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Modulating Phase Encoding with Amplitude Compensation for Hologram Reconstruction

Jinbin Gui, Xiandong Ma, Junchang Li, and Qinghe Song

***Abstract*—In holographic three-dimensional (3D) display, phase-only spatial light modulators (SLMs) can only accept pure phase modulation signals, often leading to amplitude distortion in the reconstructed image and inaccurate object reconstruction. This paper presents a new modulating phase encoding method with amplitude compensation. Based on the imaging characteristics of phase digital holograms, the object light amplitude variation is extended to the nonlinear region of the first-order Bessel function to maximize the intensity of the reconstructed 3D image. The amplitude distortion of the reconstructed image is effectively eliminated by pre-distortion treatment of the light amplitude. Theoretical simulations and optical experiments validate the presented method.**

***Index Terms*—Amplitude compensation; Computer generated hologram; Holographic display.**

I. INTRODUCTION

IN the study field of holographic 3D display, phase spatial light modulators (SLMs) are widely used due to their high transmission and diffraction efficiency [1],[2],[3],[4]. The continuous development of material manufacturing and packaging technologies has improved the resolution and pixel number of commercial SLMs, facilitating their application in holographic 3D display systems [5],[6],[7],[8],[9],[10]. However, in phase holograms, the amplitude is regarded as a constant, retaining only the phase information of the object light and not completely preserving all object light information. Based on the basic theory of traditional phase holograms [11], we proposed a method to transform holographic interference intensity into a digital hologram with modulating phase encoding [12]. Theoretical and experimental research shows that this method can reconstruct images with both object amplitude and phase information. However, phase distortion exists in the reconstructed optical field, resulting in poor image quality. One solution to achieve reconstruction without phase distortion is complex amplitude modulation. A 4F filtering system can remove disturbing information and reconstruct images without phase distortion [15]. However, theoretical analysis reveals that the reconstructed image suffers from amplitude distortion because the object light is proportional to the first-order Bessel expression with the actual light amplitude as its parameter. To reduce distortion, the variation interval of the Bessel function must be compressed to achieve an approximately linear change, but this reduces the intensity of the reconstructed image. In this paper, we study the imaging characteristics of phase digital holograms and present two improvements to address the distortion. First, the object light amplitude is changed to the nonlinear region of the first-order Bessel function to maximize the intensity of the reconstructed 3D image. Second, the amplitude distortion of the reconstructed image is effectively eliminated by pre-distortion processing of the light amplitude. The results demonstrate that this modulating phase encoding method with amplitude compensation can improve the intensity of the reconstructed image and accurately reconstruct the 3D image with both amplitude and phase information.

II. HOLOGRAPHIC IMAGING BASED ON PHASE SPATIAL LIGHT MODULATOR

In the Cartesian coordinate system, $o-xyz$, $z=0$ represents the hologram plane, $\mathrm{j}=\sqrt{-1}$, the object wave in the hologram plane is denoted by $O(x,y)=o(x,y)\exp[\mathrm{j}\varphi(x,y)]$, and The reference wave is expressed as $R(x,y)=A_r\exp[\mathrm{j}\varphi(x,y)]$. Let the reference light be a light wave that propagates parallel to the *o-xz* plane and has an angle θ with the *z*-axis, where $\phi_r(x,y)=k\theta_x$ ($k=2\pi/\lambda$, and λ is the wavelength). The digital hologram can be expressed by:

$$I(x,y)=o^2(x,y)+A_r^2 \\ +2A_r o(x,y)\cos[\varphi(x,y)-\varphi_r(x,y)] \tag{1}$$

Since the photosensitive amount of the phase hologram recorded by the photosensitive material is proportional to the above equation, the phase digital hologram can be expressed as:

$$t_H(x,y)=\exp[jgI(x,y)] \tag{2}$$

Where, g is an undetermined constant called the phase hologram modulation parameter [12]. Let:

$$K=\exp[jg(o^2(x,y)+A_r^2)] \tag{3}$$

$$\alpha=2\mathrm{g}A_r o(x,y) \tag{4}$$

$$\psi(x,y)=\frac{\pi}{2}-\varphi(x,y)+\varphi_r(x,y) \tag{5}$$

Then (2) can be rewritten as:

$$t_H(x,y)=K\exp[j\alpha\sin(\psi(x,y))] \tag{6}$$

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The authors are with the Kunming University of Science and Technology, Chenggong, Yunnan, China, (e-mail: guijinbin@kust.edu.cn; 3153834066@qq.com; jcli99@163.com; songqinghe@kust.edu.cn)

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Due to the variation range of $\sin(\psi(x,y))$ function between ±1, to ensure that the transmission function value is single-valued when forming a phase-type digital hologram, the selection of the composition parameter g must satisfy:

$$\alpha \leq \pi \tag{7}$$

According to the properties of the integer-order Bessel function $J_n(\alpha)$, formula (6) can be expanded as:

$$t_H(x,y) = K\sum_{n=-\infty}^{\infty} J_n(\alpha)\exp\,[jn\psi(x,y)] \tag{8}$$

The above formula indicates that when a phase hologram is illuminated with a unit amplitude plane wave, there are zero-order diffracted waves (n=0) propagating along the optical axis z in the transmitted light, and diffracted waves of n=1, 2, ... are symmetrically distributed on both sides. Due to the ability to select light waves that contribute to imaging through a gating filter composed of a lens system [13],[14], for simplicity, only diffraction waves with n=0, ±1 will be discussed.

When the phase hologram is loaded onto the SLM (Spatial Light Modulator) and illuminated by the original reference light, the complex amplitude of the transmitted wave modulated by the SLM is:

$$\begin{aligned} U_H(x,y) &= R(x,y)t_H(x,y) \\ &= U_{H0}(x,y) + U_{H+}(x,y) + U_{H-}(x,y) \end{aligned} \tag{9}$$

Here,

$$U_{H0}(x,y) = R(x,y)KJ_0(\alpha) \tag{10}$$

$$U_{H+}(x,y) = R(x,y)KJ_1(\alpha)\exp\,[j\psi(x,y)] \tag{11}$$

$$U_{H-}(x,y) = R(x,y)KJ_2(\alpha)\exp\,[-j\psi(x,y)] \tag{12}$$

The results reveal that the transmitted light waves become three beams of light that are tilted in the direction of the reconstruction light. Where, $U_{H0}(x,y)$ represents a zero-order diffracted light wave, $U_{H+}(x,y)$ represents the conjugate light wave, and the last $U_{H-}(x,y)$ represents the object light that can form real images. Because of $\alpha = 2gA_r o(x,y)$, to obtain strong light, the parameter g should be suitably designed so that the value of $-J_{-1}(\alpha)$ has a wide interval with the variety of the object light amplitude. Because of $J_1(\alpha) = -J_{-1}(\alpha)$, the curve of the Bessel function $J_1(\alpha)$ with a dashed line is shown in Fig.1, and the first maximum position of $J_1(\alpha)$ is marked. The function $\phi(\alpha)$ is the straight-line equation that connects origin and the point $(\alpha_{max},\ J_1(\alpha_{max}))$.

To analyze the properties of the reconstructed image, $U_{H-}(x,y)$ can be expressed as:

$$U_{H-}(x,y) = -AKJ_{-1}(\alpha)\exp\,[j\varphi(x,y)] \tag{13}$$

The propagation direction of this light wave is the same as the optical axis; the diffracted wave $U_{H-}(x,y)$ can reconstruct the real image of the object light field. However, since $\alpha = 2gA_r o(x,y)$, $A_r J_{-1}(\alpha)$ is not proportional to the change in the magnitude of the object light amplitude $o(x,y)$, resulting in both amplitude and phase distortion in the reconstructed object image.

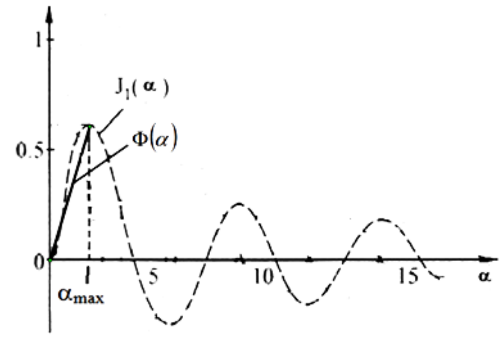

Fig. 1. $J_1(\alpha)$ and $\phi(\alpha)$ function curves

An improvement is presented in [16], where the phase hologram is formed using the last item in (1):

$$\begin{gathered} t_H(x,y) = \\ \exp\,[j2gA_r o(x,y)\cos\,(\varphi(x,y) - \varphi_r(x,y))] \end{gathered} \tag{14}$$

When the SLM loaded with this phase hologram is irradiated with the original reference light, the transmitted light becomes:

$$U_{H-}(x,y) = -A_r J_{-1}(\alpha)\exp\,[j\varphi(x,y)] \tag{15}$$

It is evident that the effect of the object light phase with the complex function K is eliminated, which is an important improvement.

III. Amplitude Compensated Modulation Phase Coding Method

A. Maximize the intensity of the reconstructed image

Observe (15) and the curve of function $J_1(\alpha)$, to obtain the maximum intensity of the reconstructed image, the parameter α should vary from zero to the point α_{max}, as shown in Fig.1. The maximum value of the light amplitude $o(x,y)$ is o_{max}. To ensureo_{max} meet α_{max}, let $\alpha_{max} = 2go_{max}A_r$. According to (4), the constituent parameter g is obtained:

$$g = \frac{\alpha_{max}}{2o_{max}A_r} \tag{16}$$

B. Eliminate the amplitude distortion of the reconstructed image

Since the imaging object light described by (15) has amplitude distortion when α falls in the range of 0-α_{max}, the 3D image of the object cannot be accurately displayed in theory. To solve this problem, the following improvements are presented:

First, let $\phi(\alpha) = \alpha J_1(a_{max})/\alpha_{max}$. Substituting with (4), we can obtain the expression of the pre-distortion function:

$$\phi(2gA_r o(x,y)) = 2gA_r o(x,y)J_1(a_{max})/a_{max} \tag{17}$$

Then, the object light amplitude arriving at the plane of the phase hologram is subjected to pre-distortion treatment according to the following formula:

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$$\hat{o}(x,y) = \frac{\phi(2gA_r o(x,y))}{J_1(2gA_r o(x,y))} o(x,y) \qquad (18)$$

Finally, replace $o(x,y)$ in formula (11) with function $\hat{o}(x,y)$. It can be readily demonstrated that the phase-type digital hologram produced using the described method encapsulates the object light information. This information is directly proportional to the actual complex amplitude of the object light, thereby allowing for an accurate reconstruction of the object's 3D image.

IV. Theoretical simulations and optical experimental

A. Optical system with filter

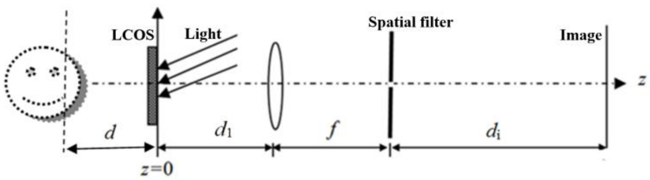

Fig. 2. Optical system with filter.

To eliminate other order reconstructive beams, an optical system with a filter is presented in Fig. 2. A plane light beam illuminates the LCOS where a phase hologram is loaded. The -1^{st} order of the reflective wave is selected by passing through the filter located on the focal plane of the lens. Since the diffraction calculation of a 3D object surface can be obtained by calculating a series of planar light sources perpendicular to the optical axis, we can study the quality of the reconstructed image of different spatial planes perpendicular to the optical axis to evaluate the quality of the reconstructed image of 3D objects.The object is a two-dimensional image in the $z = -d$ plane, a lens is laid in the $z = d_1$ plane, and an aperture is laid in the back focal plane of the lens. The distance between the aperture and the observation screen is d_i:

$$d_i = (\frac{1}{f} - \frac{1}{d_1 - d})^{-1} \qquad (19)$$

The lateral magnification of the image is expressed as:

$$M = -\frac{d_i}{d_1 + d} \qquad (20)$$

B. Theoretical simulation

We can calculate the optical diffraction field behind the optical system using scalar diffraction theory. To verify that the amplitude distortion can be eliminated by the improved method, the object is designed as a two-dimensional transmission screen on the $z = -d$ plane, with the transmittance varying in the horizontal direction from 0 to 255. By illuminating the transmission screen with a unit amplitude plane wave and comparing the amplitude of the reconstructed image with the transmitted light, we can determine if the amplitude of the reconstructed image is distorted. The simulation steps are as follows:

1) Calculate the object light on the $z = 0$ plane, which is diffracted along the z-axis by a distance d. The width of the object plane aperture is ΔL_0 and the amplitude transmittance is $A_0(x_0, y_0)$. Illuminating the aperture with a unit amplitude uniform plane wave, the object wave field after the aperture can be written as:

$$O_0(x_0, y_0) = rect(\frac{x_0}{\Delta L_0}, \frac{y_0}{\Delta L_0}) A_0(x_0, y_0) \qquad (21)$$

2) Find the maximum value o_{max} of the object light amplitude at the $z = 0$ plane.
3) Let the reference light amplitude $A_r = o_{max}$, and select the composition parameter g according to (18).
4) Obtain$\hat{o}(x,y)$ by applying the object light amplitude pre-distortion processing according to (18).
5) Because the pixel size of LCOS is $\Delta x = 6.4\mu m$, to satisfy the sampling theorem in the simulation calculation, the angle between the reference light and the optical axis must satisfy $\lambda/\theta \geq 2\Delta x$. The light source is YAG laser with a wavelength of 532nm; the calculation yields $\theta = 1.2° < \lambda/(2\Delta x)$. Replace $o(x,y)$ with $\hat{o}(x,y)$. The phase digital hologram is obtained according to (14):

$$\hat{t}_H(x,y) = \exp\,[j2gA_r \hat{o}(x,y) \cos\,(\varphi(x,y) - \varphi_r(x,y))] \qquad (22)$$

6) Illuminating the phase digital hologram with reference light $R(x,y) = o_{max} \exp\,[j\varphi_r(x,y)]$, we can calculate the light wave field at the diaphragm plane. The result is the Fourier transform of the input planar light wave field multiplied by a quadratic phase factor:

$$O_1(x,y) = \frac{\exp[jk(d_1 + f)]}{j\lambda f} \exp\left[\frac{jk}{2f}\left(1 - \frac{d_1}{f}\right)(x^2 + y^2)\right] \times \iint_{-\infty}^{\infty} R(x_0, y_0)\, \hat{t}_H(x_0, y_0) \exp\,[-j2\pi(x_0 \frac{x}{\lambda f} + y_0 \frac{x}{\lambda f})] dx_0 dy_0 \qquad (23)$$

7) Suppose the transmittance of the selective filter is , and the light wave field at the image plane is:

$$O_i(x_i, y_i) = \frac{\exp(jkd_i)}{j\lambda d_i} \times \iint_{-\infty}^{\infty} \Theta(x,y) O_1(x,y) \exp\,[j\frac{k}{2d_i}((x - x_i)^2 + (y - y_i)^2)] dx dy \qquad (24)$$

In the simulation, the lens focal length is $f = 300mm$, and the number of samples is $N = 1024$, $\Delta L = N\Delta x = 6.5536mm$. Fig. 3 depicts the reconstructed images using three different coding methods.

Figure 4 displays the curves along the x-axis of the reconstructed images (b), (c), and (d) from Figure 3. It is evident that the reconstructed images using the coding method referenced in [12,13] exhibit amplitude distortion. In contrast, the enhanced coding method more effectively eliminates this distortion, resulting in higher quality reconstructed images.

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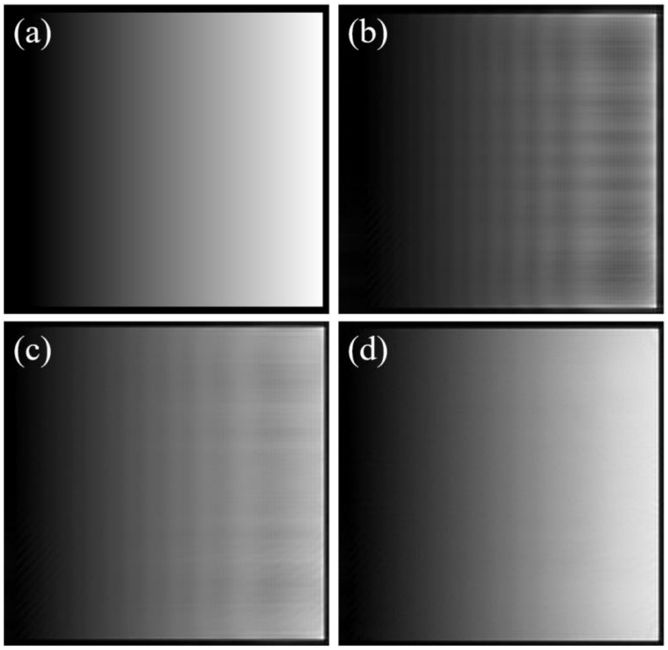

Fig. 3. Reconstructed images with a horizontal transmittance of 0-255 using three methods. (a) Original image. (b) The method in [12]. (c) The method in [13]. (d) The method in this article.

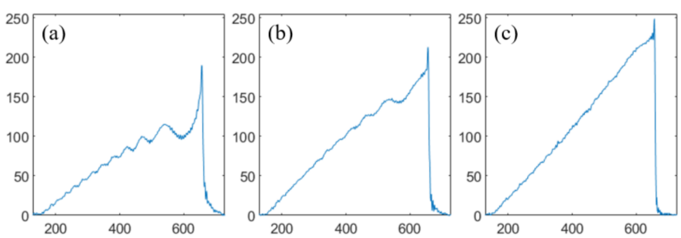

Fig. 4. Fig. 4. X-axis amplitude curves of the reconstructed images using three different methods. (a) The method in [12]. (b) The method in [13]. (c) The method in this article

It is easy to observe from the image of the Bessel function $J_1(\alpha)$ that if a smaller constituent parameter g is chosen at the cost of reducing the intensity of the reconstructed image, the encoding methods in [12] and [13] can also reconstruct images with smaller amplitude distortion. To confirm this analysis, let the constituent parameters determined in (16) be g_{max}, and $g = 0.2g_{max}$, $0.4g_{max}$, $0.6g_{max}$, $0.8g_{max}$, $1.0g_{max}$. Figure 5 shows the axial direction amplitude distribution curves of the reconstructed images using the three methods (each curve from bottom to top corresponds to the added value of g).

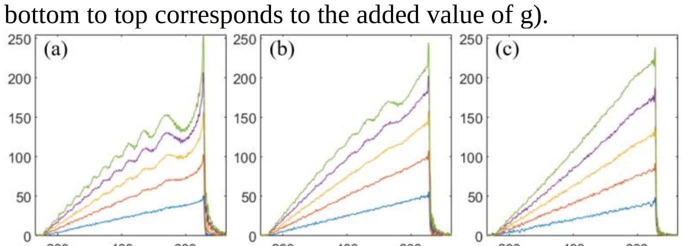

Fig. 5. The influence of modulation parameter g on the reconstructed image amplitude distribution of three methods. (a) The method in [12]. (b) The method in [13]. (c) The method in this article.

Evidently, the choice of a smaller constituent parameter g can improve the quality of the reconstructed image in the coding methods proposed in references [12] and [13]. However, this improvement comes at the expense of reducing the intensity of the reconstructed image. In contrast, the improved coding method of amplitude compensation modulation phase proposed in this paper can reconstruct the image without amplitude distortion, regardless of the composition parameter g, as long as it does not exceed g_{max}.

C. Experimental

To obtain experimental proof of the theoretical analysis and intuitively understand the effect of amplitude distortion on the quality of reconstructed images, three different methods were implemented to generate the corresponding phase holograms using MATLAB on a Windows 10 system. The hologram resolution was set to 1024×1024, with a pixel size of 0.0045mm, a light wavelength of 532nm, and a propagation distance of 200mm. The generated phase holograms were then reconstructed sequentially. During reconstruction, the filtering system shown in Fig. 2 was simulated in MATLAB to eliminate the other orders of reconstructed beams, with parameters $d = 200$, $d_1 = 200$, and $f = 200$ set accordingly. Fig. 6 illustrates a comparison between the original image and the reconstructed images simulated using the three different reconstruction methods.

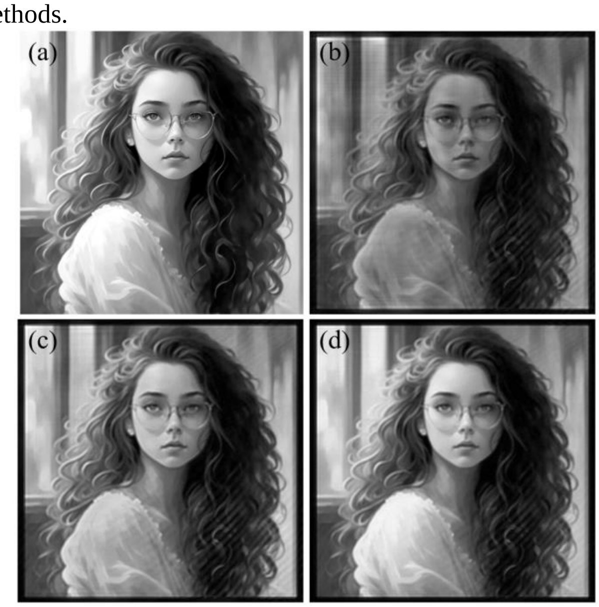

Fig. 6. Comparative Analysis of Reconstructed and Original Images Using Three Different Methods. (a) Original image. (b) Method described in reference [12]. (c) Method described in reference [13]. (d) Method developed in this article.

An optical path based on the filtering system of Fig. 2 was built, as schematized in Fig. 7. Computer 1, connected to the SLM, controls the loading of phase holograms onto the SLM. The reconstructed image is received through a CCD (Charge-coupled Device) and displayed on computer 2, which is connected to the CCD.

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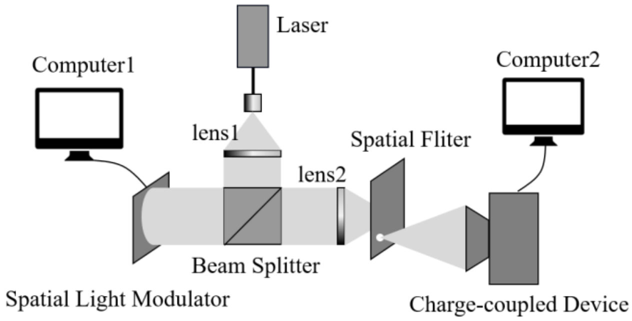

Fig. 7. Schematic Representation of the Optical Reconstruction Light Path.

The SLM used in the experiment is a phase type with a resolution of 1920x1080 and a pixel size of 4.5 um. The laser wavelength is 532 nm, the SLM is placed 200 mm from lens2, which has a focal length of 200 mm, and the CCD is 200 mm from the filter. Fig. 8 shows the original image and the reconstructed images of phase holograms generated by the three methods.

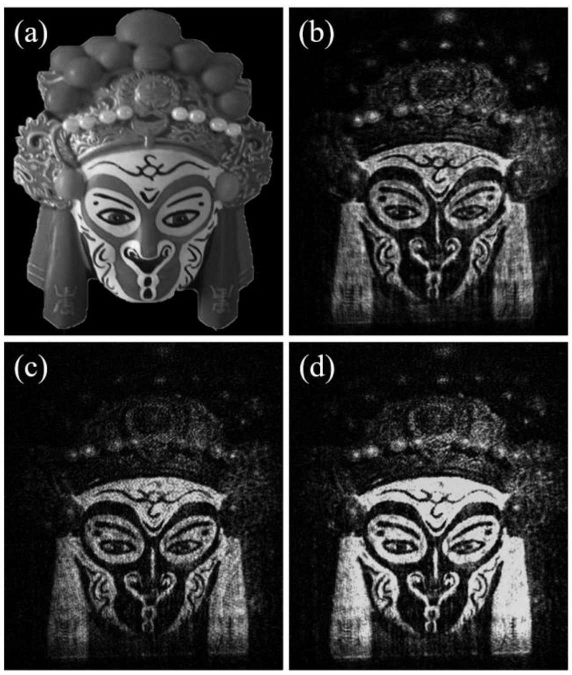

Fig. 8. Comparative Display of Optical Reconstructions of Holograms Produced by Three Distinct Methods. (a) The method described in reference [12]. (b) The method described in reference [13]. (c) The method described in this article.

It can be seen that the experimental results demonstrate good agreement with the theoretical simulations, confirming the superiority of the improved coding method in terms of image quality.

It is worth noting that although the improved method enhances image quality, it necessitates pre-distortion processing of the image amplitude, which increases computational complexity. From the perspective of human eye observation of monochromatic light imaging quality, the method proposed in reference [13], despite containing amplitude changes, remains a simple and suitable coding approach. However, when displaying color images, since the imaging points are formed by stacking different color dots, distortion of the dot color components leads to distortion of the synthesized color. Accurately displaying the object's color 3D image with high brightness and quality is always the desired goal.

To verify the effectiveness of the improved coding method for holograms of 3D objects, experiments were conducted using the 3D model depicted in Fig. 9. As with processing 2D pictures, after calculating the diffraction distribution of the 3D object in the hologram plane, its amplitude is pre-distorted following the steps in section III(c) and encoded as a phase hologram.

Fig. 9. Schematic Overview of the Optical Reconstruction Light Path.

The alignment between our theoretical simulations and experimental measurements is clearly evident, underscoring the superior image quality achieved through our refined encoding method. These experimental outcomes are depicted in Fig. 10.

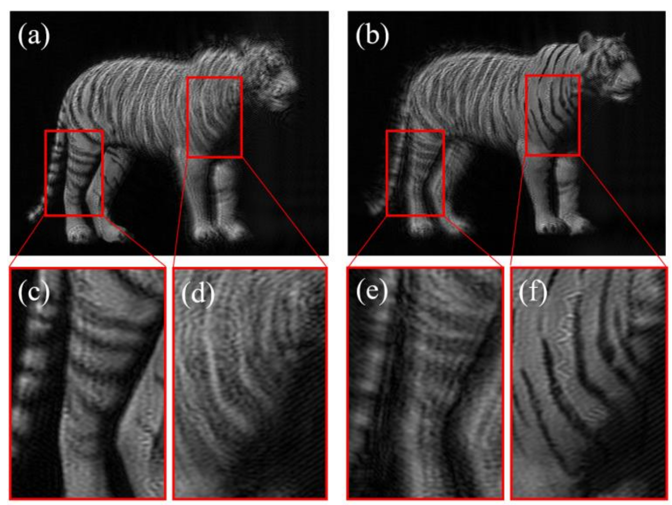

Fig. 10. Numerical Reconstructions from Holograms Created Using the Proposed Method. (a) Focused numerical reconstruction on the tiger's hind legs.(b) Focused numerical reconstruction on the tiger's forelimbs. (c), (d), (e), (f) Enlargements of specific areas in (a) and (b) respectively.

The results demonstrate that encoding holograms of three-dimensional objects allows for selective focus at varying distances, effectively validating the efficacy of the enhanced holographic encoding technique for three-dimensional objects.

V. CONCLUSION

In the study of holographic 3D display based on the phase-type spatial light modulator (SLM), reconstructing object images with high brightness and quality remains an active research topic. This paper presents improvements to two recently proposed SLM control coding methods, resulting in a coding approach that achieves high diffraction efficiency and accurate reproduction of 3D object images, supported by theoretical simulations and experimental evidence. It is hoped that the work presented in this paper will serve as a valuable reference for the research and application of digital holographic 3D displays.

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