# Outage Probability Analysis and Joint Optimization for UAV-Aided FSO/RF Systems With Nonlinear Power Amplifiers

Hwi-Sung Park<sup>®</sup>, Jeongju Jee<sup>®</sup>, and Hyuncheol Park<sup>®</sup>, Senior Member, IEEE

Abstract—The unmanned aerial vehicle (UAV)-aided free-space optical/radio frequency (FSO/RF) technique has recently attracted significant attention, which offers new possibilities thanks to its dynamic deployment capability. However, many researchers have not considered the fading characteristic of both RF and FSO links and the limited UAVs' size, weight, and power (SWAP) simultaneously. This paper focuses on a mixed FSO/RF system that facilitates the communication between a base station and mobile stations via a hovering UAV acting as a decode-and-forward relay. Considering the fading effects and the power amplifier (PA) non-linearity caused by the SWAP constraint, we mathematically analyze the closedform and asymptotic outage probabilities. Especially in high transmit power regions, we present the destructive impact of nonlinear PAs on the outage probability and investigate modulation schemes to mitigate this degradation. The derived outage expressions match well with numerical results. Moreover, we propose a joint placement and transmit power optimization algorithm that minimizes the end-to-end outage probability with round-robin and absolute signal-to-noise ratio-based scheduling schemes. Finally, simulation results show that the performance of the proposed algorithm is comparable to that of the brute-force method.

*Index Terms*—Unmanned aerial vehicle (UAV), free-space optics (FSO), multiple-input single-output (MISO), nonlinear power amplifier (PA), outage probability, placement, equal gain transmission (EGT).

# I. INTRODUCTION

**R** ECENTLY, unmanned aerial vehicle (UAV) communication systems have gained significant attention as a new relay station [1], [2]. The UAV-aided systems are beneficial for combining free space optics (FSO) and radio frequency (RF) technologies. They have the capability to dynamically support large amounts of data from a base station (BS) while avoiding

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interference from other UAVs and multiple mobile stations (MSs) [3].

To further enhance the performance of the UAV-aided FSO/RF systems, many researchers focused on the UAV's parameter optimization algorithms and the analysis of the performance metrics [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15]. A list of existing works on UAV-aided FSO/RF systems is provided in Table I. The authors proposed the optimization algorithms considering throughput [4], latency [5], the number of MSs satisfying throughput [6], and spectral efficiency (SE) [7]. As the unlimited capacity of the FSO backhaul link was assumed in [4], [5], [6], [7], the authors in [8] presented an algorithm to jointly optimize bandwidth allocation and UAV's 3D placement by considering the capacity constraint of the FSO link. In [9], the authors optimized the trajectory of a UAV-based FSO/RF system with UAV buffer constraints and delay considerations. However, the authors in [4], [5], [6], [7], [8], [9] did not consider fading effects, which is a significant characteristic of RF and FSO channels.

On the other hand, the impact of fading on the UAV-aided FSO/RF system has been investigated in [10], [11], [12], [13], [14], [15]. In [10], the authors analyzed the outage probability, ergodic capacity (EC), and average bit-error-rate (BER) when each RF and FSO channel follows  $\kappa - \mu$  fading and Málaga- $\mathcal{M}$  fading, respectively. The EC was derived in [11] for an UAV operating as both a buffer-aided and non-buffer-aided relay while accounting for the UAV's instability. By extending the system model to satellite-UAV-terrestrial networks, the authors in [12] and [13] derived various closed-form performance metrics, such as outage probability, EC, average BER, and average symbol-error-rate (SER). The authors in [14] and [15] proposed a multi-user scheduling scheme and a beamforming scheme based on the derived closed-form equation for EC, respectively. However, authors in [10], [11], [12], [13], [14], [15] did not address the practical hardware limitations of the UAV-aided relay systems.

When utilizing UAVs in mixed FSO/RF communication systems, it encounters some crucial challenges. Due to UAVs' limited size, weight, and power (SWAP), UAVs cannot adopt complex, power-intensive, and linear hardware components [1]. This constraint can result in hardware limitations on the UAV. As Table I reveals, the existing works on UAV-aided FSO/RF systems do not focus on the practical limitations and unique characteristics of UAVs. A crucial challenge that needs to be

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Dof	Channel	UAV's hardware	Performance	Optimization	
Kel.	fading limitations		analysis metrics	parameters	
				Bandwidth allocation,	
[4]	X	Limited transmit power	Throughput	Transmit power	
				3D Placement	
	~			Bandwidth allocation,	
[5]		~	Latency	User association,	
				3D Placement	
	~	~		Bandwidth allocation,	
[6]		^	The No. of MSs	User association,	
				3D Placement	
[7]	×	×	SE	User association,	
				3D Placement	
[8]	×	×	The No. of MSs	Bandwidth allocation,	
	~			3D Placement	
[9]	<u> </u>	<u> </u>	Throughput	Trajectory	
[10]	0	×	Outage probability,	×	
[]	-		EC, BER		
[11]	0	×	EC	×	
[12]	0	×	Outage probability,	×	
[12]		-	EC, BER		
[13]	0	×	Outage probability,	×	
[15]			EC, SER	-	
[14]	$\mathcal{O}$	×	EC	Multiuser scheduling	
[15]	0	×	EC	Beamforming	
Proposed		Nonlinear PAs,		Transmit nower	
work	O	Limited battery	Outage probability	3D Placement	
WOIK		Limited transmit power		SD Placement	

 TABLE I

 COMPARISON OF THE PROPOSED WORK WITH OTHER EXISTING STUDIES

addressed is the non-linearity of power amplifiers (PAs), which can degrade the overall performance of UAV-aided relay systems [16]. Moreover, UAVs are equipped with onboard batteries that have limited capacity, which can restrict their travel distance and transmit power to support MSs [17], [18], [19]. Lastly, both RF and FSO systems consider outage probability as a crucial factor in determining the optimal location for UAV [20], [21]. Therefore, optimizing the joint UAV placement and transmit power based on outage probability is essential for practical UAVaided FSO/RF systems, considering fading effects, nonlinear PAs, and limited battery.

In this paper, we focus on a mixed FSO/RF system that employs a single UAV-aided decode-and-forward (DF) relay with nonlinear PAs. The UAV relay serves multiple MSs with conventional scheduling schemes such as round-robin (RR) or absolute signal-to-noise ratio-based (ASB) scheduling. The FSO link model considers channel factors such as atmospheric loss, turbulence, angle of arrival (AoA) fluctuation, and pointing error loss [22], [23]. For the RF link of our system model, we consider multiple-input single-output (MISO) under nonlinear PAs and equal gain transmission (EGT) beamforming. Furthermore, the RF links are modeled as frequency flat Nakagami-*m* fading channels, which is a practical assumption for the hovering UAVto-ground communication scenario in a line-of-sight environment [11], [24]. The contributions of this paper are summarized as follows:

• Considering nonlinear PAs and fading effects in UAVaided FSO/RF systems, we mathematically derive the closed-form expressions of the end-to-end outage probability. We confirm that our mathematical analysis matches with numerical results. Moreover, our results reveal that optimizing transmit power and 3D placement is necessary to enhance the outage probability of the UAV-aided FSO/RF system.

- We derive an asymptotic outage probability to observe the effect of nonlinear PAs with per-antenna transmit power. In the high-transmit power region, the end-to-end outage probability approaches one due to the presence of signal-to-noise-and-distortion ratio (SNDR) ceilings. This observation reveals that the nonlinear characteristics of the PAs fundamentally limit the system's performance.
- Our paper indicates that the modulation schemes significantly affect the performance of the systems with nonlinear PAs. We derive that the outage probability and the SNDR ceilings are quantified explicitly using the moments of modulated symbols. By implementing suitable modulation schemes, we can effectively mitigate the effect of nonlinear PAs.
- In terms of minimizing the derived outage probability, we finally propose an algorithm for optimizing the UAV's transmit power and 3D placement concerning the battery and transmit power limitations. Specifically, we divide the optimization problem into two subproblems and propose an iterative algorithm to solve the transmit power and the 3D placement alternately. The proposed algorithm can adopt conventional RR and ASB scheduling schemes. Numerical



Fig. 1. UAV-aided FSO/RF relaying system with  $N_{\rm M}$  MSs.

results verify that our proposed algorithm performs comparably to the Brute-force Search, which is the optimal baseline scheme.

The rest of the paper is organized as follows. First, Section II presents the system model for UAV-aided FSO/RF systems. In Section III, we analyze closed-form and asymptotic outage probability. Section IV formulates the outage probability minimization problem and proposes an algorithm that optimizes the UAV's transmit power and 3D placement. In Section V, we offer numerical simulation results on the proposed algorithm. Finally, our conclusions are summarized in Section VI.

*Notations:* Bold lowercase letters denote vectors,  $(\cdot)^{\mathrm{T}}$  and  $(\cdot)^*$  indicate the transpose and conjugate operation, respectively. diag(**x**) denotes the diagonal matrix with the elements of **x**;  $|\cdot|$  and  $||\cdot||$  denote the absolute value and Euclidean norm, respectively;  $\mathbb{E}[\cdot]$  represents the expectation;  $\mathbb{C}^{M \times N}$  denotes the complex space of  $M \times N$ ;  $\mathcal{CN}(\mu, \sigma^2)$  represents the complex Gaussian distributions with mean  $\mu$  and covariance  $\sigma^2$ ;  $Nakagami(m, \omega)$  denotes the Nakagami-*m* distribution with severity parameter *m* and average power  $\omega$ ; and  $\operatorname{Re}(x)$  represents the real part of *x*.

#### II. SYSTEM MODEL

In this section, we provide a system model of a UAV-aided DF relaying system in a mixed FSO/RF environment as illustrated in Fig. 1. The BS transmits an optical signal to the UAV via an FSO link. The UAV, acting as the DF relay node, receives the optical signal and converts it into electrical signals. Then, the UAV transmits RF signals to  $N_{\rm M}$  MSs via  $N_{\rm T}$  antennas with nonlinear PAs. The MSs, having a single antenna each due to size limitations, are served by the UAV using either the RR or the ASB scheduling scheme. Note that our system model has no direct communication link between the BS and the MSs due to severe blockage and the directionality of the FSO link.

# A. FSO Link

First, the BS transmits the optical signal  $x_f$  to the UAV through the FSO link. The received FSO signal at the UAV can

be expressed as

$$y_f = I_f x_f + n_f, \tag{1}$$

where  $x_f$  is the transmit optical signal with a total power constraint of  $\mathbb{E}[|x_f|^2] \leq P_f$  and  $n_f$  is an additive white Gaussian noise (AWGN) with  $n_f \sim C\mathcal{N}(0, \sigma_f^2)$ . The noise variance on the FSO link is given as  $\sigma_f^2 = N_f B_f$  where  $N_f$  is the noise power spectral density and  $B_f$  is the FSO bandwidth. The FSO channel gain  $I_f$  in (1) can be expressed as

$$I_f = I_p I_a I_{pg} I_{pa}, (2)$$

where  $I_p = \exp(-D_f\xi)$  is an atmospheric loss with scattering coefficient  $\xi$  and FSO link distance  $D_f = ||\mathbf{r}_u - \mathbf{r}_s||$ . Here,  $\mathbf{r}_s = (x_s, y_s, z_s)$  and  $\mathbf{r}_u = (x_u, y_u, z_u)$  are the 3D position of BS and UAV, respectively [22], [23]. Moreover,  $I_a$  represents atmospheric turbulence,  $I_{pg}$  is the pointing error loss due to the deviation between the receiver lens center and the received beam center, and  $I_{pa}$  is a link interruption due to AoA fluctuations. The turbulence-induced fading  $I_a$  is modeled as Gamma-Gamma distribution with the effective numbers of large-scale and smallscale turbulence cells given as

$$\alpha_f = \left[ \exp\left(\frac{0.49\sigma_R^2}{\left(1 + 1.11\sigma_R^{\frac{12}{5}}\right)^{\frac{7}{6}}}\right) - 1 \right]^{-1}, \qquad (3)$$

and

$$\beta_f = \left[ \exp\left(\frac{0.51\sigma_R^2}{\left(1 + 0.69\sigma_R^{\frac{12}{5}}\right)^{\frac{5}{6}}} \right) - 1 \right]^{-1}, \qquad (4)$$

respectively [25]. The Rytov variance is defined as  $\sigma_R^2 = 1.23C_n^2 k_f^{7/6} D_f^{11/6}$  where  $k_f$  is the wave number defined as  $k_f = 2\pi/\lambda_f$  with the wavelength for the FSO link  $\lambda_f$ . The refractive-index structure parameter  $C_n^2$  depends on atmospheric altitude and is evaluated from the Hufnagel-Valley (H-V) model described by  $C_n^2 \approx C_n^2(0) \exp(\frac{-z_u}{100})$ , where  $C_n^2(0)$  denotes the nominal value at the ground [11], [26].

# B. RF Links With Nonlinear PAs

Next, the DF-based UAV relay decodes the received signal  $y_f$  and re-encodes it to s, which is a complex data symbol with arbitrary modulation satisfying  $\mathbb{E}[ss^*] = 1$ . Before amplification by the nonlinear PAs, the RF transmit signal for the *k*th MS can be expressed as

$$\mathbf{u}_k = \sqrt{P_r} \mathbf{w}_k s,\tag{5}$$

where  $P_r$  is the transmit power of RF links. The EGT beamforming vector denotes  $\mathbf{w}_k = [w_{1,k}, \ldots, w_{N_{\mathrm{T}},k}]^{\mathrm{T}}$  satisfying  $||\mathbf{w}_k||^2 = 1$ , where  $w_{n,k} = \frac{1}{\sqrt{N_{\mathrm{T}}}} \exp(-\iota \angle h_{n,k})$  with  $\angle h_{n,k}$ denoting the phase of  $h_{n,k}$  and  $\iota = \sqrt{-1}$ . After passing through the nonlinear PAs, the signal  $\mathbf{x}_k$  can be expressed as

$$\mathbf{x}_{k} = \sum_{p=1}^{Q} \alpha_{p} \operatorname{diag}\left(\left|\mathbf{u}_{k}\right|^{p-1}\right) \mathbf{u}_{k},\tag{6}$$

where  $\alpha_p$  denotes the *p*th-order coefficient for the input-output characteristic of the PA, and Q is the number of nonlinear components [16], [27]. Finally, the received signal at the *k*th MS is described as

$$y_k = g_k \mathbf{h}_k^{\mathrm{T}} \mathbf{x}_k + n_r, \tag{7}$$

where  $g_k$  represents the free-space path loss which is typically modeled as  $g_k = \frac{1}{c_r D_k}$ . Here,  $c_r = \frac{f}{23.85}$  with the center frequency f in MHz and  $D_k$  denote the RF link distance satisfying  $D_k = ||\mathbf{r}_k - \mathbf{r}_u||$  with the position of the kth MS  $\mathbf{r}_k =$  $(x_k, y_k, z_k)$  [11]. The RF fading coefficient  $\mathbf{h}_k$  in (7) denotes as  $\mathbf{h}_k = [h_{1,k}, \ldots, h_{N_{\mathrm{T}},k}]^{\mathrm{T}}$  where  $h_{n,k}$  is the complex channel gain satisfying  $|h_{n,k}| \sim Nakagami(m_k, \omega)$ . We consider a relationship between  $m_k$  and  $\theta_k \in [0, \pi/2]$  characterized by  $m_k =$  $m_{\min} \exp(m_p \theta_k)$  where  $m_p = \frac{2}{\pi} \ln(\frac{m_{\max}}{m_{\min}})$ ,  $m_{\min}$  and  $m_{\max}$  are the minimum and maximum values of  $m_k$ , respectively. Also,  $\theta_k$ denotes the elevation angle defining  $\theta_k = \arcsin(\frac{|z_u-z_k|}{D_k})$ . Note that when the multipath fading gets its highest level (i.e.,  $\theta_k = 0$ ),  $m_k$  reaches  $m_{\min}$ , whereas for  $\theta_k = \pi/2$ ,  $m_k$  has approaches  $m_{\max}$ . In addition,  $n_r$  in (7) is an AWGN with  $n_r \sim \mathcal{CN}(0, \sigma_r^2)$ . The noise variance in the RF channel is given as  $\sigma_r^2 = B_r N_r N_0$ where  $B_r$  is the RF bandwidth,  $N_r$  is the receiver noise figure, and  $N_0$  is the noise power spectral density.

#### III. OUTAGE PROBABILITY

This section presents the outage probability analysis for the mixed FSO/RF UAV-aided systems. Based on the received signals of individual links, we derive the end-to-end and asymptotic outage probabilities. To validate our analysis, we simulate the outage probability and compare the results with our numerical calculations.

#### A. FSO Link

By assuming intensity modulation/direct detection (IM/DD) modulation, the signal-to-noise-ratio (SNR) at UAV can be expressed from (1) as

$$\gamma_f = \frac{P_f^2 |I_f|^2}{\sigma_f^2} = \frac{\bar{\gamma}_f |I_f|^2}{(I_p A_0)^2},\tag{8}$$

where  $\bar{\gamma}_f = \frac{(P_f I_p A_0)^2}{\sigma_f^2}$  and  $A_0$  is the fraction of the collected power at the center of the receiver lens. Then, the cumulative distribution function (CDF) of  $\gamma_f$  is given as [23]

$$F_{\gamma_f}(\gamma) \simeq a_1 + (1 - a_1) \sum_{n=0}^{N_c} \left( \frac{c_1}{\tau} \left( \frac{\gamma}{\bar{\gamma}_f} \right)^{\frac{\tau}{2}} + \frac{c_2}{n + \alpha_f} \left( \frac{\gamma}{\bar{\gamma}_f} \right)^{\frac{n+\alpha_f}{2}} - \frac{c_3}{n + \beta_f} \left( \frac{\gamma}{\bar{\gamma}_f} \right)^{\frac{n+\beta_f}{2}} \right),$$
(9)

where  $a_1 = \exp(-\frac{\theta_{\text{FOV}}^2}{2(\sigma_{so}^2 + \sigma_{ro}^2)})$  with field-of-view (FoV) angle  $\theta_{\text{FOV}}$ , the standard deviations for orientation of the transmit node  $\sigma_{so}^2$  and the receiver node  $\sigma_{ro}^2$ . Additionally,  $N_c$  is the coefficient depending on the Rytov variance  $\sigma_R^2$  and  $\tau$  is the ratio between

the equivalent beam radius and the pointing error displacement standard deviation at the receiver given as  $\tau = \frac{w_{zeq}^2}{4(\sigma_{sp}^2 + \sigma_{rp}^2)}$ , where  $w_{zeq}^2$  is the equivalent beam waist,  $\sigma_{sp}$  and  $\sigma_{rp}$  are the standard deviations for the position of the transmit and the receiver node, respectively. The coefficients  $c_1, c_2$  and  $c_3$  in (9) are represented in Appendix A.

# B. RF Links With Nonlinear PAs

By using the Bussgang theorem [28], we can decompose  $g_k \mathbf{h}_k^{\mathrm{T}} \mathbf{x}_k$  in (7) into the component linearly correlated with s and the rest uncorrelated terms as

$$y_k = G_k s + d_k + n_r, \tag{10}$$

where  $G_k$  is the linearity coefficient following  $G_k = \frac{\mathbb{E}[y_k s^*]}{\mathbb{E}[ss^*]}$  and  $d_k$  denotes the residual distortion uncorrelated with s. Considering the nonlinear PAs and EGT beamforming, the SNDR of the RF links is as follows [16]

$$\gamma_k = \frac{|G_k|^2}{\sigma_k^2 + \sigma_r^2} = \frac{g_k^2 H_k^2 K}{g_k^2 H_k^2 L + \sigma_r^2},$$
(11)

where  $\sigma_k^2$  is the variance of  $d_k$ ,  $H_k = \sum_{n=1}^{N_T} |h_{n,k}|$ ,  $K = \sum_{i=1}^Q \sum_{j=1}^Q \alpha_i \alpha_j^* S_{i,j}^S P_a^{\frac{i+j}{2}}$ , and  $L = \sum_{i=1}^Q \sum_{j=1}^Q \alpha_i \alpha_j^* S_{i,j}^D P_a^{\frac{i+j}{2}}$  with per-antenna transmit power  $P_a = \frac{P_r}{N_T}$ ,  $S_{i,j}^S = S_{i+1}S_{j+1}$ ,  $S_{i,j}^D = S_{i+j} - S_{i+1}S_{j+1}$ , and  $S_i = \mathbb{E}[|s|^i]$ . Note that K and L result in a non-monotonic relationship between  $\gamma_k$  and  $P_a$ . Specifically, the uncorrelated distortion L is smaller than the desired signal K in the low-power regions. However, L increases faster than K, which reduces the SNDR in the high-power regions [29], [30]. Moreover, for the system with nonlinear PAs,  $\gamma_k$  is a function of the modulation schemes since  $S_i$  is determined by the constellation points of modulation schemes. On the other hand, for the system with linear PAs, i.e.,  $\alpha_1 = 1$ ,  $\alpha_i = 0 \forall i \ge 2$ , we can obtain  $K = P_a$  and L = 0 with  $S_2 = 1$ , implying that  $\gamma_k$  and modulation schemes are unrelated. In the following proposition, we derive the CDF of SNDR  $\gamma_k$  based on (11).

*Proposition 1:* The CDF of  $\gamma_k$  under the nonlinear PAs, EGT beamforming and independent and identically distributed (i.i.d.) Nakagami-*m* fading channels can be approximated as

$$F_{\gamma_{k}}(\gamma) \approx \begin{cases} 1 - \frac{1}{\Gamma(M_{k})} \Gamma\left(M_{k}, \frac{M_{k} \gamma \sigma_{r}^{2}}{\Omega_{k} g_{k}^{2}(K - \gamma L)}\right), & \gamma < \frac{K}{L}, \\ 1, & \gamma \geq \frac{K}{L}, \end{cases}$$
(12)

where  $M_k = m_k N_{\rm T}$ ,  $\Omega_k = \omega N_{\rm T} (1 + \frac{(N_{\rm T}-1)\Gamma^2(m_k + \frac{1}{2})}{m_k \Gamma^2(m_k)})$ ,  $\Gamma(\cdot)$ and  $\Gamma(\cdot, \cdot)$  are the gamma function and the upper incomplete gamma function, respectively.

*Proof:* The CDF of  $\gamma_k$  can be expressed as

$$F_{\gamma_k}(\gamma) = \Pr\left(\gamma_k \le \gamma\right)$$
$$= \Pr\left(H_k^2 \le \frac{\sigma^2 \gamma}{g_k^2 \left(K - \gamma L\right)}\right). \tag{13}$$

If  $K - \gamma L \leq 0$ , then the inequality is satisfied for any realization of the non-negative variable  $H_k^2$  [31]. Since the sum of i.i.d. Nakagami-*m* random variables  $H_k$  can be approximated accurately by  $H_k \sim Nakagami(M_k, \Omega_k)$  [32], the CDF of  $H_k^2$  can be expressed as

$$F_{H_k^2}(x) \approx 1 - \frac{1}{\Gamma(M_k)} \Gamma\left(M_k, \frac{M_k x}{\Omega_k}\right).$$
 (14)

Substituting (14) into (13), we can obtain (12).

In the special case of the single-input single-output (SISO) system with the ideal PA, i.e.,  $\alpha_1 = 1$ ,  $\alpha_i = 0 \quad \forall i \ge 2$ , and  $N_{\rm T} = 1$ , we can reduce (12) to the conventional result of [32],  $F_{\gamma k}^{id}(\gamma) = 1 - \frac{1}{\Gamma(m_k)} \Gamma(m_k, \frac{m_k \gamma}{\bar{\gamma}_{r,s}^{id}})$  where  $\bar{\gamma}_{r,s}^{id} = \frac{g_k^2 \omega P_r}{\sigma_r^2}$ . The UAV serves multiple MSs by employing multi-user scheduling schemes, and the resulting CDF can be expressed as follows. For the RR scheduling, each MS has an equal chance to receive the data from the UAV. Thus, the CDF of the RF link for the RR scheme is expressed as [33]

$$F_{\gamma_r^{\text{RR}}}(\gamma) = \frac{1}{N_{\text{M}}} \sum_{k=1}^{N_{\text{M}}} F_{\gamma_k}(\gamma) \,. \tag{15}$$

The ASB scheduling is to select the *k*th MS, where  $k = \arg \max_k \{\gamma_k\}$ , experiencing the largest instantaneous received SNR during a particular time slot. The CDF of the RF link for the ASB scheduling is obtained as [13]

$$F_{\gamma_r^{\text{ASB}}}(\gamma) = \prod_{k=1}^{N_{\text{M}}} F_{\gamma_k}(\gamma) \,. \tag{16}$$

# C. Outage Probability Analysis

This subsection presents the closed-form expression of the end-to-end outage probability. Moreover, we analyze the asymptotic outage probability in high transmit power regions and demonstrate how the nonlinear PAs and modulation schemes affect the outage probability.

1) Closed-Form Analysis: The overall CDF of SNDR for the UAV-aided FSO/RF DF relaying system is obtained as [34]

$$F_{\gamma_{\text{end}}}(\gamma) = \Pr[\min(\gamma_f, \gamma_r) \le \gamma]$$
  
=  $F_{\gamma_f}(\gamma) + F_{\gamma_r}(\gamma) - F_{\gamma_f}(\gamma)F_{\gamma_r}(\gamma),$  (17)

where  $F_{\gamma_r}(\gamma)$  denotes the CDF of the SNDR  $\gamma_r$  of the RF link after multi-user scheduling schemes. Note that we can substitute  $F_{\gamma_r}(\gamma)$  with (15) or (16) according to the multi-user scheduling. An outage of the system is encountered when the end-to-end SNDR  $\gamma$  falls below a predetermined threshold  $\gamma_{th}$ . By setting  $\gamma = \gamma_{th}$  in (17), the outage probability can be rewritten as

$$p_{\text{out}} = p_{\text{out}}^{\text{FSO}} + p_{\text{out}}^{\text{RF}} - p_{\text{out}}^{\text{FSO}} p_{\text{out}}^{\text{RF}}, \qquad (18)$$

where  $p_{out}^{FSO}$  and  $p_{out}^{RF}$  are the outage probability of the FSO link and the RF link, respectively.

2) Asymptotic Analysis: For high FSO transmit power  $P_f$ , as the summation terms in (9) go to zero, we first express  $p_{out}^{FSO}$  as

$$\lim_{P_f \to \infty} p_{\text{out}}^{\text{FSO}} = a_1. \tag{19}$$

Note that in high transmit regions, the outage probability of the FSO link is determined by link interruption due to the movement of the UAV [22]. Next, for the RF link, we consider high regimes of per-antenna transmit power  $P_a$  to show the fundamental impact on our system model with nonlinear PAs.

*Proposition 2:* For the case of high  $P_a$ , the outage probability of RF link under nonlinear PAs utilizing the EGT beamforming satisfies

$$\lim_{P_a \to \infty} p_{\text{out},k}^{\text{RF}} = \begin{cases} 0, & \gamma_{th} \le \nu, \\ 1, & \gamma_{th} > \nu, \end{cases}$$
(20)

where  $\nu$  is the SNDR ceiling defined as

$$\nu = \frac{S_{Q+1}^2}{S_{2Q} - S_{Q+1}^2}.$$
(21)

*Proof:* By taking the limit  $P_a \to \infty$ , we can calculate the dominant terms of  $K - \gamma_{th}L$  in (13) with i = j = Q as

$$K - \gamma_{th} L \approx |\alpha_Q|^2 P_a^Q \left( S_{Q+1}^2 - \gamma_{th} \left( S_{2Q} - S_{Q+1}^2 \right) \right).$$
 (22)

When  $S_{Q+1}^2 - \gamma_{th}(S_{2Q} - S_{Q+1}^2) \ge 0$ ,  $F_{\gamma_k}(\gamma_{th})$  in (13) goes to zero. With some basic algebra, we rewrite the condition in terms of  $\gamma_{th}$  as  $\gamma_{th} \le \frac{S_{Q+1}^2}{S_{2Q} - S_{Q+1}^2}$ . Note that we can reveal  $S_{2Q} - S_{Q+1}^2 \ge 0$  by Cauchy-Schwarz inequality. Otherwise, since  $F_{\gamma_k}(\gamma_{th})$  goes to one when  $S_{Q+1}^2 - \gamma_{th}(S_{2Q} - S_{Q+1}^2) < 0$ , we can obtain the results of Proposition 2.

Note that every MS has the same value  $\nu$  since the SNDR ceiling  $\nu$  is determined by the characteristics of the transmitter, such as modulation schemes and the non-linearity of PAs. Finally, substituting (19) and (20) into (18), regardless of multiuser scheduling schemes, we can obtain the end-to-end outage probability in high transmit power regions as

$$\lim_{P_f, P_a \to \infty} p_{\text{out}} = \begin{cases} a_1, & \gamma_{\text{th}} \le \nu, \\ 1, & \gamma_{\text{th}} > \nu. \end{cases}$$
(23)

For the linear PAs (i.e., Q = 1), the SNDR ceiling  $\nu$  in (21) tends to infinity, implying no ceilings exist. However, for the nonlinear PAs (i.e.,  $Q \ge 3$ ),  $p_{out}$  goes to one for  $\gamma_{th}$  larger than  $\nu$ . Since the ceiling can occur with any i.i.d. fading distributions in the high  $P_a$  regime, the performance under nonlinear PAs is limited by the SNDR ceiling. Moreover, we observe that modulation schemes affect  $\nu$  in (21) since the constellation points of modulation schemes can affect the system with nonlinear PAs in the following subsection.

*3)* Special Cases for SNDR Ceilings: To better understand SNDR ceilings, we assume the data symbol *s* with particular modulations. First, we present the result of *s* with M-ary phase shift keying (PSK) modulations.

*Corollary 1:* M-ary PSK modulations are not affected by the SNDR ceilings.

*Proof:* For M-ary PSK modulations,  $\mathbb{E}[|s|^i] = 1 \ \forall i \in \mathbb{N}$ . As  $S_{2p} - S_{p+1}^2 = 0 \ \forall p \in \mathbb{N}$ , the SNDR ceilings for M-ary PSK is infinity.

Note that as L = 0 in (11) regardless of  $P_a$ , utilizing M-ary PSK modulations can reduce the impact of nonlinear PAs.

	Symbol Value		Symbol	Value	
link	ξ	$1.6 \ {\rm km}^{-1}$	$A_0$	$5 \times 10^{-3}$	
	$w_{zeq}$	2.0026 m	$\lambda_f$	1.55 um	
	$B_f$	100 MHz	$N_f$	$10^{-14} \text{ A}^2/\text{Hz}$	
SO	$\theta_{\rm FoV}$	20 mrad	$C_{n}^{2}(0)$	$3 \times 10^{-13} \mathrm{m}^{-\frac{2}{3}}$	
н	$\sigma_{sp}$	0.05 m	$\sigma_{so}$	0 mrad	
	$\sigma_{rp}$	0.1 m	$\sigma_{ro}$	0.3 mrad	
×	f	5 GHz	$B_r$	100 MHz	
RF link	$N_0$	-114  dBm	N	5	
		/MHz	1 V T		
	$m_{\min}$	1	$m_{\rm max}$	4	
	$E_{\rm max}$	639.36 kJ	$T_{\rm UAV}$	500 s	
	$\alpha_{\rm up}$	315	$\beta_{\rm up}$	-211.261	
ers	$\alpha_{\rm hm}$	308.709	$\beta_{\rm hm}$	0.852	
nete	$\alpha_{ m ho}$	4.917	$\beta_{\rm ho}$	275.204	
ran	$\alpha_{\rm dn}$	68.956	$\beta_{\rm dn}$	-65.183	
UAV pa	$\alpha_1$	0.9018	$\alpha_2$	0	
	$\alpha_3$	-0.4093		0	
		$-0.1382\iota$	4	0	
	$\alpha_5$	0.0085	0.0	0	
		$+0.0035\iota$	μ α <sub>6</sub>	0	

TABLE II Simulation Parameters

Next, we elaborate on the data symbol *s* following Gaussian distribution to attain fundamental insights into nonlinear PAs.

Lemma 1: When  $s \sim \mathcal{CN}(0, 1)$ , the SNDR ceiling is obtained as

$$\nu = \frac{\Gamma^2\left(\frac{Q+3}{2}\right)}{\Gamma\left(Q+1\right) - \Gamma^2\left(\frac{Q+3}{2}\right)}.$$
(24)

*Proof:* As |s| follows Rayleigh distribution, the moments are given by  $\mathbb{E}[|s|^i] = \Gamma(1 + \frac{i}{2})$ . After some mathematical manipulations and simplifications with (21), we can obtain (24).

Corollary 2: As the number of nonlinear components Q increases, the SNDR ceiling decreases when the data symbol s follows  $\mathcal{CN}(0,1)$ . Moreover, the SNDR ceiling approaches zero when Q goes to infinity.

*Proof:* Please refer to Appendix **B**.  $\Box$ 

When dealing with highly complex nonlinear systems that have Gaussian signals, it is only possible that the outage probability is less than one by reducing transmit power or increasing the number of antennas. However, other effective strategies for managing nonlinear systems, such as M-ary PSK, can significantly reduce the outage probability, avoiding the SNDR ceiling.

#### D. Numerical Results for Outage Probability

In this subsection, we present simulation results to verify the analytical and asymptotic expressions in Section III-C. We assume  $r_s = (0, 0, 30)$ m,  $\omega = 1$ , and  $\gamma_{th} = 10$  dB. The other system parameters are listed in Table II and Table III. In Fig. 2, we compare the closed-form equation for the outage probability to Monte Carlo simulations to verify its accuracy when varying with modulation schemes, the number of antennas, and transmit

TABLE III COEFFICIENTS  $c_0$  and  $N_c$  for  $\sigma_R^2$ 

$\sigma_R^2$	0-0.54	0.54-0.58	0.58-0.65	0.65-0.9	0.9-2	2-15
$c_0$	3.2	3.3	3.5	4	4.8	5.2
$N_c$	35	34	32	29	26	21



Fig. 2. Outage probabilities versus the transmit power  $P_r$  for  $\mathbf{r}_u = (250, 250, 100)$ m, and  $\mathbf{r}_1 = (600, 600, 0)$ m.

power. We can observe that the analysis result in (18) matches the Monte-Carlo simulation results. However, a slight difference is due to the approximation used for a sum of Nakagami-*m* distribution. The outage probability of linear PAs converges to  $p_{out}^{FSO}$ in (9) with  $\gamma_{th}$ , regardless of modulation schemes. When  $P_r$  is low, the performance of nonlinear PAs is similar to that of linear PAs. However, as the transmit power increases, the end-to-end outage probability of nonlinear PAs is non-monotonic against transmit power due to K and L in (12). Unlike linear PAs, modulation schemes affect the outage probability of nonlinear PAs. Specifically, 16-quadrature amplitude modulation (QAM) modulation is more affected by nonlinear PAs than 16-PSK modulation. Hence, the system with nonlinear PAs needs proper transmit power control and modulation schemes to prevent system outages.

Fig. 3 represents the end-to-end outage probability as a function of varying UAV and MS positions. We consider only one MS with the position as  $r_1 = (x_1, 0, 0)$ m and set the y-axis position of UAV to zero. The UAV's optimal position varies depending on the MS position. Specifically, when the position of x-axis for the MS is 500 m, the outage probability is minimized at the UAV position #1. Otherwise, when  $x_1 = 700$  m and  $x_1 = 900$  m, the UAV position #2 and #3 minimize the outage probability, respectively. Note that we can effectively minimize the outage probability by determining the optimal UAV position depending on the MS position.

In Fig. 4, we represent the end-to-end outage probability at different positions of the UAV for the multiuser scheduling



Fig. 3. Outage probability performance versus the x-axis position of the MS  $x_1$  and the UAV position for  $P_f = 38$  dBm,  $P_r = 31.5$  dBm,  $\theta_{\text{FoV}} = 20$  mrad,  $N_{\text{T}} = 2$  and 16-QAM. The UAV position #1, #2, and #3 are  $r_u = (321, 0, 106)$  m,  $r_u = (427, 0, 83)$  m, and  $r_u = (576, 0, 51)$  m, respectively.



Fig. 4. Outage probability performance versus the x-axis position of the UAV  $x_u$  for  $P_f = 41.5$  dBm,  $P_r = 31.5$  dBm,  $\theta_{\text{FoV}} = 14$  mrad,  $N_{\text{T}} = 2$ ,  $\mathbf{r}_1 = (350, 0, 0)$ m,  $\mathbf{r}_2 = (900, 0, 0)$ m and 16-QAM.

schemes, such as the RR and the ASB cases. The ASB scheduling scheme performs better than the RR scheduling scheme since the ASB scheme utilizes the diversity of multiple MSs in RF links. In the ASB case at  $\mathbf{r}_u = (300, 0, 150)$ m, the end-to-end outage probability is minimized and asymptotically approaches outage lower bound  $a_1$  in (23). Otherwise, the outage probability of the RR case is minimized when the UAV is located at  $\mathbf{r}_u = (600, 0, 150)$ m. Note that the optimal position of the UAV varies depending on the scheduling scheme, implying that determining the UAV position according to the scheduling scheme can significantly improve outage probability performance.

Fig. 5 shows the relationship between the outage probabilities and the SNDR ceilings in (23) with different modulations,



Fig. 5. Outage probability performance versus the SNDR thresholds for  $P_f = 45 \text{ dBm}$ ,  $P_r = 61 \text{ dBm}$ ,  $\theta_{\text{FoV}} = 25 \text{ mrad}$ ,  $\mathbf{r}_u = (100, 100, 100) \text{m}$ , and  $\mathbf{r}_1 = (2000, 2000, 0) \text{m}$ .

i.e., Gaussian signaling, 16-QAM, 64-QAM, and 64-PSK as a function of the threshold  $\gamma_{th}$ . As expected, increasing the number of antennas decreases the outage probability for low  $\gamma_{th}$  regions. When  $N_{\rm T} = 2$ , the outage probability saturates to the outage lower bound  $a_1$  in (23), implying that the dominant factor of end-to-end outage probability is the FSO links. For  $\gamma_{th}$  larger than v, the outage rapidly converges to one except for 64-PSK, regardless of  $N_{\rm T}$ . The SNDR ceilings v vary depending on the modulation schemes. As the number of constellation points increases, the SNDR ceiling decreases, making it difficult to use high-order modulations in high-threshold regions. However, as mentioned in Corollary 1, M-ary PSK modulations are unaffected by the ceiling. This means M-ary PSK modulations are more reliable than other modulations in high-threshold regions.

Figs. 2, 3, and 4 show that the derived closed-form of the outage probability matches the Monte-Carlo simulation. Moreover, optimizing transmit power and 3D placement according to scheduling schemes is necessary to minimize the outage probability of the UAV-aided FSO/RF system. To this end, we propose a joint optimization algorithm, which will be discussed in the following section.

# IV. JOINT 3D PLACEMENT AND TRANSMIT POWER OPTIMIZATION

In this section, we propose an iterative algorithm that jointly optimizes the 3D placement and transmit power in terms of minimizing the derived outage probability. Specifically, the algorithm can support both RR and ABS scheduling schemes by adjusting the parameters. We alternately optimize the transmit power  $P_r$  and 3D placement  $\mathbf{r}_u$  by solving the problems while keeping the other variables fixed.

#### A. Problem Formulation

First, in order to optimize the UAV's 3D placement and transmit power, we formulate the end-to-end outage minimization problem as

$$\min_{\mathbf{r}_u, P_r} \quad p_{\text{out}} \tag{25a}$$

s.t. 
$$E_{\text{UAV}} \le E_{\text{max}}$$
 (25b)

$$P_r \le P_{\lim},$$
 (25c)

where  $E_{\text{UAV}}$  is the total energy consumption of the UAV,  ${}^{1}E_{\text{max}}$  denotes the maximum available energy considering the battery capacity of the UAV, and  $P_{\text{lim}}$  is the transmit power constraint for the UAV. The problem (25) is hard to solve directly since (25) is a non-convex problem due to the non-convex objective function  $p_{\text{out}}$  and the non-convex constraint (25b). To tackle this issue, we solve the original problem (25) by partitioning the entire optimization variables into two blocks for the transmit power and the 3D placement.

# B. Transmit Power Optimization

In this subsection, we convert (25) into the transmit power optimization problem for the fixed UAV's position. Then, as  $p_{out}^{FSO}$  becomes constant, the transmit power optimization problem can be simplified as

$$\min_{P_r} \quad p_{\text{out}}^{\text{RF}} \tag{26a}$$

s.t. 
$$P_r \le P_{\max}, (25c),$$
 (26b)

where  $P_{\text{max}}$  denotes the maximum available battery power and can be obtained by converting the constraint (25b) for  $P_r$ . To solve the non-convex problem (26), we utilize an analytical approach and employ the optimal power of a single MS case for both the RR and the ASB scheduling schemes.<sup>2</sup> As  $\Gamma(a, x)$ is a monotonically decreasing function for  $x \ge 0$ , (26a) can be rewritten for *k*th MS as

$$\operatorname{argmin}_{P_{r}} p_{\operatorname{out},k}^{\operatorname{RF}} = \operatorname{argmax}_{P_{r}} \Gamma\left(M_{k}, \frac{M_{k}\gamma_{th}\sigma_{r}^{2}}{\Omega_{k}g_{k}^{2}\left(K-\gamma_{th}L\right)}\right)$$
$$= \operatorname{argmax}_{P_{r}} \sum_{i=1}^{Q} \sum_{j=1}^{Q} \alpha_{i}\alpha_{j}^{*}S_{i,j}^{SD} P_{a}^{\frac{i+j}{2}}, \qquad (27)$$

where  $S_{i,j}^{SD} = S_{i,j}^S - \gamma_{th} S_{i,j}^D$ . To solve (27), we define  $q(P_a)$  as a derivative of (27) and find solutions  $\hat{P}_a = \min\{P_a \in \mathbb{R}^+ | q(P_a) = 0\}$ .<sup>3</sup> Note that  $\hat{P}_a$  is only affected by nonlinear PAs and modulation schemes, regardless of the UAV's placement. Finally, with updating  $P_{\text{max}}$  at the given UAV's position, we efficiently determine the optimal transmit power considering the constraints (26b) as

$$P_r = \min\left\{N_{\rm T}\hat{P}_a, P_{\rm max}, P_{\rm lim}\right\}.$$
(28)

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Algorithm 1: Proposed Algorithm for the Prob	lem (2	25)	
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1:	Initialize $\mathbf{r}_u \in \mathcal{C}$ and calculate $\hat{P}_a$ .
2:	while $p_{out}$ has not converged <b>do</b>
3:	Determine $P_r$ with (28) for given $\mathbf{r}_u$ .
4:	while $\mathbf{r}_u$ has not converged <b>do</b>
5:	Calculate $\nabla p_{\text{out}}$ with (30) for given $P_r$ .
6:	Update $\mathbf{r}_u$ with (31).
7:	end while
8:	Calculate $p_{\text{out}}(\mathbf{r}_u, P_r)$ with (18).
9:	end while
10:	return $\mathbf{r}_u, P_r$ .

# C. 3D Placement Optimization

Given the fixed transmit power  $P_r$ , we determine the 3D placement of the UAV while minimizing the outage probability. In this circumstance, the optimization problem of the UAV can be formulated as

$$\begin{array}{ll} \min & p_{\text{out}} \\ \mathbf{r}_u \\ \text{s.t.} & (25b). \end{array} \tag{29a}$$

Since (29a) is non-convex with respect to  $\mathbf{r}_u = \{x_u, y_u, z_u\}$ , we alternatively adopt the gradient descent-based algorithm. The gradient of the outage probability  $\nabla p_{\text{out}}$  with respect to  $\mathbf{r}_u$  can be expressed as

$$\nabla p_{\text{out}} = \nabla p_{\text{out}}^{\text{FSO}} + \nabla p_{\text{out}}^{\text{RF}} - p_{\text{out}}^{\text{RF}} \cdot \nabla p_{\text{out}}^{\text{FSO}} - p_{\text{out}}^{\text{FSO}} \cdot \nabla p_{\text{out}}^{\text{RF}},$$
(30)

where  $\nabla p_{out}^{FSO}$  denotes the gradient of the FSO link.<sup>4</sup> In addition,  $\nabla p_{out}^{RF}$  is the gradient of the RF links, which varies with multi-user scheduling schemes.<sup>5</sup> Considering constraint (25b), we update the position  $\mathbf{r}_u$  using the gradient descent technique as follows

$$\mathbf{r}_{u} = P_{\mathcal{C}} \left( \mathbf{r}_{u} - \nabla p_{\text{out}} \right), \tag{31}$$

where  $P_{\mathcal{C}}(x)$  is the projection operator defined as  $P_{\mathcal{C}}(\mathbf{a}) = \operatorname{argmin}_{\{x_a, y_a, v\} \in \mathcal{C}} ||z_a - v||$  and  $\mathcal{C} = \{x, y, z \mid E_{\text{UAV}}(x, y, z) \leq E_{\max}\}$ .<sup>6</sup> For satisfying constraint (25b), we fix the horizontal position and update only  $z_u$  since increasing  $z_u$  typically consumes more energy than moving horizontally.

The overall algorithm for optimizing the 3D placement and transmit power is summarized in Algorithm 1. The algorithm supports both the RR and the ASB scheduling schemes by substituting  $\nabla p_{\text{out}}$  for the 3D placement. The proposed algorithm iteratively works until the outage probability is converged.

<sup>4</sup>Please refer to Appendix F for the derivations of  $\nabla p_{out}^{FSO}$ .

<sup>5</sup>The RF gradient of the outage probability with the RR and ASB schemes are expressed as  $\nabla p_{out}^{RF} = \{d_x^{RF}, d_y^{RF}, d_z^{RF}\}$ , with  $d_j^{RF} = \frac{1}{N_M} \sum_{k=1}^{N_M} d_{k,j}^{RF}$  and  $d_j^{RF} = \sum_{k=1}^{N_M} d_{k,j}^{RF} \prod_{i=1,i\neq k}^{N_M} p_{out,i}^{RF}$  for  $j \in \{x, y, z\}$ , respectively. Please refer to Appendix G for the derivations of  $d_{k,j}^{RF}$ .

<sup>6</sup>To reduce the impact of the gradient value (30) can be adjusted as  $\nabla p_{\text{out}} = \frac{\epsilon_{pl} \nabla p_{\text{out}}}{||\nabla p_{\text{out}}||}$  if  $||\nabla p_{\text{out}}|| < \epsilon_{pl}$ .

<sup>&</sup>lt;sup>1</sup>For the detail definition of  $E_{\text{UAV}}$ , please refer to Appendix C.

<sup>&</sup>lt;sup>2</sup>For detailed proof, please refer to Appendix **D**.

<sup>&</sup>lt;sup>3</sup>For a detailed method for obtaining  $\hat{P}_a$ , please refer to Appendix E.



Fig. 6. Outage probability performance versus  $P_{\text{lim}}$  for  $N_{\text{T}} = 3$ ,  $\mathbf{r}_1 = (800, 500, 0)$ m and  $\mathbf{r}_2 = (300, 500, 0)$ m.

### V. SIMULATION RESULTS

In this section, we present simulation results to show the performance of the algorithm proposed in Section IV for the mixed FSO/RF UAV-aided relaying systems. We consider  $\epsilon_{pl} = 10^{-2}$ and the other simulation parameters listed in Section III-D. For the comparison, we offer several comparable schemes.

- Brute-force search with the EGT beamforming: Bruteforce search finds the minimum outage probability for every position over a 3D lattice with 1-m spacing, and every transmit power with 0.1-dBm spacing. It serves as an optimal baseline in this paper.
- Optimized 3D placement with the RF constraint power (OP-FT): OP-FT denotes the scheme optimizing the 3D placement with the Brute-force search while considering the RF constraint power as the transmit power, i.e.,  $P_r = P_{\text{lim}}$ .
- Fixed 3D placement with the optimized transmit power (FP-OT): FP-OT denotes the scheme optimizing the transmit power with the Brute-force search when UAV's position is fixed as  $\mathbf{r}_u = (200, 200, 100)$ m.

Fig. 6 illustrates the outage probability as a function of the transmit power constraint varying with modulation and multi-user scheduling schemes. We observe that the proposed algorithm performance is comparable to the Brute-force Search, regardless of the modulation type and multi-user scheduling schemes. The OP-FT degrades severely in high  $P_{\text{lim}}$  regions as power control is not adequately performed. With the proposed algorithm, when  $P_{\text{lim}}$  and  $P_f$  increase, the outage probability decreases. However, the outage probability remains constant when  $P_{\text{lim}}$  is larger than 31 dBm for 16-QAM and 34 dBm for 16-PSK since the transmit power is fixed to an optimal value according to the modulation scheme. As mentioned in Corollay 1, 16-PSK modulation performs better in high  $P_{\text{lim}}$  regions since 16-PSK modulation is less affected by nonlinearity.

In Fig. 7, we evaluate the outage probability verses  $N_{\rm T}$ . The results indicate that the proposed algorithm and the brute-force



Fig. 7. Outage probability performance verse  $N_{\rm T}$  for  $P_f = 40$  dBm,  $P_{\rm lim} = 36$  dBm,  $\mathbf{r}_1 = (800, 500, 0)$ m, and  $\mathbf{r}_2 = (300, 500, 0)$ m.



Fig. 8. Optimized 3D placement of the UAV varying with  $N_{\rm T}$  when the RF links modulation is 16-QAM.

search technique have similar outage probabilities for all  $N_{\rm T}$ . In comparison with the RR scheduling schemes, the ASB scheme has a lower outage probability since the ASB scheme serves an MS with the best channel condition. Especially, the ASB scheme for 16-PSK reaches an outage lower bound  $a_1$  of (23) when  $N_{\rm T} = 5$ . Moreover, the FP-OT shows a poor outage probability compared to the proposed algorithm, implying that optimizing the 3D placement of the UAV can further improve the outage probability.

Fig. 8 illustrates the optimized 3D placement of the UAV as a function of  $N_{\rm T}$ , when the proposed algorithm achieves the minimized outage probability as shown in Fig. 7. As increasing the number of antennas improves the performance of both the ASB and RR scenarios for the RF link, the UAV's position is closer to the BS in order to improve the FSO links. In the ABS case, the UAV is positioned close to the MS<sub>2</sub>, which has better channel conditions on average. However, in the RR case,



Fig. 9. Outage probability performance verse  $N_{\rm M}$  for  $N_{\rm T}=3, P_f=40~{\rm dBm},$  and  $P_{\rm lim}=36~{\rm dBm}.$ 

TABLE IV LOCATIONS OF MSS

k	$x_k$	$y_k$	$z_k$	$\mid k$	$x_k$	$y_k$	$z_k$
1	800 m	500 m	0 m	4	600 m	600 m	0 m
2	300 m	500 m	0 m	5	300 m	800 m	0 m
3	800 m	800 m	0 m	6	800 m	700 m	0 m

the UAV is positioned equidistant from all MSs to provide fair support.

The outage probabilities are plotted as a function of the number of MSs in Fig. 9. The positions of MSs are given in Table IV. Regardless of  $N_{\rm M}$ , the outage probability of the proposed algorithm similarly achieves that of the Brute-force search. Similar to Fig. 7, it is noted that the outage probability does not decrease in the FP-OT with both the RR and ASB cases, which results from the outage probability of the FSO link remaining constant. Moreover, regardless of  $N_{\rm M}$  and modulation scheme, the outage probability is saturated in the RR case. However, as increasing  $N_{\rm M}$ , the outage probability of the ASB scheme approaches the lower bound  $a_1$ . This is because the RR scheme utilizes the diversity of multiple MSs.

The outage probability of the proposed algorithm is equivalent to the Brute-force Search for various simulation environments, implying that the algorithm effectively finds the optimal UAV's transmit power and 3D placement. Moreover, the proposed algorithm performs in the RR and ASB cases and can be applied to various multiuser schemes.

#### VI. CONCLUSION

This paper investigated the mixed FSO/RF relaying systems where the UAV serves as a DF relay with nonlinear PAs. Considering the channel characteristics of FSO and RF links, we analyzed closed-form and asymptotic expressions for the outage probabilities of the system where the multiple MSs are served with the RR and ASB scheduling schemes. Moreover, in MISO systems with EGT beamforming, we analyzed the effects of nonlinear PAs and demonstrated the existence of SNDR ceilings determined by modulation schemes. Based on the derived outage probability, we formulated the optimization problem and proposed the algorithm to find the UAV's 3D placement and transmit power. Specifically, we have designed an iterative algorithm that alternately solves the transmit power and the 3D placement. The proposed algorithm found the UAV's 3D placement and transmit power, which achieved the outage probability equivalent to the minimum outage probability. From the proposed algorithm, it is possible to enhance the system performance by fully utilizing the benefits of UAV-aided relaying systems.

# APPENDIX A COEFFICIENTS OF EQUATION (9)

In (9), the coefficients  $c_1$ ,  $c_2$ , and  $c_3$  are defined as  $c_1 = c_3 c_0^{n+\beta_f-\tau} - c_2 c_0^{n+\alpha_f-\tau}, c_2 = c_4 c_6$ , and  $c_3 = c_4 c_5$ , where

$$c_4 = \frac{\pi\tau}{\Gamma(\alpha_f)\Gamma(\beta_f)\sin\left(\pi\left(\alpha_f - \beta_f\right)\right)},\tag{32}$$

$$c_5 = \frac{\left(\alpha_f \beta_f\right)^{n+\beta_f}}{\left(n+\beta_f-\tau\right) \Gamma \left(n-\alpha_f+\beta_f+1\right) n!},\qquad(33)$$

and

$$c_6 = \frac{\left(\alpha_f \beta_f\right)^{n+\alpha_f}}{\left(n+\alpha_f-\tau\right) \Gamma\left(n+\alpha_f-\beta_f+1\right) n!}.$$
 (34)

In addition,  $c_0$  is the coefficient correlated with Rytov variance  $\sigma_R^2$  [23].

# APPENDIX B PROOF OF COROLLARY 2

The SNDR ceiling in (24) can be rewrite as  $\nu = \frac{1}{g(Q)-1}$ , where  $g(Q) = \frac{\Gamma(Q+1)}{\Gamma^2(\frac{Q+3}{2})}$ . The first derivatives of g(Q) is obtained as

$$g'(Q) = \frac{\Gamma(Q+1)}{\Gamma^2\left(\frac{Q+3}{2}\right)} \left(\psi^0(Q+1) - \psi^0\left(\frac{Q+3}{2}\right)\right), \quad (35)$$

where  $\psi^0(x)$  denote the digamma function. Since  $\psi^0(x)$  is strictly increasing on  $(0, \infty)$ ,  $\psi^0(Q+1) \ge \psi^0(\frac{Q+3}{2})$  for  $Q \ge 1$ . Thus, g(Q) is an increasing function for  $Q \ge 1$  (i.e.,  $g'(Q) \ge 0$ ). Moreover, we obtain that  $\nu$  is a decreasing function of g(Q) since  $\nu'(g(Q)) < 0$ . Therefore, as Q increases for  $Q \ge 1$ , g(Q) increases and  $\nu$  decreases. Moreover, when Q approaches infinity, g(Q) tends to infinity and  $\nu$  goes to zero.

# APPENDIX C The Definition of $E_{\text{UAV}}$

 $E_{\text{UAV}}$  is the total energy consumed by the UAV, which is the sum of the energy required for data transmission and the energy consumed by the mechanical parts of the UAV during hovering, upward, downward, and horizontal movement.  $E_{\text{UAV}}$  is defined

as

E

$$UAV = \underbrace{\left(\alpha_{up}(z_u - z_s) + \beta_{up}\right)}_{E_{Upward}} + \underbrace{\left(\alpha_{dn}(z_u - z_s) + \beta_{dn}\right)}_{E_{Downward}} + \underbrace{2\left(\alpha_{hm}\sqrt{(x_u - x_s)^2 + (y_u - y_s)^2} + \beta_{hm}\right)}_{E_{Horizontality}} + \underbrace{\left(\alpha_{ho}(z_u - z_s) + \beta_{ho}\right)T_{UAV}}_{E_{Hovering}} + \underbrace{T_{UAV}P_r}_{E_{Transmission}}$$
(36)

where  $T_{\text{UAV}}$  is the time when UAV provides communication services. Moreover,  $\alpha_{\text{up}}$ ,  $\beta_{\text{up}}$ ,  $\alpha_{\text{hm}}$ ,  $\beta_{\text{hm}}$ ,  $\alpha_{\text{ho}}$ ,  $\beta_{\text{ho}}$ ,  $\alpha_{\text{dn}}$ , and  $\beta_{\text{dn}}$ are the constants related to the mechanical movement of the UAV [35]. The UAV flies from BS and provides communication services for  $T_{\text{UAV}}$  seconds after moving to a location that minimizes the outage probability. After providing the communication service for the predetermined time  $T_{\text{UAV}}$ , the UAV returns to the location of the BS.

# APPENDIX D

#### THE OPTIMAL POWER FOR THE MULTIUSER SCHEMES

For the RR scheduling schemes, we can rewrite the objective function (26a) and introduce inequality as

$$\min_{P_r} \frac{1}{N_{\rm M}} \sum_{k=1}^{N_{\rm M}} p_{{\rm out},k}^{\rm RF} \ge \frac{1}{N_{\rm M}} \sum_{k=1}^{N_{\rm M}} \min_{P_r} p_{{\rm out},k}^{\rm RF}.$$
 (37)

Since equality occurs in (37) when every MS has the same optimal transmit power, i.e.,  $\hat{P}_r = \operatorname{argmin}_{P_r} p_{\operatorname{out},k}^{\operatorname{RF}} \forall k$ , we utilize (28) as the optimal power for the RR case. For the ASB scheduling schemes, the objective function (26a) can be approximated with the incomplete gamma function as

$$\underset{P_r}{\operatorname{argmin}} \prod_{k=1}^{N_{\mathrm{M}}} p_{\mathrm{out},k}^{\mathrm{RF}} \approx \underset{P_r}{\operatorname{argmin}} \prod_{k=1}^{N_{\mathrm{M}}} \left( \frac{A_k}{K - \gamma_{th}L} \right)^{M_k}, \quad (38)$$

where  $A_k = \frac{M_k \gamma_{th} \sigma_r^2}{\Omega_k g_k^2}$ . Since  $A_k$  and  $M_k$  are positive values, the solution of (38) is the same as the optimal power for a single MS in (27).

# $\begin{array}{c} \text{Appendix E} \\ \text{The Detailed Method for Obtaining } \hat{P}_a \end{array}$

The derivative of (27) is obtained as

$$q(P_a) = \sum_{i=1}^{Q} \sum_{j=1}^{Q} \left(\frac{i+j}{2}\right) P_a^{\frac{i+j}{2}-1} \alpha_i \alpha_j^* S_{i,j}^{SD}.$$
 (39)

Since  $q(P_a)$  is complicated by the polynomials with high degrees, we utilize 3rd-degree Taylor series expansion at  $P_a = 0$ as

$$q(P_a) \approx q(0) + q'(0)P_a + \frac{1}{2}q''(0)P_a^2 + \frac{1}{6}q'''(0)P_a^3,$$
 (40)

where

$$q(0) = |\alpha_1|^2 S_{1,1}^{SD},\tag{41}$$

$$q'(0) = 4 \cdot \operatorname{Re}(\alpha_1 \alpha_3^*) S_{1,3}^{SD}, \tag{42}$$

$$q''(0) = 12 \cdot \operatorname{Re}\left(\alpha_1 \alpha_5^*\right) S_{1,5}^{SD} + 6|\alpha_3|^2 S_{3,3}^{SD}, \qquad (43)$$

and

$$q'''(0) = 48 \left( \operatorname{Re} \left( \alpha_1 \alpha_7^* \right) S_{1,7}^{SD} + \operatorname{Re} \left( \alpha_3 \alpha_5^* \right) S_{3,5}^{SD} \right).$$
(44)

The algebraic formula of the polynomial equation can find the solutions of  $q(P_a) = 0$ . Then, we take the real positive solution closest to zero as the solution  $\hat{P}_a$ .

# Appendix F The Derivation of $\nabla p_{\text{out}}^{\text{FSO}}$

The gradient of the FSO link is represented as  $\nabla p_{\text{out}}^{\text{FSO}} = \{d_x^{\text{FSO}}, d_y^{\text{FSO}}, d_z^{\text{FSO}}\}$ . To calculate  $d_x^{\text{FSO}}$  using the chain rule, we can express the gradient of the outage probability  $p_{\text{out}}$  with respect to  $x_u$  as follows

$$d_x^{\text{FSO}} = \frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \bar{\gamma}_f} \frac{\partial \bar{\gamma}_f}{\partial x_u} + \frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \alpha_f} \frac{\partial \alpha_f}{\partial \sigma_R} \frac{\partial \sigma_R}{\partial x_u} + \frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \beta_f} \frac{\partial \beta_f}{\partial \sigma_R} \frac{\partial \sigma_R}{\partial x_u}.$$
(45)

With some mathematical transformation, we can obtain  $\frac{\partial \bar{\gamma}_f}{\partial x_u} = -\xi \bar{\gamma}_f D_f^{-1}(x_u - x_s)$  and  $\frac{\partial \sigma_R}{\partial x_u} = \frac{11}{12} \sigma_R D_f^{-2}(x_u - x_s)$ . Moreover,  $\frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \bar{\gamma}_f}$ ,  $\frac{\partial \alpha_f}{\partial \sigma_R}$ ,  $\frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \beta_f}$ , and  $\frac{\partial \beta_f}{\partial \sigma_R}$  are represented in (50) on the top of the next page, where  $\operatorname{csch}(\cdot)$  and  $\cosh(\cdot)$  denote the hyperbolic cosecant function and the hyperbolic cosine function, respectively. Additionally,  $s_\alpha$ ,  $s_\beta$ ,  $s_2$  and  $s_3$  in (50) are given as

$$s_{\alpha} = c_3 k_{3,\alpha} c_0^{n+\beta_f - \tau} - c_2 c_0^{n+\alpha_f - \tau} \left( k_{2,\alpha} + \log c_0 \right)$$
(46)

$$s_{\beta} = c_3 c_0^{n+\beta_f-\tau} \left(k_{3,\beta} + \log c_0\right) - c_2 k_{2,\beta} c_0^{n+\alpha_f-\tau}, \quad (47)$$

$$s_2 = k_{2,\alpha} + \frac{1}{2} \log \frac{\gamma_{th}}{\bar{\gamma}_f} - \frac{1}{n + \alpha_f},$$
(48)

and

$$s_3 = k_{3,\beta} + \frac{1}{2} \log \frac{\gamma_{th}}{\bar{\gamma}_f} - \frac{1}{n + \beta_f},$$
(49)

respectively. The parameters  $k_{i,\kappa}$  with  $i \in \{2,3\}$  and  $\kappa \in \{\alpha,\beta\}$  are given in (50). By similar approach to  $d_x^{\text{FSO}}$ , we can derive  $d_y^{\text{FSO}}$  with  $\frac{\partial \tilde{\gamma}_f}{\partial y_u} = -\xi \bar{\gamma}_f D_f^{-1}(y_u - y_s)$  and  $\frac{\partial \sigma_R}{\partial y_u} = \frac{11}{12} \sigma_R D_f^{-2}(y_u - y_s)$ . Moreover,  $d_z^{\text{FSO}}$  can be obtained with  $\frac{\partial \tilde{\gamma}_f}{\partial z_u} = -\xi \bar{\gamma}_f D_f^{-1}(z_u - z_s)$  and  $\frac{\partial \sigma_R}{\partial z_u} = \sigma_R(\frac{11}{12}D_f^{-2}(z_u - z_s) - \frac{1}{200})$ .

# APPENDIX G THE DERIVATION OF $d_{k,j}^{\text{RF}}$

Using the multi-variable chain rule, we reveal  $d_{k,x}^{\text{RF}} = \left(\frac{\partial p_{\text{out},k}^{\text{RF}}}{\partial M_k}\frac{\partial M_k}{\partial m_k} + \frac{\partial p_{\text{out},k}^{\text{RF}}}{\partial \Omega_k}\frac{\partial \Omega_k}{\partial m_k}\right)\frac{\partial m_k}{\partial \theta_k}\frac{\partial \theta_k}{\partial x_u} + \frac{\partial p_{\text{out},k}^{\text{RF}}}{\partial D_k}\frac{\partial D_k}{\partial x_u}$ . After some mathematical transformation, we can obtain  $d_{k,x}^{\text{RF}}$  as  $d_{k,x}^{\text{RF}} = s_{d,k}(x_u - x_k)$ , where

$$s_{d,k} = s_{o,k} \left( 2 - \frac{s_{r,k} m_p |z_u - z_k|}{\sqrt{D_k^2 - (z_u - z_k)^2}} \right),$$
(51)

$$\begin{split} \frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \bar{\gamma}_{f}} &= -\frac{(1-a_{1})}{2} \sum_{n=0}^{N_{c}} \left( \frac{c_{1}}{\bar{\gamma}_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{\pi}{2}} + \frac{c_{2}}{\bar{\gamma}_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\alpha_{f}}{2}} - \frac{c_{3}}{\bar{\gamma}_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\beta_{f}}{2}} \right), \\ \frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \alpha_{f}} &= (1-a_{1}) \sum_{n=0}^{N_{c}} \left( \frac{s_{\alpha}}{\tau} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{\pi}{2}} + \frac{c_{2}s_{2}}{n+\alpha_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\alpha_{f}}{2}} - \frac{c_{3}k_{3,\alpha}}{n+\beta_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\beta_{f}}{2}} \right), \\ \frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \beta_{f}} &= (1-a_{1}) \sum_{n=0}^{N_{c}} \left( \frac{s_{\beta}}{\tau} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{\pi}{2}} + \frac{c_{2}k_{2,\beta}}{n+\alpha_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\alpha_{f}}{2}} - \frac{c_{3}s_{3}}{n+\beta_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\beta_{f}}{2}} \right), \\ \frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \sigma_{R}} &= (1-a_{1}) \sum_{n=0}^{N_{c}} \left( \frac{s_{\beta}}{\tau} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{\pi}{2}} + \frac{c_{2}k_{2,\beta}}{n+\alpha_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\alpha_{f}}{2}} - \frac{c_{3}s_{3}}{n+\beta_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\beta_{f}}{2}} \right), \\ \frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \sigma_{R}} &= (1-a_{1}) \sum_{n=0}^{N_{c}} \left( \frac{s_{\beta}}{\pi} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{\pi}{2}} + \frac{c_{2}k_{2,\beta}}{n+\alpha_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\alpha_{f}}{2}} - \frac{c_{3}s_{3}}{n+\beta_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\beta_{f}}{2}} \right), \\ \frac{\partial p_{\text{out}}^{\text{FSO}}}{\partial \sigma_{R}} &= (1-a_{1}) \sum_{n=0}^{N_{c}} \left( \frac{s_{\beta}}{\pi} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{\pi}{2}} + \frac{c_{2}k_{2,\beta}}{n+\alpha_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\alpha_{f}}{2}} - \frac{c_{3}s_{3}}}{n+\beta_{f}} \left( \frac{\gamma_{th}}{\bar{\gamma}_{f}} \right)^{\frac{n+\alpha_{f}}{2}} \right), \\ \frac{\partial p_{0}^{\text{FSO}}}{\partial \sigma_{R}} &= (1-a_{1}) \sum_{n=0}^{N_{c}} \left( \frac{s_{2}}{2,02} \frac{s_{2}}{\sigma_{f}} \left( \frac{s_{2}}{1+1,11} \frac{s_{2}}{\sigma_{f}}^{\frac{13}{2}} \right)^{\frac{n}{2}}}, \\ \frac{\partial p_{1}}^{\frac{n}{2}} \left( \frac{s_{2}}{1+1,11} \frac{s_{2}}{\sigma_{f}}^{\frac{13}{2}} \right)^{\frac{n}{2}}} - \frac{c_{3}s_{3}}}{1-(\cos \left( \left( \frac{s_{1}}{2,0} \frac{s_{1}}{\sigma_{f}} \right)^{\frac{n}{2}} \right), \\ \frac{\partial p_{1}}^{\frac{n}{2}} \left( \frac{s_{2}}{2,0} \frac{s_{1}}{\sigma_{f}} \right)^{\frac{n}{2}} \left( \frac{s_{1}}{1+1,11} \frac{s_{1}}{\sigma_{f}}^{\frac{13}{2}} \right)^{\frac{n}{2}}}, \\ \frac{\partial p_{1}}^{\frac{n}{2}} \left( \frac{s_{2}}{2,0} \frac{s_{1}}{\sigma_{f}$$

with

$$s_{o,k} = \frac{1}{D_k^2 \Gamma(M_k)} \left(\frac{\gamma_{th}}{\bar{\gamma}_k}\right)^{M_k} \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_k}\right), \quad (52)$$

and

$$s_{r,k} = \exp\left(\frac{\gamma_{th}}{\bar{\gamma}_{k}}\right) \left(\log\left(\frac{\gamma_{th}}{\bar{\gamma}_{k}}\right) - \psi^{0}\left(M_{k}+1\right) + 1\right) + \frac{2\left(\Omega_{k}-N_{\mathrm{T}}\omega\right)}{m_{k}^{-1}\Omega_{k}} \left(\psi^{0}\left(m_{k}\right) - \psi^{0}\left(m_{k}+\frac{1}{2}\right) + \frac{1}{2m_{k}}\right).$$
(53)

Note that since calculating  $\frac{\partial p_{\text{out},k}^{\text{RF}}}{\partial M_k}$  is complicated by the upper incomplete gamma function, we utilize  $\frac{\partial p_{\text{out},k}^{\text{RF}}}{\partial M_k} \approx \frac{\partial p_{a,k}^{\text{RF}}}{\partial M_k}$ , where

$$\begin{aligned} \frac{\partial p_{a,k}^{\mathsf{RF}}}{\partial M_k} &= \frac{1}{\Gamma\left(M_k + 1\right)} \\ &\times \left(\log\left(\frac{\gamma_{th}}{\bar{\gamma}_k}\right) - \psi^0\left(M_k + 1\right) + 1\right) \left(\frac{\gamma_{th}}{\bar{\gamma}_k}\right)^{M_k} \end{aligned}$$

with  $\bar{\gamma}_k = \frac{\Omega_k g_k^2 (K - \gamma_{th} L)}{M_k \sigma_r^2}$ . Similar to  $d_{k,x}^{\text{RF}}$ , we can calculate  $d_{k,y}^{\text{RF}} = s_{d,k}(y_u - y_k)$  with the multi-variable chain rule after some mathematical manipulations and simplifications. Moreover, by similar approach to  $d_{k,x}^{\text{RF}}$ , we can calculate  $d_{k,z}^{\text{RF}} = s_{z,k}(z_u - z_k)$  with

$$s_{z,k} = s_{o,k} \left( 2 + \frac{s_{r,k} m_p \sqrt{D_k^2 - (z_u - z_k)^2}}{|z_u - z_k|} \right).$$
(54)

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