Competitions Among Service Providers in Cloud Computing: A New Economic Model

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***Abstract*—Cloud computing has emerged as a new computing paradigm, with provisioning model generally consisting of cloud service providers (CSPs), network service providers (NSPs), and end users. The associated economics has opened up a new research area; and with the expansion of the cloud computing market, the relationship between CSPs and NSPs, is changing profoundly. In addition to providing the default network services, traditional NSPs, in attempt to compete with CSPs, have started offering cloud services to end users. Though much progress has been made toward addressing competitions among CSPs themselves or among NSPs themselves, few studies have focused on the competition between CSPs and NSPs. In this paper, we investigate the problem of insufficient studies on the competition between CSPs and NSPs by presenting a new economic model to characterize the competition between CSPs and NSPs, and by conducting thorough theoretical analysis as well as numeric experiments to validate the proposed model. We believe, based on results, that the proposed economic model is general and feasible, and thus is applicable to modeling the competition among service providers in cloud computing market.**

***Index Terms*—Cloud computing, Cloud service providers, network service provider, competition, non-cooperative game.**

I. Introduction

CLOUD computing is a recently emerged paradigm in the information technology field that reshapes the way of service management and provisions. Cloud computing is driven by economies of scale, in which a pool of abstracted, virtualized, dynamically scalable computing functions and services are delivered on demand to external customers over the Internet [1]–[3]. A cloud service provisioning model generally consists of three types of participants [4]: cloud service providers (CSPs), network service providers (NSPs), and end users, where CSPs are the suppliers of cloud services, NSPs provide networking facilities or access services to both CSPs and end users, and end users are the consumers of cloud and networking services. In a rough sense, end users submit service requests to CSPs via NSPs, while CSPs lease infrastructures from NSPs and deliver back requested services to end users. As such, customers are able to obtain the cloud services on a "pay-as-you-go" basis [5].

In cloud service provisioning, CSPs and NSPs are usually supposed to be co-dependent [6]. Since the revenue of an NSP mainly comes from CSPs and end users who use its networking services, it would receive a higher profit if CSPs gain more shares in the cloud computing market. With the expansion of the cloud computing market, the relationship between CSPs and NSPs, however, is changing drastically. In addition to providing their default network services, some NSPs start offering cloud services to end users as well. For example, the Verizon Communications Inc. has released Verizon Cloud to provide IaaS-based storage services [7]. Compared with the same storage services offered by Amazon or Rackspace, Verizon Cloud storage services can guarantee the network performance to customers much easier than Amazon and Rackspace through their own Verizon network facilities. Changes like this lead to a new trend in cloud computing market and more complicated interplays between CSPs and NSPs.

When CSPs and NSPs provide mutually *substitutable* cloud services such as storage space or service computing, an NSP, on one hand, may gain a higher revenue if CSPs own more cloud storage market shares; that is, an NSP can make a higher profit as the result of CSPs and end users utilizing more network services. On the other hand, an NSP may lose some of its revenue due to its competition with CSPs that provide the same cloud services as the NSP does. A CSP, for the best interests of itself, however, needs to not only cooperate with NSPs for service delivery, but also compete with them for more cloud market shares. Therefore, interactions and competitions between CSPs and NSPs play a vital role in shaping the cloud computing market, which calls for a new economic model to characterize such a phenomenon.

Although numerous studies have been carried out to model the competitions either among CSPs or among NSPs, few studies have focused on the interactions between CSPs and NSPs. Zhang *et al.* [8] modeled the competition between service providers using Cournot games, the competition between network providers using Bertrand games, and then tied them together in a two-stage Stackelberg game. To our knowledge, their work is the first attempt to address the relationship between service providers and network providers. However, in [8], competitions between service providers and network providers when they offer

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the same type service are not addressed in the context of cloud computing.

In this paper, we use Game Theory to model the competition between CSPs and NSPs. We analyze the elements that will affect the market share and the profit of CSPs and NSPs, and investigate the ensuing Nash equilibrium in the competition. The contributions of this work are summarized as follows.

- We investigate the competition between CSPs and NSPs and develop a game-theoretic model, in which the payoff of a service provider is formulated by the gain that is proportional to its market share and the defined utility.
- We formally prove the existence and the uniqueness of the Nash equilibrium (NE) of the model, based on which we propose an iterative algorithm to compute the NE. The algorithm iteratively updates the service rate of a service provider (either CSP or NSP) and terminates when the currently generated value of service rate is greater than one or the difference of the service rates yielded by the last two consecutive iterations is less than a certain small number.
- We conduct thorough theoretical analysis and numeric experiments to determine the factors such as replacement coefficient, connectivity rate, and service coefficient that will affect the market share and the profit of CSPs and NSPs. In addition, the impacts of cloud service connectivity rate, service rates of CSPs, and service rates of NSPs on the social utility are also examined. We believe, based on results, that the proposed economic model is general and feasible, and thus is applicable to modeling the competition among service providers in cloud computing market.

The remainder of the paper is organized as follows. Section II reviews the related work; Section III presents the system model of the competition using Game Theory; and Section IV addresses the issue of Nash equilibrium of the game. Experimental results are shown in Section V to demonstrate the correlations among various parameters in the competition as well as the validity of the game model. Section VI concludes the paper.

II. Related Work

Network economics has become an active research area, but it is still in its infancy in the context of cloud computing. For example, Feng *et al.* [9] investigated the price competition in a cloud market formed by multiple IaaS cloud providers, and presented an analytical study on monopoly, duopoly, and oligopoly markets where multiple IaaS cloud providers are competing with one another. Roh *et al.* [10] formulated a resource pricing problem in geo-distributed clouds by considering the game-playing between CSPs and application service providers (ASPs), where an ASP acts as a CSP's agent who provides cloud services without constructing its own private data centers. Models that are capable of predicting revenues and utilizations achieved under admission-control policy based revenue management and stochastic demands were developed by Pueschel *et al.* [11]. Their work allows CSPs to significantly increase their revenues by choosing the optimal policy. While Petri *et al.* [12] examined the risks in service-level agreements (SLAs) in service provider communities, Allon and Federgruen [13] studied the scenario in which multiple service providers compete for customers using various price- and time-guarantee strategies. Cohen and Echabbi [14] proposed a price determination mechanism for CSPs by using the cooperative game model, which shows the potential that a shared profit increase may encourage the CSPs to collaborate with one another. Moon *et al.* [15] studied the SLA-based resource allocation problem for CSPs by using Game Theory and seeking the ensuing Nash equilibrium point. Similarly, Do *et al.* [16] examined the issue of resource allocation for two (or more) multimedia service providers, aiming to maximize the service providers' profits.

Also, Wei *et al.* [17] studied the coordination of cloud service supply chains in a duopoly market toward improving the overall operation efficiency; and Rahimi *et al.* [18] conducted a survey of the mobile cloud computing, which integrates cloud computing with smart mobile devices and turns out to be one of the currently most popular research topics. A Nash bargaining was presented by Feng *et al.* [19] to maximize the resource utilization in the context of video streaming. While Xu and Li [20] attempted to find the optimal pricing strategy for cloud service providers by using a revenue management framework, Mihailescu and Teo [21] proposed a dynamic pricing mechanism to more efficiently allocate the shared resources, and evaluated the proposed work by theoretical analyses and simulation experiments. Furthermore, a game-theoretic approach to resource allocation in cloud computing was studied in [22] by Srinivasa *et al.*, which exhibits that the issue of resource supply and allocation should be dealt with by considering the maximization of utilities.

Given the extensive studies on network economics and cloud computing in the literature, we notice that most of them only consider the competitions among CSPs themselves or among NSPs themselves [9]–[22]. The competition between CSPs and NSPs is, however, substantially less addressed and essentially overlooked. The first attempt aiming to relate service providers and network providers occurs in [8] where the competition between CSPs and NSPs is, however, not studied in the context of cloud computing. In our work, we address such competitions by developing a non-cooperative game model, investigate the impacts exerted by the internal and external elements on market shares and profits of CSPs and NSPs, and analyze the ensuing Nash equilibrium of the developed game model. Note, in particular, that our work is different from [9]. While the study in [9] primarily focuses on the competition among CSPs and its objective is to find the optimal price, our work deals with the competitions between CSPs and NSPs and our objective is to find the optimal service rates of service providers. Also, the modeling mechanism in our work is more comprehensive than that of [9] in the sense that more parameters and different formulations are used in our work.

III. System Model

We in this section describe the model of the system. Throughout the paper, we make the following assumptions.

- There are only two types of service providers in the cloud service market: CSPs that offer cloud services merely, and NSPs that offer both cloud services and network services.
- The set of CPSs and NSPs in the market is the set of players in the game model, and the strategy space of each player is the set of attainable values of the service rate of that player.
- Each user can only choose one service provider: either a CSP or an NSP.
- Service providers can always meet the marketing demands.
- The cloud services offered by CSPs and NSPs are of the same network performance due to the network neutrality [23].
- There exists a minimum value in terms of user utility. If the service rendered to a user by a service provider fails to generate a user utility larger than or equal to this minimum value, then the user will no longer choose this service provider.

Both CSPs and NSPs are configured with an *M*/*M*/1 queue [9], serving a common pool of potential cloud users with one "super" server. We use $u_k^{c,i}, u_k^{n,j}$ to denote the service rates (i.e., the ability of a service provider to deal with users' requests) of CSP *i* and NSP *j* in the *k*-th round of the competition, respectively. Note that the service rate of a service provider relies on its resource capacity; a stronger resource capacity would certainly lead to a higher service rate. Therefore CSPs and NSPs usually have different service rates. Considering the randomness of user service request arrivals in cloud service systems and the characteristics of cloud service delivery, the arrival rate of user requests can be statistically regarded as a Poisson distribution [24], [25]. We use λ_m to represent the amount of information that needs to be processed for an arbitrary user *m*. Assuming the total number of users is *M*, then the sum of all users' service requests equals to the total cloud service market size; that is, $\Lambda = \sum_{m=1}^{M} \lambda_m$. Then by [9], the service time $t_k^{c,i}$ of CSP *i* and the service time $t_k^{n,j}$ of NSP *j* for user *m* in the *k*-th round of the competition would be

$$\begin{cases} t_k^{c,i} = \frac{\lambda_m}{q \cdot u_k^{c,i}} \\ t_k^{n,j} = \frac{\lambda_m}{q \cdot u_k^{n,j}}, \end{cases} \tag{1}$$

where q is the connectivity rate indicting the probability of users' service requests being successfully processed by the service provider. Note that the connectivity rate depends on the quality and availability of the communication network infrastructure — a higher quality communication network infrastructure will give rise to a higher connectivity rate, and that CSPs need to use the network facilities provided by NSPs to serve the users. We assume, without loss of generality, that the connectivity rates offered by CSPs and NSPs are equal to signify the idea that they have equal technical competing powers in this respect. The experience values $Q_k^{c,i}(\lambda_m)$ and $Q_k^{n,j}(\lambda_m)$ of a cloud service user *m* with CSP *i* or NSP *j*, in the *k*-th round of the competition, can be formulated as follows

$$\begin{cases} Q_k^{c,i}(\lambda_m) = g\lambda_m - pt_k^{c,i} \\ Q_k^{n,j}(\lambda_m) = g\lambda_m - pt_k^{n,j}, \end{cases} \tag{2}$$

where $g > 0$ is the benefit factor of the user utility, and $p > 0$ is the time cost factor of the user utility. Note that formula (2) reveals the fact that the experience value of a user with a service provider is closely related to the service time – a shorter service time means a higher service efficiency and a higher (i.e., better) user experience value. For the convenience of references, all notations used in this paper together with their definitions are listed in Table I.

A service provider will charge a user for the services it provides. If the price charged by a service user, in the *k*-th round of the competition, is *P*, and the service request amount of user *m* is λ_m, then the fee that the user needs to pay to the service provider would be $P\lambda_m$. For user *m*, we define its utilities $\mathcal{U}_k^{c,i}(\lambda_m)$ and $\mathcal{U}_k^{n,j}(\lambda_m)$, when being served by CSP *i* or NSP *j* in the *k*-th round of the competition, to be

$$\begin{cases} \mathcal{U}_k^{c,i}(\lambda_m) = Q_k^{c,i}(\lambda_m) - P^c \cdot \lambda_m \\ \mathcal{U}_k^{n,j}(\lambda_m) = Q_k^{n,j}(\lambda_m) - P^n \cdot \lambda_m, \end{cases} \tag{3}$$

where P^c and P^n are the service prices charged by CSPs and NSPs, respectively. The scenario considered in this paper is that the service provided by any one CSP (or NSP respectively) can be completely replaced by that of any other CSPs (or NSPs respectively), and partially replaced by any NSPs (or CSPs respectively). So, here, we use P^c and P^n to denote the price charged by any CSPs and any NSPs, respectively.

Users are an indispensable part of the cloud service market. User utility is the direct indication of the competence of a service provider in the market in the sense that a higher user utility for a service provider would likely attract more users to this service provider. In our work, we stipulate that the expected minimal value for user utilities is *v*. If a user's utility (with respect to a service provider) is lower than *v*, then this user would choose to be served by other service providers rather than its current service provider. Thus, the condition for a user *m* to choose a service provider to do business is

$$\begin{cases} \mathcal{U}_k^{c,i}(\lambda_m) = g\lambda_m - \frac{p\lambda_m}{qu_k^{c,i}} - P^c \cdot \lambda_m \geq v \\ \mathcal{U}_k^{n,j}(\lambda_m) = g\lambda_m - \frac{p\lambda_m}{qu_k^{n,j}} - P^n \cdot \lambda_m \geq v. \end{cases} \tag{4}$$

Pricing also plays an important role in the competition of service providers, and alterations of prices may affect user utilities and thus the market shares of service providers. It can be derived from formula (4) that the service rates $u_k^{c,i}, u_k^{n,j}$ must satisfy the following conditions

$$\begin{cases} \frac{p}{q\left[g - P^c - \frac{v}{\lambda_m}\right]} \leq u_k^{c,i} \\ \frac{p}{q\left[g - P^n - \frac{v}{\lambda_m}\right]} \leq u_k^{n,j}. \end{cases} \tag{5}$$

In other words, if the service rate of a service provider in the *k*-th round of the competition does not meet the condition specified in formula (5), then this service provider will no longer be able to meet the demands of users and will be rejected by the users.

TABLE I
NOTATIONS AND THEIR DEFINITIONS

Notation	Definition
$u_k^{c,i}, u_k^{n,j}$	The service rates of CSP i and NSP j in the k-th round of the competition.
$(u^{c,i})^*, (u^{n,j})^*$	The optimal service rates of CSP i and NSP j.
$f_k^{c,i}, f_k^{n,j}$	The market shares of CSP i and NSP j in the k-th round of the competition.
λ_m	The service request amount of user m.
Λ	The market size.
$t_k^{c,i}, t_k^{n,j}$	The expected time of processing the user request λ_m at CSP i or NSP j in the k-th round of the competition.
q	The connectivity rate of cloud service delivery network.
$\mathcal{Q}_k^{c,i}(\lambda_m), \mathcal{Q}_k^{n,j}(\lambda_m)$	The experience values of cloud userwhen being served by CSP i or NSP j in the k-th round of the competition.
g	The benefit factor of cloud service users.
p	The time cost factor of cloud users' utility.
$\mathcal{U}_k^{c,i}(\lambda_m), \mathcal{U}_k^{n,j}(\lambda_m)$	The utility of user m when being served by CSP i or NSP j in the k-th round of the competition.
P^c, P^n	The prices charged by all CSPs and all NSPs.
M	The total number of cloud service users.
v	The lower bound of user utilities.
k_1^c, k_1^n	The service cost factors of all CSPs and all NSPs.
k_2^c, k_2^n	The networking cost factors of all CSPs and all NSPs.
$c_k^{c,i}, c_k^{n,j}$	The operating costs of the CSP i and the NSP j in the k-th round of the competition.
$U_k^{c,i}, U_k^{n,j}$	The utilities of CSP i and NSP j in the k-th round of the competition.
$\pi_k^{c,i}, \pi_k^{n,j}$	The profits of CSP i and NSP j in the k-th round of the competition.
a_i, a_j^*	The service coefficients of CSP i and NSP j.
b_{ij}	The replacement coefficient of CSP i with respect to NSP j.
b_{ji}^*	The replacement coefficient of NSP j with respect to CSP i.
U_k^s	The grand total social utility in the k-th round of the competition.
U_k^{su}	The social utility contributed by all users in the k-th round of the competition.
U_k^{sc}	The social utility contributed by all CSPs in the k-th round of the competition.
U_k^{sn}	The social utility contributed by all NSPs in the k-th round of the competition.
N	The set of service providers on the market.
r	The number of service providers on the market.
m	The number of CSPs on the market.

In the cloud service market, the operational cost of one service provider is likely to be different from that of another service provider due to their individual ways of running business. We use $c_k^{c,i}$ and $c_k^{n,j}$ to represent the operational costs of CSP *i* and NSP *j* in the *k*-th round of the competition, and compute them as follows

$$\begin{cases} c_k^{c,i} = k_1^c u_k^{c,i} + k_2^c q \\ c_k^{n,j} = k_1^n u_k^{n,j} + k_2^n q, \end{cases} \tag{6}$$

where k_1^c and k_2^c are the service cost factor and networking maintenance cost factor of any CSPs, and k_1^n and k_2^n are the service cost factor and networking maintenance cost factor of any NSPs. Clearly, a larger value of the service rate or the connectivity rate would lead to a higher value of operational cost for any service providers. Moreover, note that a CSP typically needs to rent the networking facilities from an NSP in order to provide cloud services to its customers, it is thus reasonable to stipulate that $k_2^c > k_2^n$.

Based on the descriptions above, we can calculate the utility $U_k^{c,i}$ of CSP *i* and the utility $U_k^{n,j}$ of NSP *j* in the *k*-th round of the competition as follows

$$\begin{cases} U_k^{c,i} = P^c - c_k^{c,i} \\ U_k^{n,j} = P^n - c_k^{n,j}, \end{cases} \tag{7}$$

which can be rewritten as

$$\begin{cases} U_k^{c,i} = P^c - k_1^c u_k^{c,i} - k_2^c q \\ U_k^{n,j} = P^n - k_1^n u_k^{n,j} - k_2^n q. \end{cases} \tag{8}$$

Formula (8) clearly indicates that a larger value of service rate or a larger value of connectivity rate *q* of service providers would decrease the utility of service providers. This makes sense because a larger service rate or connectivity rate means that a service provider needs to increase its operational costs to improve its customer service quality, which would lead to an increased user utility and a decreased service provider utility simultaneously.

Also, formula(8) signifies the difference between the utilities of CSPs and NSPs. This can be seen by comparing the counterparts of these two utilities. Specifically, due to their own technical strength and business features (e.g., NSPs have sufficient bandwidth resources which allow them to offer high download/upload speed to customers, and CSPs having a longer history than NSPs in providing cloud services business and tend to offer a better customer service quality), CSPs and NSPs are most likely to offer different prices; that is, P^c and P^n will be different. Regarding the service rates $u_k^{c,i}$ and $u_k^{n,j}$, since the service rates of NSPs and CSPs rely on their service capabilities which, in turn, depend on their individual business characteristics, it is thus not difficult to see that $u_k^{c,i}$ and $u_k^{n,j}$ will also be different. Besides, note that CSPs typically need to rent networking resources from NSPs to provide cloud services to customers, so the associated cost in connectivity for CSPs is usually higher than that of NSPs, that is, $k_2^c > k_2^n$.

Now we define the profit $\pi_k^{c,i}$ of CSP i and the profit $\pi_k^{n,j}$ of NSP j in the k-th round of the competition as follows

$$\begin{cases} \pi_k^{c,i} = f_k^{c,i} \cdot U_k^{c,i} \\ \pi_k^{n,j} = f_k^{n,j} \cdot U_k^{n,j}, \end{cases} \tag{9}$$

where $f_k^{c,i}$ and $f_k^{n,j}$ are the market shares of CSP i and NSP j in the k-th round of the competition, respectively. It can be seen that the profit of a service provider is decided by and proportional to its market share and utility.

IV. The Competition Among Service Providers

The competition among service providers can be seen from the fact that if the user utility that a user receives from a service provider is lower than the expected minimum value, then the user would reject this service provider and select another service provider. Therefore each service provider will try to lower its price to attract more users, resulting in a price competition among service providers. However, the competition will eventually come to an end with an equilibrium. Considering that each service provider always pursues its utility and profit maximization during the competition, we may model this competition as a non-cooperative game.

In the discussion and analysis below, we assume that there are $r \geq 2$ service providers on the market with $0 < m < r$ CSPs and $r - m$ NSPs. By theories in economics, we know that the service quality of a service provider is proportional to its market demands [26]. That is, a superior service quality will lead to a higher demand for that service in the market. Also, we notice that the cloud service provided by a CSP can be replaced by that provided by an NSP, and vice versa. Specifically, the cloud service provided by a CSP or an NSP can be replaced by the cloud service provided by another CSP or another NSP. Thus, the market share $f_k^{c,i}$ of CSP i and $f_k^{n,j}$ of NSP j in the k-th round of the competition are defined as follows

$$\begin{cases} f_k^{c,i} = a_i u_k^{c,i} - \sum_{l=1, l\neq i}^{m} u_k^{c,l} - \sum_{j=1}^{r-m} b_{ij} u_k^{n,j} \\ f_k^{n,j} = a_j^* u_k^{n,j} - \sum_{h=1, h\neq j}^{r-m} u_k^{n,h} - \sum_{i=1}^{m} b_{ji}^* u_k^{c,i}, \end{cases} \tag{10}$$

where $i \in \{1, 2, \ldots, m\}$, $j \in \{1, 2, \ldots, r-m\}$. The first term on the right-hand side of Eqs. (10) indicates that the market share of a CSP (or an NSP, respectively) is proportional to its service rate; that is, a larger service rate will result in a higher market share for a CSP (or an NSP, respectively). The second term indicates that the service (rate) provided by a CSP (or an NSP, respectively) can be replaced by that of other CSPs (or NSPs, respectively), and the third term indicates that the service (rate) provided by a CSP (or an NSP, respectively) can be replaced by that of all NSPs (or CSPs, respectively). a_i and a_j^* are the service coefficients of CSP i and NSP j respectively, which indicate the correlation between service rate of a service provider and its market share; $b_{ij} \in (0, 1)$ is the replacement coefficient of CSP i with respect to NSP j which indicates the probability that CSP i can be replaced by NSP j; similarly, $b_{ji}^* \in (0, 1)$ is the replacement coefficient of NSP j with respect to the CSP i which indicates the probability that NSP j can be replaced by CSP i. A larger replacement coefficient of a service provider indicates a stronger possibility that this service provider will be replaced by another one; that is, its market share is more easily affected by other service providers. Based on the discussions we have so far, the profit $\pi_k^{c,i}$ of CSP i and the profit $\pi_k^{n,j}$ of NSP j in the k-th round of the competition can be recalculated as follows

$$\begin{cases} \pi_k^{c,i} = \left[a_i u_k^{c,i} - \sum_{l=1, l\neq i}^{m} u_k^{c,l} - \sum_{j=1}^{r-m} b_{ij} u_k^{n,j} \right] \\ \qquad \left[P^c - k_1^c u_k^{c,i} - k_2^c q \right] \\ \pi_k^{n,j} = \left[a_j^{*u_k^{n,j}} - \sum_{h=1, h\neq j}^{r-m} u_k^{n,h} - \sum_{i=1}^{m} b_{ji}^* u_k^{c,i} \right] \\ \qquad \left[P^n - k_1^n u_k^{n,j} - k_2^n q \right]. \end{cases} \tag{11}$$

For example, suppose there are only two CSPs in the market. For the first CSP, if the ratio of the increase of its market share over the increase of its service rate is 1.2, its service charge to its customers is \$1300 per month, its service cost is 40% of its service rate offered to customers, and its networking cost is 80% of its network connectivity rate, then the profit of this CSP would be

$$\begin{aligned} \pi_k^{c,1} &= \left(1.2 u_k^{c,1} - u_k^{c,2}\right)\left(1.3 - 0.4 u_k^{c,1} - 0.8q\right) \\ &= -0.48\left(u_k^{c,1}\right)^2 + \left(1.56 - 0.96q + 0.4 u_k^{c,2}\right) u_k^{c,1} \\ &\quad - 1.3 u_k c, 2 + 0.8 q u_k^{c,2}. \end{aligned}$$

Clearly, the curve of the profit of the CSP is a parabola opening downward with respect to its service rate, and thus can be maximized.

A. The Existence and Uniqueness Proofs of the Nash Equilibrium of the Proposed Model

We now discuss the existence and the uniqueness of the Nash equilibrium associated with the proposed model. Note that the Glicksberg-Fan fixed point theorem [27], [28] states that if (1) a game has a finite number of players, (2) each player's pure strategy space is a non-empty, compact, and convex set, and (3) each player's profit function is continuous and is quasi-concave over its strategy space, then this game has a pure strategy Nash equilibrium.

Theorem 1: The game model specified by formulas (1) - (11) together with Table I has a Nash equilibrium.

Proof: Given the set of players N, each player's strategy $u_k^{c,i}$ or $u_k^{n,j}$, and each player's profit $\pi_k^{c,i}$ or $\pi_k^{n,j}$, at the k-th round of the competition for any natural number k, we observe the following fact:

1) Since there are $r = |N|$ service providers on the market for some natural number r, N is clearly finite.
2) For any $u_k^{c,i}, u_k^{n,j}$, we know $u_k^{c,i}, u_k^{n,j} \in (0, u_m)$ where u_m is the largest possible service rate that can be offered by any service provider. Since $(0, u_m)$ is a bounded

domain, it must be a non-empty, compact, and convex set (in Euclidean space).

3) For any $\pi_k^{c,i}, \pi_k^{n,j}$, we have $\frac{\partial^2 \pi_k^{c,i}}{\partial (u_k^{c,i})^2} = -2a_i k_1^c < 0$ and $\frac{\partial^2 \pi_k^{n,j}}{\partial (u_k^{n,j})^2} = -2a_j^* k_1^n < 0$, which indicates that the profit functions $\pi_k^{c,i}$ and $\pi_k^{n,j}$ are concave (downward), and therefore quasi-concave. Also, both $\pi_k^{c,i}$ and $\pi_k^{n,j}$ are clearly continuous.

Combining these observations, the desired result follows immediately by the Glicksberg-Fan fixed point theorem. ■

Theorem 2: Let *r* be the number of service providers and *m* be the number of CSPs. The Nash equilibrium of the proposed game model is unique, if $a_i, a_j^* \in (\frac{r-1}{2}, r)$ for all $i \in \{1, 2, \ldots, m\}$ and all $j \in \{1, 2, \ldots, r-m\}$.

Proof: By using equation (11), the service rate of any particular service provider can be expressed by the service rates of other service providers via a response function *h*, as shown in formula (12), as shown at the bottom of this page.

According to Cachon and Netessine [29], if the response function in a game is contractive over the entire strategy space, then the relevant game Nash equilibrium must be unique. In our game model, note that showing *h* is contractive is equivalent to showing that the Hessian matrix expressed in (13) as shown at the bottom of this page, is diagonally dominant [30], [31].

So, we need to show that

$$\begin{cases} \left|\frac{\partial^2 \pi_k^{c,i}}{\partial \left(u_k^{c,i}\right)^2}\right| > \sum_{j=1, j\neq i}^{m} \left|\frac{\partial^2 \pi_k^{c,i}}{\partial u_k^{c,i} \partial u_k^{c,j}}\right| + \sum_{j=1}^{r-m} \left|\frac{\partial^2 \pi_k^{c,i}}{\partial u_k^{c,i} \partial u_k^{n,j}}\right| \\ \left|\frac{\partial^2 \pi_k^{n,j}}{\partial \left(u_k^{n,j}\right)^2}\right| > \sum_{i=1}^{m} \left|\frac{\partial^2 \pi_k^{n,j}}{\partial u_k^{n,j} \partial u_k^{c,i}}\right| + \sum_{i=1, i\neq j}^{r-m} \left|\frac{\partial^2 \pi_k^{n,j}}{\partial u_k^{n,j} \partial u_k^{n,i}}\right| \end{cases} \tag{14}$$

for any $i \in \{1, 2, \ldots, m\}, j \in \{1, 2, \ldots, r-m\}$. Since

$$\sum_{j=1, j\neq i}^{m} \left|\frac{\partial^2 \pi_k^{c,i}}{\partial u_k^{c,i} \partial u_k^{c,j}}\right| + \sum_{j=1}^{r-m} \left|\frac{\partial^2 \pi_k^{c,i}}{\partial u_k^{c,i} \partial u_k^{n,j}}\right| = k_1^c \Big[m - 1 + b_{i1} + \cdots + b_{i(r-m)}\Big]$$

and

$$\sum_{i=1}^{m} \left|\frac{\partial^2 \pi_k^{n,j}}{\partial u_k^{n,j} \partial u_k^{c,i}}\right| + \sum_{i=1, i\neq j}^{r-m} \left|\frac{\partial^2 \pi_k^{n,j}}{\partial u_k^{n,j} \partial u_k^{n,i}}\right| = k_1^n \Big[b_{j1}^* + \cdots + b_{jm}^* + r - m - 1\Big]$$

with $b_{ij}, b_{ji}^* \in (0, 1)$ for all *i* and all *j*, we have

$$k_1^c \Big(m - 1 + b_{i1} + \cdots + b_{i(r-m)}\Big) < k_1^c (m - 1 + r - m) = k_1^c (r-1)$$

and

$$k_1^n \Big(b_{j1}^* + \cdots + b_{jm}^* + r - m - 1\Big) < k_1^n (m + r - m - 1) = k_1^n (r-1).$$

Note also that

$$\left|\frac{\partial^2 \pi_k^{c,i}}{\partial \left(u_k^{c,i}\right)^2}\right| = 2a_i \cdot k_1^c, \quad \left|\frac{\partial^2 \pi_k^{n,j}}{\partial \left(u_k^{n,j}\right)^2}\right| = 2a_j^* \cdot k_1^n.$$

For the sake of the validity of data, we assume the service coefficients a_i, a_j^* are less than the number of service providers *r* on the market. Hence, the inequalities in (14) will hold as long as $a_i, a_j^* \in (\frac{r-1}{2}, r)$. This implies the contractiveness of the response function *h*, which in turn implies the uniqueness of the Nash equilibrium. ■

B. The Computation of the Nash Equilibrium

With the existence and uniqueness of Nash equilibrium being addressed, we now turn our attention to the finding of the expected Nash equilibrium. Based on the earlier discussion, we know that the profit (functions) $\pi_k^{c,i}, \pi_k^{n,j}$ of service providers are concave downward with respect to their respective service rates $u_k^{c,i}, u_k^{n,j}$, and the service rate of a particular service provider is fixed when the service rates of all other service providers are given. Therefore, the optimal strategy for a service provider would be the service rate that maximizes its profit. In other words, the problem of maximizing all service

$$\begin{cases} u_k^{c,i} = h\Big(u_k^{c,1}, \ldots, u_k^{c,i-1}, u_k^{c,i+1}, \ldots, u_k^{c,m}, u_k^{n,1}, \ldots, u_k^{n,r-m}\Big) = \frac{P^c - k_2^c q}{2k_1^c} + \frac{\sum_{l=1, l\neq i}^{m} u_k^{c,l} + \sum_{j=1}^{r-m} b_{ij} u_k^{n,j}}{2a_i} \\ u_k^{n,j} = h\Big(u_k^{c,1}, \ldots, u_k^{c,m}, u_k^{n,1}, \ldots, u_k^{n,j-1}, u_k^{n,j+1}, \ldots, u_k^{n,r-m}\Big) = \frac{P^n - k_2^n q}{2k_1^n} + \frac{\sum_{h=1, h\neq j}^{r-m} u_k^{n,h} + \sum_{i=1}^{m} b_{ji}^* u_k^{c,i}}{2a_j^*} \end{cases} \tag{12}$$

$$H = \begin{vmatrix} \frac{\partial^2 \pi_k^{c,1}}{\partial \left(u_k^{c,1}\right)^2} & \cdots & \frac{\partial^2 \pi_k^{c,1}}{\partial u_k^{c,1} \partial u_k^{c,m}} & \frac{\partial^2 \pi_k^{c,1}}{\partial u_k^{c,1} \partial u_k^{n,1}} & \cdots & \frac{\partial^2 \pi_k^{c,1}}{\partial u_k^{c,1} \partial u_k^{n,r-m}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 \pi_k^{c,m}}{\partial u_k^{c,m} \partial u_k^{c,1}} & \cdots & \frac{\partial^2 \pi_k^{c,m}}{\partial \left(u_k^{c,m}\right)^2} & \frac{\partial^2 \pi_k^{c,m}}{\partial u_k^{c,m} \partial u_k^{n,1}} & \cdots & \frac{\partial^2 \pi_k^{c,m}}{\partial u_k^{c,m} \partial u_k^{n,r-m}} \\ \frac{\partial^2 \pi_k^{n,1}}{\partial u_k^{n,1} \partial u_k^{c,1}} & \cdots & \frac{\partial^2 \pi_k^{n,1}}{\partial u_k^{n,1} \partial u_k^{c,m}} & \frac{\partial^2 \pi_k^{n,1}}{\partial \left(u_k^{n,1}\right)^2} & \cdots & \frac{\partial^2 \pi_k^{n,1}}{\partial u_k^{n,1} \partial u_k^{n,r-m}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 \pi_k^{n,r-m}}{\partial u_k^{n,r-m} \partial u_k^{c,1}} & \cdots & \frac{\partial^2 \pi_k^{n,r-m}}{\partial u_k^{n,r-m} \partial u_k^{c,m}} & \frac{\partial^2 \pi_k^{n,r-m}}{\partial u_k^{n,r-m} \partial u_k^{n,1}} & \cdots & \frac{\partial^2 \pi_k^{n,r-m}}{\partial \left(u_k^{n,r-m}\right)^2} \end{vmatrix} \tag{13}$$

Algorithm 1: Computation of the Nash Equilibrium

Input: $P^c, k_1^c, k_2^c, a_i, b_{ij}, P^n, k_1^n, k_2^n, a_j^*, b_{ji}^*, v_i^c, v_j^n$ for all $i = 1, 2, \ldots, m, j = 1, 2, \cdots, r-m$.

Output: $(u^{c,i})^*, (u^{n,j})^*$ for all *i, j*.

Initialization
$u_0^{c,i} \leftarrow v_i, u_0^{n,j} \leftarrow v_j, k \leftarrow 1, flag_i = 1, flag_j = 1$ for all *i, j*.
while *until* $flag_i = 0$ *and* $flag_j = 0$ *for all i, j.* **do**
  for $i = 1$ *to* m **do**
    if $flag_i = 1$ **then**
      $u_k^{c,i} \leftarrow \frac{P^c - k_2^c q}{2k_1^c} + \frac{\sum_{l=1, l\neq i}^{m} u_{k-1}^{c,l} + \sum_{j=1}^{r-m} b_{ij} u_{k-1}^{n,j}}{2a_i}$
      if $\left|u_k^{c,i} - u_{k-1}^{c,i}\right| < \varepsilon$ **then**
        $(u^{c,i})^* \leftarrow u_k^{c,i}, flag_i \leftarrow 0$
      end
    else
      $u_k^{c,i} \leftarrow (u^{c,i})^*$
    end
  end
  for $j = 1$ *to* $r - m$ **do**
    if $flag_j = 1$ **then**
      $u_k^{n,j} \leftarrow \frac{P^n - k_2^n q}{2k_1^n} + \frac{\sum_{h=1, h\neq j}^{r-m} u_{k-1}^{n,h} + \sum_{i=1}^{m} b_{ji}^* u_{k-1}^{c,i}}{2a_j^*}$
      if $\left|u_k^{n,j} - u_{k-1}^{n,j}\right| < \varepsilon$ **then**
        $(u^{n,j})^* \leftarrow u_k^{n,j}, flag_j \leftarrow 0$
      end
    else
      $u_k^{n,j} \leftarrow (u^{n,j})^*$
    end
  end
  $k \leftarrow k + 1$
end

providers' profits simultaneously (i.e., reaching the state of Nash equilibrium)

$$\begin{cases} \max\left(\pi_k^{c,i} = f_k^{c,i} \cdot U_k^{c,i}\right), \text{for all } i \in \{1, 2, \ldots, m\} \\ \max\left(\pi_k^{n,j} = f_k^{n,j} \cdot U_k^{n,j}\right), \text{for all } j \in \{1, 2, \ldots, r-m\} \\ \text{with } a_i, a_j^* \in \left(\frac{r-1}{2}, r\right) \end{cases} \tag{15}$$

amounts to finding the respective optimal service rates $(u^{c,i})^*, (u^{n,j})^*$ such that

$$\begin{cases} \left(u^{c,i}\right)^* = \arg\left\{\max\left(\pi_k^{c,i} = f_k^{c,i} \cdot U_k^{c,i}\right)\right\} \\ \left(u^{n,j}\right)^* = \arg\left\{\max\left(\pi_k^{n,j} = f_k^{n,j} \cdot U_k^{n,j}\right)\right\} \\ a_i, a_j^* \in \left(\frac{r-1}{2}, r\right) \end{cases} \tag{16}$$

for all *i* and *j*.

At this point, we can devise an iterative algorithm to compute the Nash-equilibrium leading $(u^{c,i})^*, (u^{n,j})^*$. The outline of the algorithm is presented below, and the pseudocode of the algorithm is given in Algorithm 1.

1) The service rate of each service provider is set to an initial value. Specifically, $u_0^{c,i} = v_i^c$ and $u_0^{n,j} = v_j^n$, for each $i \in \{1, 2, \ldots, m\}$ and each $j \in \{1, 2, \ldots, r-m\}$.
2) The service rate of each service provider is iteratively computed via formula (12). We use $u_k^{c,i}, u_k^{n,j}$ to represent the k-th computation of the service rates of CSP *i* and NSP *j*, respectively.
3) The iterative computation of any service rate $u_k^{c,i}$ (respectively, $u_k^{n,j}$) stops when $|u_k^{c,i} - u_{k-1}^{c,i}| < \varepsilon$ (respectively, $|u_k^{n,j} - u_{k-1}^{n,j}| < \varepsilon$) for some pre-defined value ε. In this case, the Nash equilibrium service rate is found, that is, $(u^{c,i})^* = u_k^{c,i}$(respectively, $(u^{n,j})^* = u_k^{n,j}$).

V. Experimental Results

In this section, on the basis of the model established earlier in the paper, we investigate the internal correlations between various properties of service providers. Note that due to the practical limitations in obtaining the real data for cloud computing market, we test our model by simulating the realistic situations. However, the efficacy of this test remains in the sense that a realistic analysis can be carried out in a similar fashion by incorporating the authentic data into the model when they are available, and the ensuing analysis can of marketing guidance for service providers.

A. Replacement Coefficients, Connectivity Rates, Service Rates, Market Shares, and Profits

We first investigate the impacts of replacement coefficients, connectivity rates, and service rates on the market shares and profits of service providers. Without loss of generality, we typify our investigation by examining properties associated with CSP 1 and NSP 1.

1) Correlations Between $b_{1j}, q, u_k^{n,j}$ and $f_k^{c,1}, \pi_k^{c,1}$ With Respect to $u_k^{c,1}$: For the sake of computational convenience, data used in the experiments are uniformed and simplified. At the k-th round of the competition, by letting $a_1 = 3.5, P^c = 1.5, k_1^c = 0.5, k_2^c = 0.7$, the correlations between the values of replacement coefficient b_{1j}, connectivity rate q, as well as service rate $u_k^{n,j}$, and the values of market share $f_k^{c,1}$ and profit $\pi_k^{c,1}$ of CSP 1, with respect to different values of the service rate $u_k^{c,1}$ of CSP 1, are depicted in Fig. 1.

Figs. 1(a) and 1(b) show the correlations between the replacement coefficient of CSP 1 and its market share as well as its profit with respect to its service rate when $u_k^{c,2} + \cdots + u_k^{c,m} = 1.8, u_k^{n,1} + u_k^{n,2} + \cdots + u_k^{n,r-m} = 1.6$, and $q = 0.7$. It can be seen that for a fixed value of the service rate $u_k^{c,1}$, both market share $f_k^{c,1}$ and profit $\pi_k^{c,1}$ decrease as the replacement coefficient b_{1j} increases. Fig. 1(c) shows the correlation between the connectivity rate of cloud service and the profit of CSP 1 with respect to its service rate, when $b_{11} = \cdots = b_{1(r-m)} = 0.5$, $u_k^{c,2} + \cdots + u_k^{c,m} = 1.8$, and $u_k^{n,1} + u_k^{n,2} + \cdots + u_k^{n,r-m} = 1.6$. Clearly, we see that for a fixed value of the service rate, the profit of CSP 1 decreases when its connectivity rate increases. This can be understood by the fact that a higher connectivity rate means that the cloud service provider needs to invest more to improve its customer service quality which would inevitably increase its operational costs and decrease its profits. Figs. 1(d) and 1(e) depict the correlation between the service rates of NSPs and the market share as well as the profit of CSP 1, with respect to the service

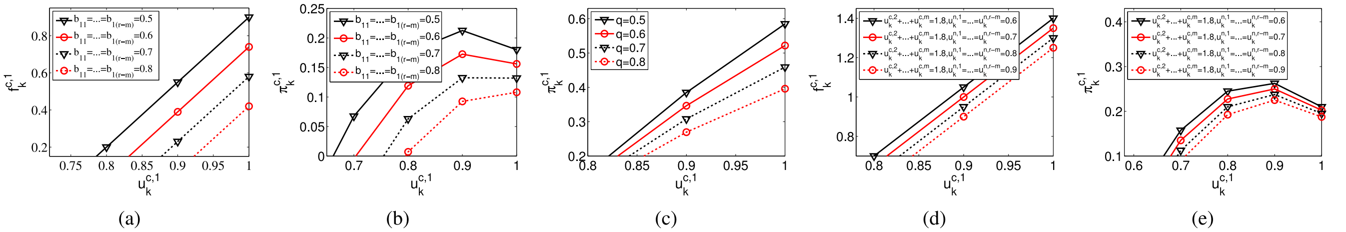


Fig. 1. Correlations between $b_{1j}, q, u_k^{n,j}$ and $f_k^{c,1}, \pi_k^{c,1}$. (a) Correlation between CSP 1's replacement coefficient and its market share wrt its service rate. (b) Correlation between CSP 1's replacement coefficient and its profit wrt its service rate. (c) Correlation between connectivity rate and CSP 1's profit wrt CSP 1's service rate. (d) Correlation between NSPs' service rates and CSP 1's market share wrt CSP 1's service rate. (e) Correlation between NSPs' service rates and CSP 1's profit wrt CSP 1's service rate.

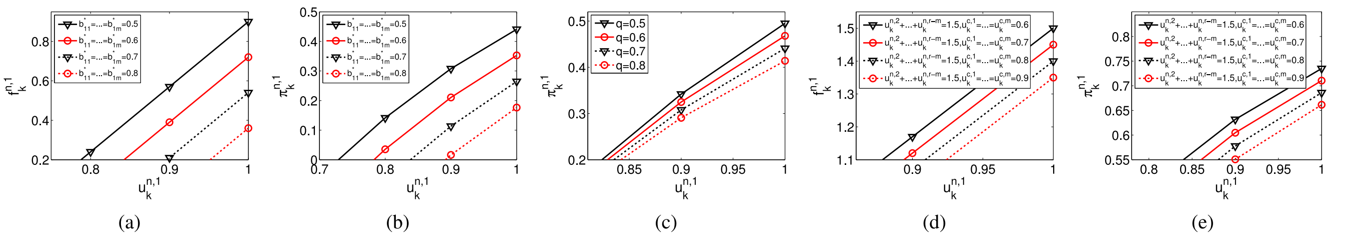


Fig. 2. Correlations between $b_{1i}^*, q, u_k^{c,i}$ and $f_k^{n,1}, \pi_k^{n,1}$ with respect to $u_k^{n,1}$. (a) Correlation between replacement coefficient and NSP 1's market share wrt NSP 1's service rate. (b) Correlation between replacement coefficient and NSP1's profit wrt NSP 1's service rate. (c) Correlation between connectivity rate and NSP1's profit wrt NSP 1's service rate. (d) Correlation between CSPs' service rates and NSP 1's market share wrt NSP 1's service rate. (e) Correlation between CSPs' service rates and NSP 1's profit wrt NSP 1's service rate.

rate of CSP 1, when $q = 0.7, b_{11} = \cdots = b_{1(r-m)} = 0.5$, and $u_k^{c,2} + u_k^{c,3} + \cdots + u_k^{c,m} = 1.8$. These two figures suggest that a larger NSP service rate would yield a smaller CSP profit or market share, when the CSP's service rate is unchanged.

2) Correlations Between $b_{1i}^*, q, u_k^{c,i}$ and $f_k^{n,1}, \pi_k^{n,1}$ With Respect to $u_k^{n,1}$: By letting $a_1^* = 3.3, P^n = 1.2, k_1^n = 0.5, k_2^n = 0.3$, we depict in Fig. 2 the correlations between the values of replacement coefficient b_{1i}^*, connectivity rate q, as well as service rate $u_k^{c,i}$ of CSP i, and the values of the market share $f_k^{n,1}$ and profit $\pi_k^{n,1}$ of NSP 1, with respect to different values of the service rate $u_k^{n,1}$ of NSP 1.

Figs. 2(a) and 2(b) reveal the correlation between the replacement coefficient of NSP 1 and its market share as well as its profit, with respect to its service rate, when $q = 0.7$, $u_k^{n,2} + \cdots + u_k^{n,r-m} = 1.5$, and $u_k^{c,1} + \cdots + u_k^{c,m} = 1.8$. Fig. 2(c) shows the correlation between the connectivity rate and the profit of NSP 1, with respect to NSP 1's service rate, when $b_{11}^* = \cdots = b_{1m}^* = 0.5$ $u_k^{n,2} + \cdots + u_k^{n,r-m} = 1.5$, and $u_k^{c,1} + \cdots + u_k^{c,m} = 1.8$. Figs. 2(d) and 2(e) exhibit the impact of the service rate of CSP 1 on the market share and profit of NSP 1, with regard to NSP 1's service rate. Just like the case of CSP 1, for any fixed service rate of NSP 1, we can see that both the market share and the profit of NSP 1 decrease as its replacement coefficient increases, and that the profit of NSP 1 drops as its connectivity rate goes up. Also in a similar fashion to that of CSP 1, Figs. 2(d) and 2(e) demonstrate that a larger value of the service rates of CSPs will result in lower values of market share and profit of NSP 1, when the service rate of NSP 1 remains unchanged.

B. Nash Equilibrium Between Service Providers

We now study of the Nash equilibrium in the competition of the service providers. Nash equilibrium is a critical notion in Game Theory, which represents the "best scenario" in the interests of all game players. Here, we focus on the impacts of the internal elements of game players (in this case, the service coefficient and the connectivity rate of service providers) on the status of the Nash equilibrium, and investigate the following cases: CSPs vs. CSPs, CSPs vs. NSPs, and NSPs vs. NSPs. Throughout the experiments, we set $P^c = 1, k_1^c = 0.5, k_2^c = 1, P^n = 0.8, k_1^n = 0.4, k_2^n = 0.8$.

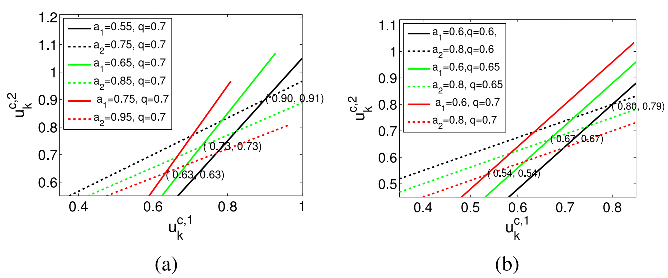


Fig. 3. Correlation between the service coefficient a_1, a_2, q and Nash equilibrium. (a) Comparison of $((u^{c,1})^*, (u^{c,2})^*)$ wrt service coefficients. (b) Comparison of $((u^{c,1})^*, (u^{c,2})^*)$ wrt the connectivity rate.

1) CSPs vs. CSPs: We typify the investigation on the competition between cloud service providers by assuming that there are only two service providers (CSP 1 and CSP 2) on the market. By earlier results in this paper, we know that the profits of CSP 1 and CSP 2 are

$$\begin{cases} \pi_k^{c,1} = f_k^{c,1} \cdot U_k^{c,1} = \left(a_1 u_k^{c,1} - u_k^{c,2}\right)\left(P^c - k_1^c u_k^{c,1} - k_2^c q\right) \\ \pi_k^{c,2} = f_k^{c,2} \cdot U_k^{c,2} = \left(a_2 u_k^{c,2} - u_k^{c,1}\right)\left(P^c - k_1^c u_k^{c,2} - k_2^c q\right), \end{cases} \tag{17}$$

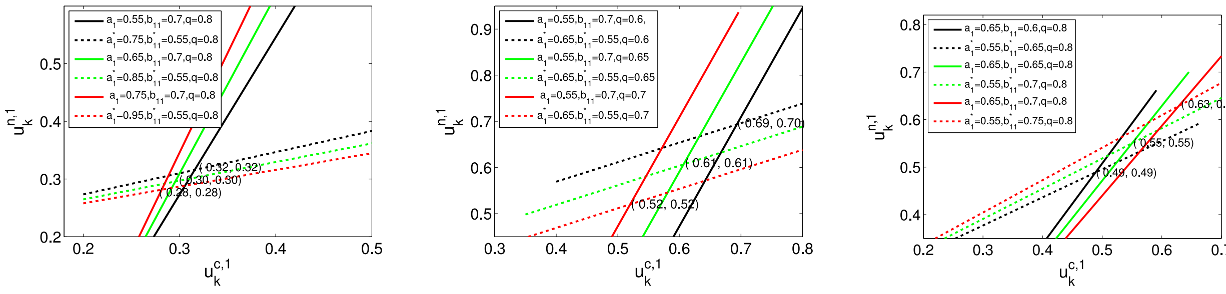


Fig. 4. Correlation between the service coefficient $a_1, a_1^*, q, b_{11}, b_{11}^*$ and Nash equilibrium points. (a) Comparison of $((u^{c,1})^*, (u^{n,1})^*)$ wrt service coefficients. (b) Comparison of $((u^{c,1})^*, (u^{n,1})^*)$ wrt the connectivity rate. (c) Comparison of $((u^{c,1})^*, (u^{n,1})^*)$ wrt replacement coefficients.

and their service rates are

$$\begin{cases} u_k^{c,1} = \dfrac{P^c - k_2^c q}{2k_1^c} + \dfrac{u_k^{c,2}}{2a_1} \\ u_k^{c,2} = \dfrac{P^c - k_2^c q}{2k_1^c} + \dfrac{u_k^{c,1}}{2a_2}. \end{cases} \tag{18}$$

The graphs of the equations in (18) with various values of a_1, a_2 and q are shown in Figs. 3(a) and 3(b). The intersection points of the two straight (color-wised) lines in these two figures denote the Nash-equilibrium enabling service rates of CSP 1 and CSP 2, and are represented by $((u^{c,1})^*, (u^{c,2})^*)$. Fig. 3(a) shows the correlation between the service coefficient and the equilibrium point where the connectivity rate is kept unchanged. Clearly, we can see that the Nash equilibrium point $((u^{c,1})^*, (u^{c,2})^*)$ goes down as the values of the service coefficients a_1 and a_2 go up.

2) CSPs vs. NSPs: We examine the competition of one cloud service provider (CSP 1) and one network service provider (NSP 1) on the market. By the results obtained earlier in the paper, we have

$$\begin{cases} \pi_k^{c,1} = f_k^{c,1} U_k^{c,1} = \left(a_1 u_k^{c,1} - b_{11} u_k^{n,1}\right)\left(P^c - k_1^c u_k^{c,1} - k_2^c q\right) \\ \pi_k^{n,1} = f_k^{n,1} U_k^{n,1} = \left(a_1^* u_k^{n,1} - b_{11}^* u_k^{c,1}\right)\left(P^n - k_1^n u_k^{n,1} - k_2^n q\right), \end{cases} \tag{19}$$

and

$$\begin{cases} u_k^{c,1} = \dfrac{P^c - k_2^c q}{2k_1^c} + \dfrac{b_{11} u_k^{n,1}}{2a_1} \\ u_k^{n,1} = \dfrac{P^n - k_2^n q}{2k_1^n} + \dfrac{b_{11}^* u_k^{c,1}}{2a_1^*}. \end{cases} \tag{20}$$

The graphs of the equations in (20) with various values of service coefficient, connectivity rate and replacement coefficient are shown in Figs. 4(a), 4(b), and 4(c). Analogous to the case of CSPs vs. CSPs, we see that when other parameters are fixed, the equilibrium point $((u^{c,1})^*, (u^{n,1})^*)$ decreases as the service coefficient a_1, a_1^* increases, decreases as the connectivity rate q increases, and increases as the replacement coefficient b_{11}, b_{11}^* increases.

3) NSPs vs. NSPs: In this case, we look into the matter of the Nash equilibrium in the competition between NSPs themselves by assuming there are only two network service providers (NSP 1 and NSP 2) on the market. Again, by earlier results in the paper, we have

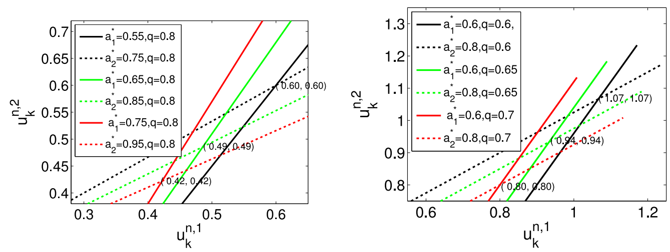


Fig. 5. Correlation between a_1^*, a_2^*, q and Nash equilibrium points. (a) Comparison of $((u^{n,1})^*, (u^{n,2})^*)$ wrt service coefficients. (b) Comparison of $((u^{n,1})^*, (u^{n,2})^*)$ wrt the connectivity rate.

$$\begin{cases} \pi_k^{n,1} = f_k^{n,1} U_k^{n,1} = \left(a_1^* u_k^{n,1} - u_k^{n,2}\right)\left(P^n - k_1^n u_k^{n,1} - k_2^n q\right) \\ \pi_k^{n,2} = f_k^{n,2} U_k^{n,2} = \left(a_2^* u_k^{n,2} - u_k^{n,1}\right)\left(P^n - k_1^n u_k^{n,2} - k_2^n q\right) \end{cases} \tag{21}$$

and

$$\begin{cases} u_k^{n,1} = \dfrac{P^n - k_2^n q}{2k_1^n} + \dfrac{u_k^{n,2}}{2a_1^*} \\ u_k^{n,2} = \dfrac{P^n - k_2^n q}{2k_1^n} + \dfrac{u_k^{n,1}}{2a_2^*}. \end{cases} \tag{22}$$

Figs. 5(a) and 5(b) show the graphs of the equations in (22) with different values of service coefficients a_1^*, a_2^* and the connectivity rate q. Evidently, the equilibrium point $((u^{n,1})^*, (u^{n,2})^*)$ decreases when the service coefficients a_1^*, a_2^* increase and the connectivity rate q is unchanged; and decreases as well when the connectivity rate q increases and the service coefficients a_1^*, a_2^* are unchanged.

C. Uniqueness of the Nash Equilibrium

We continue the investigation on Nash equilibrium in this section by demonstrating its uniqueness. With loss of generality, we typify our investigation by studying the competitions of service providers when $r = 2$ or 3.

1) Competition Between Two Service Providers: We assume that there are two service providers on the market. By letting parameters be set as shown in Table II, the iterative computations of these two service providers' service rates for CSP 1 vs. CSP 2, NSP 1 vs. NSP 2, and CSP 1 vs. NSP 1 are depicted in Figs. 6(a), 6(b), and 6(c), respectively.

TABLE II
PARAMETER SETTINGS FOR FIG. 6

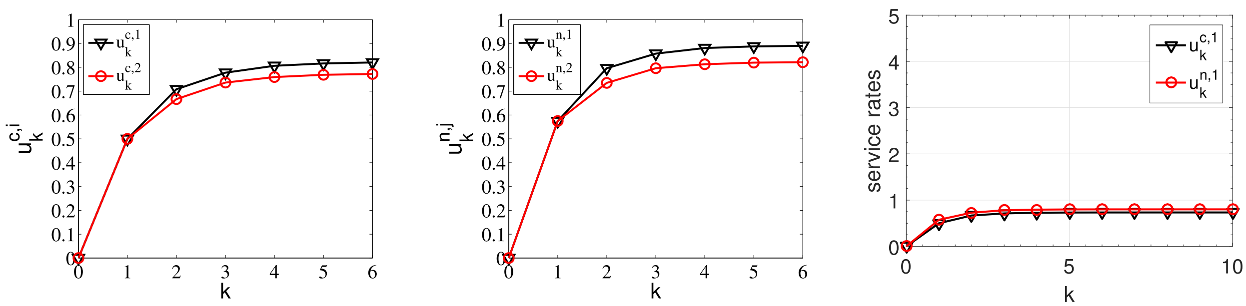
Fig.	P^c	P^n	k_1^c	k_2^c	k_1^n	k_2^n	q	a_1	a_2	a_1^*	a_2^*	b_{11}	b_{11}^*	ε
6(a)	1.3	n/a	0.5	1	n/a	n/a	0.8	1.2	1.5	n/a	n/a	n/a	n/a	0.008
6(b)	n/a	1.1	n/a	n/a	0.4	0.8	0.8	n/a	n/a	1.3	1.8	n/a	n/a	0.008
6(c)	1.3	1.1	0.5	1	0.4	0.8	0.8	1.2	n/a	1.3	n/a	0.7	0.8	0.008

TABLE III
PARAMETER SETTINGS FOR FIGS. 7(a) AND 7(b)

Fig.7(a)	$P^c = 1.3$	$k_1^c = 0.5$	$k_2^c = 1$	$q = 0.8$	$a_1 = 2.0$	$a_2 = 2.5$	$a_3 = 2.8$	$\varepsilon = 0.008$
Fig.7(b)	$P^n = 1.1$	$k_1^n = 0.4$	$k_2^n = 0.8$	$q = 0.8$	$a_1^* = 2.3$	$a_2^* = 2.5$	$a_3^* = 2.8$	$\varepsilon = 0.008$

TABLE IV
PARAMETER SETTINGS FOR FIGS. 7(c) AND 7(d)

Fig.	P^c	P^n	k_1^c	k_2^c	k_1^n	k_2^n	q	a_1	a_2	a_1^*	a_2^*	b_{11}	b_{12}	b_{21}	b_{11}^*	b_{12}^*	b_{21}^*	ε
7(c)	1.3	1.1	0.5	1	0.4	0.8	0.8	2.0	2.3	2.5	n/a	0.7	n/a	0.7	0.7	0.8	n/a	0.008
7(d)	1.3	1.1	0.5	1	0.4	0.8	0.8	2.0	n/a	2.5	2.8	0.7	0.8	n/a	0.7	n/a	0.7	0.008



(a) (b) (c)

Fig. 6. Uniqueness of the Nash equilibrium with two service providers. (a) Service rate computations for CSP 1 vs. CSP 2. (b) Service rate computations for NSP 1 vs. NSP 2. (c) Service rate computations for CSP 1 vs. NSP 1.

2) Competition Among Three Service Providers: By assuming that there are three service providers on the market, and letting parameters be set as shown in Table III, the iterative computations of the service rates for CSP 1 vs. CSP 2 vs. CSP 3 and NSP 1 vs. NSP 2 vs. NSP 3 are depicted in Figs. 7(a) and 7(b), respectively. Similarly, by letting parameters be set as shown in Table IV, the iterative computations of the service rates for CSP 1 vs. CSP 2 vs. NSP 1, and CSP 1 vs. NSP 1 vs. NSP 2 are depicted in Figs. 7(c) and 7(d), respectively.

We can see in Figs. 6 and 7 that the service rates of service providers approach their respective constants as the computation iterates, which evidently suggests the uniqueness of the Nash equilibrium. The cases where there exist more than 3 service providers on the market are similar to the ones in Figs. 6 and 7, and are simply omitted in this paper. Also, note that the service rate of CSP 1 in Fig. 6(c) may exceed that of NSP 1 if k_1^c is less than k_1^n. This is evident mathematically from Eq. (20), and also has a sensible economic explanation as higher (lower, respectively) service cost factor of a CSP would certainly decrease (increase, respectively) its service rate. Figs. 7(c) and (d) bare the similar situations.

D. Social Utility Analysis

Finally, we investigate the impacts of cloud service connectivity rate, service rates of CSPs, and service rates of NSPs on the social utility. Recall that $\mathcal{U}_k^{c,i}(\lambda_m)$, $\mathcal{U}_k^{n,j}(\lambda_m)$, $U_k^{c,i}$, $U_k^{n,j}$ denote the utility of user *m* when served by CSP *i*, the utility of user *m* when served by NSP *j*, the utility of CSP *i*, and the utility of NSP *j*, at the *k*-th round of the competition, respectively. The social utility then can be formulated as the sum of all these utilities. Specifically, let U_k^s, U_k^{su}, U_k^{sc}, U_k^{sn} be the entire social utility, the social utility contributed by all users, the social utility contributed by all CSPs, and the social utility contributed by all NSPs in the k-th round of the competition, respectively. Then,

$$U_k^s = U_k^{su} + U_k^{sc} + U_k^{sn} \tag{23}$$

with

$$\begin{cases} U_k^{su} = \sum\limits_{i=1}^{m} \sum\limits_{m=1}^{M} \mathcal{U}_k^{c,i}(\lambda_m) + \sum\limits_{j=1}^{r-m} \sum\limits_{m=1}^{M} \mathcal{U}_k^{n,j}(\lambda_m) \\ U_k^{sc} = \sum\limits_{i=1}^{m} U_k^{c,i} \\ U_k^{sn} = \sum\limits_{j=1}^{r-m} U_k^{n,j}. \end{cases} \tag{24}$$

Again, without loss of generality, we study the social utility impacts generated by the service rates of the service providers CSP 1, CSP 2, NSP 1, and NSP 2, among others, with respect to the service request of all users. By setting $g = 3.5, p = 1.0, P^c = 1.5, k_1^c = 0.5, k_2^c = 0.7, P^n = 1.2, k_1^n = 0.5, k_2^n = 0.3$, the correlations between U_k^s and $q, u_k^{c,1}, u_k^{c,2}, u_k^{n,1}, u_k^{n,2}$

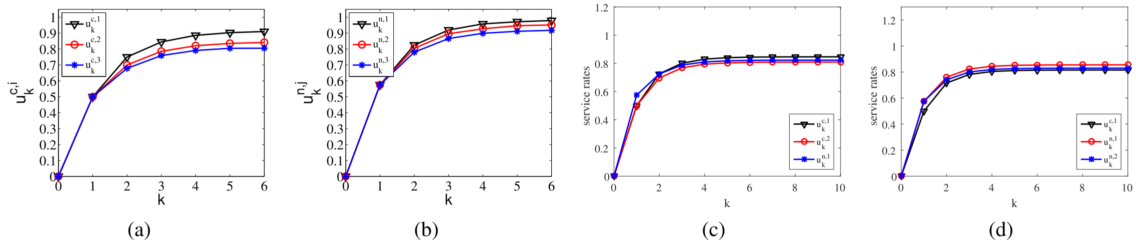


Fig. 7. Uniqueness of the Nash equilibrium with three service providers. (a) Service rate computations for CSP 1, CSP 2, and CSP 3. (b) Service rate computations for NSP 1, NSP 2, and NSP 3. (c) Service rate computations for CSP 1, CSP 2, and NSP 1. (d) Service rate computations for CSP 1, NSP 1, and NSP2.

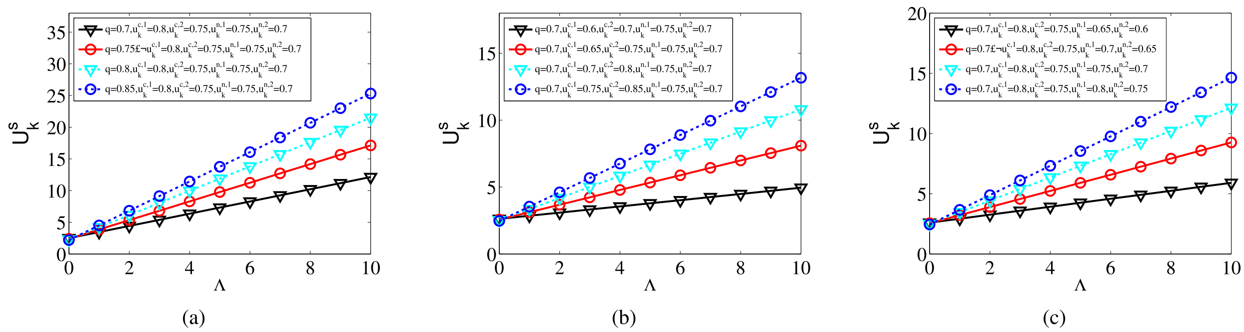


Fig. 8. Impacts of service rates and connectivity rate on the social utility. (a) Correlation between social utility and the connectivity of cloud service. (b) Correlation between social utility and the service rates of CSPs. (c) Correlation between social utility and the service rates of NSPs.

with respect to $\Lambda = \sum_{m=1}^{M} \lambda_m$ are depicted in Figs. 8(a), 8(b), and 8(c), respectively.

All three charts in Fig. 8 indicate an increase in social utility. That is, for a fixed user request, there would be a higher social utility, if the connectivity rate, CSPs' service rates, or NSPs' service rates increase. It can also be noticed that the social utility will increase as the total amount of user requests increase when other parameters are fixed. Therefore, social utility can be intentionally boosted by increasing the service rates of CSPs and NSPs, the connectivity rate, the total amount of user requests, or a combination of them.

VI. Conclusion and Future Work

We have in this paper leveraged the Game Theory to model the competition between cloud service providers and network service providers on the cloud computing market. Issues such as Nash equilibrium, correlations of replacement coefficients and connectivity rates with respect to market shares and profits of service providers, and the impacts of replacement coefficients, service coefficients, as well as connectivity rates on the Nash equilibrium point, are thoroughly investigated. Our main findings are as follows:

- A service provider with a small replacement coefficient has a competitive edge and a relatively high profit. Thus, service providers should strive to make their business unique and multifarious to reduce the chance of being replaced by their peers.
- A higher connectivity rate means that a service provider would need to invest more on improving its customer service quality and therefore would result in more costs and less profits. As such, a service provider may increase its profit by somehow lowering the connectivity rate.
- A larger service coefficient indicates that a service provider's market share and profit depend more on its service rate, and that the service provider is more vulnerable in the fluid competition over the cloud computing market.
- The Nash equilibrium point would increase when the service coefficient decreases, or the connectivity rate decreases, or the replacement coefficient increases.
- The total social utility may be increased by properly increasing the service rates of CSPs and NSPs, the connectivity rate of cloud services, or the amount user service requests.

In our future work, we plan to address the following issues:

- Investigation on the time and space complexities of the Nash equilibrium point computation algorithm. Results of such analyses may help improve the performance of the algorithm.
- Completeness of the verification for Nash equilibrium uniqueness. We plan to evaluate convergence speed (number of iterations).
- Games of incomplete information. We assumed in this paper that each game player (service provider) knows the information of every other player. This, unfortunately, will not be the case if some service provider hides its information purposely for making a higher profit. We, therefore, plan to investigate the scenario of incomplete information games, where the probability of each service provider's strategy can be dealt with by using Bayesian laws, and the ensuing Bayesian-Nash equilibrium can be probed.

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