Consonance of Vibrotactile Chords

Yongjae Yoo, *Student Member, IEEE*, Inwook Hwang, *Student Member, IEEE*, and Seungmoon Choi, *Member, IEEE*

Abstract—This paper is concerned with the perception of complex vibrotactile stimuli in which a few sinusoidal vibrations with different frequencies are superimposed. We begin with an observation that such vibrotactile signals are analogous to musical chords in which multiple notes are played simultaneously. A set of so-called "vibrotactile chords" are designed on the basis of musical chords, and their degrees of consonance (harmony) that participants perceive are evaluated through a perceptual experiment. Experimental results indicate that participants can reliably rate the degrees of consonance of vibrotactile chords and establish a well-defined function that relates the degree of consonance to the base and chordal frequency of a vibrotactile chord. These findings have direct implications for the design of complex vibrotactile signals that can be produced by current wideband actuators such as voice-coil, piezoelectric, and electroactive polymer actuators.

Index Terms—Vibrotactile perception, consonance, superposition, vibrotactile chord, beat

1 Introduction

The past five years have seen rapid progress in the technology of vibrotactile actuators, particularly owing to the need for mobile devices. Small, inexpensive commercial actuators using voice-coil, piezoelectric, or electroactive polymer (EAP) technologies are already available. These high-performance actuators have a wide frequency bandwidth, which can greatly help diversify the vibrotactile stimuli we can create. In contrast, our understanding of the perception of complex vibrotactile stimuli is rather limited, and this makes it difficult to fully exploit the improved performance of vibrotactile actuators.

This paper is concerned with the perception of *superimposed* vibrations. A superimposed vibration is a combination of several sinusoidal vibrations with different frequencies into one signal. Examples are provided in Fig. 1. In our experience, the perceptual impression of superimposed vibrations can be substantially different from that of simple sinusoidal vibrations (see Section 2.1). We address the perceptual characteristics of superimposed vibrations in terms of *consonance*, adapted from a similar concept for music perception. In music, a chord is any set of notes (with different frequencies) that are sounded simultaneously. When a musical chord is perceived as harmonious, its degree of consonance is said to be high. The degree of *dissonance* reflects the opposite concept. We apply the same idea to investigating the perceptual characteristics of superimposed vibrations. If each single frequency component of a multiple-frequency signal is regarded as one note, then the mixed signal can be called a *vibrotactile chord*.

The concept of vibrotactile consonance can be used in ways similar to those in which audio consonance is utilized. For example, in the case of a mobile device, a vibrotactile stimulus with high consonance can be appropriate for a gentle reminder of appointments, while one with high dissonance can be useful for emergency alarm.

The present study addresses the following three fundamental questions about vibrotactile consonance:

- *Q1: Can we reliably evaluate the degree of consonance for vibrotactile chords?*
- *Q2: If the answer to Q1 is positive, does the frequency difference in a vibrotactile chord affect its degree of consonance in systematic manner?*
- *Q3: If the answers to Q1 and Q2 are both positive, can we find a measure that maps the physical parameters of a vibrotactile chord to its degree of consonance?*

To find answers to the above three questions, we conducted a perceptual experiment using 80 vibrotactile dyads (chords consisting of two notes) that were designed based on musical dyads. Forty participants evaluated the degrees of consonance of the 80 vibrotactile chords in a 0-100 scale. A set of adjectives that are suited to describing the perceived attributes of vibrotactile consonance or dissonance were also collected in a post-experimental survey. Further details are presented in the remainder of this paper.[1]

2 Related Work and Background

2.1 Superimposed Vibrations

Vibrotactile signals that include multiple frequency components can form various waveforms. Such superimposed

- *Y. Yoo and S. Choi are with the Haptics and Virtual Reality Laboratory, Department of Computer Science and Engineering, Pohang University of Science and Technology, 77 Cheongam-ro, Nam-gu, Pohang, Gyeongbuk 790-784, Korea. E-mail: {dreamseed, choism}@postech.ac.kr.*
- *I. Hwang is with the Electronics and Telecommunications Research Institute, Gajeong-ro 218, Yuseong-gu, Daejeon City 305-700, Korea.*

Manuscript received 6 Dec. 2012; revised 6 Aug. 2013; accepted 18 Oct. 2013. Date of publication 19 Nov. 2013; date of current version 14 May 2014.
Recommended for acceptance by G. Baud-Bovy.
For information on obtaining reprints of this article, please send e-mail to: reprints@ieee.org, and reference the Digital Object Identifier below.
Digital Object Identifier no. 10.1109/TOH.2013.57

1. Some preliminary results of the present study were presented at the 2011 International Workshop on Haptic and Audio Interaction Design [1]. Major extensions of this paper include a much higher number of participants for Q1 and an extensive data analysis for Q3. We also provide more comprehensive experimental data that could not be reported in [1] due to space constraints.

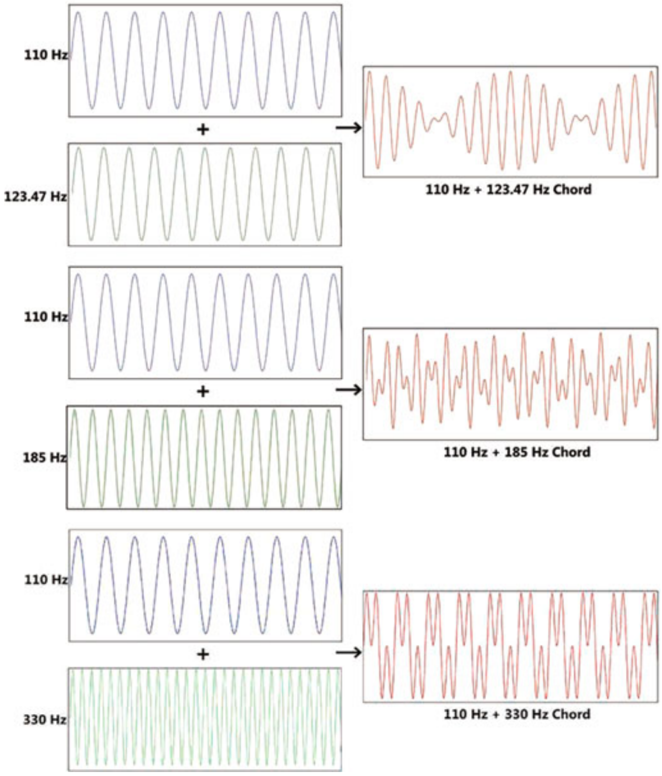


Fig. 1. Examples of superimposed vibrations (vibrotactile chords). (Top) 110 + 123.47 Hz, (Middle) 110 + 185 Hz, and (Bottom) 110 + 330 Hz. Both frequency components in each plot have the same amplitude, and all waveforms are about 0.08 s long.

signals may have a waveform envelope that varies much more slowly than the individual signals, as shown in the top panel of Fig. 1. In contrast, the property of one signal can be dominant over those of the others, as shown in the bottom panel of Fig. 1. The former signal delivers low-frequency pulsatory sensations, while the latter retains the smooth vibrational sensations of the original signals. The signal shown in the middle panel of Fig. 1 is between the two cases. For these reasons, superimposed vibrotactile signals can be perceived to be disparate from single-frequency vibrations. For example, Bensmaïa et al. measured the discriminability between several simple sinusoids and superimposed vibrations and demonstrated that they can be distinguished very well (sensitivity index d' as high as 3.0) [2].

In applications, amplitude-modulated vibrotactile signals are frequently used. They are a special case of signal superposition in which two signals have the same amplitude. Examples that utilize amplitude modulation include tactile aids for the hearing-impaired [3], tactile icons [4], [5], [6], virtual texture rendering through vibrotactile stimuli [7], and button-click sensations for virtual buttons [8]. In addition, our group examined the perceptual space of amplitude-modulated vibrations with a carrier frequency of 150 Hz and modulation frequencies in the range 0-80 Hz [9]. This perceptual space allows us to visualize the perceptual dissimilarities between various amplitude-modulated vibrations, which can contribute to designing tactile stimuli with high discriminability.

Based on the above, we expect that the recent advent of small, inexpensive, wideband actuators will further expedite the use of superimposed vibrotactile stimuli in a variety of applications.

2.2 Musical Chords and Consonance

In Western music, chords are commonly classed by their root note. For instance, the chord C major can be described as a chord of major quality built upon the note C, and it can be either a two-note chord (called an interval or dyad) or a three-note chord (called a triad). An octave consists of 12 (semi-)tones, so 12 different intervals belong to an octave. Each semitone is separated by one key on a piano, and the frequency ratio of two adjacent semitones is always $2^{\frac{1}{12}} : 1$ (about $1.059 : 1$) in the standard tuning. Thus, the frequency of an octave-higher pitch is always twice higher than that of the original pitch.

Perception of musical chords has been extensively studied since the late 19th century. Established theories pertaining to consonance in sound include Helmholtz's theory [10] and the critical band theory [12]. Helmholtz argued that consonance and dissonance in sound are determined by the level of acoustic *beat*[2] [10]. Two simultaneously played sound waves interfere with each other, and the human auditory system perceives them as a single combinational tone. When two pure tones are mixed, their frequency difference creates beats. In Helmholtz's theory, if the beats between the frequency components are so intense that humans hear them as rough and unpleasant, then the chords are regarded as dissonant. Otherwise, they are considered as consonant. This theory can account for the perception of consonance in music to a large extent. However, it is not suitable for a very low or high frequency band where the frequency differences between semitones are too small or large.

In the mid-20th century, the critical band theory was introduced by Plomp and Levelt to fully explain the perception of consonance [12]. According to their theory, a sound pitch determines an area of the basilar membrane that is vibrated in the human cochlea. When the frequency difference between the two pitch components in a chord is small, the two pitches vibrate two nearby areas of the basilar membrane, and the human perceives the chord as dissonant. Otherwise, they vibrate two distant areas and are perceived as consonant. Here, a *critical band* means a frequency band that is mediated by the same region of the human cochlea. If two or more frequency components are within the critical band, they stimulate the same region of the human cochlea, being perceived as dissonant.

There have been several attempts to describe auditory consonance with pairs of adjectives such as beautiful-ugly, euphonious-cacophonous, and pleasant-unpleasant [13], [14]. Smoothness was also frequently used to describe consonance [15], [16]. These studies showed that these adjectives can represent the degree of auditory consonance in consistent manner.

2. A pulsation caused by the coincidence of the amplitudes of two oscillations of unequal frequencies, which has a frequency equal to the difference between the frequencies of the two oscillations (dictionary.com).

Our experimental results are compared later with these theories of auditory consonance.

2.3 Perception of Vibrotactile Pitch

Superimposed vibrations include several different frequency components. Hence, the perception of vibrotactile pitch, which refers to the perceived frequency or rapidity of vibration, is relevant to our study.

An early study of von Bèkèsy [17] showed that vibrotactile pitch increases with vibration frequency, but under the influence of other factors such as duration and amplitude. In his experiment that used a 100-Hz vibrotactile stimulus, longer duration increased the perceived pitch in a logarithmic form, whereas higher amplitude decreased the perceived pitch. He also suggested the possibility of the existence of a few separate neural channels associated with vibrotactile pitch perception.

Later studies attempted to relate vibrotactile pitch to the activation levels of associated mechanoreceptive channels. Using 30- and 150-Hz vibrations, Morley and Rowe showed that vibrotactile pitch is correlated with the activation levels of the rapidly adapting (RA) and PC channels [18]. Based on this, they suggested that temporal coding in the impulse activity of the RA and PC channels could be responsible for pitch perception. Unlike this study, Roy and Hollins presented evidence that a ratio code can be a better candidate [19]. They demonstrated that the loudness of the PC channel normalized by the sum of the loudnesses of the PC, RA, and SA-I channels can be an appropriate descriptor of vibrotactile pitch in a power function form.

In spite of these previous studies, little is known about the pitch perception of superimposed vibrations. It is even unclear whether two frequency components are perceived as two individual pitches or one unified pitch.

2.4 Perception of Vibrotactile Roughness

A large body of research pertaining to the vibrotactile perception of roughness has been carried out, motivated from research interests in the tactile perception of textures. A general consensus is that two types of sensory cues, spatial and temporal cues, enable texture perception [20]. The spatial cues, mediated by the SA-I channel, result from spatial features on the surface and are dominant when the features are relatively coarse (feature size larger than 100 μm [21]). The temporal cues are the vibratory cues that occur when the skin is moved against the surface and thus mediated by the RA and PC channels. They are responsible for the perception of fine features (feature size smaller than 100 μm [21]). Hence, it is accepted that vibratory cues alone can elicit rough sensations, and this has been a subject of recent research. In particular, Bensmaïa and Hollins investigated the viability of temporal and intensive codes for vibrotactile roughness perception [22]. Using a set of surfaces with fine features (spatial periods from 16 to 416 μm), they demonstrated that the Pacinian-weighted power of a stimulus is highly correlated with the subjective judgment of roughness, supporting the intensive code than the frequency code.

Application-driven research on vibratory roughness perception has also emerged owing to the prevalence of vibrotactile displays [23]. This line of research does not assume relevance to texture perception but rather emphasizes the subjective experience of users in HCI applications. For example, Kyung and Kwon showed that perceived roughness increases as vibratory amplitude and frequency are increased using a vibrotactile pin-array [24]. The latter result is not consistent with the general tendency that vibrotactile roughness decreases with frequency. This might have caused from their experimental design: a constant vibrotaction amplitude was used for different frequencies, which effectively increased perceived intensity with frequency. In one of our previous studies [25], a 'bumpy-smooth' adjectival axis was mapped into a 2D perceptual space of sinusoidal vibrations. Bumpy sensations, similar to rough ones, were associated with low-frequency vibrations, while smooth sensations were with high-frequency vibrations. In our recent study [26], we superimposed two sinusoidal vibrations of different frequencies (175 and 210 Hz) with various amplitude combinations and estimated their perceived roughnesses by a magnitude estimation procedure. It was found that perceived roughness increases as the amplitude mixing ratio becomes even.

Fig. 2. Mini-shaker with a mobile device mockup.

At the moment, what relation the perception of vibrotactile consonance and roughness has each other is unclear. We discuss this issue later based on our experimental results (Section 5.4).

3 Methods

This section presents the methods used in our perceptual experiment carried out to answer our three research questions about vibrotactile consonance.

3.1 Participants

Forty participants (28 male and 12 female; 19 to 27 years old with an average 21.57; 38 right-handed, 1 left-handed, and 1 ambidextrous) participated in this experiment. All were native Koreans, and no one reported to have any known somatosensory disorder. Four had considerable experience of haptic devices, while the other 36 had little or none. They were each paid (about 15 USD) after the experiment.

3.2 Apparatus

A mini-shaker (Brüel & Kjær; model 4810) with a power amplifier (Brüel & Kjær; model 2718), shown in Fig. 2, produced all vibrotactile chords used in this experiment. The mini-shaker is a voice-coil actuator with a very wide frequency bandwidth (DC to 18 kHz) and high linearity. A mobile device mockup made of acrylic resin ($11.5 \times 4.5 \times 1.5$ cm; 91.7 g) was attached to the mini-shaker using a screw-type aluminum bracket. Participants grasped the mockup with their left hand to feel the vibrations produced by the shaker. A triaxial accelerometer (Kistler; model 8765C) was attached to the mockup to measure the vibrations. A computer with a data acquisition board (National Instruments; model PCI-6251) controlled the shaker system at a 10 kHz sampling rate. The linear relationships between input voltage amplitude and output vibration amplitude were calibrated for the frequencies between 40 and 330 Hz in 10 Hz steps, following the procedure detailed in [27]. Gains for in-between frequencies were linearly interpolated from the calibrated gains.

3.3 Stimuli

All vibrotactile stimuli used in the experiment were 2 s long. They consisted of two frequency components. Their base frequency was one of 40, 55, 80, and 110 Hz. The frequency of 40 Hz was chosen as a representative of the vibrations mediated by the rapidly adapting channel that give fluttering sensations [28]. Its double, 80 Hz, is an analogy with musical octaves. The frequency 110 Hz is one of the tuning standards for musical pitch (A2) for low-pitched musical instruments, such as the contrabass and tuba. This frequency is also on the boundary at which the role of the PC (Pacinian) channel begins to become dominant in glabrous skin [29]. Lastly, 55 Hz was selected as a half of 110 Hz to constitute a one-octave difference.

In each vibrotactile chord, the chordal frequency was one of the 19 vibrotactile semitones with respect to the base frequency. For example, the chords for the 110 Hz base frequency were the superimposed vibration pairs of $110 + 116.54$ Hz, $110 + 123.47$ Hz, $110 + 130.81$ Hz, $\ldots$, and $110 + 329.63$ Hz. For comparison, the stimulus set also included single-frequency vibrations with the base frequencies and doubled intensities. Thus, a total of 80 (4×20) vibrotactile chords were used in the experiment. For the base frequencies of 40, 55, 80 and 110 Hz, the ranges of the chordal frequencies were 40-119.86, 55-164.81, 80-239.73, and 110-329.63 Hz, respectively.

For each stimulus, the amplitudes of the two frequency components were set so that each resulted in the same perceived intensity. For this purpose, we used a psychophysical magnitude function presented in [30]. This magnitude function was measured with the same shaker under closed-loop control. Each frequency component had a perceived intensity of 10 according to this magnitude function, which allowed clear perception of the superimposed signals.

3.4 Procedure

During the experiment, the participants were seated in front of a computer. They grasped the mobile device mockup with their left hand in a comfortable posture. They also wore headphones that played pink noise to preclude any auditory cue. In each trial, the participant perceived a vibrotactile chord and assessed its degree of consonance using a slider bar displayed on a monitor screen. The participant could perceive the chord repeatedly by clicking a play button. The relative position of the slider bar was mapped to a number in the range 0-100. Two more buttons were provided on the screen, for proceeding to the next trial or going back to the previous trial, respectively.

The experiment consisted of nine sessions, and each session was finished within 5 min. The first session was for training, and it presented 20 vibrotactile chords evenly sampled from the entire 80 chords. During this session, the participants were instructed to establish consistent criteria for assessing the degree of consonance of vibrotactile chords. The data of this session were not used in data analysis. Each of the next eight sessions presented 20 vibrotactile chords. As a result, each participant was tested with the 80 vibrotactile chords twice, resulting in 160 consonance scores. The presentation order of the chords was randomized for each participant. The participants were required to take a 5- min rest between sessions to prevent tactile adaptation and fatigue. The entire procedure took 70-90 min per participant.

In addition, all participants completed pre- and post-experimental surveys. The pre-experimental survey was to collect the demographic information about the participants, such as their gender, handedness, and experience of haptic devices. In the post-experimental survey, the participants were asked to write down the criteria that they had used for rating vibrotactile consonance in a free form. They also assessed the difficulty of establishing the criteria on a five-point Likert scale. Moreover, the participants were provided with two sets of adjectives, and they evaluated the agreement of each adjective to vibrotactile consonance and dissonance, respectively, on a seven-point Likert scale. Some of the adjectives were extracted from [25], which listed a set of adjectives suitable for describing the perceptual impressions of sinusoidal vibrations. The other adjectives were chosen by the experimenter from among those commonly used to account for musical perception. The adjectives for consonance generally had positive meanings, while those for dissonance had negative meanings. These adjectives were presented in Korean (the mother tongue of the participants) to the participants; their English translations are shown in Table 1.

4 Results

4.1 Consistency of Consonance Evaluation

The first objective of this study was to determine whether the participants could assess the consonance of vibrotactile chords with consistency. To this end, we examined an inter-repetition correlation coefficient and an average deviation of the consonance scores of each participant. If the correlation coefficient was less than 0.5 or the average deviation is greater than 20, the participant was regarded as incapable of reliable consonance evaluation. The experimental data of such participants were excluded from further analysis. The histograms of the inter-repetition correlation coefficients and the average deviations are provided in Fig. 3.

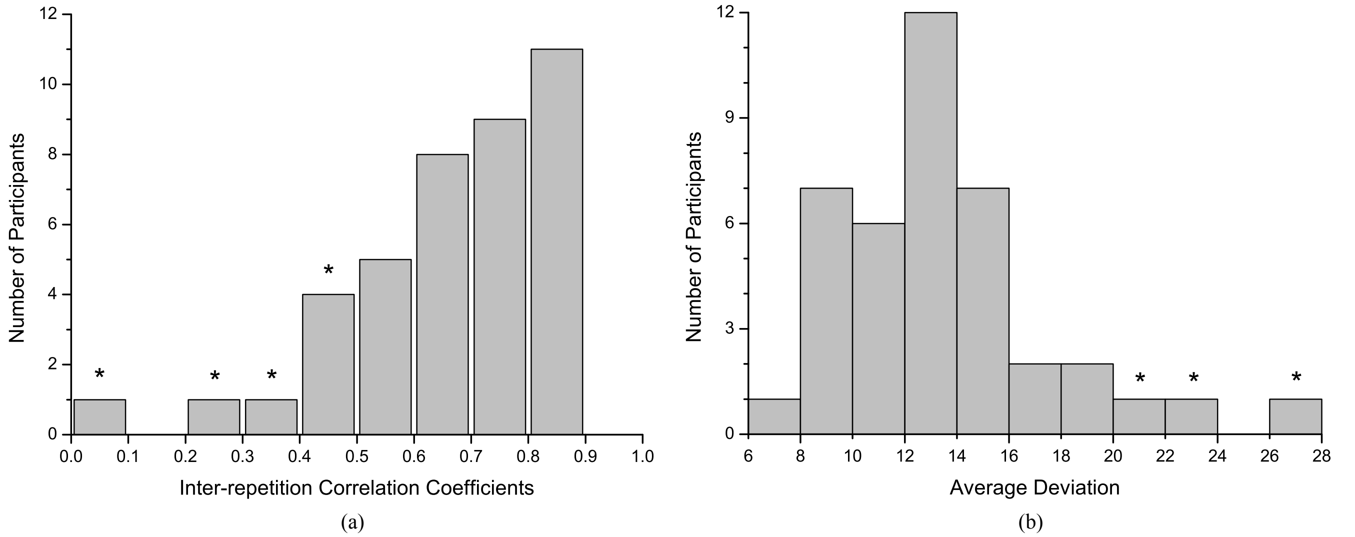


Fig. 3. Histograms of (a) the inter-repetition correlation coefficients and (b) the average deviations. Asterisks indicate data of the seven screened-out participants.

This procedure detected seven participants out of the 40 participants. Three of the seven did not meet either criterion, while the other four satisfied the criterion for the average deviation but not that for the correlation coefficient. Their correlation coefficients and average deviations ranged from 0.021 to 0.485 and from 13.90 to 27.21, respectively.

The participants who satisfied the screening criteria showed much higher consistency in the vibrotactile consonance scores. Their average inter-repetition correlation coefficient was 0.725 (SD 0.103), and their average deviation was 12.28 (SD 2.58). This was also despite the training session that was intentionally made short only using one quarter of the 80 vibrotactile chords used in the main sessions. Therefore, it can be said that 82.5 percent of the participants (33 out of 40) were able to consistently evaluate the degree of consonance.

In addition, the difficulty in establishing consistent criteria had a mean of 2.88 (SD 0.98) on a five-point Likert scale, where score 1 represented “very difficult” and score 5 “very easy.” A histogram of these difficulty scores is shown in Fig. 4. This result suggests that the participants were able to assess the degree of consonance of vibrotactile chords with moderate difficulty.

4.2 Degrees of Consonance

The average degrees of vibrotactile consonance scored by the 33 participants who passed the consistency test are shown in Fig. 5, for the ratio of chordal frequency to base frequency (a) and for the chordal frequency itself (b). Individual standard deviations are not shown for visibility, but they ranged from 9.7 to 20.3 with a mean of 13.8.

Important observations drawn from these plots are as follows. First, the single frequency (base-frequency only) stimuli resulted in very high consonance scores compared with the superimposed stimuli. The maximum degrees of consonance of the superimposed stimuli with the base frequencies of 80 and 110 Hz were comparable to the degrees of consonance of the corresponding single frequency stimuli. On the other hand, for the superimposed vibrations with the base frequencies of 40 and 55 Hz, their maximum degrees of consonance, which appeared at the highest chordal frequencies, were greater than the degrees of consonance of the corresponding single frequency signals. Second, vibrotactile chords with very small frequency ratios (slightly greater than 1.0) were evaluated as the most dissonant, as can be seen in the abrupt drops of both consonance plots. Further increases of the frequency ratio improved the degree of consonance. Third, the increasing behavior of the degree of consonance for the frequency ratios greater than about 2.0 depended on the base frequency. While the degree of consonance continued to increase with the frequency ratio for the 40- and 55-Hz base frequencies, it eventually saturated for the 80- and 110-Hz base frequencies. Lastly, vibrotactile chords with higher base frequencies showed the greater degrees of consonance for the same frequency ratio.

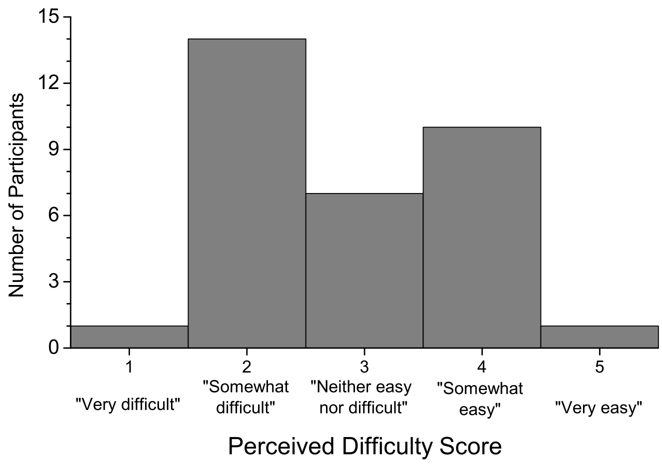
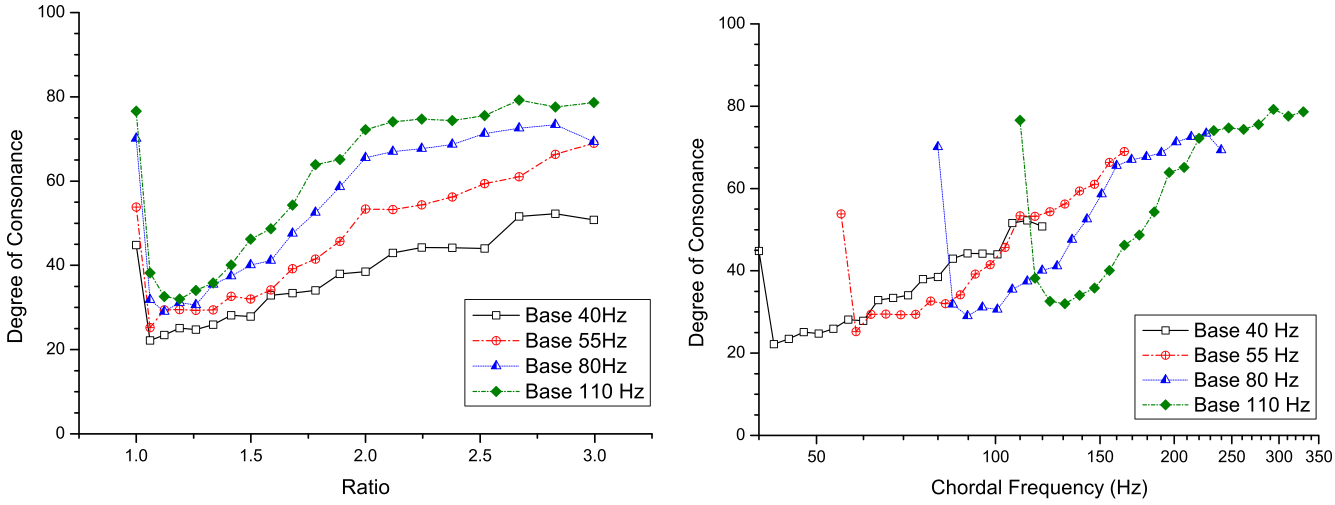


Fig. 4. Histogram of the difficulty in evaluation criteria establishment for vibrotactile consonance.



(a) (b)

Fig. 5. Degree of consonance (a) versus the ratio of chordal frequency to base frequency and (b) versus chordal frequency.

4.3 Model for the Degree of Consonance

Our next analysis was to find a measure function that maps the base and chordal frequencies of a vibrotactile chord to its degree of consonance. To this end, we tested a number of functions of known forms and evaluated their goodness of fit to the collected degrees of consonance. However, we were unable to find a function that accounts for the entire consonance data with a high degree of fit. This was because the consonance data included only a very small number of base and chordal frequencies at which the degree of consonance decreased abruptly with chordal frequency (the dropping regions in Fig. 5). Furthermore, the general shapes of the two regions with decreasing and increasing consonance were extremely asymmetric. Thus, we partitioned the data to two sets, one for the decreasing consonance region and the other for the increasing consonance region, found two best-fitting functions, one for each region, and then connected the two functions together.

The chordal frequencies that belonged to the decreasing consonance region were 40-42.37, 55-58.25, 80-89.72, and 110.0-123.47 Hz for the base frequencies of 40, 55, 80, and 110 Hz, respectively (recall that each pair of the same base and chordal frequencies means a single sinusoidal vibration at that frequency). The other chordal frequencies were included in the decreasing consonance region with each base frequency. Denoting the base and chordal frequencies by f_b and f_c, the boundary between the two regions was determined to be:

$$f_B(f_b) = f_b\, 2^{\frac{1}{12}\lfloor \frac{f_b}{30} \rfloor}, \tag{1}$$

where $\lfloor \cdot \rfloor$ is the floor function. If $f_c < f_B(f_b)$, (f_b, f_c) is in the decreasing consonance region. Otherwise, the point is in the increasing consonance region.

In the decreasing consonance region, we used a power function to take care of the abrupt decreases in the degree of consonance. A measure function for the degree of consonance C in this region was:

$$\begin{aligned} C = {} & K + 15.60 + 0.1678 f_b \\ & + 0.7611\, f_b\, 60.01^{-\frac{10(f_c - f_b)}{f_b} + 0.001570 f_b}, \end{aligned} \tag{2}$$

where $K = \Sigma_{i=0}^{3} \alpha_i {f_b}^i$ with $\alpha_0 = 22.76$, $\alpha_1 = -1.070$, $\alpha_2 = 0.01560$, and $\alpha_3 = 7.014 \times 10^{-5}$. Here, K has the role of equating the values of the two measure functions at their boundary points. This function was suitable for modeling the abrupt drops of the degree of vibrotactile consonance.

To determine a measure in the increasing consonance region, we used a generalized logistic function:

$$C = 61.97 + 0.1434 f_b - \frac{42.99}{1 + \left(\frac{f_c - f_b}{0.4371 f_b}\right)^{0.1175 f_b}}. \tag{3}$$

This generalized logistic model well fitted the relatively slow increases of the degree of consonance with chordal frequency.

A 3D plot of the two measure functions is provided in Fig. 6, where a good match between the consonance data and the measure functions can be seen. The goodness of fit of this model was sufficiently high (Pearson's $R = 0.9929$).

4.4 Post-Experimental Survey

Among the participants' consonance criteria reported in the free-form questionnaire, those frequently mentioned were "smoothness" (10 of 33; 30 percent), "regularity" (21 percent), "perceived as one" (18 percent), "high frequency" (18 percent), "continuity" (15 percent), and "weakness" (15 percent). The criteria for dissonance were associated with "high intensity" (12 of 33; 36 percent), "fluctuation" (24 percent), "discontinuity" (24 percent), "roughness" (21 percent), and "irregularity" (18 percent).

The average relevances of adjectives to vibrotactile consonance and dissonance rated by the participants are shown in Table 1. The sensations described as "pleasant," "smooth," "even," and "clear" were highly relevant to consonance (scores higher than 5 on a 1-7 Likert scale), whereas

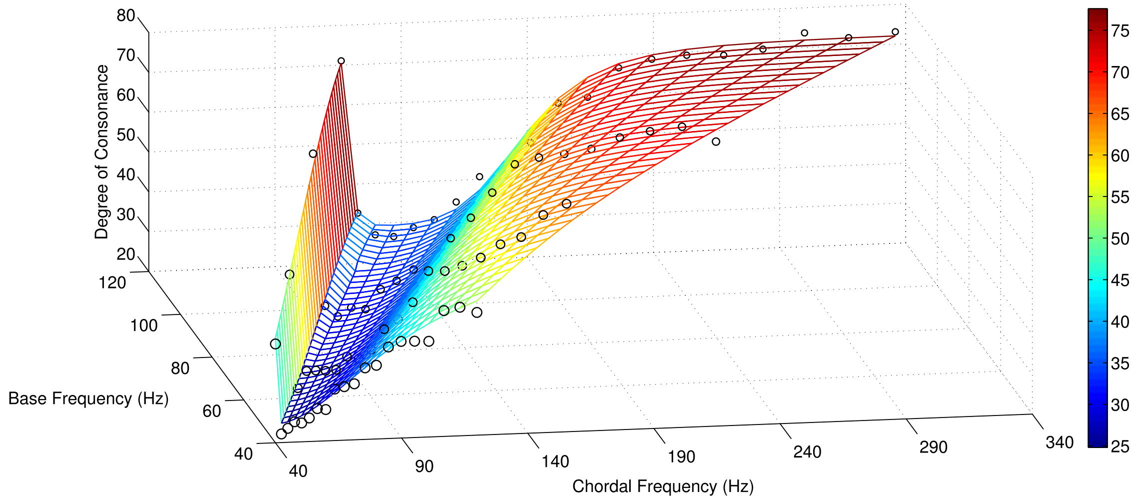


Fig. 6. A 3D plot of the two measure functions (shown connected at the minimum points) obtained from the collected degrees of vibrotactile consonance. The measured points are represented by black open circles.

those described as “jagged,” “bumpy,” “rough,” “sparse,” and “muddy” were closely associated with dissonance. Hence, theses adjectives can be regarded as decision criteria for the degrees of consonance of vibrotactile chords.

4.5 Summary of Results

Based on the experimental results reported above, the three research questions we raised earlier can now be answered.

- Q1: *Can we reliably evaluate the degree of consonance for vibrotactile chords?*
- A1: The answer to Q1 is affirmative to a large extent, supported by the high rate (82.5 percent) of the participants who passed the consistency test (Section 4.1), the moderate perceived difficulty (2.88/5) of consonance assessment (Section 4.1), and the existence of the adjectives highly correlated with the degree of consonance (Section 4.4).
- Q2: *If the answer to Q1 is positive, does the frequency difference in a vibrotactile chord affect its degree of consonance in systematic manner?*
- A2: Fig. 5a shows a well-defined functional relationship between the degree of consonance and the ratio of chordal frequency to base frequency, as detailed earlier in Section 4.2. Hence, the answer to Q2 is also affirmative.
- Q3: *If the answers to Q1 and Q2 are both positive, can we find a measure that maps the physical parameters of a vibrotactile chord to its degree of consonance?*
- A3: The functions given in (2) and (3) provide a compact representation of the degrees of consonance collected in the experiment. However, a unified measure that accounts for the entire data with a high degree of fit was unavailable. Therefore, the answer to Q3 is positive, but to limited extent.

5 Discussion

5.1 Consonance Evaluation

In general, people have relatively little life-time exposure to diverse vibrotactile stimuli with different frequencies, and it is more so for superimposed vibrations. The emphasis of our experiment was on looking into the immediate responses of participants about superimposed vibrations when they were still unfamiliar with such complex vibrations. For this reason, the training session was made brief, presenting only 20 superimposed vibrations out of the total 80 vibrations. Despite the short training session, a high proportion (82.5 percent) of the participants could consistently evaluate the degrees of consonance of the vibrotactile chords. However, the participants considered the task as moderately difficult, as demonstrated in the subjective difficulty scores. Most participants also reported in post-experimental interviews that establishing decision criteria for vibrotactile consonance was not straightforward.

The experimental data of the seven screened-out participants showed three tendencies. First, the consonance scores of four participants were generally similar to those of the other 33 participants who passed the screening test, but their data lacked consistency. It is possible that these four participants could show better agreement in consonance judgments if provided with more training. Second, two other participants resulted in rather erratic consonance scores, wherein no clear patterns could be found. Third, the other one participant showed clear patterns in the consonance judgements, but the patterns were disparate from

TABLE 1
Relevance of Adjectives to Vibrotactile Consonance and Dissonance (on a Scale of 1-7)

Consonance		Dissonance	
Adjective	Score	Adjective	Score
Even	5.788	Jagged	6.424
Smooth	5.636	Bumpy	6.152
Pleasant	5.455	Rough	5.970
Clear	5.424	Sparse	5.670
Clean	4.970	Muddy	5.667
Gentle	4.878	Unpleasant	4.970
Dense	4.697	Dark	3.424
Bright	3.455	Blasphemous	2.607
Beautiful	2.818	Dirty	2.182
Divine	2.091	Ugly	2.121

The adjectives that received high scores greater than 5 are highlighted.

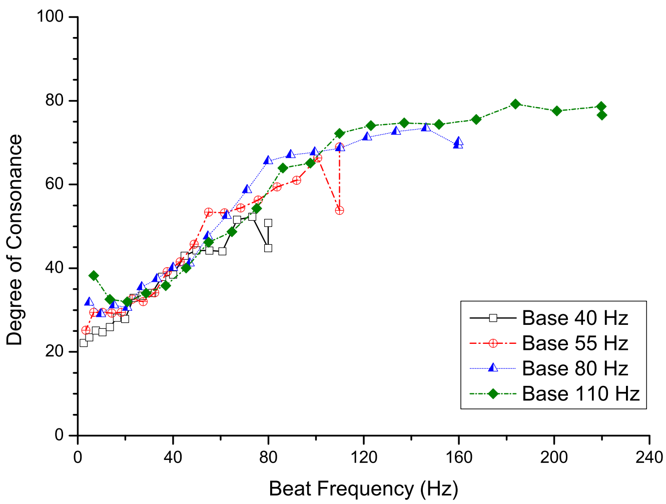


Fig. 7. The degree of consonance versus the beat frequency of vibrotactile chords.

those of the other 37 participants who showed the same patterns. The data of this participant satisfied the average deviation criterion, but the inter-repetition correlation coefficient was 0.49, which was slightly below the threshold (0.5). The reasons for the second- and third-type behaviors are unknown at the moment; we only expect that such behaviors might disappear with further exposure to superimposed vibrations.

5.2 Effects of Base and Chordal Frequencies

Given a base frequency, the degree of consonance produced the lowest score when the frequency ratio was slightly greater than 1.0 (Fig. 5a). The chordal frequencies in this range usually create intense low-frequency beats, which contribute to the fluttering, rough sensations. An example of such waveforms is shown in the top panel of Fig. 1. After this minimum point, the degree of consonance tended to increase monotonically with the chordal frequency. As the chordal frequency increases, the beats become higher in frequency as demonstrated in the bottom panel of Fig. 1. This may explain the consonance improvements for the frequency ratios between 1.0 and 2.0.

When the frequency ratios exceeded 2.0, two patterns were observed. For the two lower base frequencies (40 and 55 Hz), the degree of consonance was apt to increase further with the chordal frequency. These low-frequency components produce very strong pulsating sensations, which might have become further neutralized by the high frequency chordal components that impart smooth vibrational sensations. For the higher base frequencies (80 and 110 Hz) that corresponded to rather smooth vibrations, the degrees of consonance were saturated at high chordal frequencies. This suggests that the sensations from these vibrotactile chords with high base frequencies were already sufficiently smooth, and the use of even higher chordal frequencies did not further affect consonance perception.

5.3 Subjective Descriptions

Based on subjective descriptions, Tan classified sinusoidal vibrations into three groups by frequency [31]. Vibrations between 1 and 3 Hz were described as a slow kinesthetic motion, those between 10 and 70 Hz as a rough motion or a fluttering, and those between 100 and 300 Hz as a smooth vibration. In addition, we have previously shown that the perceptual space of sinusoidal vibrations perceived via a mobile device consists of the two perceptual dimensions that depend on frequency: one for 40-100 Hz and the other for 100-250 Hz [25].

TABLE 2
Correlation (r^2) between the Beat Frequency and the Degree of Consonance

Base Frequency	40 Hz	55 Hz	80 Hz	110 Hz
r^2 (Linear Fit)	0.9279	0.9094	0.8775	0.8550
r^2 (Logistic Fit)	0.9548	0.9455	0.9883	0.9892

The results of the present study appear to be consistent with these earlier findings. Fig. 5a shows that the degree of consonance improved as the base frequency increased from 40 to 110 Hz. The chordal frequencies were varied in the range 42.38-120 Hz for the 40-Hz base frequency and 116.59-330 Hz for the 110-Hz base frequency. The post-experimental surveys (both the questionnaire and the adjective rating) indicated that smooth and pleasant stimuli were considered as highly consonant. This implies that vibrotactile chords sensed as smooth and high-pitch vibrations are regarded as consonant. In contrast, the signs of dissonance seem to be low-frequency, fluttering, pulsatory, rough, and low-pitch sensations.

5.4 Effects of Beat Frequency

Some superimposed vibrations deliver apparent low-frequency beat sensations. To look into this issue further, we performed additional analysis focusing on the beat frequency of superimposed vibrations. Given a superimposed vibration, its beat frequency is the primary frequency of its envelope. It can be found by constructing the envelope using Hilbert transform [32], transforming the envelope into the frequency domain using FFT, and then finding the peak frequency.

A plot that shows the beat frequencies and the degrees of consonance of the vibrotactile chords is presented in Fig. 7. This plot indicates that the degree of consonance increased with the beat frequency almost monotonically. Correlations between them were also very high, as summarized in Table 2. It is also notable that even the degrees of consonance between vibrotactile chords with different base frequencies were very similar if their beat frequencies were close. This is somewhat in agreement with the general guideline for tactile icon design that rhythm, which is determined by the envelope of a vibrotactile stimulus, is the most effective feature for improving the discriminability of the icons [5].

Further, the beat frequencies that corresponded to the neutral consonance score (50) were between 52 and 66 Hz for the four base frequencies. This suggests that if the beat frequency is less than 52 Hz, thereby stimulating the RA channel and eliciting rough, fluttering sensations, then the vibrotactile chord begins to feel dissonant. If the beat frequency is above 66 Hz and the PC channel is more activated, the sensation turns into smooth vibration, increasing the degree of consonance.

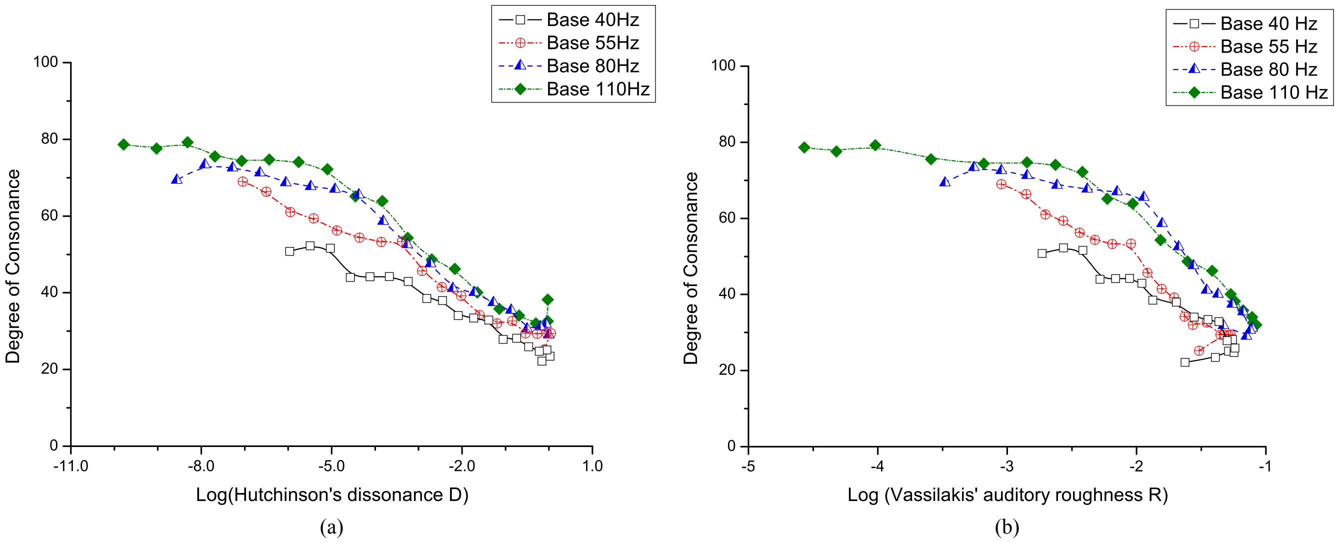


Fig. 8. Degree of vibrotactile consonance versus (a) Hutchinson's dissonance factor and (b) Vassilakis' roughness factor.

This analysis indicated that beat frequency is the dominant sensory cue for consonance perception. The roles of other higher spectral components for consonance perception seem to be far less significant, although they may contribute to other perceptual tasks such as discrimination. This finding reminds us of Helmholtz's theory [10] in which acoustic beat is the central concept; vibrotactile beat may have the same role. This is also in line with the concept of critical band in audio perception in that interference is limited to a range of neighbor frequencies [12].

5.5 Measure for the Degree of Consonance

The measure of the degree of consonance, which consists of the two functions given in (2) and (3), allows for the estimation of the degree of consonance in the parameter range used in our experiment. This measure, however, has two limitations. First, we were not able to find a function that fits to the measured data on the entire frequency range tested. This was mostly due to the fact that only a very small number of data points were available in the narrow region of sharply decreasing degree of consonance. Strengthening this region with more data points may improve the situation. Second, the measure is only for the case in which the two frequency components of vibrotactile chords have the same perceived intensity. A more general measure that can handle arbitrary combinations of different vibratory amplitudes is left as future work.

5.6 Comparison with Auditory Consonance

There exist several computational models for auditory consonance and roughness developed by fitting a function to empirical data. These models are frequently used to analyze audio signals using computers. We noticed that the form of the degree of vibrotactile consonance, when inverted, closely resembles that of the degree of auditory dissonance and roughness. To assess their similarity, we computed Parncutt's approximation of Hutchinson's auditory dissonance D [33] and Vassilakis' auditory roughness R [34] using the frequencies and amplitudes of the vibrotactile chords used in our experiment.

The estimated dissonance D and roughness R of our vibrotactile chords are plotted against the measured degree of vibrotactile consonance in Figs. 8a and 8b, respectively. $\log D$ and $\log R$ are used in the plots as they had reasonably linear relationships with the degree of vibrotactile consonance. Both $\log D$ and $\log R$ were highly correlated with the degree of vibrotactile consonance. The correlation coefficients were -0.942 and -0.874, respectively (note the negative correlations).

This comparison, along with the finding that the effect of beat is similar between auditory and vibrotactile perception (Section 5.4), suggests that noticeable similarity between the two modalities seems to exist in consonance perception. Even the subjective descriptions for consonant/dissonant vibrotactile chords were similar to those of auditory chords. This is despite the disparate neurophysiological mechanisms of vibrotactile and auditory perception, although it was recently reported that crosstalk exists between auditory and tactile stimuli in frequency perception [35]. In this regard, the computational models of auditory consonance and roughness may be clues for finding more general vibrotactile counterparts that account for both vibration frequencies and amplitudes.

5.7 Practical Implications

Our experimental results also have some implications for practical applications. First, vibrotactile chords can be easily rendered using wideband actuators, such as voice-coil, piezoelectric, and electroactive polymer actuators. While these actuators enable designers to use sinusoidal vibrations of various frequencies, superimposed vibrations may provide an alternative means for expressive vibrotactile rendering with well-understood perceptual characteristics. Second, vibrotactile chords can also be produced by narrow-band actuators, e.g., linear resonance actuators (LRAs) popular in commercial smartphones. The actuators can generate vibrations in only a small range of frequencies, but our results indicate that superimposing two vibrations in that frequency range (e.g., those around the resonance frequency of an LRA) suffices to deliver distinctively rough and fluttering sensations with a high degree of dissonance. Since such vibrations feel immediately different from smooth high-frequency vibrations, they can contribute to diversifying the perceptual impression of vibrotactile stimuli and the subsequent design of meaningful vibrotactile signals for information transmission.

6 Conclusions

In this paper, we have investigated the consonance perception of vibrotactile chords, superimposed vibrations with two different frequencies, as a way of studying the perception of complex vibrotactile stimuli that include multiple spectral components. An experiment with a large number of participants (40) demonstrated that people are able to reliably evaluate the degree of consonance of various vibrotactile chords. A well-defined functional relationship was observed between the degree of consonance and vibration frequencies, suggesting that vibrotactile consonance can be a good metric to represent the perceptual characteristics of superimposed vibrations. The beat frequency of the envelope of superimposed vibrations was shown highly relevant to consonance perception. The subjective impressions associated with vibrotactile consonance and dissonance were smooth vibrational feeling and rough, fluttering sensations, respectively. In addition, our analysis indicated that vibrotactile consonance perception is correlated to auditory consonance perception on the basis of the resemblance between their physical measures. All of these results can contribute to broadening our knowledge of the perceptual characteristics of complex vibrations, as well as facilitating the design of diverse vibrotactile stimuli with well-understood perceptual properties.

In the future, we plan to extend this study by investigating the effect of vibration amplitudes on consonance perception and the perception of more complex vibrotactile stimuli that include more than two spectral components. We will also investigate the relationship between vibrotactile consonance and roughness.

Acknowledgments

This work was supported by NRF through a Pioneer Program 2011-0027994, a BRL 2010-0019523, an ITRC Program NIPA-2013-H0301-13-3005 and a grant 2013R1A2A2A01016907. The first author was also supported by a global PhD fellowship program 2011-0008614 from NRF. The authors thank Dr. Hyesu Shin for her advice on the psychological theories of music.

References

[1] Y. Yoo, I. Hwang, and S. Choi, “Consonance Perception of Vibrotactile Chords: A Feasibility Study,” *Proc. Int’l Conf. Haptic and Audio Interaction Design (HAID ’11)*, pp. 42-51, 2011.

[2] S. Bensmaïa, M. Hollins, and J. Yau, “Vibrotactile Intensity and Frequency Information in the Pacinian System: A Psychophysical Model,” *Perception & Psychophysics*, vol. 67, no. 5, pp. 828-841, 2005.

[3] M.H. Goldstein Jr. and A. Proctor, “Tactile Aids for Profoundly Deaf Children,” *J. Acoustical Soc. of Am.*, vol. 77, no. 1, pp. 258-265, 1985.

[4] L.M. Brown, S.A. Brewster, and H.C. Purchase, “A First Investigation into the Effectiveness of Tactons,” *Proc. World Haptics Conf.*, pp. 167-176, 2005.

[5] D. Ternes and K.E. MacLean, “Designing Large Sets of Haptic Icons with Rhythm,” *Proc. Int’l Conf. Haptic: Perception, Devices and Scenarios (EuroHaptics ’08)*, vol. 5024, pp. 199-208, 2008.

[6] E. Hoggan, S. Anwar, and S.A. Brewster, “Mobile Multi-Actuator Tactile Displays,” *Proc. Int’l Conf. Haptic and Audio Interaction Design (HAID ’07)*, vol. 4813, pp. 22-33, 2007.

[7] T. Ahmaniemi, J. Marila, and V. Lantz, “Design of Dynamic Vibrotactile Textures,” *IEEE Trans. Haptics*, vol. 3, no. 4, pp. 245-256, Oct.-Dec. 2010.

[8] G. Park, S. Choi, K. Hwang, S. Kim, J. Sa, and M. Joung, “Tactile Effect Design and Evaluation for Virtual Buttons on a Mobile Device Touchscreen,” *Proc. Int’l Conf. Human-Computer Interaction with Mobile Devices and Services (MobileHCI)*, pp. 11-20, 2011.

[9] G. Park and S. Choi, “Perceptual Space of Amplitude-Modulated Vibrotactile Stimuli,” *Proc. IEEE World Haptics Conf.*, pp. 59-64, 2011.

[10] H. von Helmholtz, *On the Sensations of Tone*. Dover Publications, Inc., 1954.

[11] C. Stumpf, “Konsonanz und Dissonanz,” *Beitrage zur Akustic und Musikwissenschaft*, vol. 7, pp. 1-108, 1898.

[12] R. Plomp and W.J.M. Levelt, “Tonal Consonance and Critical Bandwidth,” *J. Acoustic Soc. of Am.*, vol. 38, pp. 548-560, 1965.

[13] J.P. Van de Geer and W.J.M. Levelt, “The Connotations of Musical Consonance,” *Acta Psychologica*, vol. 20, pp. 308-319, 1962.

[14] C. Valentine, “The Method of Comparisons in Experiments with Musical Intervals and the Effect of Practice on the Appreciation of Discords,” *British J. Psychology*, vol. 7, pp. 118-135, May 1914.

[15] M. Guernsey, “The Role of Consonance and Dissonance in Music,” *Am. J. Psychology*, vol. 40, pp. 173-204, 1928.

[16] C.F. Malmberg, “Perception of Consonance and Dissonance in Music,” *Psychology Monograph*, vol. 3, no. 2, pp. 93-133, 1918.

[17] G. von Bèkèsy, “Synchronism of Neural Discharges and Their Demultiplication in PTCH Perception on the Skin and in Hearing,” *J. Acoustic Soc. of Am.*, vol. 31, pp. 338-349, 1959.

[18] J.W. Morley and M.J. Rowe, “Perceived Pitch of Vibrotactile Stimuli: Effects of Vibration Amplitude, and Implications for Vibration Frequency Coding,” *J. Physiology*, vol. 431, pp. 403-416, 1990.

[19] E.A. Roy and M. Hollins, “A Ratio Code for Vibrotactile Pitch,” *Somatosensory and Motor Research*, vol. 15, no. 2, pp. 134-145, 1998.

[20] M. Hollins and S.R. Risner, “Evidence for the Duplex Theory of Tactile Texture Perception,” *Perception & Psychophysics*, vol. 62, pp. 695-705, 2000.

[21] M. Hollins, S.J. Bensmaïa, and E.A. Roy, “Vibrotaction and Texture Perception,” *Behavioural Brain Research*, vol. 135, pp. 51-56, 2002.

[22] S.J. Bensmaïa and M. Hollins, “The Vibrations of Textures,” *Somatosensory and Motor Research*, vol. 20, no. 1, pp. 33-43, 2003.

[23] S. Choi and K.J. Kuchenbecker, “Vibrotactile Display: Perception, Technology, and Applications,” *Proc. IEEE*, vol. 101, no. 9, pp. 2093-2104, Sept. 2013.

[24] K.-U. Kyung and D. Kwon, “Perceived Roughness and Correlation with Frequency and Amplitude of Vibrotactile Stimuli,” *Proc. Eurohaptics*, pp. 277-282, 2006.

[25] I. Hwang and S. Choi, “Perceptual Space and Adjective Rating of Sinusoidal Vibrations Perceived via Mobile Device,” *Proc. IEEE Haptics Symp.*, pp. 1-8, 2010.

[26] J. Lee and S. Choi, “Real-Time Perception-Level Translation from Audio Signals to Vibrotactile Effects,” *Proc. ACM SIGCHI Conf. Human Factors in Computing Systems (CHI ’13)*, pp. 2567-2576, 2013.

[27] A. Israr, S. Choi, and H.Z. Tan, “Detection Threshold and Mechanical Impedance of the Hand in a Pen-Hold Posture,” *Proc. IEEE/RSJ Int’l Conf. Intelligent Robots and Systems (IROS ’06)*, pp. 472-477, 2006.

[28] L.A. Jones and S.J. Lederman, *Human Hand Function*. Oxford Univ. Press, 2006.

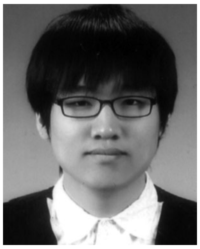
[29] S.J. Bolanowski Jr., G.A. Gesheider, R.T. Verrillo, and C.M. Checkosky, “Four Channels Mediate the Mechanical Aspects of Touch,” *J. Acoustical Soc. Am.*, vol. 84, no. 5, pp. 1680-1694, 1988.

[30] I. Hwang, J. Seo, M. Kim, and S. Choi, “Perceived Intensity of Tool-Transmitted Vibration: Effects of Amplitude and Frequency,” *Proc. IEEE Haptic Audio-Visual Environments and Games (HAVE ’12)*, pp. 1-6, 2012.

[31] H.Z. Tan, “Information Transmission with a Multi-Finger Tactual Display,” PhD dissertation, Massachusetts Inst. of Technology, 1996.

[32] D. Rice, V. Venkatachalam, and M. Wegmann, “A Simple Envelope Detector,” *IEEE Trans. Instrumentation and Measurement*, vol. 37, no. 2, pp. 223-226, June 1998.

[33] R. Parncutt, *Harmony: A Psychoacoustical Approach*. Springer-Verlag, 1989.
[34] P.N. Vassilakis and R.A. Kendall, "Psychoacoustic and Cognitive Aspects of Auditory Roughness: Definitions, Models and Applications," *Proc. SPIE Human Vision and Electronic Imaging*, vol. 7527, article 75270, 2010.
[35] J.M. Yau, J.B. Olenczak, J.F. Dammann, and S.J. Bensmaia, "Temporal Frequency Channels Are Linked across Audition and Touch," *Current Biology*, vol. 19, pp. 561-566, 2009.



Yongjae Yoo received the BS degree in computer science and engineering from the Pohang University of Science and Technology in 2011. He was awarded a Global PhD fellowship program of Korean National Research Foundation in 2011. Since 2011, he has been working toward the PhD degree at the Haptics and Virtual Reality Laboratory, Department of Computer Science and Engineering, Pohang University of Science and Technology. His main research interests include haptic perception, applications and human-computer interactions. He is a student member of the IEEE.

Inwook Hwang received the BS and PhD degrees in computer science and engineering from the Pohang University of Science and Technology (POSTECH) in 2006 and in 2013, respectively. He is currently a researcher at the Electronics and Telecommunications Research Institute (ETRI). His main research interests include haptic perception and haptic feedback in mobile devices. He is a student member of the IEEE.

Seungmoon Choi received the BS and MS degrees in control and instrumentation engineering from Seoul National University in 1995 and 1997, respectively, and the PhD degree in electrical and computer engineering from Purdue University in 2003. He is an associate professor of computer science and engineering at the Pohang University of Science and Technology (POSTECH). He is an associate editor of the *IEEE Transactions on Haptics* and an editorial board member of *Virtual Reality*. He was a co-chair of the IEEE Technical Committee on Haptics in 2009-2011. He received a 2011 Early Career Award from IEEE Technical Committee on Haptics and several best paper awards from premium international conferences. His research interests include haptic rendering and perception, with emphasis on kinesthetic rendering of hardness and texture, tactile rendering, skill modeling and transfer, haptic augmented reality, mobile haptic interface, data haptization, and applied perception. His basic research has been applied to mobile devices, automobiles, virtual prototyping, and motion-based remote controllers. He is a member of the IEEE.