Effect of Self-Focusing on Propagation of a Laser Beam in the Atmosphere Along a Slant Path

Liyun Pu, Xiaoqing Li, and Xiaoling Ji

***Abstract*—The equivalent B-integral phase screen (EBPS) method of numerical simulation is proposed to study the propagation of a high-power laser beam in the atmosphere along a slant path, and the computational accuracy and efficiency of the EBPS method are confirmed. It is shown that a decrease of the self-focusing caused by the atmosphere refraction effect. However, the refraction correction for the elevation angle is hardly affected by the self-focusing effect. On the other hand, the analytical expressions of the B integral and the optimal initial beam power of a high-power laser beam propagating in the atmosphere along a slant path are derived. It is found that the optimal beam power and the target maximum intensity decrease as the apparent elevation angle decreases, while the optimal beam power decreases but the target maximum intensity increases as the wavelength decreases. A method to optimize the beam quality on the target is presented.**

***Index Terms*—Atmospheric propagation, slant path, nonlinear self-focusing, equivalent B-integral phase screen (EBPS) method.**

I. Introduction

THE propagation of high-power laser beams from the ground through the atmosphere to space orbits is a topic that is of considerable theoretical interest and practical interest (e.g., laser ablation propulsion's applications in space) [1]. For numerous low-earth-orbit (LEO) debris with cm scales, the ground-based laser space-debris removal may play an important role [1], [2], [3], [4], [5]. The beam quality on the target is significantly affected by the physical features of high-power laser beams propagating through the inhomogeneous atmosphere [6], [7], [8], [9], [10], [11], [12]. The nonlinear self-focusing effect is one of the main physical features because the beam power is well above the self-focusing critical power in the atmosphere. It was shown that the intensity on the debris target decreases due to self-focusing [6], [7], and the self-focusing can be eliminated by applying the initial beam defocusing or phase mask or adaptive optics [6], [7], [8]. Furthermore, the influence of the beam order, the beam spatial coherence and the hard aperture on the self-focusing and the target beam quality were investigated, respectively [9], [10], [11]. Very recently, our group studied the optimal momentum coupling between the ground-based laser impulse and space debris [12]. However, until now, all studies have been carried out only concerning the vertical propagation of high-power laser beams from the ground through the inhomogeneous atmosphere to space orbits [6], [7], [8], [9], [10], [11], [12], and the slant propagation of high-power laser beams in the inhomogeneous atmosphere has not been involved.

A repetitively pulsed laser is focused on a space debris, making a plasma jet, which results in slowing the debris [3], [5], [13]. Hundreds of pulses are needed to do this, but they can be applied during one pass overhead for the small debris [3]. It means that the slant beam pointing will be encountered in practice. However, the propagation behavior of a slant laser beam in the inhomogeneous atmosphere is quite different from that of a vertical one. It is known that the light bending phenomenon will take place due to the atmosphere refraction effect when a laser beam propagates upwards in the inhomogeneous atmosphere along a slant path, and it is necessary to correct the elevation angle because of atmosphere refraction effect. Furthermore, the atmosphere refraction effect is related to the wavelength because the refraction index is a function of the wavelength (i.e., atmospheric dispersion) [14], and the self-focusing effect is also related to the wavelength. Thus, some interesting questions arise: Is the beam self-focusing affected by the atmosphere refraction in a slant path? Does the refraction correction for the elevation angle change due to the self-focusing effect? How is the beam quality affected by the elevation angle and the wavelength? On the other hand, the amount of numerical simulation calculation increases greatly for the slant beam propagation in the atmosphere. It is difficult to perform the numerical simulation by using the uniform phase screen (UPS) method for the beam propagation in the atmosphere along a slant path. Therefore, it is important to propose an optimized numerical simulation method to improve the computational accuracy and efficiency.

In this paper, a numerical simulation method, i.e., the equivalent B-integral phase screen (EBPS) method is proposed to study the propagation of a high-power laser beam in the atmosphere along a slant path, and the computational accuracy and efficiency of the EBPS method are confirmed. Note that the EBPS method is also valid for the vertical propagation of laser beams in the atmosphere because the vertical beam propagation is a special case of the slant beam propagation. On the other hand, the interplay between the atmosphere refraction and self-focusing is studied. Furthermore, the analytical expressions of the B integral and the optimal initial beam power of a high-power laser beam

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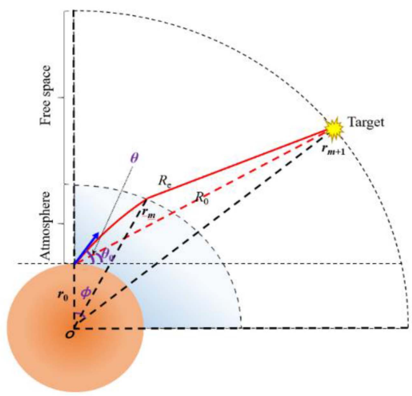

Fig. 1. Schematic depiction of a laser beam propagation in the atmosphere along a slant path. θ: The apparent elevation angle; θ_0: The geometric elevation angle; ϕ: The spherical angle; r_0: Radius of the earth; $r_{\rm m}$ and $r_{\rm m+1}$: The distances from the center of the earth to the points; $R_{\rm e}$: The distance of light sight path; R_0: The geometric distance from the laser source to the target.

propagating in the atmosphere along a slant path are derived. In particular, a method to optimize the beam quality on the target is presented. The main results obtained in this paper are explained in physics.

II. Equivalent B-Integral Phase Screen (EBPS) Method

For the sake of brevity, the principal features are limited to the refraction, nonlinear self-focusing and diffraction in this paper. Fig. 1 is the schematic depiction of a laser beam propagation in the atmosphere along a slant path, in which some parameters are introduced. The light bending will take place because of atmosphere refraction effect in the inhomogeneous atmosphere. Thus, the path of the laser beam is a curve (see the red solid curve in Fig. 1), rather than a straight line (see the red dashed straight line in Fig. 1). It is clear that the apparent elevation angle θ is different from the geometric elevation angle θ_0, and $\varepsilon = \theta - \theta_0$ is called the refraction correction for the elevation angle [15]. Furthermore, ε also depends on atmospheric dispersion because the refraction index is related to the wavelength.

Based on the Cauchy's dispersion formula [14] and the Gladstone-Dale formula [16], the refractive index of the atmosphere is a function of the altitude h and the wavelength λ, and it can be expressed as [17],

$$n(h,\lambda) = \rho(h)\left[28.79\times10^{-5}\left(1+\frac{5.67\times10^{-15}}{\lambda^2}\right)\right]+1, \tag{1}$$

where the atmosphere density ρ is a function of the altitude h, i.e., [18], $\rho(h) = \rho_0 \exp(-h/H_6)$, with ρ_0 being the density at sea level, and $H_6 = 6$ km.

According to (1), the refractive index n decreases as h increases, and it reaches the value 1 as h is large enough (e.g., $h = H_{40} = 40$ km). Thus, we take the laser beam propagation model as, $h = 0 \sim H_{40}$ in the inhomogeneous atmosphere (the first stage), and then $h = H_{40} \sim H_{\rm tar}$ in vacuum (the second stage), where $H_{\rm tar}$ is the altitude of the debris target. In the numerical simulation, the 40 km atmosphere is divided into m layers, and the refractive index of the ith layer is n_i. Within each layer, n_i is regarded as a constant. Thus, the beam refraction takes place at the interfaces of layer to layer. In addition, the second stage of propagation in vacuum is regarded as (m+1)th layer, and n_{m+1} = 1. In this paper, it is called the multi-layer atmosphere model.

Based on the ray theory, one can obtain the integral expressions of ϕ, $R_{\rm e}$, R_0 and θ_0, i.e., [14], [19],

$$\phi = a_1\int_{r_0}^{r_{m+1}}\frac{{\rm d}r}{r\sqrt{n^2r^2-a_1^2}}, \tag{2}$$

$$R_e = \int_{r_0}^{r_{m+1}}\frac{n^2r{\rm d}r}{\sqrt{n^2r^2-a_1^2}}, \tag{3}$$

$$\theta_0 = \arctan\left(\frac{r_{m+1}\cos\phi - r_0}{r_{m+1}\sin\phi}\right), \tag{4}$$

$$R_0 = \sqrt{r_{m+1}^2 - r_0^2\cos^2\theta_0} - r_0\sin\theta_0, \tag{5}$$

where $a_1 = n_1 r_0 \cos\theta$ According to the multi-layer atmosphere model mentioned above, (2) and (3) can be rewritten as the summation form, i.e.,

$$\phi \approx \sum_{i=0}^{m+1}\frac{1}{a_1}\left[\arccos\left(\frac{a_1}{n_i r_{i+1}}\right) - \arccos\left(\frac{a_1}{n_i r_i}\right)\right], \tag{6}$$

$$R_e \approx \sum_{i=0}^{m+1}\left(\sqrt{n_i^2r_{i+1}^2-a_1^2} - \sqrt{n_i^2r_i^2-a_1^2}\right), \tag{7}$$

where r_i is the distance from the center of the earth to the point of each layer. Based on (4) and (6), the refraction correction for the elevation angle ε can be calculated.

On the other hand, for the application of ground-based laser space-debris removal, the self-focusing effect in the atmosphere will take place because the beam power is well above the critical power of self-focusing in the atmosphere, but the filamentation can be avoided when a laser beam with large size propagates through the inhomogeneous atmosphere [6]. The features of the self-focusing and the diffraction can be described by the nonlinear Schödinger equation (NLSE), i.e., [6], [18],

$$2{\rm i}nk_0\frac{\partial A}{\partial z} + \nabla_\perp^2 A + 2nn'\,k_0^2|A|^2A = 0, \tag{8}$$

where $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial/\partial y^2$ is the transverse Laplace operator, $k_0 = 2\pi/\lambda$ is the wave number in the vacuum, A is the electric field, n' is the nonlinear refractive index of the atmosphere. The initial electric field of focused Gaussian beams can be expressed as

$$A(x,y,z=0) = \sqrt{\frac{2P}{\pi w_0^2}}\exp\left[-\left(\frac{1}{w_0^2}+\frac{{\rm i}k_0}{2F}\right)(x^2+y^2)\right], \tag{9}$$

where w_0 is the initial beam width, P is the beam power, F is the focal length of the lens located at $z = 0$ plane, and $F = R_{\rm e}$. In addition, n' is the function of the altitude h [18], and $h = z/\csc\theta$, i.e.,

$$n'(z) = n'(0)\exp\left(-\frac{z}{H_6\csc\theta}\right), \tag{10}$$

where $n'(0) = 4.2 \times 10^{-23}$ m^2/W is the nonlinear refractive index on the ground.

The NLSE (i.e., (8)) can be solved numerically by using the multi-phase screen method [20], [21]. Let $A(x, y, z_j)$ be the solution of (8) at z_j plane, and the solution of (8) at $z_{j+1} = z_j + \Delta z$ plane can be written as [20]

$$A(x,y,z_{j+1}) = \exp\left(-\frac{\mathrm{i}}{4k_0}\Delta z \nabla_\perp^2\right)\exp(-\mathrm{i}s)\exp \times \left(-\frac{\mathrm{i}}{4k_0}\Delta z \nabla_\perp^2\right) A(x,y,z_j), \quad (11)$$

where s is the phase modulation caused by the nonlinear self-focusing effect within Δz propagation. The terms on the right side of (11) denote a free space propagation of the field over a distance $\Delta z/2$, an incrementing of the phase caused by the nonlinear self-focusing, and a free space propagation of the field over a distance $\Delta z/2$, respectively.

When the beam diffraction is ignored, (8) reduces to

$$\mathrm{i}\frac{\partial A}{\partial z} + k_0 n' |A|^2 A = 0. \quad (12)$$

When a laser beam propagates in a nonlinear medium from z_j to $z_j+\Delta z$, the solution of (12) can be derived as

$$A(x,y,z_j+\Delta z) = A(x,y,z_j)\exp\left[\mathrm{i}k_0 n' |A(x,y,z_j)|^2 \Delta z\right]. \quad (13)$$

Noted that (13) is obtained approximately when $|A|^2$ does not change over a propagation distance Δz, i.e., (12) is only an ordinary differential equation. From (13), we can obtain $s = k_0 n' |A(x,y,z_j)|^2 \Delta z$. Thus, the numerical simulation of a high-power laser beam propagating in the atmosphere can be performed by using the multi-phase screen method.

The B integral is an important parameter to represent quantitatively the strength of self-focusing [22]. Furthermore, the nonlinear refractive index of the atmosphere decreases as the altitude h increases. In order to improve the computational accuracy and efficiency, in this paper we propose a method of the equivalent B-integral phase screen (EBPS) to arrange the phase screens, i.e., the value of B integral is the same for each layer, and the interval of each layer increases as the altitude h increases. Based on the definition of the B integral [22], the radial B integral for each layer can be expressed as

$$B(\Delta h_i) = \frac{2k_0 P n'(0)}{\pi w_0^2}\exp\left(-\frac{h_i}{H_6}\right)\Delta h_i, \ (i = 1, 2, 3, \ldots m), \quad (14)$$

where h_i represents the altitude of the ith layer, and Δh_i is the interval of each layer. Assume the value of $B(\Delta h_i)$ is the same constant (i.e., $B(\Delta h_i) = B_{\mathrm{con}}$) for each layer, the values of Δh_i for each layer can be calculated by using (14). It is noted that the multi-layer atmosphere model mentioned above should be arranged by using the EBPS method.

To show the computational accuracy and efficiency of the EBPS method, Fig. 2 shows the results obtained the EBPS method (see the hollow symbol) and by using the UPS method (see the solid symbol) respectively, where N_{s} denotes the number of phase screen. From Fig. 2, one can that the results by using

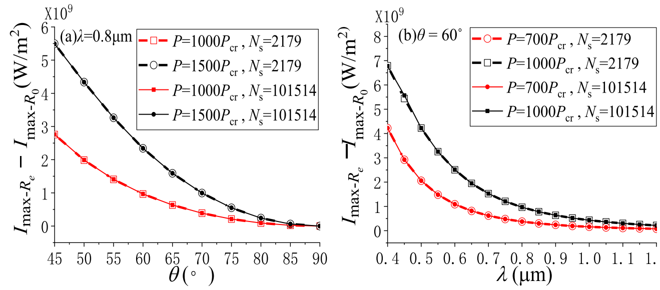

Fig. 2. Difference of the maximum intensity on the target between beam propagation along a curved path (R_{e}) and a straight path (R_0) versus the apparent elevation angle θ (a) and the wavelength λ (b). Hollow symbol: By using the EBPS method; Solid symbol: By using the UPS method.

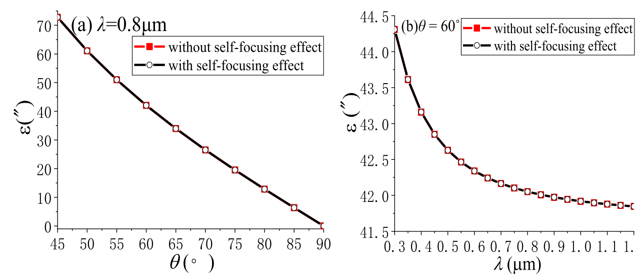

Fig. 3. Refraction correction for the elevation angle ε versus the apparent elevation angle θ (a) and the wavelength λ (b).

the EBPS method when $N_{\mathrm{s}} = 2179$ are well in agreement with those by using the UPS method when $N_{\mathrm{s}} = 101514$. Thus, the computational accuracy is guaranteed by using the EBPS method when $N_{\mathrm{s}} = 2179$, and the calculation time is much less than that by using the UPS method. Therefore, $N_{\mathrm{s}} = 2179$ is adopted in the simulations of Figs. 3–9 to save the calculation time.

Unless otherwise stated, in this paper the calculation parameters are taken as $\lambda = 0.8$ μm, $w_0 = 1.414$ m, $\theta = 60°$, $n_0 \approx 1.0004$, $H_{\mathrm{tar}} = 1000$ km (the space debris is most concentrated near 1000 km [3]), and $P = 1000P_{\mathrm{cr}}$, where P_{cr} is the self-focusing critical power on the ground, and $P_{\mathrm{cr}} = \lambda^2/(2\pi n_0 n'(0))$ = 2.42 GW.

III. Influence of the Refraction on the Self-Focusing

The light path will change due to the atmosphere refraction effect when a high-power laser beam propagates in the atmosphere along a slant path, which results in a change of the strength of self-focusing. In Fig. 2, $I_{\text{max-Re}} - I_{\text{max-R0}}$ is the difference of the maximum intensity on the target between beam propagation along a curved path (i.e., the red solid curve in Fig. 1) and a straight path (i.e., the red dashed straight line in Fig. 1). The $I_{\text{max-Re}} - I_{\text{max-R0}}$ versus the apparent elevation angle θ and the wavelength λ are shown Fig. 2(a) and (b), respectively. One can see that $I_{\text{max-Re}} - I_{\text{max-R0}} > 0$, i.e., the self-focusing effect decreases because of refraction effect. The physical reason is explained as follow. The intensity on the target decreases due to self-focusing [6]. The propagation distance in the atmosphere decreases due to atmosphere refraction effect, and then the phase modulation caused by self-focusing decreases, which results in

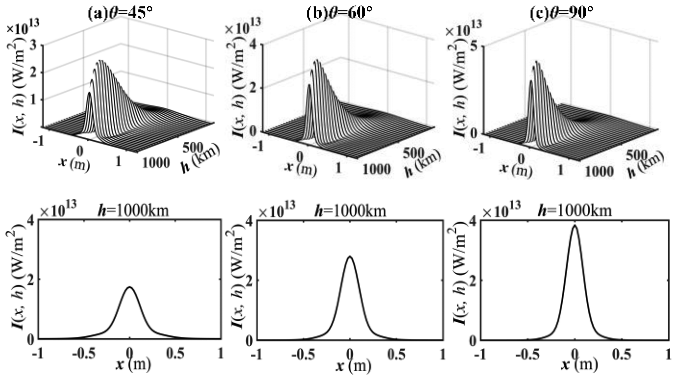

Fig. 4. For different values of the apparent elevation angle θ, the evolution of intensity distributions during beam propagation versus the altitude h, $\lambda = 0.8\ \mu$m.

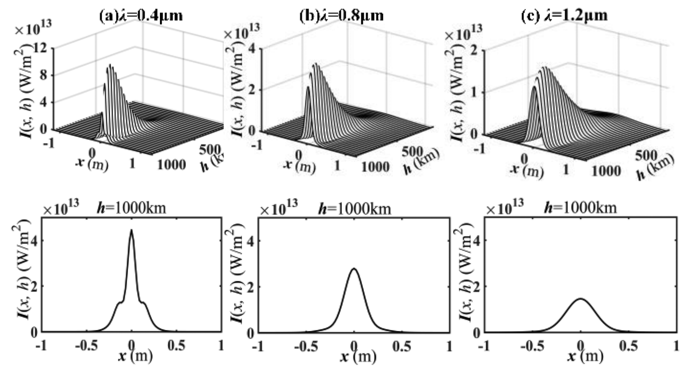

Fig. 5. For different values of the wavelength λ, the evolution of intensity distributions during beam propagation versus the altitude h, $\theta = 60°$.

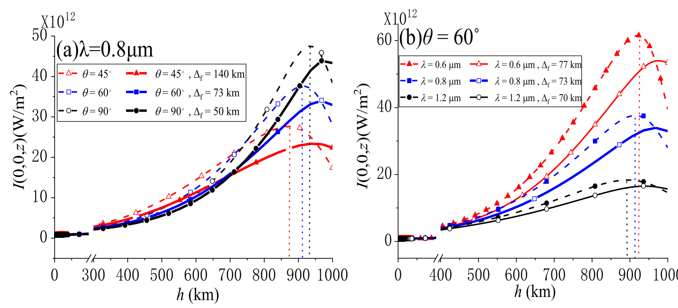

Fig. 6. Change of the on-axis intensity $I(0, 0, z)$ during beam propagation versus the altitude h. (a) for different values of the apparent elevation angle θ; (b) for different values of the wavelength λ. Solid curves: With the focal shift compensation; dashed curves: Without the focal shift compensation.

an increase of the target intensity. Furthermore, Fig. 2 indicates that $I_{\text{max-Re}} - I_{\text{max-R0}}$ increases as θ or λ decreases, i.e., as θ or λ decreases, a decrease of the self-focusing caused by the atmosphere refraction effect will further decrease. In particular, $I_{\text{max-Re}} = I_{\text{max-R0}}$ when $\theta = 90°$, and $I_{\text{max-Re}} \approx I_{\text{max-R0}}$ when the value of λ is large enough (e.g., $\lambda = 1.2\ \mu$m), i.e., a decrease of self-focusing caused by the atmosphere refraction effect can be ignored when $\theta = 90°$ or λ is large enough.

IV. Influence of the Self-Focusing on the Refraction

The refractive index of the atmosphere will change due to nonlinear self-focusing effect. It is worth to study the influence of the self-focusing on the refraction when a high-power laser beam propagates in the atmosphere along a slant path. The changes of the refraction correction for the elevation angle ε versus the apparent elevation angle θ and the wavelength λ are shown Fig. 3(a) and (b) respectively, where two cases with and without self-focusing effect are all considered. For the two cases, ε decreases as θ or λ increases because the atmosphere refraction becomes weak. In particular, the results with self-focusing effect are in agreement with those without self-focusing effect. It means that the refraction correction for the elevation angle ε is hardly affected by the self-focusing effect. The physical reason is that the change of the atmosphere refractive index caused by self-focusing effect is too small to affect the beam refraction, which is proved by our numerical calculation results.

V. Evolution of the Intensity and Compensation of the Focal Shift

For different values of the apparent elevation angle θ and the wavelength λ, the evolution of the intensity distributions during beam propagation versus the altitude h is shown in Figs. 4 and 5 respectively, where the intensity distributions on the target ($h = H_{\text{tar}} = 1000$ km) are also given. During beam propagation, the intensity distributions change, and there exists a maximum of the on-axis intensity, and its position is called the actual focus. It is clear that the actual focus is not located at the target, i.e., the focal shift takes place because of self-focusing and diffraction effects. Furthermore, the intensity distribution on the target cannot remain Gaussian profile when the values of θ and λ are small enough because the self-focusing effect is very strong (e.g., $\theta = 45°$ in Fig. 4(a), and $\lambda = 0.4\ \mu$m in Fig. 5(a)). On the other hand, the target intensity decreases as θ decreases (see Fig. 4), while it increases as λ decreases (see Fig. 5). The physical reason is that the intensity on the target depends not only on the self-focusing effect but also the diffraction effect, and the diffraction becomes weaker as λ decreases.

The intensity on the target can be improved by the compensation of the focal shift, i.e., the actual focus can be moved to the target by initial beam defocusing [6]. Thus, focal length of the lens located at $z = 0$ is modified by $F_{\text{mod}} = R_{\text{e}} + \Delta_f$, where Δ_f is called the compensation parameter of focal shift. For different values of the apparent elevation angle θ and the wavelength λ, the changes of the on-axis intensity $I(0, 0, z)$ during beam propagation versus the altitude h are shown in Fig. 6(a) and (b) respectively, where the solid and dashed curves denote the results with and without the compensation of focal shift, respectively. It is clear that the intensity on the target is improved by the compensation of focal shift, and Δ_f increases as θ and λ decrease.

VI. Optimal Initial Beam Power

For different values of the apparent elevation angle θ and the wavelength λ, the changes of the maximum intensity on the target (i.e., $I_{\max}$) versus the relative beam power P/P_{cr} are shown

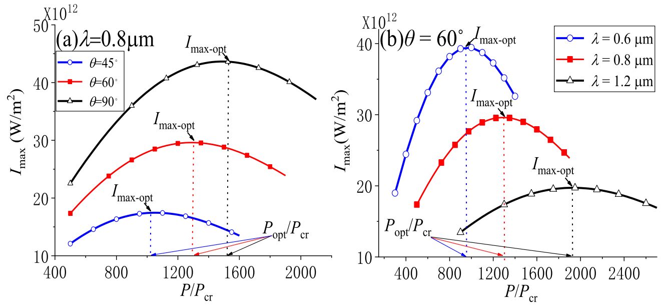

Fig. 7. For different values of the apparent elevation angle θ (a) and the wavelength λ (b), the target maximum intensity $I_{\max}$ versus the relative beam power P/P_{cr}.

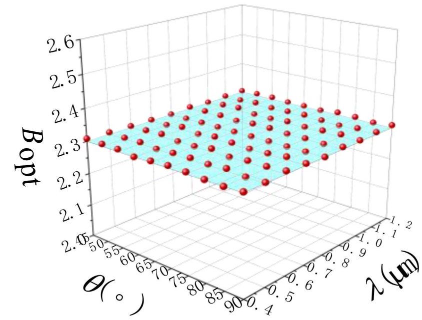

Fig. 8. Optimal B integral (B_{opt}) versus the apparent elevation angle θ and the wavelength λ.

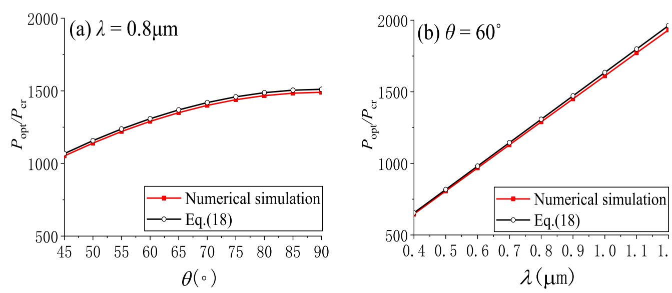

Fig. 9. Confirmation of the analytical formula of optimal initial beam power P_{opt} (i.e., (18)). P_{opt} versus the apparent elevation angle θ (a) and the wavelength λ (b).

in Fig. 7(a) and (b), respectively. One can see that there exists an optimal intensity (i.e., $I_{\text{max-opt}}$) when initial beam power $P = P_{\mathrm{opt}}$, and P_{opt} is called the optimal initial beam power [23]. Furthermore, P_{opt} and $I_{\text{max-opt}}$ decrease as θ decreases, while P_{opt} decreases but $I_{\text{max-opt}}$ increases as λ decreases. These results are caused by the competition between the nonlinear self-focusing effect and the linear diffraction effect. The target intensity increases as P increases when the linear diffraction effect dominants, while it decreases as P increases when the nonlinear self-focusing effect dominants because of focal shift. As θ decreases, the self-focusing effect becomes stronger and so the P_{opt} decreases when the $I_{\text{max-opt}}$ arrives. Furthermore, the $I_{\text{max-opt}}$ also depends on the spot size on the target except for the initial beam power. As θ decreases, the beam spot size on the target increases because the diffraction becomes stronger (see Fig. 4), which results in a decrease of $I_{\text{max-opt}}$. On the other hand, the self-focusing effect becomes stronger as λ decreases. Thus, as λ decreases, the P_{opt} decreases when the $I_{\text{max-opt}}$ arrives. Furthermore, as λ decreases, the beam spot size on the target decreases because the diffraction becomes weaker (see Fig. 5), which results in an increase of $I_{\text{max-opt}}$. Therefore, a high-power laser beam with smaller value of λ is more suitable for laser ablation propulsion's applications in space, if the transmission window of the atmosphere is satisfied. Namely, a method to optimize the beam quality on the target is presented in this paper.

The B integral of a laser beam propagating in the atmosphere along a slant path can be expressed as

$$B = k_0 \int_0^{H_{40}\csc\theta} I(0,0,z) n'(z)\mathrm{d}z. \tag{15}$$

Our numerical results show that the intensity $I(0, 0, z)$ within the altitude $h = H_{40}$ is almost the same as the initial intensity on the ground although the phase is modulated obviously by the self-focusing effect in the atmosphere. Thus, considering (9), (15) can be rewritten as

$$B = \frac{2k_0 P}{\pi w_0^2} \int_0^{H_{40}\csc\theta} n'(z)\mathrm{d}z. \tag{16}$$

On substituting (10) into (16), the analytical expression of B integral of a laser beam propagating in the atmosphere along a slant path can be derived, i.e.,

$$B = \frac{4n'(0)H_6 P\csc\theta}{\lambda w_0^2}\left[1 - \exp\left(-\frac{H_{40}}{H_6}\right)\right]. \tag{17}$$

Equation (17) indicates that the B integral increases as θ or λ decreases, namely, the self-focusing effect becomes stronger as θ or λ decreases.

The influence of self-focusing on the beam quality can be described quantitatively by the B integral. Thus, it is interesting to study the B integral corresponding to the optimal initial beam power P_{opt}, which is called the optimal B integral and is denoted by B_{opt}. The values of B_{opt} versus the apparent elevation angle θ and the wavelength λ can be obtained by using the numerical simulation method (see Fig. 8). One can see that the value of B_{opt} is the same (i.e., $B_{\mathrm{opt}} = 2.3$) for different values of θ and λ. The physical reason is that the deterioration degree of the beam quality is always the same when the optimal initial beam power arrives, although the values of the initial beam parameter or the beam propagation paths are different.

Based on (17) and B_{opt} being a constant, we derive the analytical expression of P_{opt}, i.e.,

$$P_{\mathrm{opt}} = \frac{B_{\mathrm{opt}}\lambda w_0^2}{4n'(0)H_6\csc\theta\left[1 - \exp\left(-H_{40}/H_6\right)\right]}, \tag{18}$$

Equation (18) shows that P_{opt} increases as θ or λ increases.

To confirm the validity of (18), the changes of P_{opt} versus the apparent elevation angle θ and the wavelength λ are shown in Fig. 9(a) and (b), respectively. One can see that the results by using (18) are in agreement with those by using numerical simulation.

VII. Conclusion and Remark

The atmospheric turbulence plays an important role in long range beam propagation. Furthermore, the beam quality decreases greatly due to the atmospheric turbulence, and the beam quality may be too poor to be suitable for laser ablation propulsion's applications in space. However, the strength of turbulence reduces rapidly when the altitude increases. Rubenchik et al. proposed that the turbulence effect can be greatly reduced by placing the laser on a high mountain [6]. Vaseva et al. indicated that for operating system all the detrimental effects must be small and can be evaluated independently [8]. On the other hand, the group-velocity dispersion effect in the atmosphere can be ignored when the pulse width is not too short (e.g., nanosecond pulses) [6]. Therefore, it is reasonable that the principal features are limited to the nonlinear self-focusing, refraction and diffraction in this paper.

In this paper, the EBPS method (a numerical simulation method) is proposed to study the propagation of a high-power laser beam in the atmosphere along a slant path. It is confirmed that the computational accuracy and efficiency of the EBPS method are better than those of the UPS method. It is shown that the self-focusing decreases because of light bending caused by the atmosphere refraction effect. On the other hand, a change of the atmosphere refractive index caused by self-focusing effect is too small to affect the beam refraction. Therefore, the refraction correction for the elevation angle is hardly affected by the self-focusing effect.

In this paper, the analytical expressions of the B integral and the optimal initial beam power P_{opt} of a high-power laser beam propagating in the atmosphere along a slant path are derived. It is shown that the value of B integral increases as the apparent elevation angle θ or wavelength λ decreases. Furthermore, it is found that P_{opt} and the target maximum intensity $I_{\text{max-opt}}$ decrease as θ decreases, while P_{opt} decreases but $I_{\text{max-opt}}$ increases as λ decreases.

In previous researches, it was demonstrated that the intensity on the target decreases due to self-focusing when the value of λ is fixed [6], [7], [8], [9], [10], [11], [12], [23]. However, the beam quality on the target depends on not only the self-focusing effect but also the diffraction effect. As λ decreases, the self-focusing effect becomes stronger, while the diffraction effect becomes weaker greatly. Thus, an interesting result is obtained in this paper, i.e., the better beam quality on the target can be achieved for a high-power laser beam with smaller value of λ. Namely, the self-focusing effect does not play a main role for the beam target quality when the value of λ is small. Therefore, under the condition of proper transmission window of the atmosphere, a high-power laser beam with smaller value of λ is more suitable for laser ablation propulsion's applications in space.

References

[1] C. R. Phipps, "Laser ablation propulsion and its applications in space," in *Advances in the Application of Lasers in Materials Science*. Berlin, Germany: Springer, 2018, pp. 217–246.

[2] J. W. Campbell, "Project ORION: Orbital debris removal using ground-based sensors and lasers," *NASA Tech. Memorandum*, 1996, Art. no. 108522.

[3] C. R. Phipps et al., "Removing orbital debris with pulsed lasers," *Adv. Space Res.*, vol. 49, pp. 1283–1300, 2014.

[4] A. M. Rubenchik, C. P. J. Barty, R. J. Beach, A. C. Erlandson, and J. A. Caird, "Laser systems for orbital debris removal," *AIP Conf. Proc.*, vol. 1278, 2010, Art. no. 347.

[5] B. Esmiller, C. Jacquelard, H. A. Eckel, and E. Wnuk, "Space debris removal by ground-based lasers: Main conclusions of the European project CLEANSPACE," *Appl. Opt.*, vol. 53, no. 31, pp. I45–154, 2014.

[6] A. M. Rubenchik, M. P. Fedoruk, and S. K. Turitsyn, "The effect of self-focusing on laser space-debris cleaning," *Light Sci. Appl.*, vol. 3, no. 4, 2014, Art. no. e159.

[7] A. M. Rubenchik, I. A. Vaseva, M. P. Fedoruk, and S. K. Turitsyn, "Laser space debris cleaning: Elimination of detrimental self-focusing effects," *Opt. Eng. Vol.*, vol. 58, no. 1, 2019, Art. no. 011003.

[8] I. A. Vaseva, M. P. Fedoruk, A. M. Rubenchik, and S. K. Turitsyn, "Light self-focusing in the atmosphere: Thin window model," *Sci. Rep.*, vol. 6, no. 1, 2016, Art. no. 30697.

[9] X. Fan, X. Ji, H. Wang, Y. Deng, and H. Zhang, "Self-focusing effect on the beam quality of Hermite-Gaussian beams propagating upwards through the inhomogeneous atmosphere," *J. Opt. Soc. Amer. A*, vol. 38, no. 2, pp. 168–172, 2021.

[10] Y. Deng, H. Wang, X. Ji, X. Li, H. Yu, and L. Cheng, "Characteristics of high-power partially coherent laser beams propagating upwards in the turbulent atmosphere," *Opt. Exp.*, vol. 28, no. 19, pp. 27927–27939, 2020.

[11] G. Zhang, Y. Deng, and X. Ji, "Hard-aperture effect on self-focusing of high-power laser beams propagating in the atmosphere," *J. Opt. Soc. Amer. A*, vol. 38, no. 9, pp. 1270–1275, 2021.

[12] Y. Deng, X. Ji, X. Li, T. Wang, and H. Yu, "Optimal momentum coupling between the ground-based laser impulse and space debris," *Appl. Phys. B*, vol. 128, 2022, Art. no. 138.

[13] J. T. Early, C. Bibeau, and C. Phipps, "Space debris deorbiting by vaporization impulse using short pulse laser," *AIP Conf. Proc.*, vol. 702, 2004, Art. no. 190.

[14] M. Born and E. Wolf, *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light*. Cambridge, U.K.: Cambridge Univ. Press, 2002,.

[15] L. K. Lin, X. M. Chen, R. R. Hu, and Z. W. Zhao, "The refraction correction of elevation angle for the mean annual global reference atmosphere," *Int. J. Antenna Propag.*, vol. 2020, 2020, Art. no. 2438515.

[16] J. Gladstone and J. Dale, "On the influence of temperature on the refraction of light," *Philos. Trans. Roy. Soc. London*, vol. 148, pp. 887–894, 1858.

[17] D. Gutierrez, F. J. Seron, A. Munoz, and O. Anson, "Simulation of atmospheric phenomena," *Comput. Graph.*, vol. 30, no. 6, pp. 994–1010, 2006.

[18] A. M. Rubenchik, M. P. Fedoruk, and S. K. Turitsyn, "Laser beam self-focusing in the atmosphere," *Phys. Rev. Lett.*, vol. 102, no. 23, 2009, Art. no. 233902.

[19] A. R. Lowry, C. Rocken, S. V. Sokolovskiy, and K. D. Anderson, "Vertical profiling of atmospheric refractivity from ground-based GPS," *Radio Sci.*, vol. 37, no. 3, pp. 1–21, 2002.

[20] G. P. Agrawal, *Nonlinear Fiber Optics*, 2nd ed. New York, NY, USA: Academic, 1995.

[21] T. R. Taha and M. J. Ablowitz, "Analytical and numerical aspects of certain nonlinear evolution equations. II. Numerical, nonlinear Schrodinger equation," *J. Comput. Phys.*, vol. 55, pp. 203–230, 1984.

[22] S. V. Chekalin and V. P. Kandidov, "From self-focusing light beams to femtosecond laser pulse filamentation," *Phys. Uspekhi*, vol. 56, no. 2, pp. 123–140, 2013.

[23] Z. Hang, X. Ji, Y. Deng, X. Li, and T. Wang, "Optimal power of laser beams propagating from the ground through the atmosphere to space orbits," *Opt. Commun.*, vol. 498, 2021, Art. no. 127210.