppression of charge separation [42]. In our studies, all the InGaN QW thicknesses were kept as 3 nm for comparison purposes.

## 3. Theoretical and Numerical Formulations

The calculations of the band structures and electron and hole wave functions are based on the self-consistent six-band  $\mathbf{k} \cdot \mathbf{p}$  formalism for wurtzite semiconductors [47], which takes into account the valence band mixing, strain effect, spontaneous and piezoelectric polarization, and the carrier screening effect. The spin–orbit interaction is taken into account in the six-band  $\mathbf{k} \cdot \mathbf{p}$  method. The

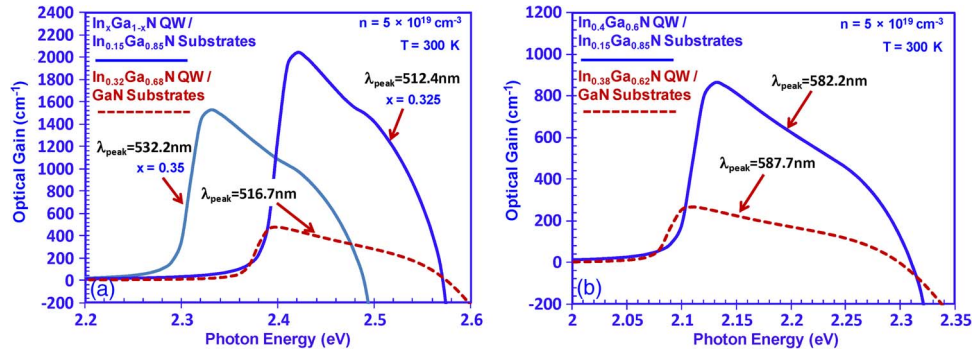


Fig. 2. Optical gain spectra for (a)  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs ( $x = 0.325, 0.35$ ) with  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  barriers on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate, and  $\text{In}_{0.32}\text{Ga}_{0.68}\text{N}$  QW with GaN barriers on GaN substrate, and (b)  $\text{In}_{0.4}\text{Ga}_{0.6}\text{N}$  QW with  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  barriers on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate, and  $\text{In}_{0.38}\text{Ga}_{0.62}\text{N}$  QW with GaN barriers on GaN substrate with  $n = 5 \times 10^{19} \text{ cm}^{-3}$  at  $T = 300 \text{ K}$ .

material parameters for GaN, AlN, and InN alloys are taken from [47]–[49] (Table 1 in [47]). The numerical model takes into account the internal electrostatic field resulting from the spontaneous and piezoelectric polarization fields with the details presented in [47]. The calculation of spontaneous and piezoelectric polarization fields follow the treatment discussed in [50] and [51], which do not include the nonlinear term discussed in [52]–[54].

The optical gain calculation is obtained based on the Fermi's Golden rule, including a Lorentzian line-shape function [44], [47]. The upper and lower  $3 \times 3$  Hamiltonian blocks from the  $6 \times 6$  diagonalized Hamiltonian matrix are denoted as  $\sigma = \text{U}$  and  $\sigma = \text{L}$ , respectively. The spontaneous emission rate for transverse-electric (TE) ( $e = x$ ) or transverse-magnetic (TM) ( $e = z$ ) polarizations can be obtained by taking into account all interband transitions between  $n$ th conduction subbands and  $m$ th valence subbands. The optical gain calculation follows the treatment in [44] and [47]

$$g_{sp}^e(\hbar\omega) = \frac{2q^2\pi}{n_r c \epsilon_0 m_0^2 \omega L_w} \sum_{\sigma=\text{U,L}} \sum_{n,m} \int \frac{k_t dk_t}{2\pi} |(M_e)_{nm}^\sigma(k_t)|^2 \cdot \frac{f_n^c(k_t)(1 - f_{\sigma m}^v(k_t))(\gamma/\pi)}{(E_{\sigma, nm}^{cv}(k_t) - \hbar\omega)^2 + \gamma^2} \quad (1)$$

where  $q$  is the magnitude of the electron charge,  $m_0$  is the electron mass in free space, and  $c$  and  $\epsilon_0$  are the velocity of light and permittivity in free space, respectively.  $e$  is the polarization vector of the optical electric field;  $n_r$  and  $L_w$  are the refractive index and thickness of the QW, respectively. The term  $\hbar\gamma$  is the half-linewidth of the Lorentzian function, and the linewidth broadening time  $\tau_s = 0.1 \text{ ps}$  ( $\gamma = (0.1 \text{ ps})^{-1}$ ) is used in our calculation [44], [47]. The inhomogeneous broadening is not taken into account here, as there has been no experimental data on this value from the growths of InGaN on ternary substrate. The term  $(M_e)_{nm}^\sigma(k_t)$  is the momentum matrix element for transitions between the  $n$ th conduction-band state and the  $m$ th valence-band state.

#### 4. Optical Gain Characteristics of InGaN QWs With Ternary Substrates

The gain properties of the InGaN QWs on ternary and GaN substrates are compared. The In-contents in the InGaN QWs were chosen for similar emission wavelengths at  $n \sim 5 \times 10^{19} \text{ cm}^{-3}$ . Fig. 2(a) shows optical gain spectra for  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs ( $x = 0.325, 0.35$ ) on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate and conventional  $\text{In}_{0.32}\text{Ga}_{0.68}\text{N}$  QW on GaN substrate for green spectral regime. For  $\text{In}_{0.32}\text{Ga}_{0.68}\text{N}$  QW on GaN substrate, relatively low material peak gain ( $g_{\text{peak}} \sim 474.1 \text{ cm}^{-1}$ ) is obtained with  $\lambda_{\text{peak}} \sim 516.7 \text{ nm}$ . The material gain is defined as the peak value of the optical gain spectrum. The material gain of the  $\text{In}_{0.325}\text{Ga}_{0.675}\text{N}$  QW/ $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate ( $g_{\text{peak}} \sim 2039.7 \text{ cm}^{-1}$ ) is  $\sim 3.3$  times higher than that of conventional method ( $g_{\text{peak}} \sim 474.1 \text{ cm}^{-1}$ ). The material gain of the  $\text{In}_{0.35}\text{Ga}_{0.65}\text{N}$  QW/ $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate ( $g_{\text{peak}} \sim 1527.7 \text{ cm}^{-1}$ ) with  $\lambda_{\text{peak}} \sim 532.2 \text{ nm}$  is  $\sim 2.2$  times higher than that of the conventional method. For yellow-emitting QWs comparison



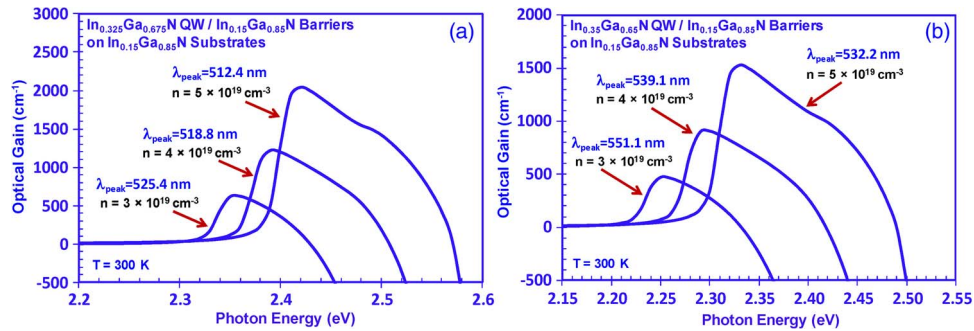


Fig. 3. Optical gain spectra for (a)  $\text{In}_{0.325}\text{Ga}_{0.675}\text{N}$  QW, and (b)  $\text{In}_{0.35}\text{Ga}_{0.65}\text{N}$  QW with  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  barriers on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate for green spectra regime at  $n = 3 - 5 \times 10^{19} \text{ cm}^{-3}$  at room temperature.

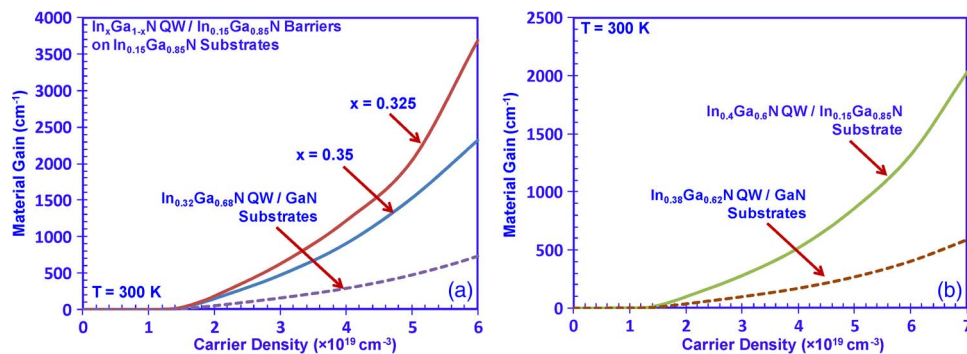


Fig. 4. Comparison of material peak gain as a function of carrier density for (a)  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs ( $x = 0.325, 0.35$ ) on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate, and conventional  $\text{In}_{0.32}\text{Ga}_{0.68}\text{N}$  QW on GaN substrate, and (b)  $\text{In}_{0.4}\text{Ga}_{0.6}\text{N}$  QW on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate, and conventional  $\text{In}_{0.38}\text{Ga}_{0.62}\text{N}$  QW on GaN substrate at room temperature.

[see Fig. 2(b)], the use of the ternary substrate leads to 3.2 times increase in the material gain ( $g_{\text{peak}} \sim 861.4 \text{ cm}^{-1}$ ) over that of the conventional method.

The optical gain spectra for green-emitting InGaN QWs on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate are shown in Fig. 3(a) and (b). The higher In-content QW exhibits lower material gain and larger wavelength blue shift, in comparison with those of lower In-content QW. By comparing the spectra at  $n = 3 \times 10^{19} \text{ cm}^{-3}$  and  $n = 5 \times 10^{19} \text{ cm}^{-3}$ , the wavelength blue shifts for the lower [see Fig. 3(a)] and higher In-content [see Fig. 3(b)] InGaN QW are  $\sim 13 \text{ nm}$  and  $\sim 19 \text{ nm}$ , respectively. The corresponding comparisons of the material gains for ternary and conventional methods were shown in Fig. 4(a) and (b) for green- and yellow-emitting QWs, respectively. At higher carrier densities, the material gains for green (yellow)-emitting InGaN QWs on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate are  $\sim 3.2$ – $5$  times ( $3.2$ – $3.5$  times) higher than that of the conventional method. The improved material gains in green and yellow spectral regimes obtained from the use of ternary substrates are attributed from the reduced charge separation issue in the QWs.

The optical gain properties for InGaN QWs on ternary substrates with various In-contents were shown in Fig. 5(a) and (b). Note that the second peak from the gain spectra of the  $\text{In}_{0.3}\text{Ga}_{0.7}\text{N}$  QW/ $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate at  $473.3 \text{ nm}$  in Fig. 5(a) is attributed to the band filling effect leading to stronger excited state transitions. The use of higher In-content ternary substrate leads to reduction in charge separation effect in the QW, which results in improved material gain. The material gain for  $\text{In}_{0.3}\text{Ga}_{0.7}\text{N}$  QW/ $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate is  $\sim 2135.9 \text{ cm}^{-1}$  ( $n = 5 \times 10^{19} \text{ cm}^{-3}$ ), which is higher than those employing lower In-content ternary substrates and conventional GaN substrate ( $g_{\text{peak}} \sim 588.5 \text{ cm}^{-1}$ ).

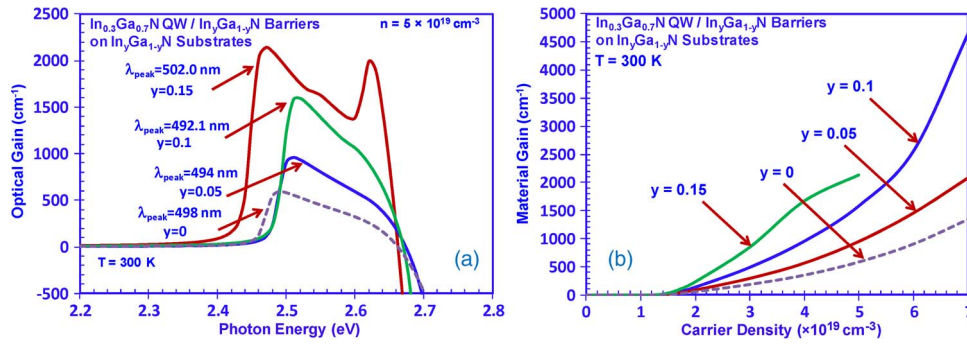


Fig. 5. (a) Optical gain spectra at  $n = 5 \times 10^{19} \text{ cm}^{-3}$  and (b) material gains as a function of carrier density for  $\text{In}_{0.3}\text{Ga}_{0.7}\text{N}$  QW with  $\text{In}_y\text{Ga}_{1-y}\text{N}$  barriers on  $\text{In}_y\text{Ga}_{1-y}\text{N}$  substrate ( $y = 0, 0.05, 0.1$  and  $0.15$ ).

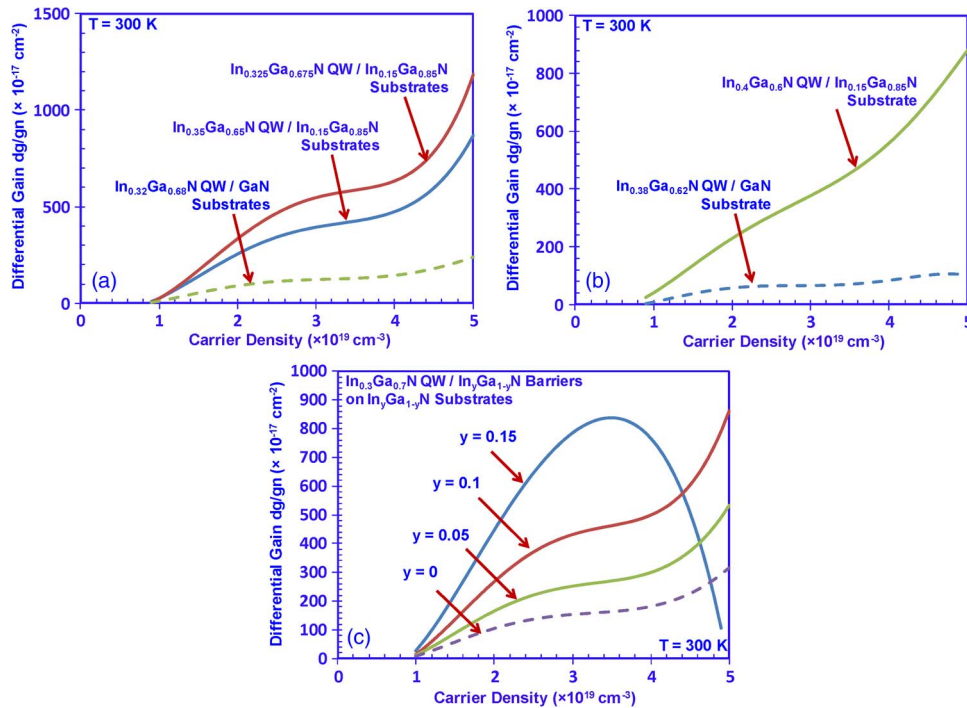


Fig. 6. Differential gain as a function of carrier density for (a)  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs ( $x = 0.325, 0.35$ ) on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate, and  $\text{In}_{0.32}\text{Ga}_{0.68}\text{N}$  QW on GaN substrate, (b)  $\text{In}_{0.4}\text{Ga}_{0.6}\text{N}$  QW on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate, and  $\text{In}_{0.38}\text{Ga}_{0.62}\text{N}$  QW on GaN substrate, and (c)  $\text{In}_{0.3}\text{Ga}_{0.7}\text{N}$  QW with  $\text{In}_y\text{Ga}_{1-y}\text{N}$  barriers on  $\text{In}_y\text{Ga}_{1-y}\text{N}$  substrate ( $y = 0, 0.05, 0.1$  and  $0.15$ ) at  $T = 300 \text{ K}$ .

## 5. Differential Gains and Wavelength Shifts Characteristics

The differential gains (dg/dn) for InGaN QWs on ternary and GaN substrates were compared in Fig. 6(a)–(c). Fig. 6(a) shows the dg/dn as a function of carrier density for  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs ( $x = 0.325, 0.35$ ) on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate and conventional  $\text{In}_{0.32}\text{Ga}_{0.68}\text{N}$  QW on GaN substrate for green spectral regime at  $T = 300 \text{ K}$ . The increasing trend of dg/dn for higher carrier density is primarily attributed to the carrier screening effect. The dg/dn for the  $\text{In}_{0.35}\text{Ga}_{0.65}\text{N}$  QW and  $\text{In}_{0.325}\text{Ga}_{0.675}\text{N}$  QW on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate are  $\sim 3.6$  and  $\sim 4.9$  times of that of the conventional  $\text{In}_{0.32}\text{Ga}_{0.68}\text{N}$  QW with  $n = 5 \times 10^{19} \text{ cm}^{-3}$ , respectively [see Fig. 6(a)]. For yellow-emitting QWs, the dg/dn for  $\text{In}_{0.4}\text{Ga}_{0.6}\text{N}$  QW/ $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate shows  $\sim 7.7$  times improvement than that of the  $\text{In}_{0.38}\text{Ga}_{0.62}\text{N}$  QW/GaN substrate with  $n = 5 \times 10^{19} \text{ cm}^{-3}$  [see Fig. 6(b)]. The dg/dn are

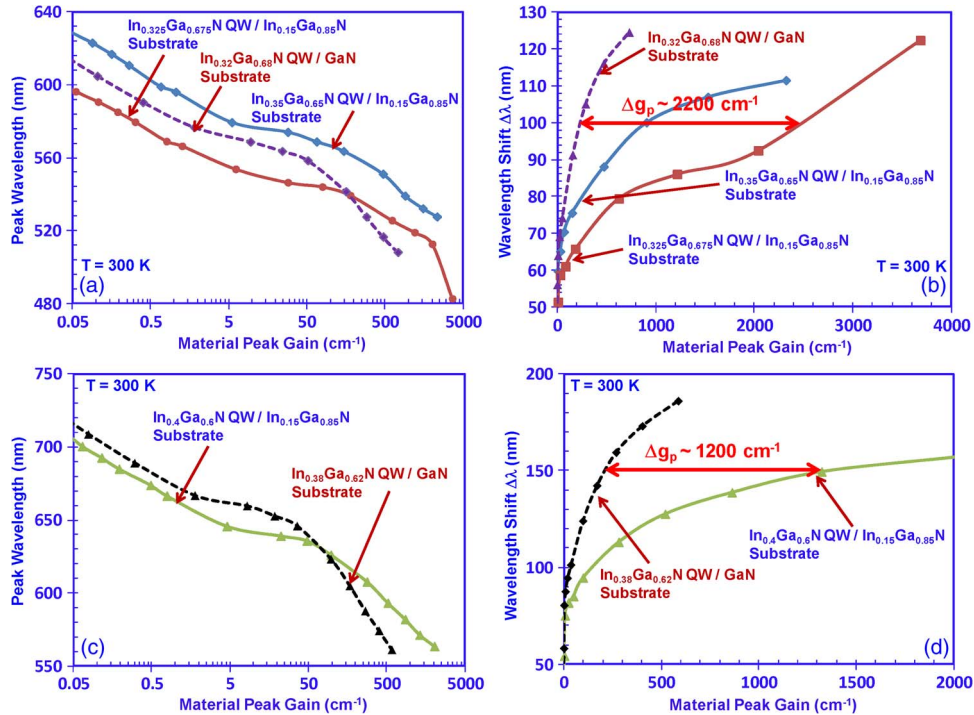


Fig. 7. (a) Peak emission wavelengths from gain spectra as a function of material peak gain, and (b) wavelength shift  $\Delta\lambda$  ( $\Delta\lambda = \lambda_0 - \lambda(n)$ ) for  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs ( $x = 0.325, 0.35$ ) on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate, and conventional  $\text{In}_{0.32}\text{Ga}_{0.68}\text{N}$  QW on GaN substrate. (c) Peak emission wavelengths as a function of material peak gain, and (d) wavelength shift  $\Delta\lambda$  ( $\Delta\lambda = \lambda_0 - \lambda(n)$ ) for  $\text{In}_{0.4}\text{Ga}_{0.6}\text{N}$  QW on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate, and conventional  $\text{In}_{0.38}\text{Ga}_{0.62}\text{N}$  QW on GaN substrate at  $T = 300$  K.

compared for  $\text{In}_{0.3}\text{Ga}_{0.7}\text{N}$  QW on  $\text{In}_y\text{Ga}_{1-y}\text{N}$  substrate ( $y = 0$  up to  $y = 0.15$ ) at  $T = 300$  K [see Fig. 6(c)]. For the use of  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate, higher differential gains are achieved for lower carrier densities ( $n \sim 3.5 \times 10^{19} \text{ cm}^{-3}$ ) due to the enhanced matrix element. The decrease in the  $dg/dn$  for the ternary substrate with 15% In-content ( $n > 3.5 \times 10^{19} \text{ cm}^{-3}$ ) can be attributed to the band filling effect.

The comparison of the wavelength shifts as a function of material gains for the green- and yellow-emitting QWs on ternary substrates are shown in Fig. 7(a) and (b). For green-emitting QWs [see Fig. 7(a)], the peak emission wavelengths show blue shift for increasing carrier densities for both QWs due to the carrier screening effect. Significantly higher material gains ( $g_{\text{peak}} > 2000 \text{ cm}^{-1}$ ) are obtained from the use of ternary substrates. In order to compare the wavelength shift quantitatively, the  $\Delta\lambda$  ( $\Delta\lambda = \lambda_0 - \lambda(n)$ ) as a function of material peak gains are extracted from Fig. 7(a), where  $\lambda_0$  is the emission wavelength at  $n = 3 \times 10^{18} \text{ cm}^{-3}$  [see Fig. 7(b)]. The use of ternary substrates leads to significantly improved material gain, accompanied with significantly reduced wavelength shift. To illustrate this improvement for  $\Delta\lambda \sim 100$  nm, the use of ternary substrate leads to increase in material gain  $\sim 2200 \text{ cm}^{-1}$  in comparison with that of conventional method. This finding illustrates the feasibility of achieving high material gain with lower carrier density from the use of ternary substrate. Similar finding is also observed for yellow-emitting QWs [see Fig. 7(c) and (d)], which indicates that the increase of  $\sim 1200 \text{ cm}^{-1}$  in material gain from the use of ternary substrate method for  $\Delta\lambda \sim 150$  nm.

The effect of employing different substrates on the screening effect is presented in Fig. 8(a) and (b). Fig. 8(a) compares the peak emission wavelengths as a function of material gain for the  $\text{In}_{0.3}\text{Ga}_{0.7}\text{N}$  QW on various  $\text{In}_y\text{Ga}_{1-y}\text{N}$  substrates ( $y = 0, 0.05, 0.1, \text{ and } 0.15$ ). Note that the use of higher In-content substrate leads to larger material gain with reduced wavelength shift [see Fig. 8(b)]. Specifically, for obtaining  $g_{\text{peak}} \sim 1000 \text{ cm}^{-1}$ , the lowest wavelength shift  $\Delta\lambda \sim 70$  nm is obtained by using  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate. In addition, an increase of  $\sim 1700 \text{ cm}^{-1}$  in material gain is

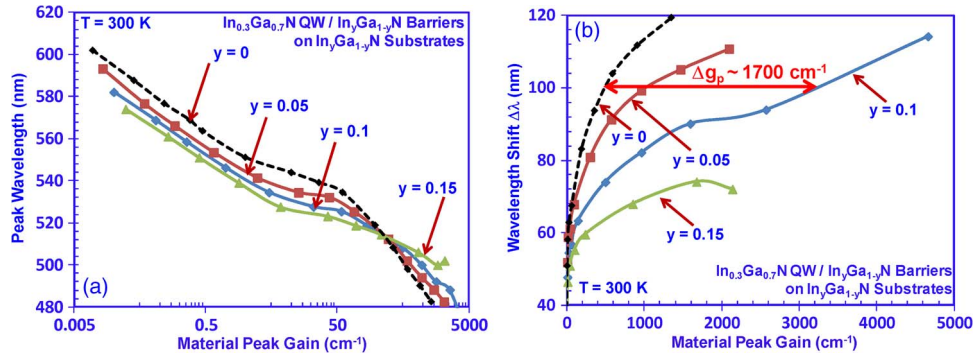


Fig. 8. (a) Peak emission wavelength from gain spectra as a function of material peak gain, and (b) wavelength shift  $\Delta\lambda$  ( $\Delta\lambda = \lambda_0 - \lambda(n)$ ) for  $\text{In}_{0.3}\text{Ga}_{0.7}\text{N}$  QW with  $\text{In}_y\text{Ga}_{1-y}\text{N}$  barriers on  $\text{In}_y\text{Ga}_{1-y}\text{N}$  substrate ( $y = 0, 0.05, 0.1$  and  $0.15$ ) at  $T = 300$  K.

obtained by employing the  $\text{In}_{0.1}\text{Ga}_{0.9}\text{N}$  substrate as compared with conventional method with  $\Delta\lambda \sim 100$  nm.

## 6. Threshold Characteristics of Green- and Yellow-Emitting Lasers

The threshold characteristics are analyzed for the green- and yellow-emitting lasers using ternary substrates. The threshold gain ( $g_{th}$ ) is estimated as  $\sim 1500$  cm<sup>-1</sup> [47], [55]. The threshold carrier densities ( $n_{th}$ ) can be obtained from the material peak gain and carrier density relation in Fig. 4(a) and (b). For green-emitting InGaN QWs on ternary substrates [see Fig. 4(a)], the threshold carrier densities are  $n_{th} \sim 4.4 \times 10^{19}$  cm<sup>-3</sup> for  $\text{In}_{0.325}\text{Ga}_{0.675}\text{N}$  QW/ $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate and  $n_{th} \sim 4.95 \times 10^{19}$  cm<sup>-3</sup> for  $\text{In}_{0.35}\text{Ga}_{0.65}\text{N}$  QW/ $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate. In contrast, the material gains obtained from the conventional active regions emitting in green spectral regime at similar carrier density ( $n \sim 4.4\text{--}6.27 \times 10^{19}$  cm<sup>-3</sup>) range from  $\sim 200\text{--}400$  cm<sup>-1</sup>, which are significantly lower than those obtained from the ternary substrate approach. For yellow-emitting InGaN QWs on ternary substrates [see Fig. 4(b)], the threshold carrier density is  $n_{th} \sim 6.27 \times 10^{19}$  cm<sup>-3</sup>. Similarly, the material gain obtained from the conventional QW at similar carrier density is as low as  $\sim 400$  cm<sup>-1</sup>, which indicates that significantly higher carrier density is required for the conventional approach in order to achieve  $g_{th} \sim 1500$  cm<sup>-1</sup>.

The threshold carrier densities of the green-emitting QWs on ternary substrates with various In-contents can be extracted from Fig. 5(b). For  $\text{In}_{0.3}\text{Ga}_{0.7}\text{N}$  QW, the  $n_{th}$  are  $\sim 6.05 \times 10^{19}$  cm<sup>-3</sup>,  $4.9 \times 10^{19}$  cm<sup>-3</sup>, and  $3.75 \times 10^{19}$  cm<sup>-3</sup>, respectively, for  $\text{In}_{0.05}\text{Ga}_{0.95}\text{N}$ ,  $\text{In}_{0.1}\text{Ga}_{0.9}\text{N}$ , and  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrates. The corresponding  $n_{th}$  for the conventional method is larger than  $7 \times 10^{19}$  cm<sup>-3</sup>. Note that the increased electron-hole wavefunction overlap in the QWs grown on ternary substrate leads to the enhancement of the radiative recombination rates and optical gains. The low threshold carrier density operation is important for suppressing monomolecular ( $\sim A \cdot n_{th}$ ) and Auger ( $\sim C \cdot n_{th}^3$ ) recombination current densities at threshold condition. The increased material gain result in reduction in  $n_{th}$ , which result in suppression of nonradiative threshold current density ( $J_{non-rad} \sim A \cdot n_{th} + C \cdot n_{th}^3$ ) [47] and reduction in total threshold current density ( $J_{th} = J_{Rad} + J_{non-rad}$ ) for nitride-based lasers on ternary substrates.

To analyze the threshold current densities ( $J_{th}$ ) for the green- and yellow-emitting lasers, the relation of the material gain as a function of  $J_{tot}$  for  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs ( $x = 0.325, 0.35,$  and  $0.4$ ) on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate are shown in Fig. 9. The monomolecular recombination rates of  $A = 6 \times 10^8$  s<sup>-1</sup>,  $1 \times 10^9$  s<sup>-1</sup> and  $1.5 \times 10^9$  s<sup>-1</sup> are used similar to [56]. For achieving  $g_{th} \sim 1500$  cm<sup>-1</sup>, yellow-emitting QWs typically have  $\sim 30\%$  higher  $J_{th}$  over that of green-emitting QWs.

The threshold comparison from the use of ternary  $\text{In}_y\text{Ga}_{1-y}\text{N}$  substrates (with  $y = 0.05, 0.1,$  and  $0.15$ ) are presented for various monomolecular recombination rates [see Fig. 10(a)–(c)]. In this comparison, the active region was chosen as  $\text{In}_{0.3}\text{Ga}_{0.7}\text{N}$  QW. The reduction in the  $n_{th}$  from the ternary substrate is important for suppressing the nonradiative current density, which, in turn,



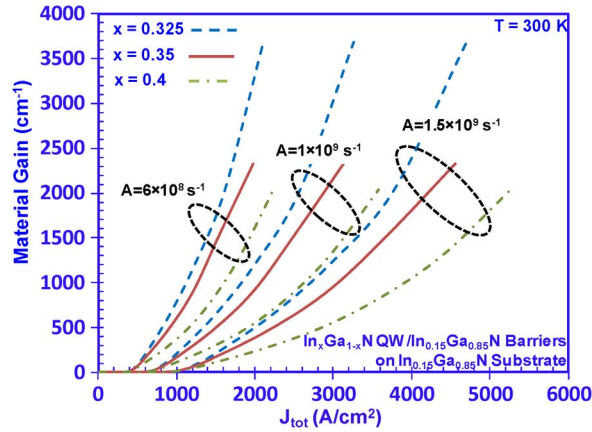


Fig. 9. Material gain as a function of total current density for  $\text{In}_x\text{Ga}_{1-x}\text{N}$  QWs ( $x = 0.325, 0.35, 0.4$ ) with  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  barriers on  $\text{In}_{0.15}\text{Ga}_{0.85}\text{N}$  substrate at  $T = 300$  K. The monomolecular recombination rates are  $A = 6 \times 10^8 \text{ s}^{-1}$ ,  $1 \times 10^9 \text{ s}^{-1}$ , and  $1.5 \times 10^9 \text{ s}^{-1}$ .

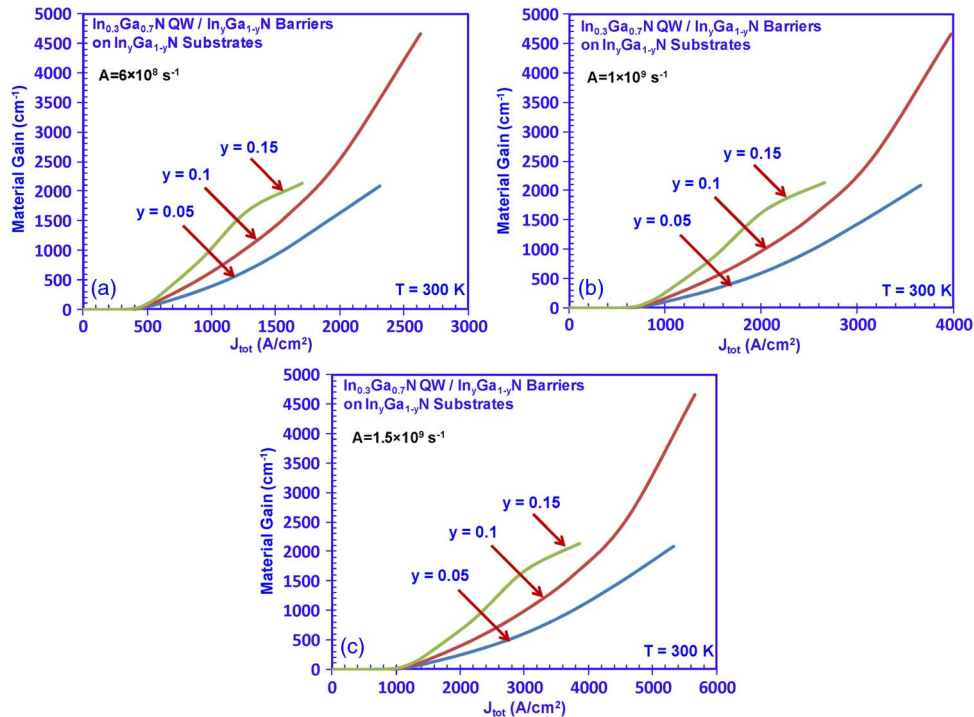


Fig. 10. Material gain as a function of total current density for  $\text{In}_{0.3}\text{Ga}_{0.7}\text{N}$  QW with  $\text{In}_y\text{Ga}_{1-y}\text{N}$  barriers on  $\text{In}_y\text{Ga}_{1-y}\text{N}$  substrate ( $y = 0.05, 0.1$  and  $0.15$ ) at  $T = 300$  K. The monomolecular recombination rates are (a)  $A = 6 \times 10^8 \text{ s}^{-1}$ , (b)  $A = 1 \times 10^9 \text{ s}^{-1}$ , and (c)  $A = 1.5 \times 10^9 \text{ s}^{-1}$ .

reduces the threshold current density. The threshold analysis here does not take into account the Auger recombination due to the low value of the Auger recombination coefficient ( $C_{\text{Auger}} \sim 10^{-32} \text{ cm}^6/\text{s}$ ) in InGaN [57], [58].

The  $C_{\text{Auger}}$  values for InGaN semiconductors had been reported with large variation [57]–[62], which range from lower limit ( $3.5 \times 10^{-34} - 1 \times 10^{-32} \text{ cm}^6/\text{s}$ ) [57], [58] up to higher limit ( $1 \times 10^{-31} - 1 \times 10^{-29} \text{ cm}^6/\text{s}$ ) [59]–[62]. For the lower limit  $C_{\text{Auger}}$  of  $1 \times 10^{-32} \text{ cm}^6/\text{s}$  [58], the Auger recombination current densities at threshold ( $J_{\text{th-Auger}} \sim n_{\text{th}}^3$ ) range from  $\sim 0.025 \text{ kA}/\text{cm}^2$  (green QW) up to  $0.118 \text{ kA}/\text{cm}^2$  (yellow QW), which are negligible (5%) in comparison with the total

threshold current densities for green and yellow lasers using ternary substrates. For the lower limit  $C_{\text{Auger}}$  of  $1 \times 10^{-31} \text{ cm}^6/\text{sec}$  [61], the  $J_{\text{th\_Auger}}$  range from  $\sim 0.253 \text{ kA/cm}^2$  (green QW) up to  $1.183 \text{ kA/cm}^2$  (yellow QW), which represent 12.3% and 27% of the total threshold current densities for green and yellow lasers using ternary substrates, respectively. Note that the reduction in  $n_{\text{th}}$  from the use of ternary substrates will be important in reducing the  $J_{\text{th\_Auger}}$  ( $\sim n_{\text{th}}^3$ ) by  $\sim 40\%$  up to 85%, in comparison with those of conventional GaN substrate method.

## 7. Summary

In summary, the optical gain and threshold characteristics of InGaN QWs on ternary InGaN substrates or templates are analyzed for green- and yellow-emitting lasers, which are also compared with the InGaN QWs on conventional GaN substrates or templates. The use of ternary InGaN substrate is expected to result in large increase in material gain and significant reduction in threshold carrier density in the active region, accompanied with less wavelength shift, which may potentially lead to high performance diode lasers emitting in the green and yellow spectral regimes. Note that, experimentally, the successful growths of bulk InGaN substrates have already been realized [37]–[39]. For the InGaN QWs grown on bulk ternary InGaN substrates, the In-contents of the InGaN QWs are lower than that of the substrates. Thus, the growth temperatures of the InGaN QWs should be lower than that of the ternary substrates, which are expected to be kept in good condition during the growths. However, it is important to note that the experimental challenge still needs to be addressed for optimized growths, as the ternary InGaN substrates have only been experimentally realized with In-contents ranging from 0.9% up to 20% [37]–[39].

The development of the ternary substrate still requires further optimization, and the availability of this substrate is the key for enabling the advantages deliberated in this study. In addition, the growths of high In-content InGaN alloy for QW active regions is still challenging, attributed to the increased phase separation issue during the epitaxy.

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