Coherent Control of the Goos-Hänchen Shift in a Cavity Containing a Five-Level Double-Ladder Atomic System

Changyou Luo, Yongqiang Kang, Xiaoyu Dai, and Yuanjiang Xiang

Abstract—**In this paper, we proposed a cavity containing an intracavity medium of five-level double-ladder-type atoms with electromagnetically induced transparency to enhance Goos-Hänchen shifts of reflected and transmitted light beams. The dependence of the Goos-Hänchen shifts has been analyzed. It is shown that Goos-Hänchen shifts can be controlled by modifying the intensity and detuning of the coherent control field without changing material and the structure of the dielectric interface. This work has considerable potential for applications such as optical devices in information processing, flexible optical-beam steering and alignment, optical sensors and optical switches.**

***Index Terms*—Goos-Hänchen shift, coherent control.**

I. Introduction

It is well known that when a light beam is totally reflected at the interface between two different media, there exists an extremely small lateral shift between the completely reflected light beam and the incident light beam. This lateral shift is known as the Goos-Hänchen (GH) shift, which was discovered by F. Goos and H. Hänchen in 1947 and theoretically explained by Artmann in 1948. Since then the GH shift has been studied in various structures containing different kinds of media [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], such as in a weakly absorbing semi-infinite medium [1], [2], negative refractive media [3], [4], photonic crystals [5], [6], [7], [8], dielectric slab [9], [10], the ballistic electrons in semiconductor quantum slabs or well [11], [12], optical biosensor [13], metamaterial absorbers [14], various level configurations quantum systems [15], [16], semiconductor structure [17], [18], structures containing graphene [19], [20], [21], and others. However, in all those aforementioned studies, the manipulation of the GH shift cannot be performed on the fixed structures. The GH shift effect has some interesting applications in optical sensor devices, which can be used to measure the refractive index, beam angle and displacement, for surface and film thickness studies [22], [23], [24], [25], [26], [27], [28], [29]. Therefore, any proposal which deals in manipulation of the GH shift using a fixed structure seems to be more appropriate. In recent years, various schemes to control the GH shift by using the temperature, electric field and light field have been proposed [17], [23], [24], [25], [26], [27], [28], [29]. Obviously, among the manipulation of the GH shift mentioned above, optical coherent control is particularly important in the applications of all-optical equipment.

More than two decades ago, in some earlier studies [30], [31], [32], it has been found that the susceptibility, refractive index or dispersion and absorption of the atomic medium can be modified by using a coherent driving field. On this basis, in 2008, Wang et al. [25] proposed a scheme to control the GH shift in a cavity containing a two-level atomic medium, via a coherent external control field. They demonstrated that without changing the structure of a cavity system, the GH shift can be easily controlled by adjusting the intensity and detuning of the external control field. Similarly, in 2010, Ziauddin et al. [29] reported that in three-level or four-level atomic systems, due to the electromagnetically induced transparency (EIT) of the medium, a coherent control of the GH shift can be found in a fixed configuration or device via superluminal and subluminal wave propagation. The negative and positive GH shifts have been observed in the reflected beam, corresponding to superluminal and subluminal propagation of the probe light beam, respectively. The transmitted beam, however, exhibits only positive GH shifts in superluminal as well as subluminal wave propagation. Furthermore they found that there is strong absorption in the three-level EIT structure during the superluminal propagation of the resonant probe light beam. This problem can be solved by substituting a three-level atomic system with a four-level atomic system.

In this proposal, we investigate the coherent control of GH shift in a cavity with two walls of some dielectric material and an intracavity medium of five-level double-ladder-type atoms. Compared with previous studies [25], [29], due to the increase of intracavity medium energy level, the control variables of Rabi frequency and frequency detuning also increase, and control methods of coherent GH shift become more flexible. We expect

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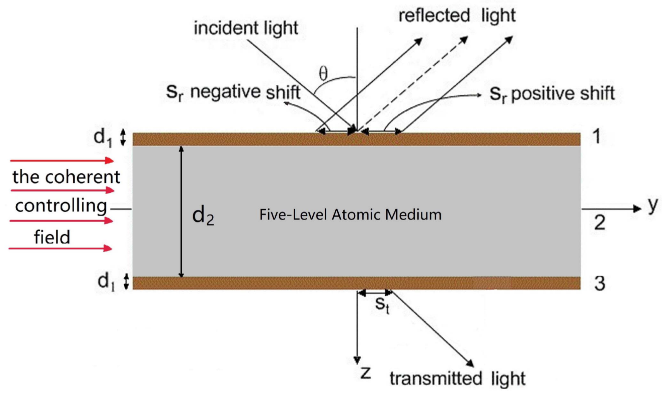

Fig. 1. Schematic diagram of the system. The incident light is incident at an angle of on the cavity wall along the z axis. The cavity walls are nonmagnetic matter with the same thickness d_1, and the width of the intracavity substance which is composed of five-level atomic medium is d_2. The coherent controlling field applied along y axis. The dotted line is the path expected from geometrical optics.

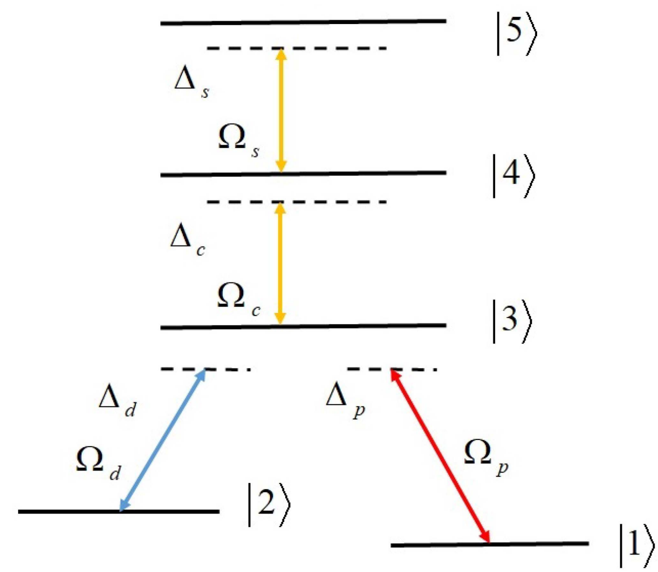

Fig. 2. Schematic diagram of a five-level atomic system in a double-ladder configuration.

a similar control of GH shifts using the coherent driving fields when the light is reflected from or transmitted through the cavity.

This paper is organized as follows: In Section II, we present the theoretical model for the GH shift in the reflected and transmitted light beams incident on a cavity containing a five-level double-ladder atomic system. Section III discusses the susceptibilities for a five-level atomic system and the numerical results for coherent-controllable GH shifts. Finally, a summary of our results will be described in Section IV.

II. Theoretical Analyses

Here, we first select a microstructure model based on previous work [25], [29] in subsection A to facilitate comparison of our present works with those of the previous studies. In subsection B, the dielectric constant model of atomic medium in microstructure is given according to [35], which is used in GH shift calculation. In subsection C, the theoretical formula of the stationary phase theory calculation of GH shift is given. In subsection D, the method of obtaining the group index of the total cavity is given. The calculated group index can be used to verify the GH shift calculated in subsection C.

A. The Model of a Cavity Structure Containing a Five-Level Double-Ladder Atomic System

The system considered in this paper is schematically illustrated in Fig. 1 [25], [29], in which a TE-polarized probe light beam with angular frequency ω_p is incident on a cavity from vacuum with permittivity $\varepsilon_0 = 1$. We can clearly observe from Fig. 1 that the cavity is a layered structure consisting of layers 1 and 3 which are acting as walls of the cavity, and layer 2 which is the intra-cavity medium. The probe light beam is incident from the air onto a slab at an angle of θ along the z axis. Layers 1 and 3, with the same thickness d_1 and permittivity ε_1, are nonmagnetic dielectric media. Layer 2 is an intracavity medium with the thickness d_2 and permittivity ε_2. The intracavity medium can be gas of five-level atoms. The strong coherent fields are applied along y axis to drive five-level atomic systems in an EIT configuration.

B. The Permittivity of a Five-Level Double-Ladder Atomic System

The atom-light interaction scheme for a five-level double-ladder-type atomic system is shown in Fig. 2. The atomic states are labeled as $|1\rangle$, $|2\rangle$, $|3\rangle$, $|4\rangle$ and $|5\rangle$. Ω_p is the one-half Rabi frequency of the weak probe applied to the transition $|1\rangle \leftrightarrow |3\rangle$. Ω_d, Ω_s and Ω_c are the one-half Rabi frequencies of the strong controlling beam applied to transitions between the states $|2\rangle \leftrightarrow |3\rangle$, $|3\rangle \leftrightarrow |4\rangle$, and $|4\rangle \leftrightarrow |5\rangle$. The parameters Δ_p, Δ_d, Δ_c, and Δ_s are the frequency detunings of the corresponding probe and controlling beam. A weak probe field with a carrier angular frequency ω_p is applied to the transition $|1\rangle \leftrightarrow |3\rangle$. Electric dipole transitions between the levels $|2\rangle \leftrightarrow |3\rangle$, $|3\rangle \leftrightarrow |4\rangle$, and $|4\rangle \leftrightarrow |5\rangle$ are coupled by control beams with carrier angular frequencies ω_d, ω_c and ω_s, respectively. The linear susceptibility of a five-level double-ladder atomic system can be written as [33]

$$\chi^{(1)} = \frac{2N|\mu_{31}|^2}{\hbar\varepsilon_0\Omega_p}\rho_{31}^{(1)} \tag{1}$$

Here we define the parameter $\beta = 2N|\mu_{31}|^2/(\hbar\varepsilon_0)$, then the (1) can be written as

$$\chi^{(1)} = \frac{\beta}{\Omega_p}\rho_{31}^{(1)} \tag{2}$$

where N is the atomic number density in the medium, μ_{31} indicates the dipole moment for the atomic transition between levels $|3\rangle$ and $|1\rangle$. $\hbar = 1$, and ε_0 is permittivity in free space. The atomic coherences ρ_{31} read as [33], [34]

$$\rho_{31}^{(1)} = -\frac{i\Omega_p}{B}c(db + |\Omega_s|^2) \tag{3}$$

where $B = abcd + bd|\Omega_d|^2 + cd|\Omega_c|^2 + ac|\Omega_s|^2 + |\Omega_d|^2|\Omega_s|^2$, $a = i\Delta_p + (\gamma_{31} + \gamma_{32})/2$, $c = i(\Delta_p - \Delta_d)$, $b = i(\Delta_p + \Delta_c) + \gamma_{43}/2$, and $d = i(\Delta_p + \Delta_c + \Delta_s) + (\gamma_{51} + \gamma_{52} + \gamma_{54})/2$. γ_{ij} are the spontaneous decay rates between level $|i\rangle$ and $|j\rangle$. Finally, we can obtain the permittivity of a five-level double-ladder atomic system $\varepsilon_2 = 1 + \chi^{(1)}$.

C. Theoretical Model for the GH Shift

When an incident probe light beam with angular frequency ω_p is incident on a cavity, the reflected and transmitted light beams will deviated from the paths usually expected from geometrical optics. For the incident light beam with a large beam waist (i.e., narrow angular spectrum, $\Delta k << k$), according to the stationary phase theory, the lateral shifts of the reflected or transmitted beam can be expressed as [25], [29]

$$S_{r,t} = -\frac{\lambda}{2\pi}\frac{d\phi_{r,t}}{d\theta} \tag{4}$$

where $\lambda = 2\pi c/\omega_p$ is the wavelength, $\phi_{r,t}$ is the phase. Generally, the reflection or transmission coefficients can be expressed as

$$r, t(\theta) = |r, t(\theta)| \exp\left[i\phi_{r,t}(\theta)\right] \tag{5}$$

where

$$\phi_{r,t}(\theta) = \arctan\frac{Im[r, t(\theta)]}{Re[r, t(\theta)]} \tag{6}$$

and then, we can give the lateral shifts of the reflected and transmitted beams as [25]

$$D_{r,t} = -\frac{\lambda}{2\pi|r, t(\theta)|^2} \times \left\{ Re[r, t(\theta)]\frac{dIm[r, t(\theta)]}{d\theta} - Im[r, t(\theta)]\frac{dRe[r, t(\theta)]}{d\theta} \right\} \tag{7}$$

Equation (7) has been adopted to investigate the GH shift for absorption materials.

We can see clearly from (7) that the reflection and transmission coefficients are necessary to find the GH shifts, in there, we use a transfer matrix to make it clear. For TE-polarized beam, the electric and magnetic fields which are propagating through the layered medium from the incident surface to the output end can be related to each other via a transfer matrix [25], [29]

$$M_j(k_y, \omega_p, d_j) = \begin{bmatrix} \cos(k_z^j d_j) & i\sin(k_z^j d_j)/q_j \\ iq_j \sin(k_z^j d_j) & \cos(k_z^j d_j) \end{bmatrix} \tag{8}$$

where $k_z^j = \sqrt{\varepsilon_j k^2 - k_y^2}$ is the z component of the wave vector $k = \omega_p/c$ in the vacuum, c is the velocity of light in vacuum. $q_j = k_z^j/k$, j represents the jth layer, d_j is the thickness of the medium. Then, the total transfer matrix of the cavity can be calculated as follows

$$Q(k_y, \omega_p) = M_1(k_y, \omega_p, d_1)M_2(k_y, \omega_p, d_2)M_3(k_y, \omega_p, d_3) \tag{9}$$

where M_j is a unimodular matrix, $j = 1, 2, 3$. Then, the reflection and transmission coefficients, using the total transfer matrix, can be obtained as [25]

$$r(k_y, \omega_p) = \frac{q_0(Q_{22} - Q_{11}) - (q_0^2 Q_{12} - Q_{21})}{q_0(Q_{22} + Q_{11}) - (q_0^2 Q_{12} + Q_{21})} \tag{10}$$

and

$$t(k_y, \omega_p) = \frac{2q_0}{q_0(Q_{22} - Q_{11}) - (q_0^2 Q_{12} - Q_{21})} \tag{11}$$

where, k_y is the y component of the wave vector, $q_0 = k_z/k$, and Q_{ij} are the elements of the matrix $Q(k_y, \omega_p)$.

D. The Group Index of the Cavity

The superluminal and subluminal wave propagation through the medium is related to the group index of the medium which is negative for superluminal wave propagation and positive for subluminal wave propagation [29]. However, the cavity includes the walls and the intra-cavity medium as Fig. 1. shows. The behavior of the group index corresponding to the total cavity may be different from the group index of the intra-cavity medium alone. Therefore, studying the dependence of the GH shifts upon the group index of the total cavity is very instructive.

The group index of the total cavity, defined as the ratio of the speed of light in vacuum to the group velocity corresponding to the reflected or transmitted light beam, can be written as [29]

$$N_g^{r,t} = \frac{c}{L}\frac{d\varphi_{r,t}}{d\omega_p} \tag{12}$$

here the superscripts or subscripts r, t correspond to reflection and transmission parts of the incident light beam, c is the speed of light in vacuum, $L = 2d_1 + d_2$, $\varphi_{r,t}$ is the phase associated with the reflection or transmission coefficients, ω_p is the angular frequency of the probe field. We expect that whether GH shift is positive or negative depends on the group index of the total cavity.

III. Numerical Results and Discussions

Now we discuss the coherent control of the GH shifts in a cavity containing a five-level double-ladder atomic system in detail on numerical calculation method according to (4)–(11). We set the thickness of the wall $d_1 = 0.2\,\mu$m, the thickness of the intracavity medium $d_2 = 5\,\mu$m, $\varepsilon_1 = 2.22$ [25], [29]. Next, we determine the parameters of the five-level double-ladder atomic system. We can consider for example the cold atoms ^{87}Rb because the corresponding driving fields with appropriate wavelengths are easy to be achieved from lasers [35]. Here, we start our numerical analysis by selecting the decay rates and detunings according to [35]. Because it is very possible to realize an intracavity medium of five-level double-ladder-type atoms with these selected parameters in the experiment according to this literature. The decay rates are chosen as follows: $\gamma_{31} = \gamma_{32} = 2\pi \times 5.3$ MHz, $\gamma_{43} = 2\pi \times 0.67$ MHz, and $\gamma_{51} = \gamma_{52} = \gamma_{54} = 2\pi \times 0.09$ MHz, respectively. The wavelength of the probe beam is 795 nm ($\omega_p = 2\pi \times 377.36$ THz). The frequency detunings of the corresponding controlling beam $\Delta_c = \Delta_s = \Delta_d = 0$. The one-half Rabi frequency $\Omega_p = 0.01\gamma_{31}$, $\Omega_c = 3\gamma_{31}$, and

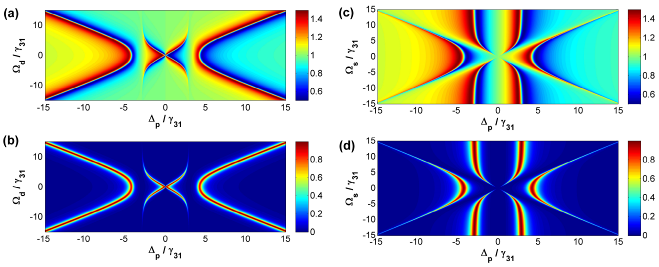

Fig. 3. Permittivity ε_2 as a function of the Rabi frequency Ω_s, Ω_d, and the detuning Δ_p: [(a) and (c) the real part; (b) and (d) imaginary part] for when (a) and (b) $\Omega_s = 3\gamma_{31}$, (c) and (d) $\Omega_d = 3\gamma_{31}$, with $\omega_p = 2\pi \times 377.36$ THz, $\gamma_{31} = \gamma_{32} = 2\pi \times 5.3$ MHz, $\gamma_{43} = 0.13\gamma_{31}$, $\gamma_{51} = \gamma_{52} = \gamma_{54} = 0.02\gamma_{31}$, $\Omega_p = 0.01\gamma_{31}$, $\Omega_c = 3\gamma_{31}$, $\beta = \gamma_{31}$, $\Delta_c = \Delta_s = \Delta_d = 0$.

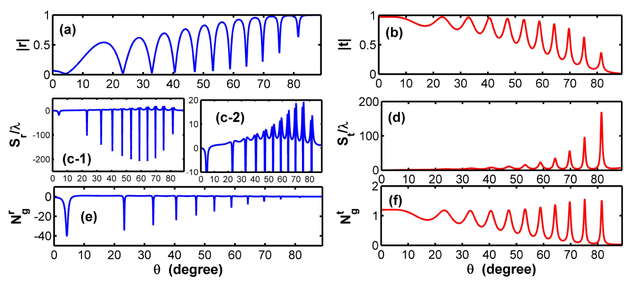

Fig. 4. (a) Absolute value of the reflection coefficient, (b) Absolute value of the transmission coefficient, (c) GH shift of reflected light (c-1 and c-2 have different Y-axis scales), (d) GH shift of transmitted light, (e) group index of reflected light, (f) group index of transmitted light, respectively, versus the incident angle θ, with $\Omega_d = \Omega_s = 3\gamma_{31}$, $\Delta_p = 0.3\gamma_{31}$, other parameters are the same as in Fig. 3.

$\beta = 3\gamma_{31}$. After the above parameters are selected, in our analysis of GH shifts in a cavity containing a five-level double-ladder-type atomic system, the control knob here can be one or more of the Rabi frequency Ω_d, Ω_p, and detuning Δ_p. First, we study the effect of these three knob on the permittivity of a five-level double-ladder atomic system. In Fig. 3, we plot the real part and the imaginary part of the permittivity ε_2 as a function of the Rabi frequency Ω_s, Ω_d, and the detuning Δ_p. As we know, the imaginary part of the permittivity indicates the absorption of the light beam propagating in it by the medium. The larger the value, the stronger the absorption, and the smaller the value, the weaker the absorption. The purpose of our plotting the permittivity ε_2 is to study whether the five-level double-ladder-type atomic system absorbs weakly or strongly when resonance occurs.Of course, we can also use the method in [36] to establish the frequency characteristic analysis model and simulate the electric field distribution, so as to more accurately analyze the transmission characteristics of the reflected or transmitted beam in the medium.

Next, we plot absolute values of the reflection and transmission coefficients in Fig. 4(a) and 4(b), and the GH shifts of the reflected and transmitted light in Fig. 4(c) and 4(d), respectively, versus incident angle of the probe light beam with $\Omega_c = \Omega_d = 3\gamma_{31}$, $\Delta_p = 0.3\gamma_{31}$, other parameters are the same as in Fig. 3. We notice that the five-level double-ladder-type atomic system exhibits weak absorption at the above parameters as Fig. 3 shown. We see that GH shifts have different behaviors

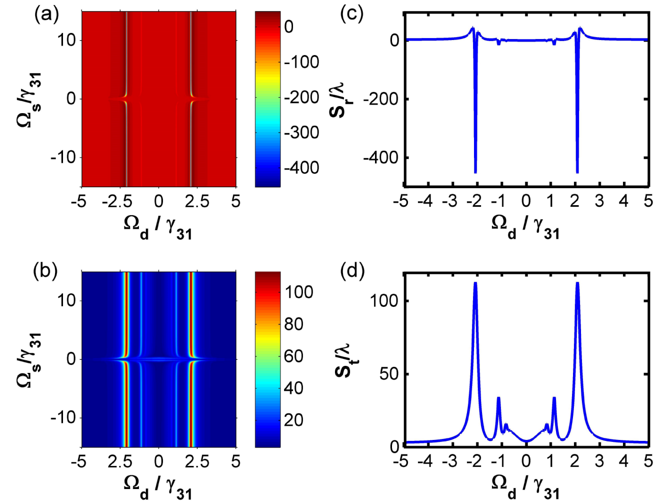

Fig. 5. (a) 2D map of GH shift of reflected light, (b) 2D map of GH shift of transmitted light, (c) GH shift of reflected light for when $\Omega_s = 10\gamma_{31}$, (d) GH shift of transmitted light for when $\Omega_s = 10\gamma_{31}$, with $\theta = 74^\circ$, $\Delta_p = 0.1\gamma_{31}$, other parameters are the same as in Fig. 3.

at different incident angle. Fig. 4 shows dips of the reflection and peaks of the transmission which correspond to the angles that satisfy the resonance condition. Here we observe the large negative and positive GH shifts of the reflected beam and the transmitted beam, respectively. We also note that the GH shift of the reflected beam is a relatively small positive value when deviating from the resonance condition, but the GH shift of the transmitted beam is a positive value at all different incident angle.

We calculate the group index of the total cavity according to (12). and plot it in Fig. 4(e) and 4(f). Fig. 4 shows that the group index is negative and the GH shift is negative. Similarly, the group index is positive and the GH shift is positive. Moreover, there has no proportional relationship between GH shift and group index. For example, at the incident angle $\theta = 23.2^\circ$, we find $N_g^r = -21.8$ and $N_g^t = 1.2$, therefore the negative GH shift of reflected beam and the positive GH shift of the transmitted beam is observed.

Now we consider the effects of the Rabi frequency Ω_d, Ω_s, and detuning Δ_p on the GH shifts of the reflected beam and the transmitted beam at a fixed incident angle. So there are three control knobs. For the convenience of studying GH shifts, we study the case that one is fixed and the other two are controlled. Fig. 5 shows the 2D graph of the GH shift of the reflected beam and the transmitted beam as a function of Ω_s and Ω_d at the incident angle $\theta = 74^\circ$, $\Delta_p = 0.1\gamma_{31}$. We found that the GH shift of the reflected beam and the transmitted beam has a negative or positive maximum when $\Omega_d = \pm 2.08\gamma_{31}$, respectively. But the controlled effect of Ω_s is not obvious except around $\Omega_s = 0$. Moreover, we also find that the behavior of GH shift is symmetric with respect to Ω_s or Ω_d because, as seen from (1)–(3), the permittivity ε_2 is a function of the square of the modulus of Ω_s or Ω_d.In fact, in this case of $\Omega_s > 2$, the real and imaginary parts of ε_2 change little, so the GH shift does not change. We have also drawn the curves of the real and

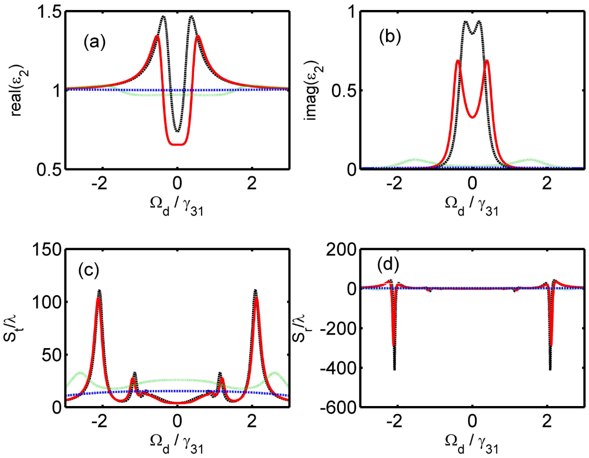

Fig. 6. (a) The real part of permittivity ε_2, (b) the imaginary part of permittivity ε_2, (c) GH shift of transmitted light, (d) GH shift of reflected light for $\Omega_s = 2\gamma_{31}$ (black line), $\Omega_s = 0.8\gamma_{31}$ (red line), $\Omega_s = 0.2\gamma_{31}$ (green line) and $\Omega_s = 0.1\gamma_{31}$ (blue line), other parameters are all the same as in Fig. 5.

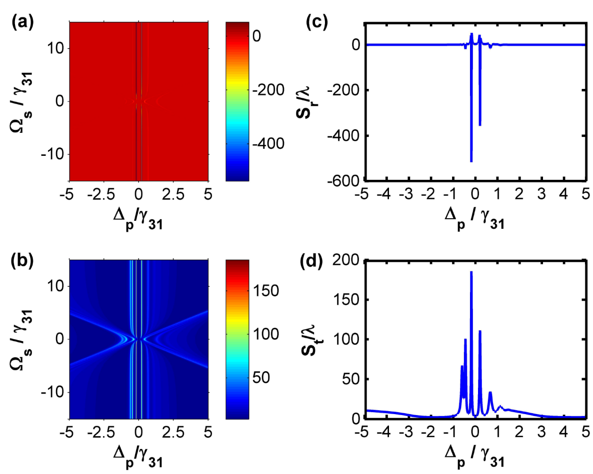

Fig. 7. (a) 2D map of GH shift of reflected light, (b) 2D map of GH shift of transmitted light, (c) GH shift of reflected light for when $\Omega_s = 10\gamma_{31}$, (d) GH shift of transmitted light for when $\Omega_s = 10\gamma_{31}$ with $\theta = 74^\circ$, $\Omega_d = 3\gamma_{31}$, other parameters are the same as in Fig. 3.

imaginary parts of the ε_2 and GH sfifts with respect to Ω_d under different Ω_s, as shown in Fig. 6, where we know that when Ω_s is increasing from zero, with the change of Ω_d, the real part and imaginary part of ε_2 will show two positive peaks symmetrically near $\Omega_d = 0$, and the larger Ω_s is, the closer the two peaks are to the center of symmetry. But the real and imaginary parts of the permittivity ε_2 behave differently in the symmetric center of $\Omega_d = 0$, when Ω_s is increasing from zero, the real part of ε_2 decreases from 1 to some value at the center of symmetry and then increases by a value less than 1, while the imaginary part of ε_2 increases from zero to near 1. However, we also note that the maximum GH shift does not occur at the maximum of the real or imaginary part of ε_2, because the all microstructure only resonates when electromagnetically induced transparency occurs, thus the considerable GH shift is obtained.

In Fig. 7, we plot the 2D graph of the GH shift of the reflected beam and the transmitted beam as a function of Ω_s and Δ_p at $\theta = 74^\circ$, $\Omega_d = 3\gamma_{31}$. Here, we can see that the controlled effect

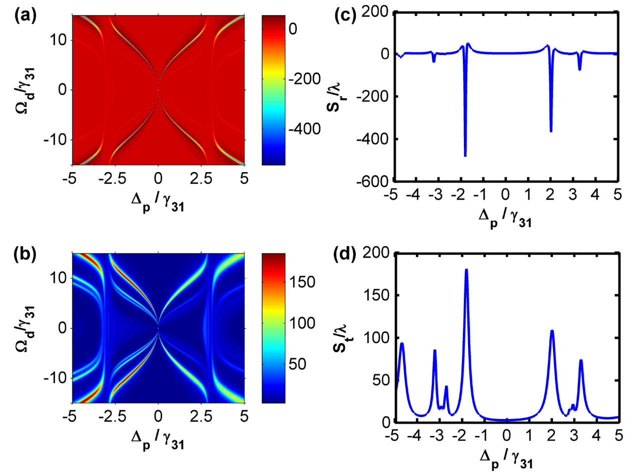

Fig. 8. (a) 2D map of GH shift of reflected light, (b) 2D map of GH shift of transmitted light, (c) GH shift of reflected light for when $\Omega_d = 10\gamma_{31}$, (d) GH shift of transmitted light for when $\Omega_d = 10\gamma_{31}$ with $\theta = 74^\circ$, $\Omega_s = 3\gamma_{31}$, other parameters are the same as in Fig. 3.

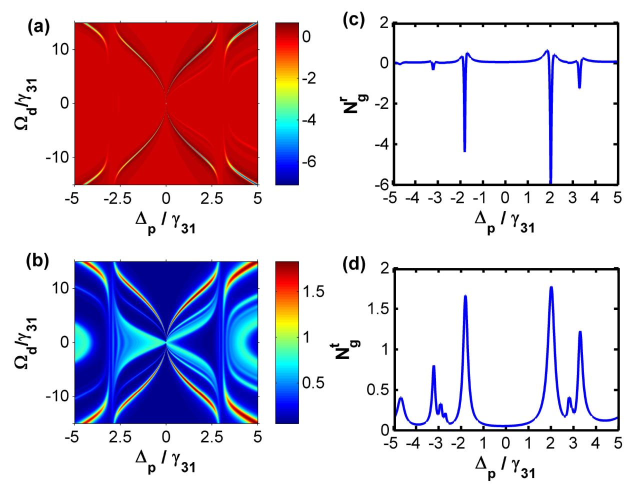

Fig. 9. (a) 2D map of N_g^r of reflected light, (b) 2D map of N_g^t of transmitted light, (c) N_g^r of reflected light for when $\Omega_d = 10\gamma_{31}$, (d) N_g^t of transmitted light for when $\Omega_d = 10\gamma_{31}$ with $\theta = 74^\circ$, $\Omega_s = 3\gamma_{31}$, other parameters are the same as in Fig. 3.

of Ω_s is the same as Fig. 5. The GH shift of the reflected beam and the transmitted beam has a large negative dips or positive peaks respectively when $\Delta_p = (\pm 0.195 + 0.015)\gamma_{31}$ and $\Delta_p = (\pm 0.57 + 0.1)\gamma_{31}$. In addition to the above control effect, there is another a GH shift peak of several tens of wavelengths in (Ω_s, Δ_p) parameter space away from the center $(\Omega_s = 0, \Delta_p = 0)$ with the increase of $|\Omega_s|$ or $|\Delta_p|$ as Fig. 7(b) shown.

Fig. 8 shows the 2D graph of the GH shift of the reflected beam and the transmitted beam as a function of Ω_d and Δ_p at $\theta = 74^\circ$, $\Omega_s = 3\gamma_{31}$. We found that the GH shift of the reflected beam and the transmitted beam always is very small at around $\Delta_p = \pm 3\gamma_{31}$, no matter how Ω_d changes. In other (Ω_d, Δ_p) parameter spaces, when one is fixed and the other is changed, the GH shift shows multi peak characteristics, and the position of these peaks gradually deviates from the central position with the increase of $|\Omega_d|$ or $|\Delta_p|$ as Fig. 8 shown.

As shown in Fig. 4, superluminal and subluminal wave propagation in a cavity containing a five-level double-ladder-type atomic was observed at various incident angles under certain conditions. Finally, we study superluminal and subluminal wave propagation again in the atomic system when the incident angle is fixed ($\theta = 74^{\circ}$) and one or two knobs (Ω_d, Δ_p) are changed. We plot the group index N_g of the total cavity at the parameters marked in Fig. 9. Under the same parameter conditions, we observe Figs. 8 and 9, and we can draw the same conclusion as before: the group index of the total cavity is negative, the GH shift is negative, the group index of the total cavity is positive, and the GH shift is positive. There has no proportional relationship between GH shift and group index of the total cavity.

IV. Conclusion

After replacing the EIT medium in the micro-cavity with a five-level double-ladder-type atomic system, we rescheduled the research plans in references [25] and [29]. Compared with previous studies, due to the increase of intracavity medium energy level, the control variables of Rabi frequency and frequency detuning also increase, and control methods of coherent GH shift become more flexible. Research has shown that by using the new micro-cavity, which contain an intracavity medium of five-level double-ladder-type atomic system, we not only obtain flexible control methods for GH shifts, but also control the GH shift of reflected beam to over 400 wavelengths and transmitted beam to over 150 wavelengths under appropriate parameter conditions. Also, our results built on stationary phase theory show that the GH shift of the reflected beam in our new micro-cavity system is negative or positive, but that of the transmitted beam is positive in a five-level double-ladder-type atomic system.

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