An Improved Result on H_∞ Load Frequency Control for Power Systems With Time Delays

Hao Shen, *Member, IEEE*, Shiyu Jiao, Ju H. Park, *Senior Member, IEEE*, and Victor Sreeram, *Senior Member, IEEE*

***Abstract*—Due to the increasingly marketized demands of power systems, the dedicated communication channel used by the traditional load frequency control (LFC) scheme is ineluctably replaced by highly open communication networks, which may introduce constant, and time-varying delays. Hereupon, for those two kinds of communication methods, this article explores the LFC problem of power systems in consideration of constant, and time-varying delays. By utilizing the Lyapunov stability theory, and an improved inequality technique, some criteria for guaranteeing the stability, and the specified H_∞ performance of the closed-loop system are obtained, and a PID-type controller with the consideration of time delays and disturbance is designed. The case studies take one-area delayed LFC scheme and the traditional/deregulated three-area LFC scheme as examples to discuss the relationships between the maximum allowable delay, and the controller gain as well as the relationships between time delays, and the minimum H_∞ performance index, respectively. Finally, the effectiveness of the method proposed is verified, and the comparison results show certain advantages of the method presented in this article over the existing literature.**

***Index Terms*—H_∞ performance, load frequency control (LFC), power systems, static output feedback control (SOFC), time delays.**

I. Introduction

LOAD frequency control (LFC) is a mechanism for maintaining frequency and power interchange with adjacent areas at a predetermined value, which occupies a special position in power systems due to its excellent ability [1]–[4]. However, with the development of the power market demand, one of the primary challenges that cannot be ignored is to integrate the three aspects including computing, communication, and control into the system operation and control with a suitable level. Among them, as the infrastructure for the integration of power systems and information technology, the communication networks have been attached increasing importance to the current development of the smart grid [5]–[9]. The traditional LFC scheme typically adopts the dedicated communication channel to realize the exchange and transmission of information. However, due to the growing size of power systems and the increasingly dispersed control services, obviously, it cannot effectively meet the requirements of power systems. Compared to the dedicated communication channel, a superior communication infrastructure is urgently needed by the market. Modern LFC scheme generally prefers open communication networks because of their low cost and flexibility [10], [11]. But there are always two sides of a coin, and the adoption of open communication networks will be also accompanied by some intractable problems. In contrast with the neglected constant delay caused by the traditional proprietary communication channel, the application of open communication networks not only brings the former but also introduces time-varying delays, which results in some gaps in the open communication networks and promotes us to probe it.

With the adoption of open communication networks, large amounts and large-scale of information exchange can be realized; however, some problems may emerge, including data loss, disorder, and updates of area control error signals, which are the main reason for delays. In addition, failure of the communication channel itself may result in data loss or unavailability, and the unpredictability and randomness of such communication failures may be treated as a kind of delays equivalently [12]–[16]. As is known, as an unreliable factor in the system, time delays cannot be ignored in some situations [17]–[19]. It may lead to a decrease of the dynamic performance, and even cause the instability of the system. Subsequently, the instability of the LFC scheme implies that the control result fails to meet the control standard, which may make a negative impact on the stable operation of power systems. The LFC scheme with communication channels can be treated as a typical delayed system. As far as the stability analysis of the system is concerned, it is of great significance to seek the maximum tolerable value of delays under the premise that power systems with the LFC scheme are stable. At present, the stable operation of power systems has attracted the attention of increasing scholars.

Observing the equation characteristics of the system, it is not difficult to know that the stability of power systems can be judged

Manuscript received February 11, 2020; revised June 20, 2020 and July 26, 2020; accepted August 3, 2020. Date of publication August 21, 2020; date of current version August 26, 2021. The work of Hao Shen was supported by the National Natural Science Foundation of China under Grant 61873002, Grant 61703004, Grant 61833005, Grant 61503002. The work of Ju H. Park was supported by the National Research Foundation of Korea grant funded by the Korea government (Ministry of Science and ICT) under Grant 2019R1A5A808029011.
(Corresponding author: Ju H. Park.)

Hao Shen is with the Anhui Provincial Key Laboratory of Power Electronics, and Motion Control, and the School of Electrical, and Information Engineering, Anhui University of Technology, Ma'anshan 243002, China (e-mail: haoshen10@gmail.com).

Shiyu Jiao is with the School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China (e-mail: syjiao430@gmail.com).

Ju H. Park is with the Department of Electrical Engineering, Yeungnam University, Kyongsan 38541, South Korea (e-mail: jessie@ynu.ac.kr).

Victor Sreeram is with the School of Electrical, Electronic, and Computer Engineering, University of Western Australia, Crawley, WA 6009, Australia (e-mail: victor.sreeram@uwa.edu.au).

Digital Object Identifier 10.1109/JSYST.2020.3014936

by calculating the eigenvalues. Apart from that, the Rekasius substitution [20], Schur–Cohn methods [21] are common methods for calculating the upper bound of delays. However, when it comes to the stability analysis and control problems of the systems with time-varying delays, some methods may fail. Naturally, time-domain analysis based on Lyapunov stability theory and linear matrix inequality (LMI) technology has received increasing attention [22]–[24]. For further reducing the conservatism of stability criteria of the systems with time-varying delays, several inequality techniques have been gradually developed, including Jensen inequality, Wirtinger inequality [25], and integral inequality based auxiliary function [26]. To reduce the conservatism of the result, a more generalized inequality technique is applied in this article.

In addition, with the development of modern power systems, their complexity and uncertainty have been rising, and many superior control methods for the LFC scheme have been proposed, such as robust control [27], [28], genetic algorithms [29], [30], and sliding-mode control [31]. However, in practical applications, the PI-type or PID-type controller is still considered as the first choice mainly because of its simplification and efficiency. What distinguishes it from other control schemes is that it does not require complex state feedback or high-order dynamic controllers. Regrettably, the effects of time delays are ignored in many studies, especially in the controller designs. Based on the above-mentioned series of discussion, the H_∞ LFC issue for power systems with time delays is studied in this article, and the main contributions are divided into the following two aspects.

1) The stability of the one-area delayed LFC scheme with a PI-type controller is analyzed, and the corresponding stability criterion is improved by using the Lyapunov stability theory and the optimized inequality technique. In addition, the relationship between the maximum allowable delay and the controller gain is discussed and summarized, which can be used as auxiliary conditions for the LFC design and adjustment.
2) For the traditional and the deregulated three-area LFC schemes, the PID-type controller is designed to ensure the stability of the system under the condition that delays are less than the present value, and to guarantee the optimal H_∞ performance index of the closed-loop system (CLS). Meanwhile, time delays introduced by communication networks are considered in the design phase of the controller, including constant and time-varying delays.

Notations: The notations used in this article are standard and consistent with [32]; hence, they are omitted here.

II. Problem Formulation

A. One-Area LFC Scheme

The common LFC scheme model of one-area can be expressed as

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}u(t) + \bar{F}w(t) \\ \bar{y}(t) = \bar{C}\bar{x}(t) \end{cases} \tag{1}$$

where

$$\bar{x}^T(t) = \left[\, \Delta f \; \Delta P_m \; \Delta P_g \,\right]$$

TABLE I
Explanation of Terminologies

Terminology	Meaning
ΔP_{di}	deviation of load
ΔP_{mki}	deviation of generator mechanical output
ΔP_{gki}	deviation of valve position
Δf_i	deviation of frequency
D_i	generator unit damping coefficient
M_i	the moment of inertia of generator unit
R_{ki}	speed droop
T_{gki}	time constant of the governor
T_{tki}	time constant of the turbine
β_i	frequency bias factor
α_{ki}	ramp rate factor
ΔP_{tie-i}	tie-line power exchange
ACE_i	area control error (ACE) of area i
$d_i(t)$	time delays of area i
T_{ij}	tie-line synchronizing coefficient between area i and area j

$$\bar{y}(t) = \text{ACE}, \bar{A} = \begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 \\ 0 & -\frac{1}{T_t} & \frac{1}{T_t} \\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_g} \end{bmatrix}, \bar{F} = \begin{bmatrix} -\frac{1}{M} \\ 0 \\ 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} \beta \\ 0 \\ 0 \end{bmatrix}^T$$

with the explanations of some terms described in Table I. Since there is no power exchange of network tie lines in one-area LFC scheme, the ACE can be expressed as

$$\bar{y}(t) = \text{ACE} = \beta \Delta f \tag{2}$$

where $\beta > 0$. With ACE being the input of the controller, consider a common PI-type controller in the following form:

$$u(t) = -K_P\text{ACE} - K_I \int \text{ACE} \tag{3}$$

in which K_P and K_I are the proportional gain and the integral gain of the controller, respectively. Defining $\hat{y}(t) \triangleq [\,\bar{y}(t) \; \int \bar{y}(t)\,]^T$, $K \triangleq [\,K_P \; K_I\,]$, and taking time delays caused by control signal transmission into account, formula (3) can be re-expressed in the following form:

$$u(t) = -K\hat{y}(t - d(t)) \tag{4}$$

where $d(t)$ represents time delays, and $0 < d(t) < d$, $\dot{d}(t) \le \mu \le 1$. By defining a new state variable $\hat{x}(t) \triangleq [\,\Delta f \; \Delta P_m \; \Delta P_g \; \int \text{ACE}\,]^T$ and considering the PI-type control problem as the output feedback control problem, introducing the controller (4) into system (1), the following CLS can be obtained:

$$\begin{cases} \dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}\hat{x}(t - d(t)) + \hat{F}w(t) \\ \hat{y}(t) = \hat{C}\hat{x}(t) \end{cases} \tag{5}$$

where

$$\hat{A} = \begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 & 0 \\ 0 & -\frac{1}{T_t} & \frac{1}{T_t} & 0 \\ -\frac{1}{\text{RT}_g} & 0 & -\frac{1}{T_g} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix}, \hat{F} = \begin{bmatrix} -\frac{1}{M} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} 0 & 0\,0 & 0 \\ 0 & 0\,0 & 0 \\ -\frac{K_p\beta}{T_g} & 0\,0 & -\frac{K_I}{T_g} \\ 0 & 0\,0 & 0 \end{bmatrix}, \hat{C} = \begin{bmatrix} \beta & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}^T.$$

B. Traditional N-Area LFC Scheme

The model of the ith area in the traditional N-area LFC scheme can be expressed as

$$\begin{cases} \dot{x}_i(t) = \tilde{A}_i x_i(t) + \tilde{B}_i u_i(t - d_i(t)) + \tilde{F}_i w_i \\ y_i(t) = \tilde{C}_{yi} x_i(t) \\ z_i(t) = \tilde{D}_i x_i(t) \end{cases} \tag{6}$$

where

$$x_i^T(t) = [\Delta f_i,\ \Delta P_{\text{tie}-i},\ \Delta P_{m1i},\ \ldots, \Delta P_{mni},\ \Delta P_{g1i},\ \ldots, \Delta P_{gni}]$$

$$w_i^T = \begin{bmatrix} w_{1i} & w_{2i} \end{bmatrix} = \begin{bmatrix} \Delta P_{di} & \sum_{j=1, j\neq i}^{N} T_{ij}\Delta f_i \end{bmatrix}$$

$$y_i(t) = \text{ACE}_i, z_i(t) = \Delta f_i, u_i(t) = \Delta P_{Ci}$$

$$\tilde{A}_i = \begin{bmatrix} \tilde{A}_{11i} & \tilde{A}_{12i} & 0 \\ 0 & \tilde{A}_{22i} & \tilde{A}_{23i} \\ \tilde{A}_{31i} & 0 & \tilde{A}_{33i} \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} 0 \\ 0 \\ \tilde{B}_{3i} \end{bmatrix}$$

$$\tilde{F}_i = \begin{bmatrix} \frac{-1}{M_i} & 0 \\ 0 & -2\pi \\ 0 & 0 \end{bmatrix}, \begin{matrix} \tilde{C}_{yi} = \begin{bmatrix} \beta & 1 & 0 & 0 \end{bmatrix} \\ \tilde{D}_i = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$\tilde{A}_{11i} = \begin{bmatrix} \frac{-D_i}{M_i} & \frac{-1}{M_i} \\ 2\pi \sum_{j=1, j\neq i}^{N} T_{ij} & 0 \end{bmatrix}, \tilde{A}_{12i} = \begin{bmatrix} \frac{1}{M_i} & \cdots & \frac{1}{M_i} \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\tilde{A}_{22i} = -\tilde{A}_{23i} = \text{diag}\left\{-\frac{1}{T_{t1i}}, ACE, -\frac{1}{T_{tni}}\right\}$$

$$\tilde{A}_{33i} = \text{diag}\left\{-\frac{1}{T_{g1i}}, \ldots, -\frac{1}{T_{gni}}\right\}$$

$$\tilde{A}_{31i} = \begin{bmatrix} -\frac{1}{T_{g1i}R_{1i}} & \cdots & -\frac{1}{T_{gni}R_{ni}} \\ 0 & \cdots & 0 \end{bmatrix}, \tilde{B}_{3i} = \begin{bmatrix} \frac{\alpha_{1i}}{T_{g1i}} & \cdots & \frac{\alpha_{ni}}{T_{gni}} \end{bmatrix}$$

and the explanations of some terms are described in Table I.

The relationship among the ACE_i of each area Δf_i and $\Delta P_{\text{tie}-i}$ is considered as a linear combination as follows:

$$\text{ACE}_i = \beta_i \Delta f_i + \Delta P_{\text{tie}-i}. \tag{7}$$

Different from the common PI-type controller in one-area LFC scheme, this article considers the following form of PID-type LFC controller:

$$u_i(t) = -K_{Pi}\text{ACE}_i - K_{Ii}\int \text{ACE}_i dt - K_{Di}\frac{d}{dt}\text{ACE}_i. \tag{8}$$

By transforming the PID control problem into the SOFC problem, the state-space model (SSM) of the CLS can be attained. Define virtual vectors $\breve{x}_i(t) \triangleq [x_i^T(t)\ \int y_i^T(t)dt]^T$, $\breve{y}_i(t) \triangleq [y_i^T(t)\ \int y_i^T(t)dt\ \frac{d}{dt}y_i^T(t)]^T$, and $\breve{z}_i(t) \triangleq [\epsilon_{1i}z_i^T(t)\ \epsilon_{2i}\int y_i^T(t)dt]^T$, where ϵ_{1i} and ϵ_{2i} are weights scalars, and by adjusting them, the desired performance can be guaranteed. The CLS ($\breve{\Sigma}$) can be rephrased as

$$\begin{cases} \dot{\breve{x}}_i(t) = \breve{A}_i\breve{x}_i(t) + \breve{A}_{di}\breve{x}_i(t - d_i(t)) + \breve{B}_{wi}w_i \\ \breve{y}_i(t) = \breve{C}_{yi}\breve{x}_i(t) + \breve{C}_i w_i \\ \breve{z}_i(t) = \breve{D}_i\breve{x}_i(t) \end{cases} \tag{9}$$

where

$$\breve{A}_i = \begin{bmatrix} \tilde{A}_i & 0 \\ \tilde{C}_{yi} & 0 \end{bmatrix}, \breve{A}_{di} = -\breve{B}_i K_i \breve{C}_{yi}$$

$$\breve{D}_i = \begin{bmatrix} \epsilon_{1i}\tilde{D}_i & 0 \\ 0 & \epsilon_{2i} \end{bmatrix}, \breve{B}_i = \begin{bmatrix} \tilde{B}_i \\ 0 \end{bmatrix}$$

$$\breve{B}_{wi} = \breve{F}_i - \breve{B}_i K_i \breve{C}_i, K_i = \begin{bmatrix} K_{Pi} & K_{Ii} & K_{Di} \end{bmatrix}$$

$$\breve{C}_{yi} = \begin{bmatrix} \tilde{C}_{yi} & 0 \\ 0 & 1 \\ \tilde{C}_{yi}\tilde{A}_i & 0 \end{bmatrix}, \breve{F}_i = \begin{bmatrix} \tilde{F}_i \\ 0 \end{bmatrix}, \breve{C}_i = \begin{bmatrix} 0 \\ 0 \\ \tilde{C}_{yi}\tilde{F}_i \end{bmatrix}.$$

C. Deregulated N-Area LFC Scheme

Due to the development of the power industry, the modern power systems are in a state of deregulation, and each Generation Company (Gencom) can sign contracts with diverse Distribution Company (Discom) within or outside the Gencom area. These signed contracts can be represented by an augmented generation participation matrix (AGPM). The LFC scheme consists of N areas, each of which covers n Gencoms and m Discoms, and its AGPM can be expressed as below for $a_i = n(i-1)$ and $b_j = m(j-1), i = 1, 2, \ldots, N, j = 1, 2, \ldots, N$

$$\text{AGPM} = \begin{bmatrix} \text{AGPM}_{11} & \ldots & \text{AGPM}_{1N} \\ \vdots & \ddots & \vdots \\ \text{AGPM}_{N1} & \cdots & \text{AGPM}_{NN} \end{bmatrix} \tag{10}$$

with

$$\text{AGPM}_{ij} = \begin{bmatrix} gpf_{a_i+1, b_j+1} & \cdots & gpf_{a_i+1, b_j+m} \\ \vdots & \ddots & \vdots \\ gpf_{a_i+n, b_j+1} & \cdots & gpf_{a_i+n, b_j+m} \end{bmatrix}$$

and $gpf_{l,k}$ represents the power generation proportion of Gencom l in the total load demand of Discom k presented in the contract.

The corresponding new load demand signals in the signed contract are represented by the red dotted lines in Fig. 1, which can be regarded as the additional disturbance of the traditional LFC scheme. They are processed as shown in [33]

$$\begin{aligned} w_{1i} &= \Delta P_{di} + \Delta P_{Li} \\ &= \sum_{j}^{m} \Delta P_{ULj-i} + \sum_{j}^{m} \Delta P_{Lj-i} \end{aligned}$$

$$w_{2i} = \sum_{j=1, j\neq i}^{N} T_{ij}\Delta f_j$$

$$w_{3i} = \sum_{k=1, k\neq i}^{N} \Delta P_{\text{tie}, ik, sch}$$

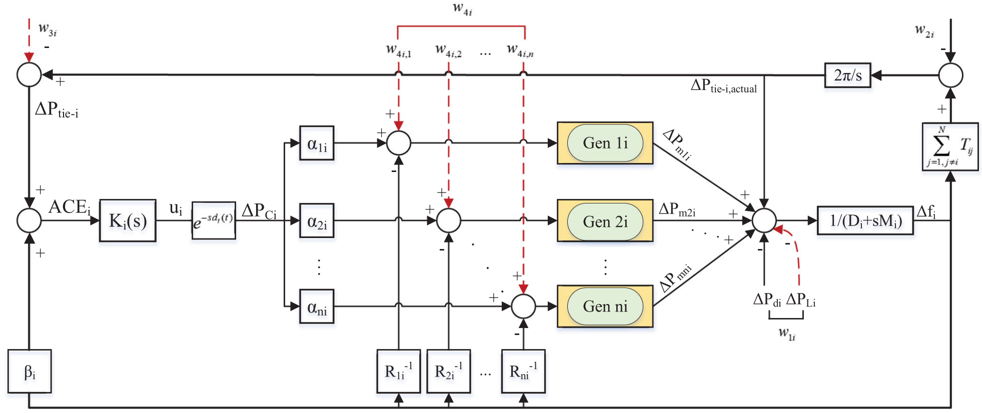

Fig. 1. LFC scheme structure of area i.

TABLE II
EXPLANATION OF TERMINOLOGIES

Terminology	Meaning
ΔP_{Li}	contracted requirements
ΔP_{di}	uncontracted requirements
ΔP_{Lj-i}	contracted requirements of Discom j in area i
ΔP_{ULj-i}	uncontracted requirements of Discom j in area i
$\Delta P_{tie,ik,sch}$	presupposed power tie-line exchange between area i and k
$\Delta P_{m,k-i}$	power generation desired of Gencom k in area i

$$\begin{aligned} w_{4i}^T &= \left[w_{4i,1} \cdots w_{4i,k} \cdots w_{4i,n} \right] \\ \Delta P_{\text{tie},ik,sch} &= \sum_{j=1}^{n}\sum_{t=1}^{m} gpf_{a_i+j,b_k+t}\Delta P_{Lt-k} \\ &\quad - \sum_{j=1}^{n}\sum_{t=1}^{m} gpf_{a_k+j,b_i+t}\Delta P_{Lt-i} \\ \Delta P_{tie-i} &= \Delta P_{\text{tie}-i,\text{actual}} - w_{3i} \\ w_{4i,k} &= \sum_{j=1}^{N}\sum_{t=1}^{m} gpf_{a_i+k,b_j+t}\Delta P_{Lt-j} \\ \Delta P_{m,k-i} &= w_{4i,k} + \alpha_{ki}\sum_{j=1}^{m}\Delta P_{ULj-i} \end{aligned}$$

where some terms are explained in Table II.

The SSM of area i is constructed as follows:

$$\begin{cases} \dot{x}_i(t) = \tilde{A}_i x_i(t) + \tilde{B}_i u(t-d_i(t)) + \tilde{F}_i w_i \\ y_i(t) = \tilde{C}_{yi} x_i(t) + \tilde{C}_{wi} w_i \\ z_i(t) = \tilde{D}_i x_i(t) \end{cases} \tag{11}$$

where $x_i(t)$, $y_i(t)$, and $z_i(t)$ are consistent with ones given in model (6), in addition

$$\tilde{F}_i = \begin{bmatrix} \tilde{F}_{i11} & 0 \\ 0 & 0 \\ 0 & \tilde{F}_{i32} \end{bmatrix}, \tilde{F}_{i32} = \text{diag}\left\{ \frac{1}{T_{g1i}}, \ldots, \frac{1}{T_{g1n}} \right\}$$

$$\tilde{F}_{i11} = \begin{bmatrix} -\frac{1}{M_i} & 0 & 0 \\ 0 & -2\pi & 0 \end{bmatrix}, \tilde{C}_{wi} = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}$$

$$w_i^T = \left[w_{1i}^T \; w_{2i}^T \; w_{3i}^T \; w_{4i}^T \right].$$

Naturally, the CLS $(\tilde{\Sigma})$ can be obtained as

$$\begin{cases} \dot{\check{x}}_i(t) = \check{A}_i \check{x}_i(t) + \check{A}_{di}\check{x}_i(t-d_i(t)) + \bar{B}_{wi} w_i \\ \check{y}_i(t) = \check{C}_{yi}\check{x}_i(t) + \bar{C}_i w_i \\ \check{z}_i(t) = \check{D}_i \check{x}_i(t) \end{cases} \tag{12}$$

where $\check{x}_i(t)$, $\check{y}_i(t)$, $\check{z}_i(t)$, $\check{A}_i$, $\check{A}_{di}$, $\check{C}_{yi}$, and $\check{D}_i$ are consistent with ones given in model (9), and other matrices are defined as below

$$\bar{B}_{wi} = \check{F}_i - \check{B}_i K_i \bar{C}_i, \check{F}_i = \begin{bmatrix} \tilde{F}_i \\ \check{C}_i \end{bmatrix}, \bar{C}_i^T = \left[\check{C}_i^T \; 0 \; \tilde{F}_i^T \check{C}_{yi}^T \right].$$

In conclusion, the CLS $(\check{\Sigma})$ and the CLS $(\tilde{\Sigma})$ can be uniformly expressed as the following CLS (Σ):

$$\begin{cases} \dot{x}(t) = \mathcal{A}x(t) - \mathcal{BKC}_y x(t-d(t)) + \mathcal{B}_w w(t) \\ z(t) = \mathcal{D}x(t) \end{cases} \tag{13}$$

where $d(t)$ represents time delays, and $0 < d(t) < d$, $\dot{d}(t) \leq \mu \leq 1$.

Remark 1: Frequency stability is an important indicator to measure the power quality in power systems. The deviation of the exchange power of the tie line between systems and the fluctuation of the system frequency may be caused by any sudden changes in load. Therefore, the LFC scheme is urgently needed in order to guarantee the power quality. In this article, the LFC issue of power systems is studied mainly aiming at the time delays caused by two communication methods.

A definition and a lemma need to be displayed here before we proceed with the next step of calculation and discussion.

Definition 1: [34] The system is considered to be asymptotically stable and meets the H_∞ performance index γ if the following conditions are met.

1) The system is asymptotically stable when the disturbance input is not taken into account (i.e., $w(t) \equiv 0$).

2) For any nonzero disturbance, given a positive scalar γ, the following inequality is satisfied under zero initial conditions ($x(t) = 0, t \in [-d, 0]$):

$$\Omega = \int_0^\infty [z^T(\alpha)z(\alpha) - \gamma^2 w^T(\alpha)w(\alpha)]d\alpha \leq 0. \tag{14}$$

Lemma 1: [32] Given a matrix $Z(\in \mathbb{R}^{n\times n}) > 0$, if there exists any matrix $T_\iota \in \mathbb{R}^{n\times m}(\iota = 0, 1, 2, \ldots, H, \text{and } H \in \mathbb{N})$ and a vector $\eta \in \mathbb{R}^m$, such that for any continuous and differentiable function $e(\cdot)$ from $[a, b]$ to $\mathbb{R}^n$, the following inequality holds:

$$-\int_a^b \dot{e}(\alpha)^T Z \dot{e}(\alpha) d\alpha \leq \sum_{\kappa=0}^{H}\left[2\lambda_H^T \theta_H^T(\kappa) T_\kappa \eta + \frac{b-a}{(2\kappa+1)}\eta^T T_\kappa^T Z^{-1} T_\kappa \eta\right] \quad (15)$$

where

$$\lambda_H \triangleq \begin{cases} \left[e^T(b)\ e^T(a)\right]^T, & H = 0 \\ \left[e^T(b)\ e^T(a)\ \frac{1}{b-a}\Psi_0^T \cdots \frac{1}{b-a}\Psi_{H-1}^T\right]^T, & H > 0 \end{cases}$$

$$\theta_H(\kappa) \triangleq \begin{cases} \left[I\ -I\right], & H = 0 \\ \left[I\ (-1)^{\kappa+1} I\ \sigma_{H\kappa}^0 I \cdots \sigma_{H\kappa}^{N-1} I\right], & H > 0 \end{cases}$$

$$\sigma_{H\kappa}^\iota \triangleq \begin{cases} -(2\iota+1)(1-(-1)^{\kappa+\iota}), & \iota \leq \kappa \\ 0, & \iota \geq \kappa + 1 \end{cases}$$

$$U_\kappa(\alpha) \triangleq (-1)^\kappa \sum_{\bar{\kappa}=0}^{\kappa}\left[(-1)^{\bar{\kappa}}\binom{\kappa}{\bar{\kappa}}\binom{\kappa+\bar{\kappa}}{\bar{\kappa}}\right]\left(\frac{\alpha-a}{b-a}\right)^{\bar{\kappa}}$$

$$\Psi_\kappa \triangleq \int_a^b U_\kappa(\alpha) e(\alpha) d\alpha.$$

Remark 2: As a powerful tool in studying the stability issue of power systems with time delays, the inequality technology cannot be underestimated. The Jensen inequality, as well as the Wirtinger-based integral inequality, is commonly used in some previous references. Differently, a further improved inequality is employed in this article to reduce the conservatism of the results. By adjusting the value of H, the conservatism brought by the above-mentioned two inequality techniques may be reduced effectively. What is worth mentioning, the selection of η also plays a significant role in reducing the conservatism. If more state variables are introduced to η, the results will be less conservative.

III. Main Results

In order to make the calculation process more concise, some expressions are given in advance as below

$$\varphi(t) \triangleq \left[\varphi_1^T(t)\ \varphi_2^T(t)\ \varphi_3^T(t)\right]^T$$

$$\varphi_1(t) \triangleq \begin{bmatrix} x(t) \\ x(t-d(t)) \\ x(t-d) \end{bmatrix}, \Pi \triangleq \begin{bmatrix} I_n & -I_n & 0 & 0 \\ I_n & I_n & -2I_n & 0 \\ I_n & -I_n & 0 & -6I_n \end{bmatrix}$$

$$\varphi_2(t) \triangleq \begin{bmatrix} \frac{1}{d-d(t)}\int_{t-d}^{t-d(t)} x(\alpha)d\alpha \\ \frac{1}{d-d(t)}\int_{t-d}^{t-d(t)}(2\frac{\alpha-t+d}{d-d(t)}-1)x(\alpha)d\alpha \end{bmatrix}$$

$$\varphi_3(t) \triangleq \begin{bmatrix} \frac{1}{d(t)}\int_{t-d(t)}^{t} x(\alpha)d\alpha \\ \frac{1}{d(t)}\int_{t-d(t)}^{t}(2\frac{\alpha-t+d(t)}{d(t)}-1)x(\alpha)d\alpha \\ w(t) \end{bmatrix}$$

$$\bar{Z}_w \triangleq \text{diag}\{\bar{Z}, 3\bar{Z}, 5\bar{Z}\}, R_w \triangleq \text{diag}\{R, 3R, 5R\}$$

$$e_\tau \triangleq \left[0_{n\times(\tau-1)n}\ I_n\ 0_{n\times(8-\tau)n}\right], \tau = 1, 2, \ldots, 8$$

$$c_\kappa \triangleq \left[0_{n\times(\kappa-1)n}\ I_n\ 0_{n\times(9-\kappa)n}\right], \kappa = 1, 2, \ldots, 9$$

$$l_\varpi \triangleq \left[0_{n\times(\varpi-1)n}\ I_n\ 0_{n\times(10-\varpi)n}\right], \varpi = 1, 2, \ldots, 10$$

$$\sum_1 \triangleq \left[c_2^T\ c_3^T\ c_4^T\ c_5^T\right]^T, \sum_2 \triangleq \left[c_1^T\ c_2^T\ c_6^T\ c_7^T\right]^T.$$

A. H_∞ Performance Analysis

Theorem 1: For given positive scalars γ, d, μ, assuming there exist matrices $P > 0$, $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$, $\bar{Z} > 0$ and matrices $\tilde{Y}_1^{3\times 10}, \tilde{Y}_2^{3\times 10}$, the system (Σ) is asymptotically stable and meets an H_∞ performance index γ, if the following conditions hold:

$$\Theta_1 \triangleq \begin{bmatrix} \Lambda & d\tilde{Y}_1^T \\ * & -\bar{Z}_w \end{bmatrix}_{13\times 13} < 0 \quad (16)$$

$$\Theta_2 \triangleq \begin{bmatrix} \Lambda & d\tilde{Y}_2^T \\ * & -\bar{Z}_w \end{bmatrix}_{13\times 13} < 0 \quad (17)$$

where

$$\Lambda \triangleq \tilde{\Lambda}_1 + \tilde{\Lambda}_2 + \tilde{\Lambda}_3 + \Lambda_4 + \Lambda_5$$

$$\tilde{\Lambda}_1 \triangleq \text{sym}\{l_1^T P\mathcal{A} l_1 - l_1^T P\mathcal{B}\mathcal{K}\mathcal{C}_y l_2 + l_1^T P\mathcal{B}_w l_8\}$$

$$\tilde{\Lambda}_2 \triangleq l_1^T \bar{Q}_2 l_1 - (1-\mu) l_2^T \bar{Q}_2 l_2 + l_1^T \bar{Q}_1 l_1 + l_3^T \bar{Q}_1 l_3$$

$$\tilde{\Lambda}_3 \triangleq \text{sym}\{d\tilde{\Gamma}_1^T \Pi^T \tilde{Y}_1 + d\tilde{\Gamma}_2^T \Pi^T \tilde{Y}_2\}$$

$$\Lambda_4 \triangleq d l_1^T \mathcal{A}^T \bar{Z} l_9 + d l_8^T \mathcal{B}_w^T \bar{Z} l_9 - d l_2^T \mathcal{C}_y^T \mathcal{K}^T \mathcal{B}^T \bar{Z} l_9$$

$$\Lambda_5 \triangleq -l_8^T \gamma^2 I l_8 - l_9^T Z l_9 + l_1^T \mathcal{D}^T l_{10} + l_{10}^T I l_{10}.$$

Proof: Choosing the appropriate Lyapunov–Krasovskii functional (LKF), we have

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (18)$$

$$V_1(t) = x^T(t) P x(t) \quad (19)$$

$$V_2(t) = \int_{t-d(t)}^{t} x^T(\alpha)\bar{Q}_2 x(\alpha)d\alpha + \int_{t-d}^{t} x^T(\alpha)\bar{Q}_1 x(\alpha)d\alpha \quad (20)$$

$$V_3(t) = d\int_{-d}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha)\bar{Z}\dot{x}(\alpha)d\alpha d\beta. \quad (21)$$

Taking the derivative of the functional, and the following formulas can be easily obtained:

$$\begin{aligned} \dot{V}_1(t) &= \text{sym}\{x^T(t) P\dot{x}(t)\} \\ &= \text{sym}\{x^T(t) P\mathcal{A} x(t) \\ &\quad - x^T(t) P\mathcal{B}\mathcal{K}\mathcal{C}_y x(t-d(t)) + x^T(t) P\mathcal{B}_w w(t)\} \\ &= \varphi^T(t)[\text{sym}\{e_1^T P\mathcal{A} e_1 + e_1^T P\mathcal{B}_w e_8 \\ &\quad - e_1^T P\mathcal{B}\mathcal{K}\mathcal{C}_y e_2\}]\varphi(t) \end{aligned} \quad (22)$$

$$\begin{aligned} \dot{V}_2(t) &= x^T(t)\bar{Q}_2 x(t) + x^T(t)\bar{Q}_1 x(t) \\ &\quad - x^T(t-d)\bar{Q}_1 x(t-d) \end{aligned}$$

$$
\begin{aligned}
&- (1-\dot{d}(t))x^T(t-d(t))\bar{Q}_2x(t-d(t)) \\
&= \varphi^T(t)[e_1^T\bar{Q}_2e_1 - (1-\dot{d}(t))e_2^T\bar{Q}_2e_2 \\
&\quad + e_1^T\bar{Q}_1e_1 + e_3^T\bar{Q}_1e_3]\varphi(t)
\end{aligned} \tag{23}
$$

$$
\dot{V}_3(t) = d^2\dot{x}^T(t)\bar{Z}\dot{x}(t) - d\int_{t-d}^{t}\dot{x}^T(\alpha)\bar{Z}\dot{x}(\alpha)d\alpha. \tag{24}
$$

Combining with Lemma 1 and setting $H = 2$, the following expressions hold:

$$
\begin{aligned}
-d\int_{t-d}^{t}\dot{x}^T(\alpha)\bar{Z}\dot{x}(\alpha)d\alpha &= -d\int_{t-d}^{t-d(t)}\dot{x}^T(\alpha)\bar{Z}\dot{x}(\alpha)d\alpha \\
&\quad - d\int_{t-d(t)}^{t}\dot{x}^T(\alpha)\bar{Z}\dot{x}(\alpha)d\alpha
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
&-d\int_{t-d}^{t-d(t)}\dot{x}^T(\alpha)\bar{Z}\dot{x}(\alpha)d\alpha \\
&\le \varphi^T(t)[d(\mathrm{sym}\{\Gamma_1^T\Pi^TY_1\} + (d-d(t))Y_1^T\bar{Z}_w^{-1}Y_1)]\varphi(t)
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
&-d\int_{t-d(t)}^{t}\dot{x}^T(\alpha)\bar{Z}\dot{x}(\alpha)d\alpha \\
&\le \varphi^T(t)[d(\mathrm{sym}\{\Gamma_2^T\Pi^TY_2\} + d(t)Y_2^T\bar{Z}_w^{-1}Y_2)]\varphi(t)
\end{aligned} \tag{27}
$$

where Y_1 and Y_2 are matrices with appropriate dimensions and

$$
\begin{aligned}
\tilde{Y}_1 &\triangleq \begin{bmatrix} Y_1 \; 0_{3\times 2} \end{bmatrix}, \tilde{Y}_2 \triangleq \begin{bmatrix} Y_2 \; 0_{3\times 2} \end{bmatrix} \\
\Gamma_1 &\triangleq \begin{bmatrix} e_2^T \; e_3^T \; e_4^T \; e_5^T \end{bmatrix}^T, \Gamma_2 \triangleq \begin{bmatrix} e_1^T \; e_2^T \; e_6^T \; e_7^T \end{bmatrix}^T \\
\tilde{\Gamma}_1 &\triangleq \begin{bmatrix} l_2^T \; l_3^T \; l_4^T \; l_5^T \end{bmatrix}^T, \tilde{\Gamma}_2 \triangleq \begin{bmatrix} l_1^T \; l_2^T \; l_6^T \; l_7^T \end{bmatrix}^T.
\end{aligned}
$$

Since $\dot{d}(t) \le \mu$, the following inequality can be obtained:

$$
\begin{aligned}
\dot{V}(t) &\le \varphi^T(t)\{\Lambda_1 + \Lambda_2 + \Lambda_3 + d[d(t)Y_2^T\bar{Z}_w^{-1}Y_2 \\
&\quad + (d-d(t))Y_1^T\bar{Z}_w^{-1}Y_1]\}\varphi(t) + d^2\dot{x}^T(t)\bar{Z}\dot{x}(t)
\end{aligned} \tag{28}
$$

where

$$
\begin{aligned}
\Lambda_1 &\triangleq \mathrm{sym}\{e_1^TP\mathcal{A}e_1 - e_1^TP\mathcal{B}\mathcal{K}\mathcal{C}_ye_2 + e_1^TP\mathcal{B}_we_8\} \\
\Lambda_2 &\triangleq e_1^T\bar{Q}_2e_1 - (1-\mu)e_2^T\bar{Q}_2e_2 + e_1^T\bar{Q}_1e_1 - e_3^T\bar{Q}_1e_3 \\
\Lambda_3 &\triangleq \mathrm{sym}\{d\Gamma_1^T\Pi^TY_1 + d\Gamma_2^T\Pi^TY_2\}.
\end{aligned}
$$

Based on the above-mentioned calculation, it is not difficult to get the following inequality:

$$
\begin{aligned}
\Xi &= \dot{V}(t) + z^T(t)z(t) - \gamma^2w^T(t)w(t) \\
&\le \varphi^T(t)\{\Lambda_1 + \Lambda_2 + \Lambda_3 - e_8^T\gamma^2Ie_8 \\
&\quad + d[(d-d(t))Y_1^T\bar{Z}_w^{-1}Y_1 + d(t)Y_2^T\bar{Z}_w^{-1}Y_2]\}\varphi(t) \\
&\quad + d^2\dot{x}^T(t)\bar{Z}\dot{x}(t) + z^T(t)z(t).
\end{aligned} \tag{29}
$$

Combining with Schur complement, one can get the following inequality from inequalities (16) and (17):

$$
\Xi < 0. \tag{30}
$$

Due to $0 < t < \infty$, associating with (14), it has

$$
\begin{aligned}
V(F) - V(0) + \int_0^\infty z^T(\alpha)z(\alpha)d\alpha - \gamma^2\int_0^\infty w^T(\alpha)w(\alpha) \\
d\alpha < 0.
\end{aligned} \tag{31}
$$

Since $V(\infty) \ge 0, V(0) = 0$, then

$$
\int_0^\infty z^T(\alpha)z(\alpha)d\alpha - \gamma^2\int_0^\infty w^T(\alpha)w(\alpha)d\alpha \le 0 \tag{32}
$$

which indicates that the system (Σ) meets the H_∞ performance index. This completes the proof of Theorem 1. ■

B. Controller Design Depends on LMIs and ICCL Algorithm

Theorem 2: For given positive scalars γ, d, μ, assuming there exist matrices $S > 0$, $\mathring{Q}_1 > 0$, $\mathring{Q}_2 > 0$, $R > 0$ and matrices $\mathring{Y}_1^{3\times 10}$, $\mathring{Y}_2^{3\times 10}$, the system ($\Sigma$) is asymptotically stable and has an H_∞ performance index γ against disturbance, if the following conditions hold:

$$
\begin{bmatrix} \mathring{\Lambda} & d\mathring{Y}_1^T \\ * & -SR_w^{-1}S \end{bmatrix}_{13\times 13} < 0 \tag{33}
$$

$$
\begin{bmatrix} \mathring{\Lambda} & d\mathring{Y}_2^T \\ * & -SR_w^{-1}S \end{bmatrix}_{13\times 13} < 0 \tag{34}
$$

where

$$
\begin{aligned}
\mathring{\Lambda} &\triangleq \mathring{\Lambda}_1 + \mathring{\Lambda}_2 + \mathring{\Lambda}_3 + \mathring{\Lambda}_4 + \mathring{\Lambda}_5 \\
\mathring{\Lambda}_1 &\triangleq \mathrm{sym}\{l_1^T\mathcal{A}Sl_1 - l_1^T\mathcal{B}K_Vl_2 + l_1^T\mathcal{B}_wl_8\} \\
\mathring{\Lambda}_2 &\triangleq l_1^T\mathring{Q}_2l_1 - (1-\mu)l_2^T\mathring{Q}_2l_2 + l_1^T\mathring{Q}_1l_1 - l_3^T\mathring{Q}_1l_3 \\
\mathring{\Lambda}_3 &\triangleq \mathrm{sym}\{d\tilde{\Gamma}_1^T\Pi^T\mathring{Y}_1 + d\tilde{\Gamma}_2^T\Pi^T\mathring{Y}_2\} \\
\mathring{\Lambda}_4 &\triangleq dl_1^TS\mathcal{A}^Tl_9 - dl_2^TK_V^T\mathcal{B}^Tl_9 + dl_8^T\mathcal{B}_w^Tl_9 - l_9^TRl_9 \\
\mathring{\Lambda}_5 &\triangleq -l_8^T\gamma^2Il_8 + l_1^TS\mathcal{D}^Tl_{10} - l_{10}^TIl_{10}
\end{aligned}
$$

with the controller gain matrix calculated by

$$
K = K_V(\mathcal{C}yS)^+ \tag{35}
$$

where the generalized inverse of $(\mathcal{C}yS)$ is expressed as $(\mathcal{C}yS)^+$.

Proof: For Θ_1 and Θ_2 given above, multiply by diag $\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, \bar{Z}^{-1}, I, P^{-1}, P^{-1}, P^{-1}\}$ and its transpose on the left-hand side and right-hand side, and define the following new matrices:

$$
\begin{aligned}
S &\triangleq P^{-1}, R \triangleq \bar{Z}^{-1}, K_V \triangleq K\mathcal{C}_yS \\
\mathring{Q}_\varkappa &\triangleq P^{-1}\bar{Q}_\varkappa P^{-1}, \mathring{Y}_\varkappa \triangleq P^{-1}Y_\varkappa P^{-1}, \varkappa = 1, 2.
\end{aligned}
$$

Through this series of means, conditions (16) and (17) can be converted to (33) and (34) successfully, and the controller gain can be calculated by (35).

Obviously, the matrices in $SR_w^{-1}S$ are unknown, and need to be determined. So the conditions in Theorem 2 are no longer conditions based on LMIs and cannot be processed simply by the convex optimization algorithm. In addition, this nonconvex problem can be transformed into the nonlinear minimization problem.

$$
\text{Minimize trace}\{S\bar{S} + R\bar{R} + O\bar{O}\} \tag{36}
$$

$$\text{subject to} \begin{cases} S > 0, \mathring{Q}_1 > 0, \mathring{Q}_2 > 0, R > 0 \\ \begin{bmatrix} \mathring{\Lambda} & d\mathring{Y}_1^T \\ * & -O \end{bmatrix} < 0, \begin{bmatrix} \mathring{\Lambda} & d\mathring{Y}_2^T \\ * & -O \end{bmatrix} < 0 \\ \begin{bmatrix} \bar{O} & \bar{S} \\ * & \bar{R} \end{bmatrix} \geq 0, \begin{bmatrix} \bar{S} & I \\ * & S \end{bmatrix} \geq 0 \\ \begin{bmatrix} \bar{R} & I \\ * & R \end{bmatrix} \geq 0, \begin{bmatrix} \bar{O} & I \\ * & O \end{bmatrix} \geq 0. \end{cases} \tag{37}$$

The controller gain matrix can be obtained by combining the ICCL algorithm and the LMIs, as described in the following algorithm. ■

Algorithm 1: Acquisition of the Controller Gain.

Input: A set of system parameters; a delay upper bound d; an H_∞ performance index γ

Output: The controller gain K

1: Construct LMIs (16), (17) and (37);
2: Solve the LMIs (37);
3: **if** condition (37) is feasible, **then** proceed to the step 6;
4: **else** reset the Input;
5: **end if**
6: Solve the nonlinear minimization problem: Minimize (36) subject to LMIs (37);
7: Solve the controller gain K by using (35);
8: Substitute the obtained gain K into LMIs (16) and (17);
9: **if** conditions (16) and (17) is feasible, **then** proceed to the step 12;
10: **else** Skip back to step 2;
11: **end if**
12: **Return** K.

Remark 3: Combining Algorithm 1 with the binary search technique shown in Fig. 2 , the controller gain can be optimized. Algorithm 2 is used to further illustrate the method, in which the most important step is to find a controller gain that meets the performance index before reaching the maximum number of iterations. In other words, a step that cannot be omitted is to check whether the obtained controller gain solves the nonlinear problem while ensuring the feasibility of conditions (16) and (17).

Remark 4: Based on the optimized Algorithm 2, the gain of the PID-type controller, in this article, is determined according to the minimum H_∞ performance index, ensuring that the CLS has a good antiinterference ability while operating stably. The controller designed in this article relies on LMIs and ICCL Algorithm, and its feasibility is verified by MATLAB software. At present, there are various methods to verify the effect of the controller [35], which is also one of the research directions we will devote to in the future.

Remark 5: It is worth mentioning that when the delay does not reach a certain value, the feasibility of the LMIs can be guaranteed. The corresponding value is usually called the maximum allowable time delay, which is difficult to be determined due to the variability of system models and parameters. In the design of the LFC scheme, the communication channel used in actual power systems largely determines the preset delay upper bound, which is generally small and less than the tolerable delay upper bound of the system. Therefore, it is natural to use the proposed method to the designed LFC scheme in power systems.

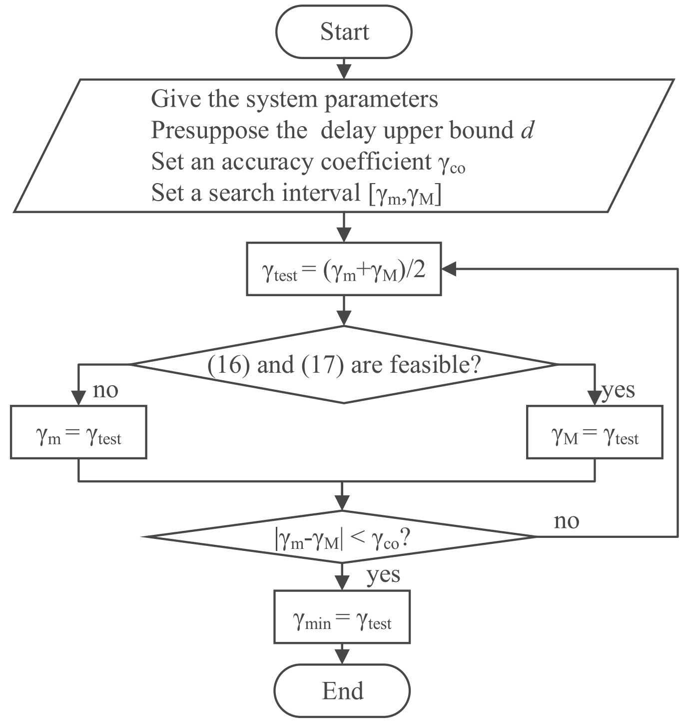

Fig. 2. Binary search optimization for solving optimal performance index.

C. Delay-Dependent Stability Criterion for One-Area System

Corollary 1: For given scalars $\varepsilon \geq 0, \theta \geq 0, \lambda \geq 0$, assuming there exist matrices $\bar{Q}_2 > 0$, $\bar{Q}_1 > 0$, $P > 0$, $\bar{Z} > 0$ and any matrices $\tilde{\Omega}_1^{3\times 9}$, $\tilde{\Omega}_2^{3\times 9}$, E, G, the system (5) is asymptotically stable if the following conditions hold:

$$\Psi_1 \triangleq \begin{bmatrix} \Upsilon & d\tilde{\Omega}_1^T \\ * & -\bar{Z}_w \end{bmatrix}_{12\times 12} < 0 \tag{38}$$

$$\Psi_2 \triangleq \begin{bmatrix} \Upsilon & d\tilde{\Omega}_2^T \\ * & -\bar{Z}_w \end{bmatrix}_{12\times 12} < 0 \tag{39}$$

where

$$\begin{aligned} \Upsilon &\triangleq \Upsilon_1 + \Upsilon_2 + \Upsilon_3 + \Upsilon_4 + \Upsilon_5 \\ \Upsilon_1 &\triangleq \text{sym}\{c_1^T P\hat{A}c_1 - c_1^T P\hat{B}c_2 + c_1^T Pc_8\} \\ \Upsilon_2 &\triangleq c_1^T\bar{Q}_2 c_1 - (1-\mu)c_2^T\bar{Q}_2 c_2 + c_1^T\bar{Q}_1 c_1 + c_3^T\bar{Q}_1 c_3 \\ \Upsilon_3 &\triangleq \text{sym}\{d\Sigma_1^T\Pi^T\tilde{\Omega}_1 + d\Sigma_2^T\Pi^T\tilde{\Omega}_2\} \\ \Upsilon_4 &\triangleq dc_1^T\hat{A}^T\bar{Z}c_9 + dc_2^T\hat{B}^T\bar{Z}c_9 + dc_8^T\bar{Z}c_9 - dc_9^T\bar{Z}c_9 \\ \Upsilon_5 &\triangleq \lambda\varepsilon^2 c_1^T E^T Ec_1 + \lambda\theta^2 c_2^T G^T Gc_2 - \lambda c_8^T Ic_8. \end{aligned}$$

Proof: The proof process of Corollary 1 is similar to Theorem 1, so it is omitted here. ■

Algorithm 2: Optimization of the Controller Gain.

Input: A set of system parameters; a delay upper bound d; a search interval $[\gamma_m, \gamma_M]$; an accuracy coefficient γ_{co}; the upper bound of the number of iterations c_{max}

Output: K_{opt}

1: Set $\gamma_{test} = \gamma_M$; $Count = 0$;
2: **if** condition (37) is feasible, **then** go to step 8;
3: **else**
4:     **if** $Count = 0$, **then** cannot find a suitable solution, end algorithm;
5:     **else** go to step 14;
6:     **end if**
7: **end if**
8: Set $Count = 1$; $c = 0$; and extract initial values $\bar{O}_c$, $\bar{S}_c$, $\bar{R}_c$, O_c, S_c, R_c from solutions of (37);
9: Solve the nonlinear minimization problem: Minimize (36) subject to LMIs (37);
10: Obtain K by (35), substitute the obtained controller gain K into LMIs (16) and (17);
11: **if** conditions (16) and (17) are feasible, **then** $\gamma_{min} = \gamma_{test}, \gamma_M = \gamma_{test}$;
12: **else**
13:     **if** $c < c_{max}$, **then** set $c = c + 1, \bar{O}_c = \bar{O}, \bar{S}_c = \bar{S}, \bar{R}_c = \bar{R}, O_c = O, S_c = S, R_c = R$, and go to step 8;
14:     **else** $\gamma_m = \gamma_{test}$;
15:     **end if**
16: **end if**
17: **if** $|\gamma_m - \gamma_M| < \gamma_{co}$, **then** output $K_{m\gamma} = K_{opt}$, go to step 20;
18: **else** $\gamma_{test} = |\gamma_m + \gamma_M|/2$, and reverse back to step 2;
19: **end if**
20: **Return** K_{opt}.

Remark 6: It is worth noting that when power systems encounter the unknown external load disturbance, it can be considered as a nonlinear perturbation of the current and delayed state vector as follows [36]:

$$\hat{F}w(t) = f(\hat{x}(t), \hat{x}(t - d(t)))$$

which meets the following condition:

$$||f(\hat{x}(t), \hat{x}(t - d(t)))|| \leq \varepsilon||\hat{x}(t)|| + \theta||\hat{x}(t - d(t))|| \quad (40)$$

where ε and θ are known nonnegative scalars. A more generalized form of the condition is adopted and shown as follows:

$$\begin{aligned} &f^T(\hat{x}(t), \hat{x}(t - d(t)))f(\hat{x}(t), \hat{x}(t - d(t))) \\ &\leq \varepsilon^2 \hat{x}^T(t)E^T E\hat{x}(t) + \theta^2 \hat{x}^T(t - d(t))G^T Gx(t - d(t)) \end{aligned} \quad (41)$$

in which E and G are both known constant matrices with appropriate dimensions. The influence of load disturbances on power systems can be concretized by nonnegative scalars ε, θ and matrices E, G.

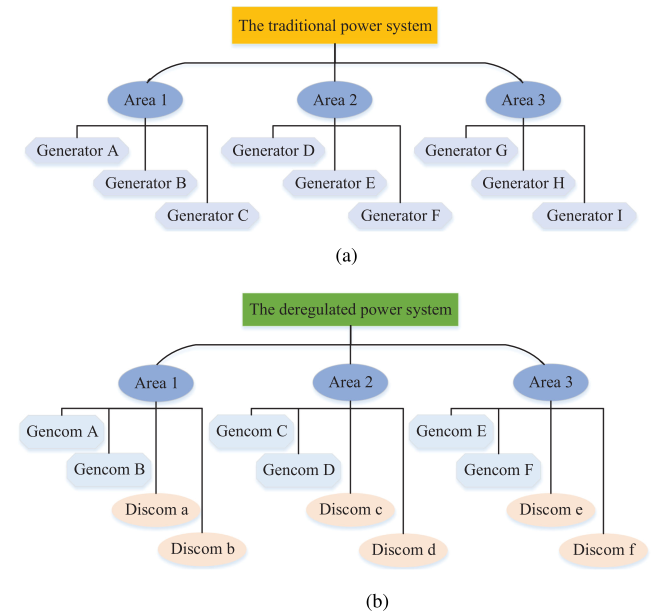

Fig. 3. Composition of power systems. (a) Traditional power system. (b) Deregulated power system.

D. Summary of Analysis Steps

The whole article mainly involves the H_∞ performance analysis of the CLS (Σ) and its PID-type controller design, as well as the improvement of stability criterion for one-area delayed LFC scheme. It can be divided into the following steps.

Step 1. Model Establishment: Considering a PID-type controller as a static output feedback controller, and the CLS model such as the system (Σ) is established.

Step 2. H_∞ Performance Analysis: By constructing an LKF and using an enhanced inequality technique, based on the above-mentioned system model, stability criterion, which can ensure that the system satisfies H_∞ performance, is obtained.

Step 3. Controller Design: Utilizing the above-mentioned results and according to Algorithm 2, an optimized controller can be obtained.

Step 4. Stability Analysis of One-Area System: By using nonnegative scalars ε, θ, and matrices E, G to quantify the impact of load disturbance on power systems, the delay-dependent stability criterion is obtained.

Step 5. Case Studies and Comparisons: The validity and superiority of our method can be fully proved by the comparisons.

IV. Case Studies

In this section, first, taking the one-area LFC scheme as an example, the relationships between different types of delays and controller gain are discussed. Then, the model of the traditional/deregulated power systems in Fig. 3 is analyzed.

A. Case 1: One-Area LFC Scheme

Consider the system parameters of one-area, as described in Table III. Suppose the time delay $d(t)$ as two cases: constant

TABLE III
PARAMETERS OF ONE-AREA LFC SYSTEM

Parameter	β	R	D	$M(s)$	$T_t(s)$	$T_g(s)$
One-area	21	0.05	1.0	10	0.3	0.1

TABLE IV
MAXIMUM ALLOWABLE DELAY d FOR LFC SYSTEM

Methods		$\mu=0$		$\mu=0.9$	
K_P	K_I	Corollary 1	[15]	Corollary 1	[15]
0.2	0.2	9.97	6.53	6.14	3.23
0.2	0.4	5.50	3.32	3.44	1.43
0.2	0.6	3.86	2.10	2.53	0.96
0.4	0.2	7.57	5.38	2.15	0.88
0.4	0.4	4.45	2.83	2.00	0.78
0.4	0.6	3.25	1.91	1.80	0.67

delays ($\mu_{\text{CDs}}=0$) and time-varying delays ($\mu_{\text{TVDs}}=0.9$). In these two cases, the maximum allowable delay is explored to ensure the stability of the system, and its relationship with the controller gain is discussed. Set the parameters $\varepsilon=0$, $\theta=0$, and $E=G=0.1I_4$. With the help of MATLAB software, the maximum allowable delay under diverse values of controller gains (K_P, K_I) can be obtained. The specific data are described in Table IV . Based on the calculation results given in Table IV, the relationship between the controller gain and the maximum allowable delay can be summarized, and the influence of different delay types on the maximum allowable delay under the same controller gain can be understood. It is not difficult to find from Table IV that for any type of delays and fixed proportional gain K_P, the maximum allowable delay decreases as the integral gain K_I increasing. And when a fixed controller gain is guaranteed, the maximum allowable delay obtained is generally relatively small when the type is time-varying delay. These changing rules can be used as auxiliary conditions for the design and adjustment of the LFC scheme. Through the comparison of the data in Table IV, the maximum allowable value calculated based on our method is larger, which reflects the effectiveness and superiority of the method proposed.

Next, take the traditional/deregulated power system model in Fig. 3 as an object and analyze its performance. The selected system parameters are as follows:

$$T_t=\begin{bmatrix}0.4 & 0.44 & 0.3\\ 0.36 & 0.32 & 0.4\\ 0.42 & 0.4 & 0.41\end{bmatrix}, T_g=\begin{bmatrix}0.08 & 0.06 & 0.07\\ 0.06 & 0.06 & 0.07\\ 0.07 & 0.08 & 0.08\end{bmatrix}$$

$$R=\begin{bmatrix}3 & 2.7273 & 2.8235\\ 3 & 2.6667 & 3\\ 3.3 & 2.5 & 2.9412\end{bmatrix}, \alpha=\begin{bmatrix}0.4 & 0.6 & 0\\ 0.4 & 0 & 0.5\\ 0.2 & 0.4 & 0.5\end{bmatrix}$$

$$T=\begin{bmatrix}0 & 0.2 & 0.25\\ 0.2 & 0 & 0.12\\ 0.25 & 0.12 & 0\end{bmatrix}$$

$$D=\begin{bmatrix}0.0440 & 0.0440 & 0.0460\end{bmatrix}$$

$$M=\begin{bmatrix}0.4867 & 0.5477 & 0.4784\end{bmatrix}.$$

TABLE V
OPTIMIZED CONTROLLER GAIN FOR TRADITIONAL LFC SYSTEM WHEN $d=2$

Area 1	K_1 in [23]			K_2		
	K_P	K_I	K_D	K_P	K_I	K_D
$\mu=0$	-0.0058	0.0822	0.0008	-0.0076	0.0985	0.0034
$\mu=0.5$	-0.0050	0.0844	0.0007	-0.0070	0.1008	0.0037

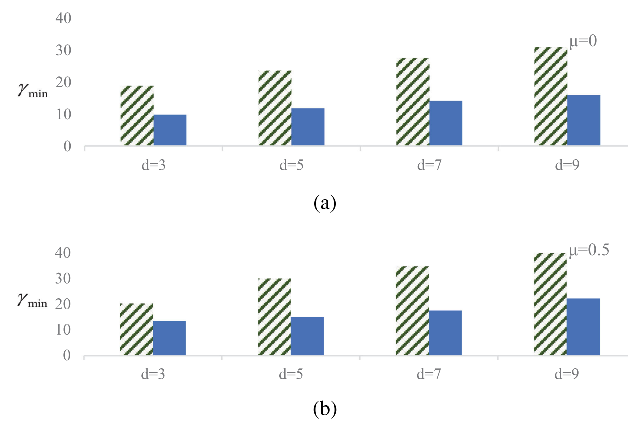

Fig. 4. Minimal $\gamma_{\min}$ relative to the delay d of traditional power system.

B. Case 2: Traditional Three-Area LFC Scheme

In this case, much attention is focused on the following two aspects.

1) Design of the Controller: Taking area one as an example, the upper bound of time delays is preset as 2 s, and $d(t)$ is considered as constant ($\mu_{\text{CDs}}=0$) and time-varying ($\mu_{\text{TVDs}}=0.5$) delays, respectively. Setting $\gamma_m=0, \gamma_M=100, \gamma_{co}=0.5, c_{\max}=20$, and by referring to the process given in Algorithm 2, the controller gain described in Table V can be obtained effortlessly.

2) Analysis of H_∞ Performance: In combination with Algorithm 2, the minimal H_∞ performance index $\gamma_{\min}$ of the CLS $(\check{\Sigma})$ is listed in Fig. 4, which can be obtained.

From Fig. 4, the variation trend of the value of $\gamma_{\min}$ is consistent with the change of delays, i.e., with the increase of time delays, the value of $\gamma_{\min}$ also increases. In addition clearly, under the same conditions (the same delay upper bound d), taking $\mu_{\text{CDs}}=0$, the value of $\gamma_{\min}$ obtained is less than the one when taking $\mu_{\text{TVDs}}=0.5$ generally. Besides, a phenomenon that cannot be ignored is that different kinds of delays have various influence on the value of the performance index $\gamma_{\min}$. Obviously, the effect of constant delays is relatively small compared to that of time-varying delays. Through the comparison in Fig. 4 (a) and (b), it is not difficult to find that the controller K_2 is better than the controller K_1 in guaranteeing H_∞ performance of the system, that is, the CLS $(\check{\Sigma})$ with controller K_2 has superior resistance to interference. This also facilitates practical applications.

C. Case 3: Deregulated Three-Area LFC Scheme

Similarly, taking area one as an example, this article will focus on the following two aspects.

TABLE VI
OPTIMIZED CONTROLLER GAIN FOR DEREGULATED LFC SYSTEM WHEN $d = 2$

Area 1	K_3 in [23]			K_4		
	K_P	K_I	K_D	K_P	K_I	K_D
$\mu = 0$	-0.0015	0.0973	-0.0020	-0.0040	0.1235	0.0008
$\mu = 0.5$	-0.0016	0.0963	-0.0016	-0.0045	0.1205	0.0015

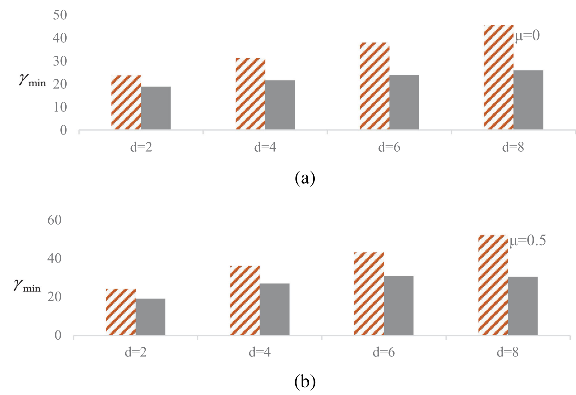

Fig. 5. Minimal $\gamma_{\min}$ relative to the delay d of deregulated power system.

1) Design of the Controller: Presetting the upper bound of the time delay as 2 s, and setting $\gamma_m = 0$, $\gamma_M = 100$, $\gamma_{co} = 0.5$, and $c_{\max} = 20$, according to the steps in Algorithm 2, it is not difficult to obtain the PID-type controller gain in Table VI. Moreover, when all parameters remain the same, the gain of the controller obtained by using the method in literature [23] is also listed in Table VI.

2) Analysis of H_∞ Performance: Setting μ_{CDs} as 0 and μ_{TVDs} as 0.5, respectively, the minimum H_∞ performance index corresponding to different time delays can be observed according to Fig. 5. It can be found that the results are similar to that in Case 2. As time delays increase, the minimum value of the performance index increases accordingly, and the influence of time-varying delays on the performance index is greater than that of constant delays under the same situation.

By combining the results of Case 2 and Case 3, and comparisons shown in Fig. 4 (a) and (b) and Fig. 5 (a) and (b), respectively, under the same delay conditions, the minimum $\gamma_{\min}$ of the robust performance index obtained by using our method is smaller than the one obtained by using the method in [23] whether in the traditional LFC system or the deregulated LFC system. Noted that a smaller value of $\gamma_{\min}$ means a better H_∞ performance of the system. Naturally, a conclusion can be clearly revealed, that is, the controller K_2 and the controller K_4 are better than the controller K_1 and the controller K_3 in guaranteeing H_∞ performance of the system, i.e., the CLS ($\check{\Sigma}$) with the controller K_2 and the CLS ($\tilde{\Sigma}$) with the controller K_4 have superior resistance to interference. According to the above-mentioned cases, the superiority of our method is verified fully.

V. CONCLUSION

In this article, the relationship between the maximum allowable delay and the controller gain has been analyzed for one-area delayed LFC scheme. Then, with the assistance of Lyapunov stability theory and a further enhanced inequality technique, a PID-type controller has been designed for the traditional/deregulated three-area LFC scheme, which can ensure that the CLS keeps the H_∞ performance index to the minimum level while ensuring that the system remains stable when the delay is less than the preset value. Furthermore, in order to approach the actual situation, time delays introduced by more open communication networks have been stressed during the design phase of the above-mentioned controller. At present, due to the importance of power systems, their stability analysis and control have always been a topic of concern, which motivates us to carry out around it in the future.

REFERENCES

[1] P. Kundur, N. J. Balu, and M. G. Lauby, *Power System Stability and Control*. New York, NY, USA: McGraw-Hill, 1994.
[2] H. Bevrani, *Robust Power System Frequency Control*. New York, NY, USA: Springer, 2009.
[3] Ibraheem, P. Kumar, and D. P. Kothari, "Recent philosophies of automatic generation control strategies in power systems," *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 346–357, Feb. 2005.
[4] W. Bi, K. Zhang, Y. Li, K. Yuan, and Y. Wang, "Detection scheme against cyber-physical attacks on load frequency control based on dynamic characteristics analysis," *IEEE Syst. J.*, vol. 13, no. 3, pp. 2859–2868, Sep. 2019.
[5] B. P. Padhy, S. C. Srivastava, and N. K. Verma, "Robust wide-area T-S fuzzy output feedback controller for enhancement of stability in multimachine power system," *IEEE Syst. J.*, vol. 6, no. 3, pp. 426–435, Sep. 2012.
[6] J. Lai, X. Lu, and X. Yu, "Stochastic distributed frequency and load sharing control for microgrids with communication delays," *IEEE Syst. J.*, vol. 13, no. 4, pp. 4269–4280, Dec. 2019.
[7] V. P. Singh, N. Kishor, and P. Samuel, "Load frequency control with communication topology changes in smart grid," *IEEE Trans. Ind. Inf.*, vol. 12, no. 5, pp. 1943–1952, Oct. 2016.
[8] X. Yu, C. Cecati, T. Dillon, and M. G. Simoes, "The new frontier of smart grids," *IEEE Ind. Electron. Mag.*, vol. 5, no. 3, pp. 49–63, Sep. 2011.
[9] X. Yu and Y. Xue, "Smart grids: A cyber-physical systems perspective," *Proc. IEEE*, vol. 104, no. 5, pp. 1058–1070, May 2016.
[10] S. Bhowmik, K. Tomsovic, and A. Bose, "Communication models for third party load frequency control," *IEEE Trans. Power Syst.*, vol. 19, no. 1, pp. 543–548, Feb. 2004.
[11] C.-K. Zhang, L. Jiang, Q. Wu, Y. He, and M. Wu, "Further results on delay-dependent stability of multi-area load frequency control," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4465–4474, Nov. 2013.
[12] Z. Zhang *et al.* "An event-triggered secondary control strategy with network delay in islanded microgrids," *IEEE Syst. J.*, vol. 13, no. 2, pp. 1851–1860, Jun. 2019.
[13] S. Wang, X. Meng, and T. Chen, "Wide-area control of power systems through delayed network communication," *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 2, pp. 495–503, Mar. 2011.
[14] H. Shen, M. Chen, Z.-G. Wu, J. Cao, and J. H. Park, "Reliable event-triggered asynchronous extended passive control for semi-Markov jump fuzzy systems and its application," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 8, pp. 1708–1722, Aug. 2020.
[15] L. Jiang, W. Yao, Q. Wu, J. Wen, and S. Cheng, "Delay-dependent stability for load frequency control with constant and time-varying delays," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 932–941, May 2012.
[16] X. Ge, Q. Han, and X. Zhang, "Achieving cluster formation of multi-agent systems under aperiodic sampling and communication delays," *IEEE Trans. Ind. Electron.*, vol. 65, no. 4, pp. 3417–3426, Apr. 2017.
[17] H. Shen, T. Wang, J. Cao, G. Lu, Y. Song, and T. Huang, "Non-fragile dissipative synchronization for Markovian memristive neural networks: A gain-scheduled control scheme," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 6, pp. 1841–1853, Jun. 2019.
[18] X.-M. Zhang, Q.-L. Han, A. Seuret, F. Gouaisbaut, and Y. He, "Overview of recent advances in stability of linear systems with time-varying delays," *IET Control Theory Appl.*, vol. 13, no. 1, pp. 1–16, 2018.
[19] H. Shen, F. Li, S. Xu, and V. Sreeram, "Slow state variables feedback stabilization for semi-Markov jump systems with singular perturbations," *IEEE Trans. Autom. Control*, vol. 63, no. 8, pp. 2709–2714, Aug. 2018.

[20] N. Olgac and R. Sipahi, "An exact method for the stability analysis of time-delayed linear time-invariant (LTI) systems," *IEEE Trans. Autom. Control*, vol. 47, no. 5, pp. 793–797, May 2002.

[21] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*. Boston, MA, USA: Birkhäuser, 2003.

[22] F. Yang, J. He, and J. Wang, "Novel stability analysis of delayed LFC power systems by infinite-series-based integral inequality," in *Proc. IEEE Conf. Control Technol. Appl.*, Aug. 2017, pp. 1384–1389.

[23] C.-K. Zhang, L. Jiang, Q. Wu, Y. He, and M. Wu, "Delay-dependent robust load frequency control for time delay power systems," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2192–2201, Aug. 2013.

[24] X.-M. Zhang, Q.-L. Han, A. Seuret, and F. Gouaisbaut, "An improved reciprocally convex inequality and an augmented Lyapunov-Krasovskii functional for stability of linear systems with time-varying delay," *Automatica*, vol. 84, pp. 221–226, 2017.

[25] A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: Application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860–2866, 2013.

[26] P. Park, W. I. Lee, and S. Y. Lee, "Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems," *J. Franklin Inst.*, vol. 352, no. 4, pp. 1378–1396, 2015.

[27] H. Shayeghi, A. Jalili, and H. Shayanfar, "A robust mixed H_2/H_∞ based LFC of a deregulated power system including SMES," *Energy Convers. Manage.*, vol. 49, no. 10, pp. 2656–2668, 2008.

[28] V. Donde, M. Pai, and I. A. Hiskens, "Simulation and optimization in an AGC system after deregulation," *IEEE Trans. Power Syst.*, vol. 16, no. 3, pp. 481–489, Aug. 2001.

[29] H. Shayeghi, A. Jalili, and H. Shayanfar, "Robust modified GA based multi-stage fuzzy LFC," *Energy Convers. Manage.*, vol. 48, no. 5, pp. 1656–1670, 2007.

[30] D. Rerkpreedapong, A. Hasanovic, and A. Feliachi, "Robust load frequency control using genetic algorithms and linear matrix inequalities," *IEEE Trans. Power Syst.*, vol. 18, no. 2, pp. 855–861, May 2003.

[31] K. Vrdoljak, N. Perić, and I. Petrović, "Sliding mode based load-frequency control in power systems," *Elect. Power Syst. Res.*, vol. 80, no. 5, pp. 514–527, 2010.

[32] H.-B. Zeng, X.-G. Liu, W. Wang, and S.-P. Xiao, "New results on stability analysis of systems with time-varying delays using a generalized free-matrix-based inequality," *J. Franklin Inst.*, vol. 356, no. 13, pp. 7312–7321, 2019.

[33] H. Shayeghi, H. Shayanfar, and A. Jalili, "Multi-stage fuzzy PID power system automatic generation controller in deregulated environments," *Energy Convers. Manage.*, vol. 47, no. 18-19, pp. 2829–2845, 2006.

[34] H. Shen, M. Xing, S. Huo, Z.-G. Wu, and J. H. Park, "Finite-time H_∞ asynchronous state estimation for discrete-time fuzzy Markov jump neural networks with uncertain measurements," *Fuzzy Sets Syst.*, vol. 356, pp. 113–128, 2019.

[35] S. Motahhir, A. El Ghzizal, S. Sebti, and A. Derouich, "MIL and SIL and PIL tests for MPPT algorithm," *Cogent Eng.*, vol. 4, no. 1, 2017, Art. no. 1378475.

[36] K. Ramakrishnan and G. Ray, "Stability criteria for nonlinearly perturbed load frequency systems with time-delay," *IEEE J. Emerg. Sel. Topic Circuits Syst.*, vol. 5, no. 3, pp. 383–392, Sep. 2015.

Hao Shen (Member, IEEE) received the Ph.D. degree in control theory and control engineering from the Nanjing University of Science and Technology, Nanjing, China, in 2011.

From 2013 to 2014, he was a Postdoctoral Fellow with the Department of Electrical Engineering, Yeungnam University, South Korea. Since 2011, he has been with the Anhui University of Technology, Ma'anshan, China, where he is currently a Professor. His current research interests include stochastic hybrid systems, complex networks, fuzzy systems and control, and nonlinear control.

Dr. Shen has served on the technical program committee for several international conferences. He is an Associate Editor/Guest Editor for several international journals, including the *Journal of The Franklin Institute*, *Applied Mathematics and Computation*, IEEE ACCESS, *Neural Processing Letters*, and *Transactions of the Institute Measurement and Control*. He was a recipient of the Highly Cited Researcher Award by Clarivate Analytics (formerly, Thomson Reuters) in 2019.

Shiyu Jiao received the M.S. degree in control theory and control engineering from the Anhui University of Technology, Ma'anshan, China, in 2020. She is currently working toward the Ph.D. degree in control science and engineering with the School of Automation, Nanjing University of Science and Technology, Nanjing, China.

Her current research interests include Markov jump systems, power systems, chaos control and synchronization, and sampled-data control.

Ju H. Park (Senior Member, IEEE) received the Ph.D. degree in electronics and electrical engineering from the Pohang University of Science and Technology (POSTECH), Pohang, South Korea, in 1997.

From May 1997 to February 2000, he was a Research Associate with the Engineering Research Center-Automation Research Center, POSTECH. He joined Yeungnam University, Kyongsan, South Korea, in March 2000, where he is currently the Chuma Chair Professor. His research interests include robust control and filtering, neural/complex networks, fuzzy systems, multiagent systems, and chaotic systems. He has authored and coauthored a number of papers in these areas.

Prof. Park is a coauthor of the monographs: *Recent Advances in Control and Filtering of Dynamic Systems With Constrained Signals* (New York, NY, USA: Springer-Nature, 2018) and *Dynamic Systems With Time Delays: Stability and Control* (New York, NY, USA: Springer-Nature, 2019) and is the Editor of an edited volume: *Recent Advances in Control Problems of Dynamical Systems and Networks* (New York, NY, USA: Springer-Nature, 2020). He serves as an Editor of *International Journal of Control, Automation and Systems*. He is also a Subject Editor/Advisory Editor/Associate Editor/Editorial Board Member for several international journals, including *IET Control Theory and Applications*, *Applied Mathematics and Computation*, *Journal of The Franklin Institute*, *Nonlinear Dynamics*, *Engineering Reports*, *Cogent Engineering*, IEEE TRANSACTIONS ON FUZZY SYSTEMS, IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, IEEE TRANSACTIONS ON CYBERNETICS, and so on. Since 2015, he is a recipient of Highly Cited Researcher Award by Clarivate Analytics (formerly, Thomson Reuters) and listed in three fields, engineering, computer sciences, and mathematics in 2019. He is a Fellow of the Korean Academy of Science and Technology.

Victor Sreeram (Senior Member, IEEE) received the Ph.D. degree in electrical engineering from the University of Victoria, Victoria, BC, Canada, in 1989.

He was a Project Engineer with the Indian Space Research Organisation from 1983 to 1985. He joined the School of Electrical, Electronic, and Computer Engineering, The University of Western Australia, Crawley WA, Australia, in 1990, where he is currently a Professor. He has held visiting appointments with the Department of Systems Engineering, Australian National University, from 1994 to 1996, and the Australian Telecommunication Research Institute, Curtin University of Technology, from 1997 to 1998. His research interests include control, signal processing, communications, smart grid, and renewable energy.

Dr. Sreeram is a Fellow of the Institution of Engineers, Australia. He was the General Chair of the Third Australian Control Conference, Perth, WA, Australia, in 2013, and the Vice Chair of the Australasian Universities Power Engineering Conference, Perth, in 2014. He is on the editorial board of many journals, including *IET Control Theory and Applications*, *Asian Journal of Control*, and *Smart Grid and Renewable Energy*.