Optical Gain in AlGaN Quantum Wells: Impact of Higher Energy States

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Abstract—Simulations of optical gain in aluminum gallium nitride (AlGaN) quantum wells are extended to the high charge carrier density regime required for achieving gain at 275 nm for UV laser diodes. Coulomb interaction is modeled using the 2nd Born approximation. We demonstrate good agreement with experimental data obtained through optical pumping, and predict gain spectra for electrical pumping. Special consideration is given to the contribution of higher bands in wide quantum wells.

Index Terms—Simulation, quantum well lasers, optoelectronic devices, aluminum gallium nitride.

I. INTRODUCTION

S HAS recently been demonstrated [2], aluminum gallium nitride (AlGaN) based laser diodes can achieve continuous wave UV-C lasing at current densities $J > J_{\text{th}} = 4.2 \text{ kA/cm}^2$.

The high threshold current density J_{th} stems from multiple issues along the lasing process. Carrier transport struggles with injection loss, most notably due to low hole mobility and thus injection [3], [4], [5]. For stimulated emission, only a fraction of the carrier density contributes to gain. Light propagation suffers from optical losses [3], [6]. Investigation and mitigation of these loss mechanisms could pave the road for a new class of UV-C lasers with applications in high-resolution manufacturing (singulation [7], polymerization [8]) and imaging (UV Raman spectroscopy [9], in-vivo fluorescence spectroscopy [10]).

To this end, we provide guidance for the design of future UV-C laser diodes, specifically the quantum well (QW). In the following, we simulate and examine stimulated emission, with a focus on maximizing peak gain.

Gain spectra are simulated for different charge densities and pumping methods. We investigate the impact of QW width, as wide QWs show different transition rates and density of states,

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width		$x_{\rm Al}$
10 nm	AlN cap	1
50 nm	waveguide	0.63
$d_{\rm QW}$	quantum well	$x_{\rm QW}$
50 nm	waveguide	0.63
900 nm	cladding	0.76
25 nm	buffer transition	:
400 nm	buffer	1

Fig. 1. Layers of investigated structures, including Al mole fraction x_{Al} .

which drastically affects the gain spectrum. From there, we analyze the influence of higher energy levels in wide QWs.

The investigated structures consist of $Al_xGa_{1-x}N$ layers epitaxially grown on an AlN/sapphire substrate (see Fig. 1); most notably a 100 nm waveguide with mole fraction x = 0.63in which the QW is embedded. Two different QW designs with width $d_{QW} = 3$ nm and $d_{QW} = 12$ nm are tested, with $x_{QW} = 43.8\%$ and 42.9% respectively, which were chosen for 275 nm emission.

II. SIMULATION MODEL

The simulation process is shown schematically in Fig. 2. Material parameters are taken from established literature [11], [12], [13], [14], [15], [16].

Gain is derived from microscopic polarization, which is solved using the semiconductor Bloch equations. We apply the $k \cdot p$ method, with valence subbands represented in a reduced $k \cdot p$ 6 × 6 basis (HH, LH, CH) i.e. heavy hole, light hole, and crystal-field split-off hole bands [17], [18]. The eigenstates $E_n(k), \psi_n(k, z)$ are derived iteratively as a self-consistent solution to Schrödinger and Poisson equation. This also yields the quasi Fermi levels E_{Fn}, E_{Fp} as a byproduct.

Due to the wurtzite crystal lattice of AlGaN, the strain at QW interfaces causes polarization charges [19]. This induces charge separation between electron and hole states in the QW, leading to lower wavefunction overlap (see Fig. 3) and thus reduced radiative recombination [18]. However, part of these interface charges can be screened by electrons and holes from trap states and external charges [20], which originate from outside the simulated region. In lieu of a full device transport simulation, we model this as a reduction of the effective polarization charge by

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Fig. 2. Flow diagram of gain simulations.



Fig. 3. Band diagrams $V_{c,v}(z)$ and eigenstates (wavefunctions and energies of ground and 1st excited states, if existent) for 3 nm and 12 nm QW, using $p_{\text{eff}} = 50\%$.

a factor p_{eff} which depends on operation mode (electrical [21], [22], [23] vs. optical pumping [20], [24]).

As we use single QWs, the boundary conditions for the electric potential are assumed as an infinite barrier setup [20] corresponding to Neumann boundary conditions at infinity.

While simpler models only consider direct transitions between bands through dipole elements, we also take Coulomb interaction and the resulting scattering into account, which is essential for the high carrier density regime that UV laser diodes operate in [26]. The combination of both interactions establishes an equilibrium which allows to calculate gain, with the scattering leading to homogeneous broadening, bandgap renormalization, and excitonic gain enhancement.

Random alloy fluctuations lead to both hole localization effects and bandgap fluctuations [27]. Neglecting the former, we can consider the variation in the bandgap through a normal distribution of band energies [28], which in turn leads to inhomogeneous broadening (IHB) of the gain spectrum. We approximate the spectral IHB through Gaussian convolution with parameter $\sigma_{\rm IHB}$.

 $\sigma_{\rm IHB}$ is calibrated by a non-linear least squares fit, which matches the convolution of simulated spectra with Gaussian broadening against measurements taken with optical pumping (see Section III). However, application in laser diodes relies on electrical pumping. Therefore, we also simulate spectra for this operation mode, using the same $\sigma_{\rm IHB}$.

The optical confinement factors to convert material gain to modal gain were obtained from 1D waveguide simulations using SiLENSe [29] and are computed as $\Gamma_{3 \text{ nm}} = 1.3\%$ and $\Gamma_{12 \text{ nm}} = 8.4\%$ respectively.

III. COMPARISON TO EXPERIMENT

Experimental net gain spectra of the structures were measured with the variable stripe length method (VSLM) using non-resonant optical pumping by a 193 nm ArF laser [30]. The measurements were subject to internal losses of about 10/cm which have been deducted to allow comparison with simulated modal gain spectra (Fig. 4). We limit the fitting range for the IHB parameter σ_{IHB} to the high-energy edge and peak (see Fig. 4). The low-energy edge (Urbach tail) is excluded as our model cannot accurately predict it.

For optical pumping above the barrier energy, carriers generated in the barriers by the pump laser screen most of the polarization charges. Both previous and current alignments between experimental and simulated spectra indicate $p_{\rm eff} \approx 10\%$. We also assume that the average electron density n and hole density p in the QW are equal.

Calculating all confined states in the 12 nm QW, we obtain $N_{\rm vb} = 20$ valence subbands and $N_{\rm cb} = 7$ conduction subbands, which are included in the gain calculation.

Both experiment and simulation show dominant TE emission.

We see that for similar optical excitation power density P, the 3 nm and 12 nm QW show very different carrier densities and spectra, with the wide QW providing higher peak gain.

The 3 nm QW shows a broader spectral width compared to the 12 nm QW even without IHB, and additionally requires more IHB to match the width of the experimental spectra, with $\sigma_{\text{IHB, 3 nm}} = 40(20)$ meV vs. $\sigma_{\text{IHB, 12 nm}} = 19(4)$ meV.

IV. ANALYSIS OF GAIN

The different spectral width between 3 nm and 12 nm QW can be understood with a simple variational approach to the bands $E_{c,v}^n(k_r) = \frac{\hbar^2}{2m^*} \left[\left(\frac{n \pi}{d_{QW}} \right)^2 + k_r^2 \right]$ of an infinite QW, with effective mass m^* of the respective charge carriers:

- Large QW widths d_{QW} lead to small subband spacing in E_c^n (compare Fig. 3). The higher density of states reduces quasi Fermi level splitting and thus energy spread: high energy states are depopulated and therefore generate no gain, leading to a sharper spectrum independently of IHB.
- Localized landscape theory predicts an energy landscape with a spread of energetically favorable quantum dot-like states [27]. If we treat their uneven distribution as a variation of $d_{\rm QW}$ of the envelope QW, then $\Delta E_v^n \propto d_{\rm QW}^{-3} \Delta d_{\rm QW}$. As such, wide QWs have their bands broadened less and show a lower $\sigma_{\rm IHB}$ which matches the behavior of the fit parameters noted in Fig. 4.

From the 12 nm QW gain characteristic in Fig. 6, we extrapolate a transparency density of $n_{\rm tr} = 16.4 \cdot 10^{18} \text{ cm}^2$ and a differential gain of $\partial G = 18.7 \cdot 10^{18} \text{ cm}^2$. This



Fig. 4. Measured [25] (dots) versus simulated (lines) modal gain spectra for optical pumping in 3 nm and 12 nm QW lasers, at different optical excitation power densities P or charge densities n = p respectively. λ range for IHB fit σ denoted by gray lines.



Fig. 5. Simulated gain spectra $G(\lambda)$ with IHB for 50% screening in 3 nm and 12 nm QW lasers.



Fig. 6. Peak gain for 12 nm QW and different band cutoffs N_b , with IHB from experimental fit (using all $N_b = 21$ bands).

suggests a notable advantage over the characteristics $n_{\rm tr} = 24.9 \cdot 10^{18} \text{ cm}^3$, $\partial G = 3.6 \cdot 10^{18} \text{ cm}^2$ of the 3 nm QW, as the wide QW achieves more gain at lower carrier densities.

Extending the measurements and simulations to $d_{\text{QW}} \in \{3, 6, 9, 12\}$ nm [25] indicates that

- $\sigma_{\text{IHB}}(d_{\text{OW}})$ sharply drops below 6 nm, then flattens out.
- $\partial G(d_{QW})$ increases slightly better than linearly.
- $n_{\rm tr}(d_{\rm QW})$ sharply drops below 6 nm, then declines linearly.

A multi-QW design could combine four 3 nm QWs to reach the same overall active volume and therefore 2D density. As such, an argument can be made to compare the characteristics with respect to 2D carrier density. As multi-QWs cannot lower the transparency density, we merely convert to 2D differential gain $\partial G_{2D, 3 \text{ nm}} = 1.2 \cdot 10^{-11}$ cm and $\partial G_{2D, 12 \text{ nm}} =$ $1.6 \cdot 10^{-11}$ cm respectively. We see that a single 12 nm QW is still preferable.

V. ELECTRICAL PUMPING

For electrical pumping, we assume $p_{\text{eff}} = 50\%$ and, due to lack of further data, n = p. As such, only p_{eff} is changed between the simulations for optical versus electrical pumping.

The simulated gain spectra for $p_{\rm eff} = 50\%$ in Fig. 5 show similar behavior to what we observed for optical pumping, though with a stronger redshift towards lower charge densities as well as a higher transparency density. Both of these effects are explained by the quantum-confined Stark effect: The distorted potential decreases both subband level distance and wavefunction overlap at low n, p where polarization charges cannot be compensated with free charges.

The 12 nm QW requires notably lower carrier density to achieve the same peak gain. The differential gain $\partial G_{3nm} = 2.8 \cdot 10^{18} \text{ cm}^2$ and $\partial G_{12nm} = 15.8 \cdot 10^{18} \text{ cm}^2$ is lower compared to $p_{\text{eff}} = 10\%$. The 12 nm QW retains its relative advantage, as screening is supplied by low energy states. This holds even in comparison with a 3 nm multi-QW setup.

VI. BAND CUTOFF

Wide QWs possess a much larger number of valence subbands $N_{\rm vb}$ and conduction subbands $N_{\rm cb}$, with Coulomb interactions arising between all of them. As a result, computational effort is



Fig. 7. Simulated gain spectra for a 12 nm QW with $n = 16.9 \cdot 10^{18}$ cm² and 90% screening ($p_{\text{eff}} = 10\%$), with and without IHB.

dominated by taking scattering into account, extending simulation time by orders of magnitude. This is typically compensated for by limiting $N_{\rm vb}$ and $N_{\rm cb}$ in the simulation, as complexity scales with $(N_{\rm vb} + N_{\rm cb})^4$ and higher-energy bands are assumed to have less impact on the spectrum. We investigate the impact of said bands quantitatively.

This is of particular interest for wide QWs, as far more bound states exist, in contrast to e.g. the 3 nm QW which only possesses $N_{\rm vb} = 4$, $N_{\rm cb} = 2$ confined bands.

For simulations of the 12 nm QW under optical pumping ($p_{\rm eff} = 10\%$), the full number of bands is $N_{\rm vb} = 20, N_{\rm cb} = 7. N_{\rm vb}$ is higher by a factor of 3 due to 6-band $k \cdot p$. We compare gain spectra where a limit N_b with $N_{\rm vb} \leq N_b, N_{\rm cb} \leq \max(\frac{1}{3}N_b, 3)$ is imposed on the bands, the results of which are plotted in Figs. 6 and 7. Note that $N_b = 21$ represents the regular simulation that considers all bands.

As can be seen in Fig. 6, while the differential gain ∂G is only marginally affected by N_b , considering more (or all) bands leads to a sharp increase in transparency density. For a fixed frequency, taking more bands into account increases the density of states, which shifts the quasi Fermi levels such that occupation of higher states is reduced. The unoccupied states contribute absorptively, thereby reducing gain.

As such, we observe that for low N_b peak gain is drastically overestimated, with $N_b = 4$ being roughly $\Delta G = 60 + -10/$ cm above the unlimited simulation. Even the $N_b = 16$ case is off by $\Delta G = 3.8 + -1.4/$ cm, which may not seem much, but still distorts the behavior near threshold.

Fig. 7 shows select spectra of a 12 nm QW for fixed charge densities, with/without IHB of $\sigma_{\text{IHB}} = 18$ meV. The spectra with IHB display that the higher-energy bands both reduce amplitude and increase wavelength of peak gain. The difference stands out even more for the spectra without IHB, as the shape of the gain peak isn't smoothed out – for $N_b = 8$ we see two distinct transitions, one of which is absorbed entirely when higher-energy bands are included. We infer that higher-energy bands affect the height, width and shape of the gain peak. Inclusion of all bands is thus required to obtain a proper fit to experimental data.

For simulations with electrical pumping ($p_{\text{eff}} = 50\%$), all these considerations are heavily amplified. Higher subbands are even more crucial as the ground states exhibit a low wavefunction overlap. Choosing $n, p = 18.9 \cdot 10^{18} \text{ cm}^3$ such that G = 5 cm, we find $N_{cb} = 6$, $N_{vb} = 9$ bands. Analyzing the contributions of different transitions to peak gain, the ground state transitions provide no gain (their own transition energy being too far below that of the peak), the bands within $N_b = 4$ only explain 52% of gain, and the higher-energy bands are essential to correctly simulate gain. Some transitions with bands beyond $N_b = 4$ provide the remaining 48% of peak gain, while others are absorptive. This is consistent with Fig. 3 and previous findings [20], [31] which show the ground state's screening effect causing gain to rely on excited states. As these investigations were limited to few bands, the major impact of high-energy bands constitutes a new insight.

VII. CONCLUSION

We present a theory of gain for AlGaN QWs that matches experimental results. Our simulations explain measured gain spectra for optically pumped QW lasers emitting at 275 nm, for multiple charge densities and QW widths. We break down which transitions contribute to gain. These findings are applied to electrically pumped QWs. Through that, we confirm previous reports [20] that wide QWs are preferable for UV-C lasers and that gain in wide QWs mainly originates from excited states.

It is shown that IHB has a larger impact on narrow wells. For wide QWs, we find that higher-energy bands have a notable impact on the gain spectra, both in terms of peak gain and spectral width. Their overall absorptive contribution is analyzed quantitatively and determined to be significant.

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