Differential Evolution With Dynamic Parameters Selection for Optimization Problems

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***Abstract*—Over the last few decades, a number of differential evolution (DE) algorithms have been proposed with excellent performance on mathematical benchmarks. However, like any other optimization algorithm, the success of DE is highly dependent on the search operators and control parameters that are often decided *a priori*. The selection of the parameter values is itself a combinatorial optimization problem. Although a considerable number of investigations have been conducted with regards to parameter selection, it is known to be a tedious task. In this paper, a DE algorithm is proposed that uses a new mechanism to dynamically select the best performing combinations of parameters (amplification factor, crossover rate, and the population size) for a problem during the course of a single run. The performance of the algorithm is judged by solving three well known sets of optimization test problems (two constrained and one unconstrained). The results demonstrate that the proposed algorithm not only saves the computational time, but also shows better performance over the state-of-the-art algorithms. The proposed mechanism can easily be applied to other population-based algorithms.**

***Index Terms*—Constrained optimization, differential evolution (DE), parameter adaptation, parameter selection.**

I. Introduction

OPTIMIZATION is a challenging area of research spanning across the fields of computer science, operations research, and engineering. Optimization problems can be either constrained or unconstrained. Constrained optimization is different from its unconstrained counterpart, as it needs to optimize the objective function while satisfying the functional constraints and variable bounds. Constrained optimization problems (COPs) can be classified into many different categories based on the nature of the problems and their mathematical properties. In any problem, the type of constraint can be either equality or inequality or both. The objective and constraint functions may possess different properties such as linear and/or nonlinear, continuous or discontinuous, and unimodal or multimodal. The feasible space could be a single bounded region or a collection of multiple disjoint regions. The feasible region could even be unbounded in some problems. The optimal solution may exist either on the boundary or in the interior of the feasible space. A large number of variables and constraints may also add to the complexity of the problem.

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Evolutionary algorithms (EAs), such as genetic algorithms (GA) [1], differential evolution (DE) [2], evolution strategies (ES) [3], and evolutionary programming (EP) [4], have a long history of successfully solving optimization problems. Among EAs, DE has shown significant success in solving different numerical optimization problems (both constrained and unconstrained, and black-box) [5], [6]. However, the choice of the control parameters plays a pivotal role in the performance of DE algorithms. While both the choice of the search operators and the control parameters affect the performance, our focus in this paper is on the control parameters (such as amplification factor, crossover rate, and the population size) and not on the search operators (such as mutation and crossover variants). In solving any optimization problem, the selection of the right combination of parameters is itself a combinatorial optimization problem. Parameter tuning, which is a trial-and-error approach, is widely used for parameter selection and is known to be tedious [7]. Using such a trial-and-error approach, a single set of parameters is usually selected based on the average performance of the algorithm over the class of test problems. Due to the variability of the underlying mathematical properties of the optimization problems, a fixed set of control parameters that suits well for one problem, or a class of problems does not guarantee that it will work well for another class, or range of problems. That is, the best set of parameters is problem dependent. To ensure best performance of the algorithm, one must choose the right set of parameters for each and every problem which is a nontrivial task. In addition, a set of parameters that works well at the early stages of the evolution process may not perform well at the later stages and vice versa.

The idea of parameter adaptation was introduced at least two decades ago in the context of GA [8]. In DE, many different mechanisms have been introduced to select and/or manage the dynamic changes of the control parameters. Based on how the control parameters are adapted, the mechanisms can be classified into three classes [9]. These classes are briefly described as follows: 1) deterministic parameter control in which parameters are changed based on some deterministic rules, regardless of any feedback from the algorithm, i.e., the time-dependent change of the mutation rates [10]; 2) adaptive parameter control in which parameters are dynamically updated, based on learning from the evolution process [11], [12]; and 3) self-adaptive parameter control in which parameters are directly encoded within individuals and undergo recombination. The adaptive and self-adaptive mechanisms outperformed the classical DE algorithms (without parameter control) in terms of the reliability and rate of convergence for many benchmark problems [11]–[13]. Some of these algorithms have been applied to unconstrained problems, where it dynamically adapted either one of the three control parameters (crossover rate, amplification factor, or the population size), or two of them together (crossover rate and amplification factor). To the best of our knowledge, only a few algorithms reported in the literature adapted all three control parameters together [14], [15]. In addition, existing investigations usually suggested a single set of parameters for all the problems under consideration. Note that some of the investigations, which determine the parameters using the traditional parametric analysis concept, require a huge number of trials.

In this paper, a DE algorithm with dynamic selection of three control parameters is proposed for solving optimization problems. We introduce the algorithm as DE-DPS. In the proposed algorithm, three sets of parameters are considered: the first set is for the amplification factor, the second is for the crossover rate, while the third is for the population size. Each individual in the population is assigned a random combination of amplification factor *(F)* and crossover rate *(Cr)*. The success rate of each combination is recorded for a certain number of generations and the better performing combinations are applied for a number of subsequent generations. This process is recognized as a cycle. Based on the success rate, the number of combinations is reduced in subsequent cycles. At the beginning of each cycle, the success rates of the current combinations are reinitialized to zero and after every few cycles, the process restarts with all combinations of parameters.

To judge the efficiency of the proposed algorithm, three well known sets of optimization test problems (from the IEEE CEC competition problem sets: CEC2006 [16] and CEC2010 constrained problem sets [17], and CEC2005 unconstrained problem set [18]) have been solved. The proposed algorithm shows consistently better performance in comparison to other state-of-the-art algorithms. Interestingly, on a single run basis, the proposed algorithm reduces the average computational time significantly in comparison to a DE with a single set of parameters. The approach outlined in this paper not only offers an excellent choice for parameter selection, but also offers a better solution with lower computational effort. The comparisons based on the performance profiles indicate that the proposed algorithm performs consistently better for not only the constrained optimization problems but also for the unconstrained problem sets considered in this paper.

This paper is organized as follows. After the introduction, Section II presents the DE algorithm with an overview of its parameters. Section III describes the design of the proposed algorithm. The experimental results and the analysis of those results are presented in Section IV. Finally, conclusions and future work are given in Section V.

II. Differential Evolution

In this section, the commonly used operators and parameters of DE are discussed.

First, we define the key terms that are used in this section. A target vector ($\vec{X}_{z,t}$) is a parent vector in generation t of an individual z. A mutant vector ($\vec{V}_{z,t}$) is the vector obtained through the mutation operation, which is also known as the donor vector. A trial vector ($\vec{U}$) is an offspring that is obtained by recombining the mutant vector with the parent vector.

A. Mutation

In the simplest form of mutation, $\vec{V}_{z,t}$ is generated by multiplying the amplification factor F by the difference of two random vectors, and the result is added to another third random vector

$$\vec{V}_{z,t} = \vec{X}_{r_1,t} + F.\left(\vec{X}_{r_2,t} - \vec{X}_{r_3,t}\right) \tag{1}$$

where r_1, r_2, r_3 are random numbers $\{1,2, ..., PS\}$, $r_1 \neq r_2 \neq r_3 \neq z$, x is a decision vector, PS is the population size, t is the current generation, and F is a positive control parameter (amplification factor) for scaling the difference vectors.

This operation enables DE to explore the search space and maintain diversity. There are many strategies for mutation, such as DE/rand-to-best/2 [12], rand/2/dir [19], DE/current-to-best/1 [20], and DE/Current-to-pbest [21]. For more details, readers are referred to [5].

B. Crossover

In DE, two crossover operators (exponential and binomial) are commonly used. These crossover operators are briefly discussed as follows.

In an exponential crossover, an integer l is randomly chosen within the range $\{1, D\}$, where D is the number of decision variables. This integer acts as a starting point in $\vec{X}_{z,t}$, from where the crossover or exchange of components with $\vec{V}_{z,t}$ starts. Another integer L is chosen from the interval $\{1, D\text{-}l\}$ [5].

The trial vector ($\vec{U}$) is formed by inheriting the values of variables in locations l to $l+L$ from the mutant vector and the remaining ones from the parent vector.

The binomial crossover is performed on each of the j^{th} variables whenever a randomly picked number (between 0 and 1) is less than or equal to a crossover rate (Cr). The generation number is indicated here by t. In this case, the number of parameters inherited from the donor has a (nearly) binomial distribution

$$u_{zj,t} = \begin{cases} v_{zj,t}, & \text{if } (rand \leq Cr \text{ or } j = j_{rand}) \\ x_{zj,t}, & \text{otherwise} \end{cases} \tag{2}$$

where $rand \in [0, 1]$, and $j_{rand} \in \{1, 2, \ldots, D\}$ is a randomly chosen index, which ensures $\vec{U}_{z,t}$ gets at least one component from $\vec{V}_{z,t}$

C. Brief Review and Analysis

In this subsection, we provide a brief review on the selection of DE parameters and strategies.

1) *DE Parameters Settings:* Various studies recommended different values for each parameter (the amplification factor F, the crossover rate Cr, and population size PS). For example, Storn and Price [2] recommended a population size of $5D$–$20D$ (D denotes the number of variables of the problem) and an F value of 0.5. Gamperle *et al.* [22] evaluated different parameter settings of DE, where they found that a plausible choice of PS is between $3D$ and $8D$, with the amplification factor $F = 0.6$ and the crossover rate Cr bounded between [0.3, 0.9]. Ronkkonen *et al.* [23] indicated that the amplification factor (F) is typically between 0.40 and 0.95 with $F = 0.9$ being a good first choice. Furthermore, the value of the crossover rate (Cr) typically lies in the range [0,0.2] when the function is separable and within the range of [0.9, 1] when the function's variables are dependent.

Abbass [24] proposed a self-adaptive operator (crossover and mutation) for multiobjective optimization problems, where the amplification factor F is generated using a Gaussian distribution N(0, 1). This technique has been modified in [25]. Zaharie [26] proposed a parameter adaptation strategy for DE based on the idea of controlling the population diversity, and implemented a multiple population approach. Qin *et al.* [12] proposed a novel differential evolution algorithm (SaDE), where the choice of the learning strategy and the two control parameters F and Cr are not required to be prespecified. The parameter F, in SaDE, is approximated by a normal distribution N (0.5, 0.3), and truncated to the interval (0, 2]. Such an approach could maintain both the intensification (with small F values) and diversity (with large F values) during the course of search. The crossover probabilities were randomly generated according to an independent normal distribution with mean Cr_m and standard deviation 0.1. The Cr_m values remain fixed for five generations before the next regeneration. Cr_m was initialized to 0.5, and it was updated every 25 generations based on the recorded successful Cr values since the last Cr_m update.

Using fuzzy logic controllers, Liu and Lampinen [27] introduced a fuzzy adaptive differential evolution, whose inputs incorporated the relative function values and individuals of successive generations to adapt the parameters for mutation and crossover.

Brest *et al.* [11] proposed a self-adaptation scheme for the DE control parameters, known as jDE. The control parameters were adjusted by means of evolution of F and Cr. In jDE, a set of F and Cr values was assigned to each individual in the population, augmenting the dimensions of each vector. In jDE, new F_{t+1} and Cr_{t+1} were calculated as follows:

$$F_{t+1} = \begin{cases} F_l + rand_1.F_u, & \text{if } rand_2 < \tau_1 \\ F_{i,t}, & \text{otherwise} \end{cases} \tag{3}$$

$$Cr_{t+1} = \begin{cases} rand_3, & \text{if } rand_4 < \tau_2 \\ Cr_{i,t}, & \text{otherwise} \end{cases} \tag{4}$$

where $rand_1, rand_2, rand_3, rand_4$ are uniform random values $\in [0, 1]$, $F_l = 0.1$, $F_u = 0.9$, $\tau_1 = \tau_2 = 0.1$.

Zhang *et al.* [21] proposed an adaptive differential evolution algorithm with optional external memory (JADE). In JADE, at each generation, the crossover probability Cr_z of each individual x_z was independently generated according to a normal distribution of mean μCr and standard deviation of 0.1, that is

$$Cr = randn_z(\mu Cr, 0.1)_z \tag{5}$$

and then truncated to [0, 1]. μCr was initialized at a value of 0.5 and updated as follows:

$$\mu Cr = (1 - c) .\mu Cr + c.meanA\,(SCr) \tag{6}$$

where c is a positive constant between 0 and 1 and mean $A(.)$ is the usual arithmetic mean, SCr is the set of all successful crossover probabilities at generation t.

Similarly, F_z of each individual x_z was independently generated according to a Cauchy distribution with location parameter μF and scale parameter 0.1

$$F_z = randci(\mu F, 0.1) \tag{7}$$

and then truncated to be 1 if $F_z > 1$, or regenerated if $F_z < 0$. SF is the set of all successful mutation factors at generation t. The location parameter μF was initialized to 0.5 and subsequently updated at the end of each generation as

$$\mu F = (1 - c) .\mu F + c.meanL(SF) \tag{8}$$

where mean $L(.)$ is the Lehmer mean

$$mean\ L\,(SF) = \frac{\sum_{F \in S_F} F^2}{\sum_{F \in S_F} F}. \tag{9}$$

Das *et al.* [28] introduced two schemes for adapting the scale factor F in DE. In the first scheme, called DERSF, [DE with a random scale factor (RSF)], F was randomly chosen between 0.5 and 1.0. In the second scheme, F was initialized with a value of 1.0, and then linearly reduced to 0.1 during the process of evolution. Zamuda *et al.* [29] introduced differential evolution with self-adaptation and local search for the constrained multiobjective optimization algorithm (DECMOSA-SQP), which incorporates a self-adaptive mechanism from DEMOwSA algorithm and a SQP local search.

2) *DE Operators:* Yong *et al.* [30] developed a composite DE algorithm (CoDE), wherein a new trial vector was created by randomly combining three trial DE strategies with three control parameter settings at each generation. Thus, three trial vectors were generated for each target vector and the best among them made its way to the population of the next generation if it was better than its target vector. The algorithm demonstrated competitive performance on a set of unconstrained test problems.

Fialho *et al.* [31] proposed an adaptive strategy selection (*AdapSS*) paradigm, in which an online selection of the mutation strategy was performed for the generation of each new individual. *AdapSS* used two components: 1) credit assignment scheme, and 2) strategy selection method. The algorithm was assessed on the BBOB test suite and demonstrated good performance.

Gong *et al.* [32] used two different techniques (probability matching and adaptive pursuit) within DE to separately select the most suitable DE strategy. Furthermore, four credit assignment methods were assessed based on their relative fitness improvements. The two variants were evaluated on a set of unconstrained problems, and showed superior performance over other state-of-the-art algorithms.

Da Silva *et al.* [33] proposed a DE algorithm with an adaptive penalty technique for solving a set of constrained problems. Moreover, they introduced a dynamic mechanism to select the best performing mutation operator, out of four, during the process of evolution.

Yang *et al.* [34] introduced a self-adaptive clustering-based DE with composite trial vector generation strategies (SaCoCDE), in which the population was divided into different subsets by a clustering algorithm. The algorithm was tested on a set of unconstrained problems and showed highly competitive performance when compared with the state-of-the-art DE algorithms.

Zamuda and Brest [35] proposed an algorithm that incorporated two multiple mutation strategies into *j*DE, and introduced a population reduction methodology in [15]. The algorithm was tested on 22 real-world applications. The algorithm showed better performance over two other algorithms.

Based on the original version in [36], Tvrdiík and Polaákovaá [37] proposed a DE based algorithm for solving a set of constrained optimization problems. In their algorithm, with a probability (q_k), one set of control parameters was selected out of 12 available sets, and during the evolution process, q_k was updated based on the success rate in previous steps. The algorithm was ranked eighth in the CEC2010 competition.

Mallipeddi and Suganthan [38] proposed an ensemble of parallel populations based on a DE algorithm, in which three different population sizes were used. Each population was assigned a prescribed number of fitness evaluations, which were adaptively updated according to the population's success in the previous generation.

Mallipeddi *et al.* [39] proposed an ensemble of mutation strategies and control parameters within DE (EPSDE). In EPSDE, a pool of distinct mutation strategies, along with a pool of values for each control parameter, coexists throughout the evolution process and competes to produce offspring. The algorithm has been used to solve a set of unconstrained problems.

TABLE I
DE-DPS ALGORITHM

```
STEP 1: At generation $t = 1$, generate an initial random population of
        size $PS_{nps}$. The variables of all individuals ($z$) must be
        within a range as shown below:
          $x_{z,j} = x_{z,j,min} + rand \times (x_{z,j,max} - x_{z,j,,min})$
        where $x_{z,j,min}, x_{z,j,max}$ are the lower and upper bounds of
        the decision variable $x_j$, and $rand$ is a random number,
        $rand \in [0,1]$.
STEP 2: Set $PS_{set} = \{PS_1, PS_2, \dots, PS_{nps}\}$, set $period = 0$,
        $PS\_period=0$, $i = nps$ and $PS = PS_i$
STEP 3: Set $F_{set} = \{F_1, F_2, \dots, F_{nf}\}$, $Cr_{set} = \{Cr_1, Cr_2, \dots, Cr_{ncr}\}$
STEP 4: Generate new offspring as follows:
        4.1 Each individual is assigned a random combination ($y$)
            of parameters, $y \in y_{set}$, and $y_{set}$ is the combination of
            all $F_{set}$ and $Cr_{set}$.
        4.2 Generate the offspring vector $\vec{V}_z$, using mutation and
            binomial crossover operators as in (10), and update the
            fitness evaluations (FEs).
        4.3 If $\vec{V}_z$ is better than its parent, then: $com.suc_y =$
            $com.suc_y + 1$
        4.4 $period = period + 1$;
        4.5 $PS\_period = PS\_period + 1$;

STEP 5: If $period$% CS=0 and $period < (\eta \times CS)$:
        5.1 Select the best half combination to be used in the
            evolution process [based on the rankings using
            equation (11)] and update $y_{set}$.
        5.2 Set each $com.suc_y = 0$, and go to Step 4.
      Else If $period$% $(\eta \times CS) = 0$
        5.3 Set each $com.suc_y = 0$ and $period = 0$
STEP 6: If $PS\_period$ % CS=0 and $i > 0$
        6.1 calculate $Rank_{PS_i}$ using:
              $Rank_{PS_i} = \frac{\sum_1^{CS} \sum_{y=1}^{tot.com} com.suc_y}{PS_i}$
        If $i < nps$
        6.2 archive the worst $(PS_i - PS_{i-1})$ individuals
        6.3 Set $i = i - 1$
        6.4 Set $PS = PS_i$
STEP 7: If $i = 0$ and $PS\_period = nps \times$ CS
        7.1 Set $PS$ to the one with the best $Rank_{PS_i}$
STEP 8: If $PS\_period = \eta \times CS$
        8.1 Set $PS\_period=0$, $i = nps$ and $PS = PS_i$
        7.2 Use individuals from the archive as required.
        7.3 Clear the archive
STEP 8: Stop if the termination criterion is met; else, set $t = t + 1$
        and go to STEP 4.
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III. DE WITH DYNAMIC PARAMETERS SELECTION

In this section, the proposed algorithm (DE-DPS) is described, followed by the details of the constraint handling technique used in this paper.

A. Algorithm

In this paper, our purpose is to find the most appropriate parameters (F, Cr, and PS) during various stages of evolution process for any given problem. The sets for the parameters F, Cr, and PS are defined as F_{set}, Cr_{set} *and* PS_{set}, where $F_{set} = \{F_1, F_2, \dots, F_{nf}\}$, $Cr_{set} = \{Cr_1, Cr_2, \dots, Cr_{ncr}\}$, and $PS_{set} = \{PS_1, PS_2, \dots, PS_{nps}\}$. Here, PS_i is assumed to be larger than PS_{i-1}, $\forall i = nps, nps - 1, \dots, 2$, and nf, ncr, and nps refer to the cardinality of the set of amplification factors, crossover rates, and population sizes, respectively. Note that the population size (PS_{i-1}) is not only smaller than (PS_i), but also a subset of PS_i. Similarly, PS_{i-2} is a subset of PS_{i-1} and so on.

The pseudocode of the algorithm is presented in Table I. In the first step, PS_{nps} (i.e., the population with the largest size considered in this paper) random individuals are generated within the variable bounds. Each individual in the population $(\vec{X}_z)$ is assigned a random F (F_z) and a random Cr (Cr_z) (see Fig. 1). The number of combinations ($tot.com$) for F and Cr is equal to a $nf \times ncr$. Note that there are nps population sizes.

For each zth individual in the population, a new offspring is generated first via a mutation operator that is further modified via a crossover operation. To perform mutation, three individuals are used, two of which are randomly selected from the population, while the third base parent is selected from

$$\overbrace{\qquad\qquad\qquad\qquad}^{\text{parameter combinations}}$$

$\vec{X}_1$	$F_1 \in rand\{F_{set}\}$	$Cr_1 \in rand\{Cr_{set}\}$
$\vec{X}_2$	$F_2 \in rand\{F_{set}\}$	$Cr_2 \in rand\{Cr_{set}\}$
…	…..	…..
$\vec{X}_{PS_{nps}}$	$F_{PS_{nps}} \in rand\{F_{set}\}$	$Cr_{PS_{nps}} \in rand\{Cr_{set}\}$

Fig. 1. *F* and *Cr* assignment (each *F* and *Cr* is randomly selected from F_{set} and Cr_{set}).

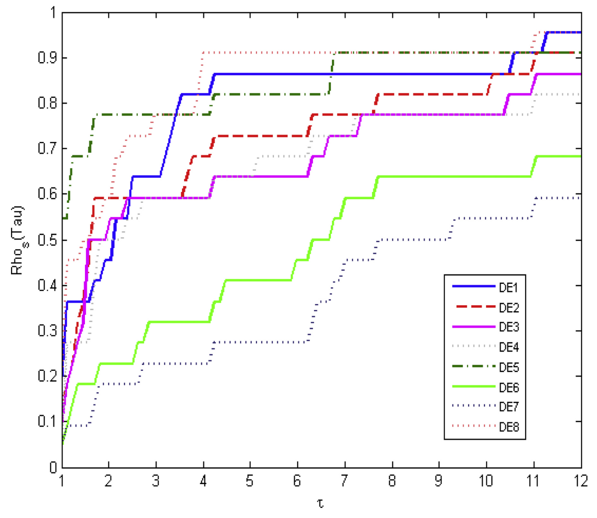


Fig. 2. Performance profiles of DE variants with eight different parameters settings.

between [a, b], where *a* is set to 10% and *b* is set to 50% in this paper, i.e., the performance of the base parent is between 10% and 50% of the individuals in the population (note that the population is sorted, based on the fitness function and/or constraint violation, before the mutation step). Following the mutation operation, the crossover is performed between the individual generated via the above mutation process and the *z*th individual in the population. Mathematically, the process can be presented as follows:

$$u_{zj,t} = \begin{cases} x_{\varphi j,t} + F_z \cdot \left(x_{r_1 j,t} - x_{r_2 j,t}\right), & \text{if } (rand \le Cr_z \text{ or } j = jrand) \\ x_{zj,t}, & \text{otherwise} \end{cases} \tag{10}$$

where φ is a random integer number within a range $[a, b]$. The introduction of these parameters (φ, *a,* and *b*), as discussed above, will have a balancing effect between two mutation strategies DE/rand/1 and DE/best/1. They are further discussed in the next section. $z \in \{1, 2, \ldots, PS\}$ and r_1 and r_2 are random integer numbers $\in\{1, 2, ..., PS\}$, $r_1 \neq r_2 \neq z$.

If the new offspring is better than its parent, i.e., the *i*th individual in the population, it will be accepted and the success of a combination *y* ($com.suc_y$) is increased by one, i.e., $com.suc_y = com.suc_y + 1$, where $y = 1, 2, \ldots, tot.com$.

The above process is repeated for *CS* generations. At the end of *CS* generations, the better performing PS_{nps-1} individuals are kept in the population, while the remaining individuals are transferred to an archive. The number of combinations

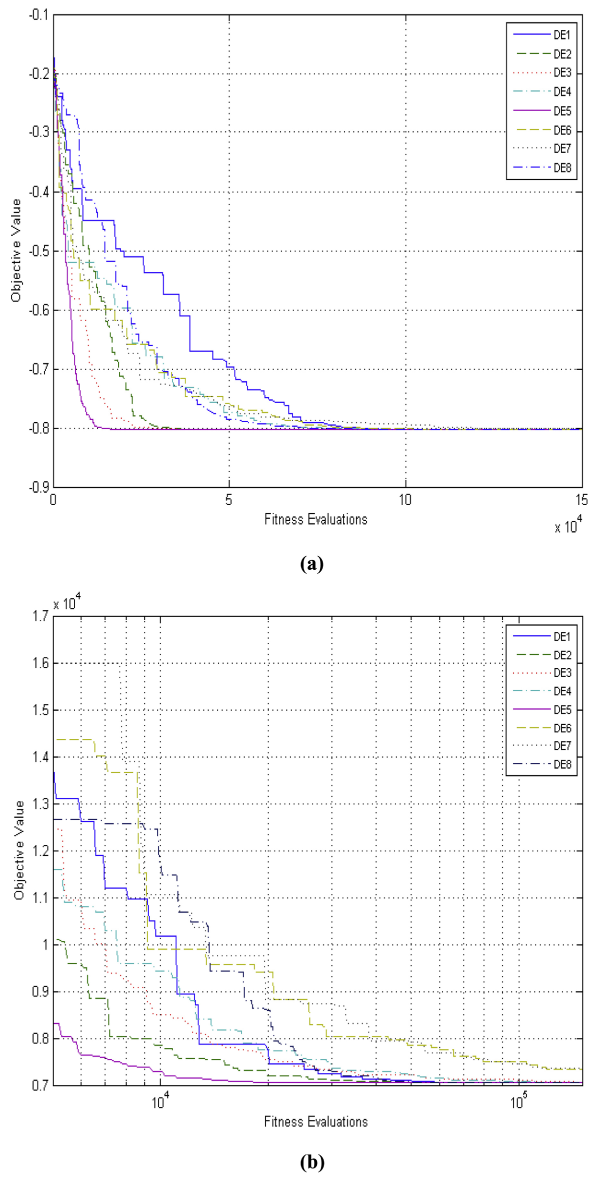


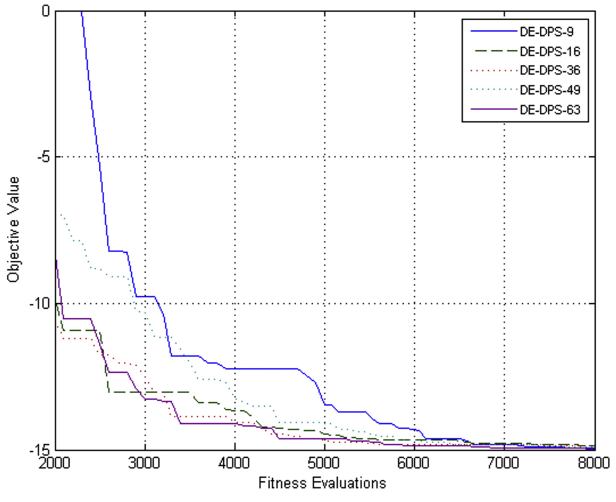
Fig. 3. Convergence plots for eight different DE variants of (a) g02 and (b) g10. The *x*-axis is in a log scale, for only the first 150 000 FEs.

of *F* and *Cr* values is also reduced to half, i.e., the better combinations of *F* and *Cr* are preserved based on the success of the combination. The ranking of any combination (*y*) is calculated using the following equation:

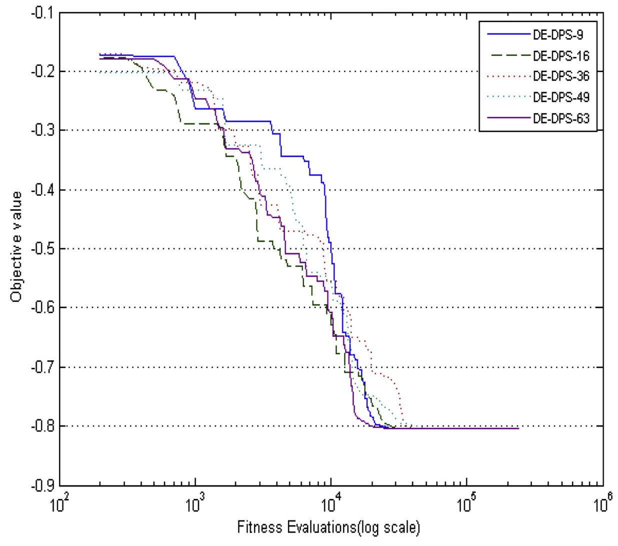
$$Rank_y = \frac{com.suc_y}{the\ number\ of\ individuals\ used\ a\ combination\ y} \tag{11}$$

where a higher value of $Rank_y$ is a better performing combination.

Individuals in the population of size PS_{nps-1} are randomly assigned *F* and *Cr* values from this reduced list. The process of evolution uses the same mutation and crossover strategy and is allowed to evolve for *CS* generations. This process is repeated until all population sizes are considered. At the end of this stage, the performance ranking of all population sizes is computed, using the equation shown in Step 6.1



(a)



(b)

Fig. 4. Convergence plots of all the proposed variants for (a) g01 and (b) g02. (a) g01 (up to 8000 FES); the x-axis is in a log scale. (b) g02; the x-axis is in a log scale.

in Table I to decide the appropriate population size. The performance ranking of any population size represents the average performance per individual in the population for all parameter combinations over CS generations.

In the event PS_{nps} is selected, the best individuals from the archive are added to make the current population size (PS_i) equal to PS_{nps}. The numbers of combinations of F and Cr values are also reduced to half based on the success as described earlier. The selected population of size (PS_i) is allowed to evolve for $(\eta - nps) \times CS$ generations wherein after every CS generation, the numbers of combinations of F and Cr are reduced to half until the number of combinations reaches 1. The above steps are referred to as a cycle. At the end of each cycle, the success tables and the archives are reset to null, the total number of combinations is reset to *tot.com*, and the population size is reset to PS_{nps}. The cycles continue until the termination criterion is met. To clarify, an example with two population sizes (100 and 75, i.e., $nps = 2$) and 64 combinations of y is considered. The population size 100 will evolve for CS generations with 64 combinations, then the population size 75 will evolve for CS generations with 32 combinations, and finally the selected population size (either 100 or 75) will be fixed for the next $(\eta - 2) \times CS = 4CS$ generations.

B. Discussions on Related Issues

In this section, we discuss a few issues relevant to the algorithm design and implementation.

In the evolution process, for a given problem, the relative performance of each combination may vary with the progression of generations. This behavior means that one combination may work well at the early (or some) stages of the search process and may perform poorly at the later (or some other) stages, or vice versa. So, it is inappropriate to give equal emphasis on all of the combinations throughout the entire process of evolution. To give a higher emphasis on the better performing combinations in a given stage of the evolution process, it is proposed that the random assignment of the parameter combinations is applied for a fixed number of generations (say CS).

The parameter combinations are assigned randomly to individuals without replacement. This means that one combination will be assigned strictly to one individual if the population size is less than or equal to the number of combinations. If the population size is larger than the number of combinations, all combinations are assigned to at least one individual. Depending on the number of combinations and population size, one combination may be assigned to more than one individual and there is a possibility that some combinations may not be assigned at all. The ranking of any assigned combination (y) is calculated using (11) and the ranking of any unassigned combination is set to zero.

C. Constraint Handling

In this paper, one of the most popular constraint handling techniques, which was proposed in [40] has been used. The method of comparison relies on the use of the following conditions: 1) between two feasible solutions, the fittest one (according to fitness function) is better; 2) a feasible solution is always better than an infeasible one; and 3) between two infeasible solutions, the one having the smaller sum of constraint violation is preferred.

In this research, the sum of constraint violation is calculated as follows:

$$\Theta(\vec{x}) = \sum_{k=1}^{K} \max(0, g_k(\vec{x})) + \sum_{e=1}^{E} \max(0, |h_e(\vec{x})| - \varepsilon) \quad (12)$$

where $g_e(\vec{x})$ is the kth inequality constraint, and $h_e(\vec{x})$ is the eth equality constraint. The equality constraints are transformed into inequalities of the form, where ε is a small value

$$-\varepsilon \le h_e(\vec{x}) \le \varepsilon \quad \text{for } e = 1, \ldots, E. \quad (13)$$

Note that the superiority of feasible solutions constraint handling technique [40] does not require user-defined parameters. However, it may lead to premature convergence [41].

TABLE II
SYMBOLS, MEANING, AND POSSIBLE VALUES OF ALL PARAMETERS USED IN PROPOSED ALGORITHM

Symbols	Meaning	Possible values
F_{set}	Set of all the possible amplification values	Up to 9 values are used here, which each value is >0 and $\leq$ 1
Cr_{set}	Set of all the crossover rates	Up to 9 values are used here, which each value is >0 and $\leq$ 1
PS_{set}	Set of all possible population sizes	Up to two values are used here.
$tot.com$	The total number of combinations	9, 16, 36, 49, 63 combinations are used.
CS	Reducing the number of combinations	Here, CS = 10, 25, 50, and 75
$\eta\times CS$	Restart point, in which the number of combinations is set as its initial setting.	η = 3, 4, 5 and 6

TABLE III
USED F AND B VALUES FOR SEVEN DE VARIANTS

Variant	F and Cr values
DE1	F=0.95 and Cr= 0.95
DE2	F is a random number between 0.4 and 0.9, Cr= 0.95
DE3	F=0.5 and Cr= 0.5
DE4	F=0.7 and Cr= 0.7
DE5	F=0.4 andCr= 0.8
DE6	F=0.9 and Cr= 0.5
DE7	F=0.9 and Cr= 0.2
DE8	F=0.99 and Cr= 0.99

IV. EXPERIMENTAL RESULTS

In this section, the computational results of different algorithms are presented and analyzed using three different benchmark problem sets. Two of them are constrained optimization problem sets taken from the IEEE-CEC Competition held in 2006 [16] and 2010 [17], while the third is an unconstrained problem set used in the 2005 [18] competition. First, we ran different variants of DE for solving 24 benchmark problems from CEC2006 [16] and analyzed their results. The effect of the number of parameter combinations, as well as the number of population sizes, is analyzed. All the algorithms have been coded using MATLAB 7.8.0 (R2009a), and have been run on a PC with a 3.0-GHz Core 2 Duo processor, a 3.5-GB RAM, and windows XP.

For convenience of reading, the parameter symbols and their possible values considered in this research are provided in Table II.

A. DE With Different Parameters

Here, a DE algorithm with eight different parameter combinations is tested. The DE uses (10) as a mutation operator and utilizes the binomial crossover. The eight variants are shown in Table III. These values are selected based on the literature discussed in Section II-C.

The motivation of this section is to show that there is no single combination of DE parameters that is always better for all types of problems. To add to this, our purpose is to show that the proposed algorithm is better than such DE algorithms in regard to the quality of solutions and the computation time across a variety of problems.

TABLE IV
FEASIBILITY RATIO OF EACH VARIANT

DE1	DE2	DE3	DE4	DE5	DE6	DE7	DE8
100 %	96.18 %	95.45 %	95.64 %	92.00 %	90.91 %	68.00 %	**100 %**

TABLE V
NUMBER OF TEST PROBLEMS THAT EACH VARIANT IS ABLE TO OBTAIN OPTIMALITY

DE1	DE2	DE3	DE4	DE5	DE6	DE7	DE8
19	14	14	14	17	10	8	**19**

Considering parameters settings, φ is a random integer number within a range [a, b], a = 10% and b = 50% of PS, where PS was set to 100 individuals. Twenty-five independent runs were conducted for each DE variant. For the equality constraints, the ε parameter was set to 0.0001. The stopping criterion was to run each test problem for up to 240 000 fitness evaluations (FEs). The detailed results [best, median, average, worst, and standard deviation ($St.d$)] are available in the supplementary document (link is provided at the end of this paper).

None of the variants could solve g20 and g22 test problems, and hence these two problems were removed from our analysis. Considering the remaining 22 test problems, the results of the proposed algorithm are the best for DE1 and DE8 with a 100% feasibility ratio. The feasibility ratios for all the variants are presented in Table IV.

Based on the number of optimal solutions obtained, DE1 and DE8 performed the best. The number of optimal solutions for all variants is shown in Table V. As can be seen from Tables IV and V, although DE5's feasibility ratio is lower than DE2, DE3, and DE4, it is able to obtain optimality in more test problems. It is also clear that the reduction of crossover rate leads to worse results, as is seen in DE7.

Considering the best and the average results, a comparative analysis is presented in Table VI. To explain the results, for example, the variants DE1 and DE2 obtained the best fitness values in nine and seven test problems, respectively. Both the variants obtained equal fitness in 13 test problems. From this table, it is clear that DE1 and DE8 are the best variants although they are worse for few different problems.

To compare the performance of these algorithms graphically, the performance profiles [42] are plotted. The performance profiles are a tool to compare the performance, of a set of algorithms using a set of test problems, using a comparison goal (such as the computational time and the average number of fitness evaluations) to obtain a certain level, of a performance indicator (such as optimal fitness). In this section, our goal is

$$g_{s,p} = \frac{FFs.Total\ runs}{Successfull\ runs}$$

TABLE VI
COMPARISON AMONG EIGHT DE VARIANTS (THE NUMBERS SHOWN IN THE TABLE ARE FOR THE FIRST ALGORITHM IN COLUMN ONE), WHERE DEC. IS THE STATISTICAL DECISION

Comparison	Fitness	Better	Equal	Worse	*Dec.*
DE1– to – DE2	Best	9	13	0	+
	Average	10	11	1	+
DE1– to – DE3	Best	8	14	0	+
	Average	9	12	1	+
DE1– to – DE4	Best	8	14	0	+
	Average	8	13	1	+
DE1– to – DE5	Best	3	15	4	≈
	Average	10	11	1	+
DE1– to – DE6	Best	12	10	0	+
	Average	12	9	1	+
DE1– to – DE7	Best	14	8	0	+
	Average	13	8	1	+
DE1– to – DE8	Best	1	19	2	≈
	Average	3	18	1	≈
DE2– to – DE3	Best	7	12	3	≈
	Average	6	11	5	≈
DE2– to – DE4	Best	7	12	3	≈
	Average	5	11	6	≈
DE2– to – DE5	Best	2	11	9	≈
	Average	5	9	8	≈
DE2– to – DE6	Best	11	9	2	+
	Average	11	9	2	+
DE2– to – DE7	Best	14	8	0	+
	Average	13	8	1	+
DE2– to – DE8	Best	1	13	8	-
	Average	2	11	9	≈
DE3– to – DE4	Best	3	14	5	≈
	Average	1	13	8	-
DE3– to – DE5	Best	2	11	9	≈
	Average	5	9	8	≈
DE3– to – DE6	Best	11	11	0	+
	Average	12	10	0	+
DE3– to – DE7	Best	13	9	0	+
	Average	12	9	1	+
DE3– to – DE8	Best	1	14	7	-
	Average	1	12	9	-
DE4– to – DE5	Best	2	11	9	≈
	Average	5	10	7	≈
DE4– to – DE6	Best	12	10	0	+
	Average	12	10	0	+
DE4– to – DE7	Best	14	8	0	+
	Average	13	9	0	+
DE4– to – DE8	Best	1	14	7	-
	Average	1	13	8	-
DE5– to – DE6	Best	13	8	1	+
	Average	12	7	3	+
DE5– to – DE7	Best	14	7	1	+
	Average	13	7	2	+
DE5– to – DE8	Best	4	15	3	≈
	Average	3	11	8	≈

which is achieved by an algorithm (s) on a test problem (p) to obtain the optimal fitness with a threshold of 0.0001. Here, the performance ratio is $\mathcal{R}_{s,p} = \frac{g_{A,p}}{B(g_{A,p}:s\in S)}$, where $B(g_{A,p})$ is the best, and $Rho_s(\tau) = \frac{1}{number\ of\ problems}|\{p \in P : \mathcal{R}_{s,p} \leq \tau\}$ is the probability for algorithm $s \in S$ that a performance ratio $\mathcal{R}_{s,p}$ is within a factor Tau (τ) $\in \mathbb{R}$ of the best possible ratio. The function (Rho_s) is the (cumulative) distribution function for the performance ratio.

Based on the abovementioned measure, the performance profiles of eight variants of DE are depicted in Fig. 2. From this figure, although DE5 has the highest probability to be the optimal variant at the beginning, DE1 and DE8 are able to reach probability of 1 first with τ of 11 and 11.2, respectively.

TABLE VII
AVERAGE COMPUTATION TIME FOR DIFFERENT VARIANTS (TIME IS IN SECONDS)

Algorithms							
DE1	**DE2**	**DE3**	**DE4**	**DE5**	**DE6**	**DE7**	**DE8**
5.902	6.729	7.063	6.913	6.337	7.540	9.493	5.905

To study the difference between any two stochastic algorithms in a more meaningful way, a test based on the statistical significance of the results is performed. A nonparametric test, the Wilcoxon signed rank test [43], is chosen that allows us to judge the difference between paired scores when assumptions required by the paired-samples t test may not be valid, such as a normally distributed population. As a null hypothesis, it is assumed that there is no significant difference between the best and/or mean values of two samples. Whereas the alternative hypothesis is that there is a significant difference in the best and/or mean fitness values of the two samples, with a significance level of 5%. Based on the test results, one of three signs (+, −, and ≈) is assigned for the comparison of any two algorithms (shown in the last column), where the "+" sign means the first algorithm is significantly better than the second, the "−" sign means that the first algorithm is significantly worse, and the "≈" sign means that there is no significant difference between the two algorithms. The results are shown in Table VI (the last column). From this table, it is clear that DE1 and DE8 are the best.

To continue our analysis, the average computational time for each algorithm is compared. The computational time was calculated as the average time consumed to reach the best known solutions with an error 0.0001, i.e., the stopping criteria is $[f(\vec{x}) - f(\overrightarrow{x^*})] \leq 0.0001]$, where $f(\overrightarrow{x^*})$ is the best known solutions, over all test problems. The summary results are shown in Table VII. From this table, DE1 is the best. Also, all variants of the proposed algorithm are faster than a DE algorithm.

Finally, as a sample, a convergence plot for all of the variants, for problems g02 and g10, is presented in Fig. 3. It can be seen from this figure that DE5 is able to converge quickly for these two problems.

B. Effect of φ

As introduced earlier, in generating individuals with (10), we used a parameter ϕ that will make a balance between DE/rand/1 and DE/best/1. Here, we would like to show the impact of this new individual generation scheme by comparing the DE variant with the proposed scheme (named DE1) with both DE/rand/1 and DE/best/1. To do this, 22 test problems from the CEC2006 set were solved by each of these three variants with $F = 0.95$, $Cr = 0.95$, 25 runs and 240K FEs.

Based on the computational results, DE1 is better than DE/rand/1 for eight and seven test problems in regard to the best and average results, respectively, and both variants are the same for the rest of test problems. Considering the best results obtained, DE1 shows its superiority to DE/best/1 for two problems, while it is worse for only one test problem. In addition, DE1 is better than DE/best/1 for 11 test problems, based on the average results, while both of them are able to obtain the same average results for the remaining test problems.

Based on the Wilcoxon test, DE1 is statistically better than DE/rand/1 for both the best and the average results. Moreover, DE1 is statistically better than DE/best/1 in regard to the average results, while there is no significant difference based on the best results. From the above analysis, it is clear that the new individual generation scheme has a positive impact on the performance of the proposed algorithm.

C. Analyzing Effect of Number of Combinations

The effect of the number of combinations (tot.com) on the performance of DE-DPS is analyzed. For this reason, the benchmark problems are solved using different variants of DE-DPS with the following parameter combinations:

1) DE-DPS-9: *tot.com*= 9, in which $F_{set} = \{0.4, 0.9, 0.99\}$ and $Cr_{set} = \{0.4, 0.9, 0.99\}$, while $\mathrm{PS}_{\mathrm{set}} = \{75, 100\}$.
2) DE-DPS-16:tot.com = 16, in which $F_{set} = \{0.4, 0.5, 0.9, 0.99\}$ and $Cr_{set} = \{0.4, 0.5, 0.9, 0.99\}$, while $\mathrm{PS}_{\mathrm{set}} = \{75, 100\}$.
3) DE-DPS-36: tot.com = 36, in which $F_{set} = \{0.4, 0.5, 0.7, 0.8, 0.9, 0.99\}$ and $Cr_{set} = \{0.4, 0.5, 0.7, 0.8, 0.9, 0.99\}$, while $PS_{set} = \{75, 100\}$.
4) DE-DPS-49: tot.com = 1, in which $F_{set} = \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}$ and $Cr_{set} = \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}$, while $PS_{set} = \{75, 100\}$.
5) DE-DPS-63: tot.com = 63, in which $F_{set} = \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}$ and $Cr_{set} = \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}$, while $PS_{set} = \{75, 100\}$.

All of these variants are compared with DE1. All of the parameter settings are set as those in the previous section, with an extra parameter *CS* set to ten generations (this means that the restart point is every 40 generations). The detailed results (best, median, average, worst, *St.d*) are presented in the supplementary document (link is provided at the end of this paper).

As seen from the results obtained, none of the versions could solve g20 and g22. From the literature, there is no feasible solution for g20, while for g22 it is rare to come across a feasible solution [16]. So these two problems are excluded from these comparisons.

As observed, all variants are able to obtain a 100% feasibility ratio. The variants DE-DPS-36, DE-DPS-49, and DE-DPS-63 obtained the optimal solution for all 22 test problems. However, DE-DPS-9 and DE-DPS-16 obtained the optimal solution for 21 test problems. All these variants are compared to each other, with respect to the best and the average fitness values, in Table VIII. Generally speaking, increasing the number of combinations leads to slightly better results.

Based on the Wilcoxon test, as is shown in Table VIII, there is no significant difference among all of the algorithms, except that DE-DPS-49 and DE-DPS-63 are statistically superior to DE-DPS-9.

The average computational time is also recorded in Table IX. From Table IX, DE-DPS-63 and DE-DPS-49 are the best with savings in time of 9.0101% and 9.0104% in comparison to DE1. It is worth mentioning here that all variants are faster than the DE1. Interestingly, with the higher the number of combinations it consumes a lower average computational time per generation. However, the overall average time to converge to a quality solution is lower than the variants with fewer parameter combinations.

TABLE VIII
COMPARISON AMONG DIFFERENT VARIANTS AND DE1 (THE NUMBERS SHOWN IN THE TABLE ARE FOR THE FIRST ALGORITHM IN COLUMN ONE)

Comparison	Fitness	Better	Equal	Worse	Dec.
DE-DPS-9–to – DE1	Best	3	19	0	≈
	Average	1	16	5	≈
DE-DPS-9–to – DE-DPS-16	Best	0	21	1	≈
	Average	0	18	4	≈
DE-DPS-9–to – DE-DPS-36	Best	0	21	1	≈
	Average	1	16	5	≈
DE-DPS-9–to – DE-DPS-49	Best	0	21	1	≈
	Average	0	17	5	-
DE-DPS-9–to – DE-DPS-63	Best	0	21	1	≈
	Average	0	17	5	-
DE-DPS-16–to– DE1	Best	3	19	0	≈
	Average	2	17	3	≈
DE-DPS-16–to– DE-DPS-36	Best	0	21	1	≈
	Average	1	18	3	≈
DE-DPS-16–to– DE-DPS-49	Best	0	21	1	≈
	Average	0	19	3	≈
DE-DPS-16–to– DE-DPS-63	Best	0	21	1	≈
	Average	0	19	3	≈
DE-DPS-36–to– DE1	Best	3	19	0	≈
	Average	4	17	1	≈
DE-DPS-36–to– DE-DPS-49	Best	0	22	0	≈
	Average	0	19	3	≈
DE-DPS-36–to – DE-DPS-63	Best	0	22	0	≈
	Average	0	19	3	≈
DE-DPS-49–to– DE1	Best	3	19	0	≈
	Average	4	17	1	≈
DE-DPS-49–to – DE-DPS-63	Best	0	22	0	≈
	Average	0	20	2	≈
DE-DPS-63–to– DE1	Best	3	19	0	≈
	Average	4	17	1	≈

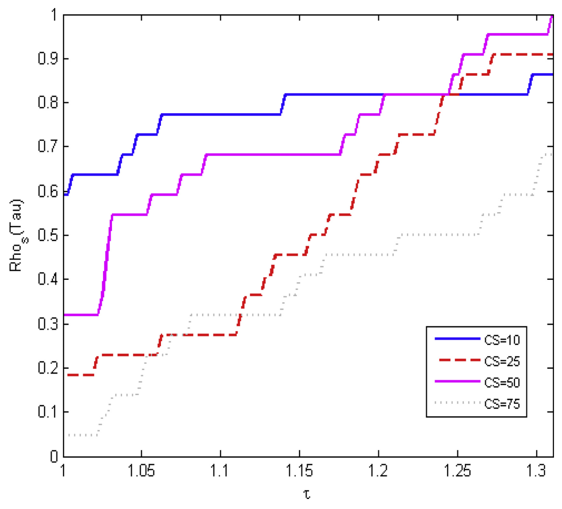


Fig. 5. Performance profiles comparing DE-DPS with different CS values.

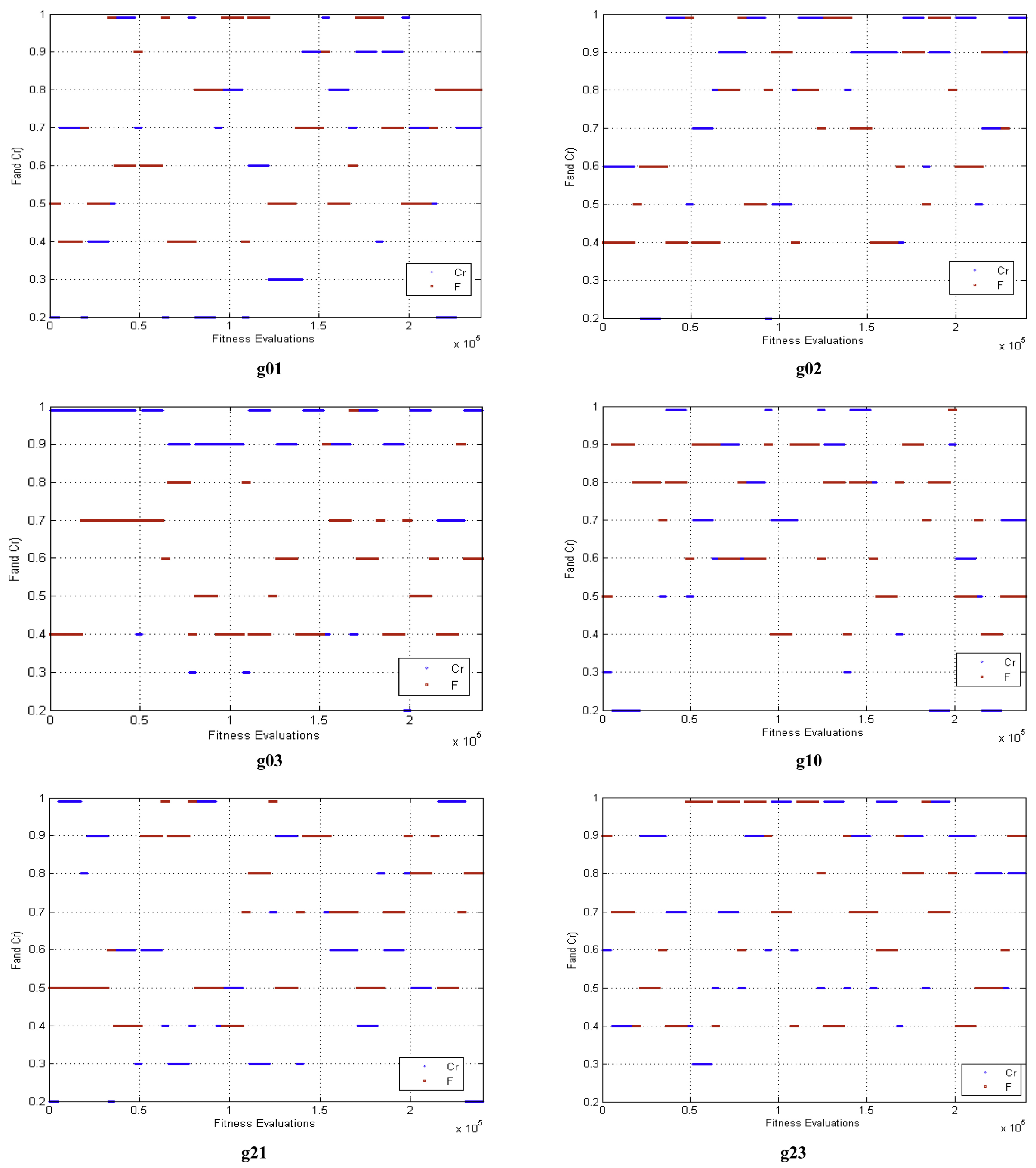


Fig. 6. Best parameter values for different test problems during the evolution process.

TABLE IX
AVERAGE COMPUTATIONAL TIME COMPARISON FOR DIFFERENT DE-DPS VARIANTS

DE1	**DE-DPS-9**	**DE-DPS-16**	**DE-DPS-36**	**DE-DPS-49**	**DE-DPS-63**
5.902	5.761	5.708	5.466	5.369802	**5.36979**

This finding justifies the research considering many parameter combinations. From the analysis, it is clear that the increase in the number of combinations leads to better results, as well as savings in the computational time.

To this end, few convergence patterns for all variants are presented in Fig. 4. This figure reveals that DE-DPS-63 is able to quickly reach the optimality. For ease of explanation, DE-DPS is referred to DE-DPS-63 in the remaining sections of this paper.

D. Analyzing the Effect of CS

In this section, the effect of *CS,* which decides the reduction of the number of combinations, is analyzed. To do this, different experiments were run by setting *CS* to 10, 25, 50,

TABLE X
COMPARISON AMONG DE-DPS WITH DIFFERENT CS VALUES (10, 25, 50, AND 75). COMPARISON IS BASED ONLY ON THE AVERAGE RESULTS (THE NUMBERS SHOWN IN THE TABLE ARE FOR THE FIRST ALGORITHM IN COLUMN ONE)

Algorithms	Better	Equal	Worse
DE-DPS (*CS*=10) –TO- DE-DPS (*CS*=25)	1	19	2
DE-DPS(*CS*=10) –TO- DE-DPS (*CS*=50)	0	19	3
DE-DPS(*CS*=10) –TO- DE-DPS (*CS*=75)	3	18	1
DE-DPS(*CS*=25) –TO- DE-DPS (*CS*=50)	0	19	3
DE-DPS(*CS*=25) –TO- DE-DPS (*CS*=75)	3	18	1
DE-DPS(*CS*=50) –TO- DE-DPS (*CS*=75)	4	18	0

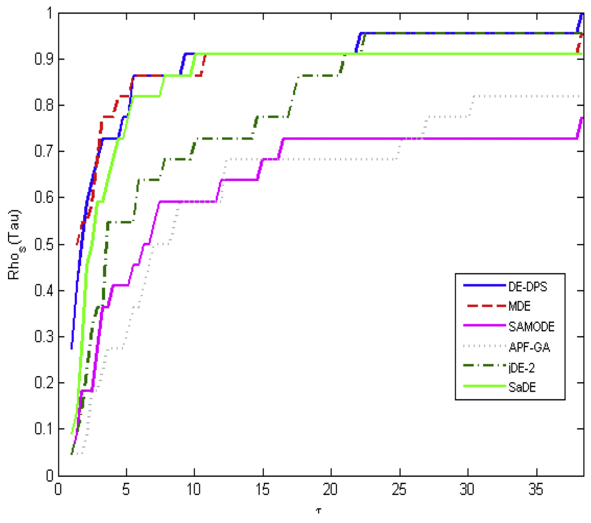
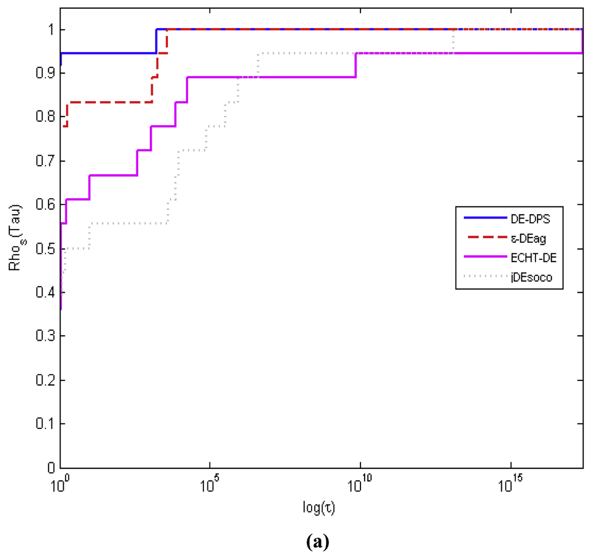


Fig. 7. Performance profiles comparing DE-DPS with different state-of-the-art algorithms based on the average results.



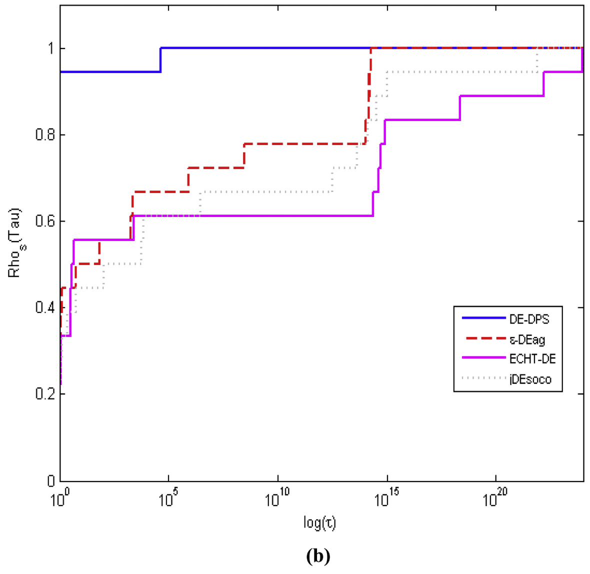


Fig. 8. Average fitness values performance profiles comparing DE-DPS with different state-of-the-art algorithms for both (a) 10-D and (b) 30-D problems, respectively. Note that the x-axis is in a log scale.

and 75 generations. All other parameters are set as those in the previous section. The detailed results are shown in the supplementary document (link is provided at the end of the paper). For all the 22 test problems considered earlier, all of those versions could obtain a 100% feasibility ratio. Based on the best results, all of the variants were able to obtain the optimal solution for 22 test problems, except DE-DPS with $CS = 10$, which obtained the optimal solution for 21 test problems. The average results are presented in Table X, while the performance profiles are depicted in Fig. 5. Based on these results, although using CS = 50 has the second highest probability at the beginning, using this value has the ability to solve all problems first, i.e., reaching a probability of 1 when τ equals to 1.31.

These results show that DE-DPS with $CS = 50$ is slightly better. However, the Wilcoxon test does not show any significant difference among the variants.

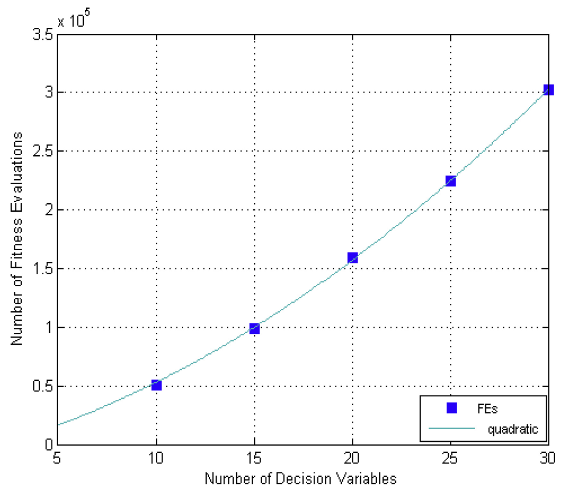
In regard to the average computational time (see Table XI), it is clear that DE-DPS with $CS = 50$ is the fastest one. This means that DE-DPS with $CS = 50$ is able to save the computational time by 14.48% in comparison to DE1.

TABLE XI
AVERAGE COMPUTATIONAL TIME FOR DE-DPS WITH DIFFERENT CS VALUES

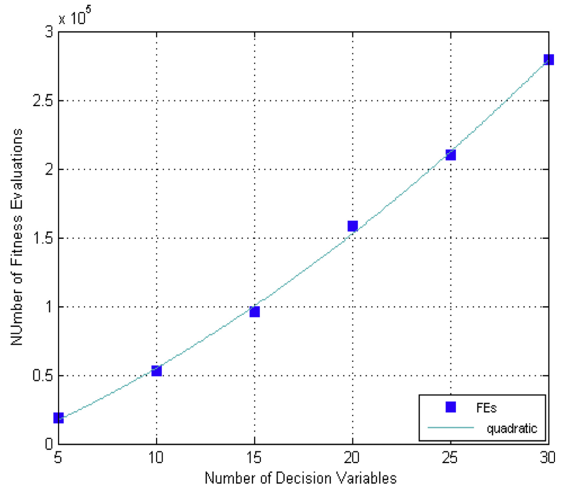
DE-DPS(*CS*=10)	DE-DPS(*CS*=25)	DE-DPS(*CS*=50)	DE-DPS(*CS*=75)
5.5818(SEC)	5.3698(SEC)	**5.0254 (SEC)**	5.1089(SEC)

E. Analyzing Effect of η

As mentioned earlier, after $\eta \times CS$ generations, all parameters are initialized to their initial settings. Therefore, in this



(a)



(b)

Fig. 9. Average number of function evaluations versus the problem dimension for (a) C07 and (b) C09 using 50 runs. The dashed lines represent the quadratic regression fitting of the data.

section, the effect of η is analyzed. Note that the maximum value of η is calculated as follows:

$$\eta \approx \frac{log(tot.com)}{log(2)}. \quad (14)$$

For instance, if *tot.com* is 64, then η is 6 (the reduction in each step is as follows: 64 $\rightarrow$32 $\rightarrow$ 16 $\rightarrow$ 8 $\rightarrow$ 4 $\rightarrow$ 2 $\rightarrow$ 1). Thus, to analyze the effect of this parameter, different experiments were run by changing η to 3, 4, 5, and 6. This means that the restart point will be at 150, 200, 250 and 300 generations, respectively, and a comparison summary among these variants is presented in Table XII. Based on these results, it is clear that DE-DPS with $\eta = 4$ is the best choice. Furthermore, both the Wilcoxon test results in Table XII, and the performance profiles based on the average results in Fig. 6, confirm this conclusion.

The average computational time of each DE-DPS with its η value is reported in Table XIII. From this table, it is clear that DE-DPS with $\eta = 4$ is the fastest one.

TABLE XII

COMPARISON AMONG DE-DPS WITH DIFFERENT VALUES OF η

Algorithms	Fitness	Better	Equal	Worse	Dec.
η = 3–TO-	Best	0	20	2	≈
η = 4	Average	0	18	4	≈
η = 3–TO-	Best	1	21	0	≈
η = 5	Average	5	17	0	+
η = 3–TO-	Best	2	20	0	≈
η = 6	Average	2	19	1	≈
η = 4–TO-	Best	1	21	0	≈
η = 5	Average	5	17	0	+
η = 4–TO-	Best	1	21	0	≈
η = 6	Average	5	16	1	≈
η = 5–TO-	Best	1	20	1	≈
η = 6	Average	3	16	3	≈

TABLE XIII

AVERAGE COMPUTATIONAL TIME OF DE-DPS WITH DIFFERENT η VALUES

η = 3	η = 4	η = 5	η = 6
5.63	**5.0254**	5.49	5.77

It is also important to observe the variation of F and Cr during the course of evolution. To do this, the corresponding best parameter values during the evolution process, for different test problems, are presented in Fig. 6. From this figure, it is clear that no fixed combination is the best during the entire evolution process for all test problems. For instance, $F \in [0.9 - 0.99]$ is favorable for g03, while it not the case for g23. As a consequence, the proposed algorithm is able to adapt the parameters based on the problem type which is the main purpose of this research.

F. Comparison to State-of-the-Art Algorithms

The first 13 problems of the CEC2006 test set have been widely used for performance testing. The detailed results of DE-DPS (DE-DPS-63, with $CS = 50$, $PS_{set} = \{75, 100\}$ and η = 4) are provided in Table XVII, along with that of the state-of-the-art algorithms such as: 1) modified differential evolution (MDE) [44]; 2) self-adaptive multioperator differential evolution (SAMO-DE) [25]; 3) adaptive penalty formulation with GA (APF-GA) [45]; 4) ensemble of constraint handling techniques based on evolutionary programming (ECHT-EP2) [46]; 5) self-adaptive differential evolution (jDE2) [47]; 6) self-adaptive differential evolution (SaDE) [13]; and 7) adaptive tradeoff model with evolution strategy (ATMES) [48].

It should be mentioned here that DE-DPS, SAMO-DE, ECHT-EP2, ATEMS used 240K FEs, while MDE, APF-GA, jDE, and SaDE use 500K FEs. The parameter ε for DE-DPS, SAMO-DE, ECHT-EP2, APF-GA, MDE, jDE2, and SaDE was set to 1.0E-04, while it was set to 5.0E-06 for ATMES. All algorithms solved 22 test problems, except ATMES, in which only the 13 test problems were solved.

From Table XVII, as indicated earlier, DE-DPS was able to obtain the optimal solutions for all of the test problems, while the algorithms MDE, SAMO-DE, APF-GA, ECHT-EP2, jDE2, and SaDE were able to obtain the optimal solutions for 20, 21, 17, 19, 20, and 22 test problems, respectively. However,

TABLE XIV
WILCOXON SIGN RANK TEST RESULTS FOR DE-DPSS AGAINST MDE, SAMO-DE, APF-GA, ECHT-EP2, JDE, SADE AND ATMES

Algorithms	Fitness	Decision
DE-DPS– to – MDE	Best	≈
	Average	≈
DE-DPS – to – SAMO-DE	Best	≈
	Average	≈
DE-DPS – to – APF-GA	Best	≈
	Average	+
DE-DPS – to – ECHT-EP2	Best	≈
	Average	≈
DE-DPS – to – jDE2	Best	≈
	Average	+
DE-DPS – to – SaDE	Best	≈
	Average	+
DE-DPS – to –ATMES	Best	≈
	Average	+
DE-DPS – to –SMES	Best	+
	Average	+

ATMES obtained the optimal solutions for 11 out of 13 test problems.

In regard to the average results, DE-DPS is superior to MDE, SAMO-DE, APF-GA, ECHT-EP2, jDE2, and SaDE for three, seven, eight, seven, nine, and seven test problems, respectively, while DE-DPS is inferior to MDE, SAMO-DE, and ECHT-EP2 for only one test problem. Moreover, DE-DPS is superior to ATMES for 8 out of 13 test problems, respectively.

Furthermore, the performance profile is presented in Fig. 7. From this figure, it is concluded that DE-DPS is the first algorithm that is able to solve all problems when τ equals 38.4. jDE2, SaDE, and MDE are competitive, while SAMO-DE and APF-GA are the worst.

It is also interesting to show the significant difference between DE-DPS and all other algorithms, based on the statistical test. For this purpose, the results' summary is presented in Table XIV. This table reveals that DE-DPS is superior to APF-GA, jDE2, SaDE, and ATMES in regards to the average results, while there is no significant difference based on the best average results. Furthermore, there is no significant difference between DE-DPS and all other algorithms based on the best and average fitness values.

G. Solving CEC2010 Constrained Problems

As indicated earlier, we also solved another set of 36 test instances (18 problems each with 10 and 30 dimensions) that were introduced in CEC2010 [17], and have compared our results with a DE algorithm which won the CEC2010 Constrained Optimization Competition, as well other two other algorithms that used dynamic mechanisms to tune the DE parameters. The algorithm was run 25 times for each test problem, where the stopping criterion was to run for up to 200K FEs for 10D instances, and 600K FEs for 30D.

The detailed computational results for both the 10D and the 30D instances are shown in Table XVIII with the detailed results of ε DEag [49] (CEC2010 competition winner), and other two DE algorithms that use a mechanism to adapt the control parameters. These two algorithms are an ensemble of constrained handling techniques based on a DE algorithm (ECHT-DE) [50] and improved jDE (*jDEsoco*) [51]. It is important to highlight that DE-DPS was able to reach 100% feasibility ratio for both the 10-D and 30-D instances, but ε DEag attained 100% feasibility ratio for only 35 out of the 36 test instances, while it only obtained a 12% feasibility ratio for C12 with 30-D. The average feasibility ratio for ECHT-DE is less than 100% for 10-D and 30-D, while *jDEsoco* obtained a 98% feasibility ratio. Note that, the "*" in Table XVIII for C12 means that it includes infeasible solutions in calculating means and other parameters.

TABLE XV
COMPARISON AMONG DE-DPS, AND ε DEAG, ECHT-DE AND *jDEsoco* (THE NUMBERS SHOWN IN THE TABLE ARE FOR THE FIRST ALGORITHM IN COLUMN 1)

D	Comparison	Fitness	Better	Equal	Worse	Dec.
10D	**DE-DPS – to – εDEag**	Best	3	14	1	≈
		Average	10	7	1	+
	DE-DPS – to – ECHT-DE	Best	4	14	0	≈
		Average	14	3	1	+
	DE-DPS – to – *jDEsoco*	Best	6	12	0	+
		Average	16	1	1	+
30D	**DE-DPS – to – εDEag**	Best	16	1	1	+
		Average	16	0	2	+
	DE-DPS – to – ECHT-DE	Best	7	3	8	≈
		Average	16	1	1	+
	DE-DPS – to – *jDEsoco*	Best	12	0	6	+
		Average	16	0	2	+

TABLE XVI
COMPARISON AMONG DE-DPS, AND JADE, JDE, SADE, EPSDE, AND CODE (THE NUMBERS SHOWN IN THE TABLE ARE FOR THE FIRST ALGORITHM IN COLUMN 1)

Algorithms	Better	Equal	Worse	Dec.
DE-DPS- to- JADE	14	9	2	+
DE-DPS- to- jDE	15	9	1	+
DE-DPS- to- SaDE	18	4	3	+
DE-DPS- to- EPSDE	17	2	6	≈
DE-DPS- to- CoDE	14	9	2	+

Considering the quality of the solutions obtained, a summary is reported in Table XV. From this table, DE-DPS shows superior performance to the other algorithms for majority of the test problems, especially for the 30-D instances.

Due to lack of appropriate data from the published algorithms, the performance profiles are defined in this section with a different comparison goal. In these cases, the optimal solutions are not known for many problems and no algorithm recorded the fitness evaluations used to obtain a predefined fitness value. For this reason, the comparison goal is defined as the average fitness value obtained after running the algorithms for a fixed number of fitness evaluations (such as 200K and 600K FEs for the 10-D and 30-D problems, respectively). The performance profiles are depicted in Fig. 8.

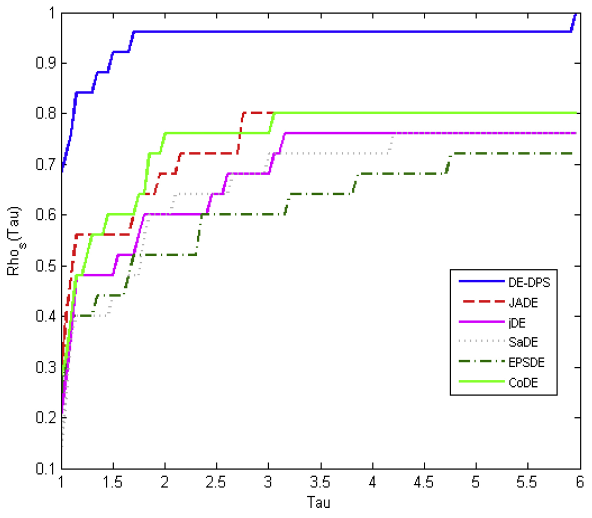


Fig. 10. Average fitness values performance profiles comparing DE-DPS with various DE algorithms.

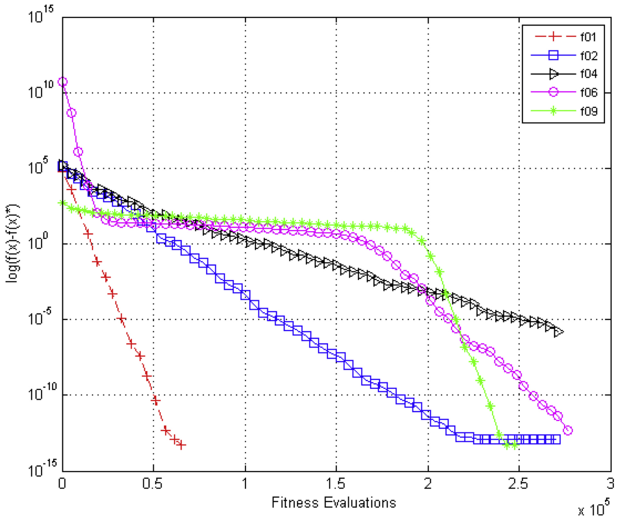
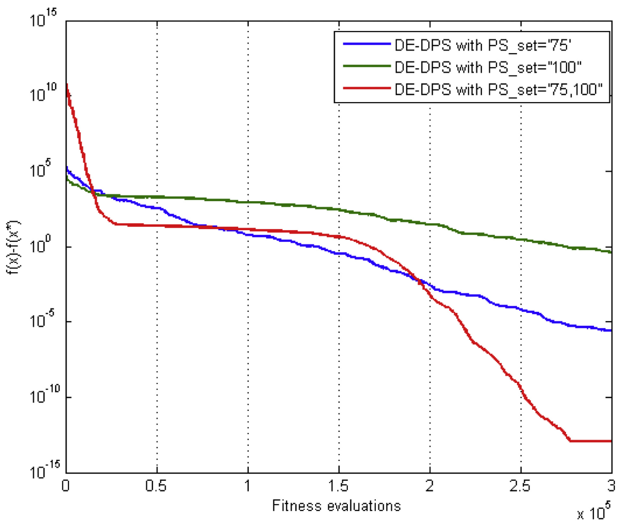
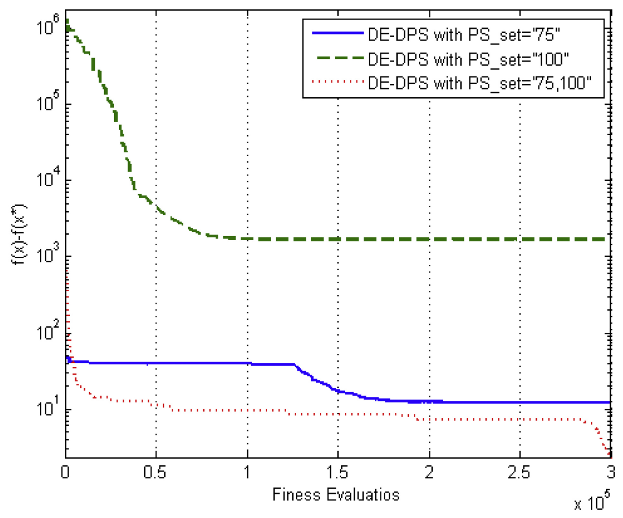


Fig. 11. Convergence plot of DE-DSP for various test problems.



(a)



(b)

Fig. 12. Convergence plots of DE-DPS with three different sets of population sizes for (a) F04 and (b) F11. The y-axis is in a log scale.

This figure shows that DE-DPS has the probability of 0.9 or more to obtain the best fitness value for both the 10-D and 30-D problems, and reached 1 with τ equals to 1700 and 45 000, respectively. It is also clear that ε DEag has the second best performing algorithm for the 10-D. In addition to this, with small values of τ, ECHT-DE shows competitive performance with jDEsoco. However, jDEsoco is better than ECHT-DE with higher value of τ.

Furthermore, the performance of DE-DPS is statistically better than ε DEag in regard to both the best and average results for the 30-D test problems, while the performance of DE-DPS is statistically better in regard to the average results for the 10-D test problems. The performance of DE-DPS is also better than ECHT-DE with regard to the average results for both the 10-D and the 30-D test problems. In comparison to *jDEsoco*, DE-DPS outperforms *jDEsoco* with regard both the best and average results.

H. Scaling Analysis

In this analysis, the relationship between the dimensionality of the test problem and the average number of function evaluations needed to find solutions with the tolerance limit (here is equal to 0.0001) is derived. Two test problems from CEC 2010, i.e., C07 and C09, have been chosen for the purpose. The problems are known to be difficult as the objective function is nonseparable, multimodal and is shifted by a matrix *shi*. Considering the constraints, C07, contains an in-equality constraint that is separable, multimodal and also shifted by the same matrix, while C09 contains an equality constraint with the same properties as in C07. The optimal solutions of these two problems are at $f(x^*) = 0$.

Both problems have been solved using different dimensions, i.e., $D = 5$, 10, 15, 20, 25, and 30 variables. For each D, the algorithm was run over 50 trials, and the average fitness evaluations were recorded to reach the stopping criteria. It must be mentioned here that up to only 30 variables are used, as the available data are up to 30 dimensions.

TABLE XVII

FUNCTION VALUES OBTAINED BY DE-DPS, MDE, SAMO-DE, APF-GA, ECHT-EP2, JDE-2, ATMES, AND SMES FOR CEC2006 TEST PROBLEMS

P.	Criteria	DE-DPS	MDE	SAMO-DE	APF-GA	ECHT-EP2	jDE-2	SaDE	ATMES
	FEs	240,000	500,000	240,000	500,000	240,000	500,000	500,000	240,000
g01	**best**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**
	Avg.	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**	**-15.0000**
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.6E-14
g02	**best**	**-0.8036191**	**-0.8036191**	**-0.8036191**	-0.803601	**-0.8036191**	**-0.8036191**	**-0.8036191**	-0.803339
	Avg.	**-0.8036191**	-0.78616	-0.79873521	-0.803518	-0.7998220	-0.8027382	-0.8015631	-0.790148
	St. d	**9.06255E-08**	1.26E-02	8.8005E-03	1.00E-04	6.29E-03	3.0488E-03	4.9786E-03	1.3E-02
g03	**best**	**-1.0005**	**-1.0005**	**-1.0005**	**-1.001**	**-1.0005**	-0.9115	**-1.0005**	**-1.000**
	Avg.	**-1.0005**	**-1.0005**	**-1.0005**	**-1.001**	**-1.0005**	-0.66242	-1.000486	**-1.000**
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.014E-01	3.4743E-005	5.9E-05
g04	**best**	**-30665.539**	**-30665.539**	**-30665.5386**	**-30665.539**	**-30665.539**	**-30665.539**	**-30665.539**	**-30665.539**
	Avg.	**-30665.539**	**-30665.539**	**-30665.5386**	**-30665.539**	**-30665.539**	**-30665.539**	**-30665.539**	**-30665.539**
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.00E-04	**0.00E+00**	**0.00E+00**	1.8550E-12	7.4E-12
g05	**best**	**5126.497**	**5126.497**	**5126.497**	**5126.497**	**5126.497**	**5126.497**	**5126.497**	**5126.498**
	Avg.	**5126.497**	**5126.497**	**5126.497**	5127.5423	**5126.497**	5127.5726	**5126.497**	5127.648
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.4324E+00	**0.00E+00**	2.4193	1.8190E-13	1.8E+00
g06	**best**	**-6961.814**	**-6961.814**	**-6961.813875**	**-6961.814**	**-6961.814**	**-6961.814**	**-6961.814**	**-6961.814**
	Avg.	**-6961.814**	**-6961.814**	**-6961.813875**	**-6961.814**	**-6961.814**	**-6961.814**	**-6961.814**	**-6961.814**
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	4.6E-12
g07	**best**	**24.3062**	**24.3062**	**24.3062**	**24.3062**	**24.3062**	**24.3062**	**24.3062**	**24.306**
	Avg.	**24.3062**	**24.3062**	24.3096	**24.3062**	24.3063	**24.3062**	**24.3062**	24.316
	St. d	**0.00E+00**	**0.00E+00**	1.5888E-03	**0.00E+00**	3.19E-05	1.7405E-15	1.4993E-05	1.1E-02
g08	**best**	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**
	Avg.	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.0E+00**	1.2580E-32	3.8426E-18	2.8E-17
g09	**best**	**680.630**	**680.630**	**680.630**	**680.630**	**680.630**	**680.630**	**680.630**	**680.630**
	Avg.	**680.630**	**680.630**	**680.630**	**680.630**	**680.630**	**680.630**	**680.630**	680.639
	St. d	**0.00E+00**	**0.00E+00**	1.1567E-05	**0.00E+00**	2.61E-08	4.2538E-14	7.9851E-14	1.0E-02
g10	**best**	**7049.24802**	**7049.24802**	**7049.24802**	**7049.24802**	7049.2483	**7049.24802**	**7049.24802**	7052.253
	Avg.	**7049.24802**	**7049.24802**	7059.81345	7077.6821	7049.2490	**7049.24802**	**7049.24802**	7250.437
	St. d	**0.00E+00**	**0.00E+00**	7.856E+00	5.1240E+01	6.60E-04	9.1279E-08	1.5244E-06	1.2E+02
g11	**best**	**0.7499**	**0.7499**	**0.7499**	**0.7499**	**0.7499**	**0.7499**	**0.7499**	**0.75**
	Avg.	**0.7499**	**0.7499**	**0.7499**	**0.7499**	**0.7499**	0.7499000091	**0.7499**	**0.75**
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	4.5540E-04	**0.00E+00**	3.4E-04
g12	**best**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**
	Avg.	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**	**-1.0000**
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.0E-03
g13	**best**	**0.053942**	**0.053942**	**0.053942**	**0.053942**	**0.053942**	0.350942	**0.053942**	**0.053950**
	Avg.	**0.053942**	**0.053942**	**0.053942**	**0.053942**	**0.053942**	0.745422	**0.053942**	0.053959
	St. d	**0.00E+00**	**0.00E+00**	1.7541E-08	**0.00E+00**	1.00E−12	2.2376E-01	2.7832E-07	1.3E-05
g14	**best**	**-47.764888**	-47.764887	**-47.76489**	-47.76479	**-47.7649**	**-47.764888**	**-47.764888**	-
	Avg.	**-47.764888**	-47.764874	-47.68115	-47.76479	-47.7648	**-47.764888**	-47.764841	-
	St. d	**0.00E+00**	1.400E-05	4.043E-02	1.00E-04	2.72E-05	3.4809E-15	6.4986E-005	-
g15	**best**	**961.71502**	**961.71502**	**961.71502**	**961.71502**	**961.71502**	**961.71502**	**961.71502**	-
	Avg.	**961.71502**	**961.71502**	**961.71502**	**961.71502**	**961.71502**	961.719424	**961.71502**	-
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	2.01E-13	2.2020E-02	**0.00E+00**	-
g16	**best**	**-1.905155**	**-1.905155**	**-1.905155**	**-1.905155**	**-1.905155**	**-1.905155**	**-1.905155**	-
	Avg.	**-1.905155**	**-1.905155**	**-1.905155**	**-1.905155**	**-1.905155**	**-1.905155**	**-1.905155**	-
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.12E-10	**0.00E+00**	**0.00E+00**	-
g17	**best**	**8853.5397**	**8853.5397**	**8853.5397**	8853.5398	**8853.5397**	**8853.5397**	**8853.5397**	-
	Avg.	8883.77467	**8853.5397**	**8853.5397**	8888.4876	**8853.5397**	**8853.5397**	8923.2097	-
	St. d	3.56207E+01	**0.00E+00**	1.1500E-05	29.0347	2.13E−08	3.6822E-17	1.6168E+01	-
g18	**best**	**-0.866025**	**-0.866025**	**-0.866025**	**-0.866025**	**-0.866025**	**-0.866025**	**-0.866025**	-
	Avg.	**-0.866025**	**-0.866025**	**-0.866024**	-0.865925	**-0.866025**	**-0.866025**	-0.850741	-
	St. d	**0.00E+00**	**0.00E+00**	7.043672E-07	0000	1.00E-09	3.6822E-17	5.2898E-02	-
g19	**best**	**32.655593**	32.655693	**32.655593**	**32.655593**	32.6591	**32.655593**	**32.655593**	-
	Avg.	**32.655593**	33.34125	32.757340	**32.655593**	32.6623	**32.655593**	**32.655593**	-
	St. d	2.52789E-07	8.475E-01	6.145E-02	**0.00E+00**	3.4E−03	1.0531E-13	7.2976E-10	-
g21	**best**	**193.72451**	**193.72451**	**193.724510**	196.63301	**193.7246**	**193.7246**	**193.72451**	-
	Avg.	**193.72451**	**193.72451**	**193.771375**	199.51581	193.7438	204.2026	193.72528	-
	St. d	**0.00E+00**	**0.00E+00**	**1.9643E-02**	2.3565E+00	1.65E-02	3.6266E+01	1.5058E-03	-
g23	**best**	**-400.0551**	**-400.0551**	-396.165732	-399.7624	-398.9731	**-400.0551**	**-400.0551**	-
	Avg.	**-400.0551**	**-400.0551**	-360.817656	-394.7627	-373.2178	-387.9531	-400.054979	-
	St. d	**0.00E+00**	**0.00E+00**	1.9623E+01	3.8656E+00	3.37E+01	5.9983E+01	3.4116E-04	-
g24	**best**	**-5.508013**	**-5.508013**	**-5.508013**	**-5.508013**	**-5.508013**	**-5.508013**	**-5.508013**	-
	Avg.	**-5.508013**	**-5.508013**	**-5.508013**	**-5.508013**	**-5.508013**	**-5.508013**	**-5.508013**	-
	St. d	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.8E-15	1.9323E-29	**0.00E+00**	-

Fig. 9 shows the average fitness functions for each dimension. For a further investigation, the regression line [52] is fitted to help readers to approximate the average fitness evaluations that are required for different dimensions.

So, the regression equation for C07 is

$$FEs = 208.58D^2 + 4147D - 9465.6 \tag{15}$$

and the coefficient of determination [52] is equal to 98.25%. This means that the line is highly fitted to predict future values.

The regression equation for C09 is

$$FEs = 146.37D^2 + 5363.2D - 13\,417 \tag{16}$$

and the coefficient of determination is equal to 98.84%; this means that the line is highly fitted.

This encouraging result demonstrates that the algorithm is efficient and scales well with increasing dimension. In addition, the presented lines appear quite smooth, suggesting that DE-DPS is well designed.

I. Solving Unconstrained Problems (CEC2005)

In this section, we solve and analyze the algorithm on 25 unconstrained problems (from the CEC2005 competition) with 30 dimensions [18]. These problems have different mathematical properties, in which $f01$ to $f05$ are shifted and/or

TABLE XVIII
FUNCTION VALUES ACHIEVED BY DE-DPS, EDEAG, ECHT-DE, AND JDESOCO FOR CEC2010 TEST PROBLEMS

Prob.	Alg.	10D			30D		
		Best	**Mean**	***St. d***	**Best**	**Mean**	***St. d***
C01	DE-DPS	**-7.473104E-01**	**-7.473104E-01**	**2.26623E-16**	**-0.8218843**	**-0.8212036**	**1.79648E-03**
	εDEag	**-7.473104E-01**	-7.470402E-01	1.323339E-03	-8.218255E-01	-8.208687E-01	7.103893E-04
	ECHT-DE	-0.7473	-0.7470	0.0014	-0.8217	-0.7994	0.0179
	jDEsoco	-0.7473103	-7.3836E-01	1.6006E-02	-0.8218841	-8.1238E-01	1.3187E-02
C02	DE-DPS	**-2.277711**	**-2.277512**	**2.54035E-04**	**-2.280671**	**-2.244631**	**5.20548E-02**
	εDEag	-2.277702E+00	-2.269502E+00	2.3897790E-02	-2.169248E+00	-2.151424E+00	1.197582E-02
	ECHT-DE	-2.2777	-2.2744	0.0067	-2.2251	-1.9943	0.2099
	jDEsoco	-1.1176240	5.6359E-01	1.1044	0.6792295	1.5603	8.4705E-01
C03	DE-DPS	**0.000000E+00**	**0.000000E+00**	**0.0000000E+00**	1.6200E-19	**1.8479E-13**	**4.17994E-13**
	εDEag	**0.000000E+00**	**0.000000E+00**	**0.0000000E+00**	2.867347E+01	2.883785E+01	8.047159E-01
	ECHT-DE	**0.00000E+00**	**0.00000E+00**	**0.00000E+00**	3.2433E-21	9.8920E+01	6.2594E+01
	jDEsoco	**0.00000E+00**	7.8105	2.9437	**9.993685E-22**	6.1447E+01	5.7577E+01
C04	DE-DPS	**-1.000000E-05**	**-1.000000E-05**	9.09633E-15	**-3.3318E-06**	**-3.3123E-06**	**2.03926E-08**
	εDEag	-9.992345E-06	-9.918452E-06	1.5467300E-07	4.698111E-03	8.162973E-03	3.067785E-03
	ECHT-DE	**-1.00000E-05**	**-1.00000E-05**	**0.00000E+00**	-3.3015E-06	-1.0257E-06	9.0135E-02
	jDEsoco	**-1.00000E-05**	**-1.00000E-05**	9.4831E-16	8.0490978E-05	3.5187E-04	2.3948E-04
C05	DE-DPS	**-4.836106E+02**	**-4.836106E+02**	1.25826E-10	**-4.836106E+02**	**-4.836106E+02**	**4.42074E-06**
	εDEag	**-4.836106E+02**	**-4.836106E+02**	**3.89035E-13**	-4.531307E+02	-4.495460E+02	2.899105E+00
	ECHT-DE	**-483.6106**	-411.4532	76.3137	-213.6844	-106.4228	167.1481
	jDEsoco	**-483.6106247**	-3.0217E+02	3.0256E+02	-19.9503700	1.0822E+02	1.5203E+02
C06	DE-DPS	**-5.786624E+02**	**-578.6621**	**8.05379E-04**	**-530.63785**	**-530.6329**	**5.89364E-03**
	εDEag	-5.786581E+02	-5.786528E+02	3.6271690E-03	-5.285750E+02	-5.279068E+02	4.748378E-01
	ECHT-DE	**-578.6624**	-562.4688	45.1479	-295.7192	-137.6152	98.8995
	jDEsoco	**-578.662366**	-5.7408E+02	1.6461E+01	-530.6377271	-4.7284E+02	1.2743E+02
C07	DE-DPS	**0.00000E+00**	**0.000000E+00**	**0.0000000E+00**	5.48786E-20	1.03359E-13	2.20540E-13
	εDEag	**0.00000E+00**	**0.000000E+00**	**0.0000000E+00**	1.147112E-15	2.603632E-15	1.233430E-15
	ECHT-DE	**0.00000E+00**	**0.00000E+00**	**0.00000E+00**	**0.00000E+00**	0.1329	0.7279
	jDEsoco	**0.00000E+00**	6.4192E-27	1.3515E-26	4.2197747E-26	**8.7396E-24**	**3.2529E-23**
C08	DE-DPS	**0.00000E+00**	3.950387E+00	**5.01904E+00**	4.575719E-13	3.447568E-09	8.86366E-09
	εDEag	**0.00000E+00**	6.727528E+00	5.560648E+00	2.518693E-14	**7.831464E-14**	**4.855177E-14**
	ECHT-DE	**0.00000E+00**	6.1566E+00	6.4527E+00	**0.00000E+00**	3.3585E+01	1.1072E+02
	jDEsoco	**0.00000E+00**	**3.7421**	1.0330E+01	7.2310934E-26	8.2585E+01	2.4395E+02
C09	DE-DPS	**0.00000E+00**	**0.000000E+00**	**0.0000000E+00**	3.81580E-23	**5.29059E-14**	**1.27939E-13**
	εDEag	**0.00000E+00**	**0.000000E+00**	**0.0000000E+00**	2.770665E-16	1.072140E+01	2.821923E+01
	ECHT-DE	**0.00000E+00**	1.4691E-01	8.0482E-01	**0.00000E+00**	4.2441E+01	1.3762E+02
	jDEsoco	**0.00000E+00**	5.2898E-01	1.4620	2.7729234E-25	2.4743	8.7782
C10	DE-DPS	**0.000000E+00**	**0.000000E+00**	**0.0000000E+00**	1.63778E-21	**2.23928E-13**	**6.24058E-13**
	εDEag	**0.000000E+00**	**0.000000E+00**	**0.0000000E+00**	3.252002E+01	3.326175E+01	4.545577E-01
	ECHT-DE	**0.00000E+00**	1.7117E+00	7.6554E+00	**0.00000E+00**	5.3381E+01	8.8308E+01
	jDEsoco	**0.00000E+00**	3.1712E+01	1.8188E+01	1.0862047E-25	2.9386E+01	7.1786
C11	DE-DPS	**-1.52271E-03**	**-1.52271E-03**	**3.14275E-12**	**-3.923438E-04**	**-3.923423E-04**	**8.77688E-10**
	εDEag	**-1.52271E-03**	**-1.52271E-03**	6.3410350E-11	-3.268462E-04	-2.863882E-04	2.707605E-05
	ECHT-DE	-0.0015	-0.0044*	0.0157	-0.0004	0.0026	0.0060
	jDEsoco	**-0.0015227**	-8.2555E-03*	2.3807E-02	-0.0003920	1.1667E-03	5.2690E-03
C12	DE-DPS	-1.9925E-01	-1.9925E-01	4.19599E-10	**-1.992635E-01**	**-1.9926E-01**	**2.60287E-08**
	εDEag	**-5.700899E+02**	**-3.367349E+02**	1.7821660E+02	-1.991453E-01	3.562330E+02	2.889253E+02
	ECHT-DE	-0.1992	-171.8714*	221.0436	**-0.1993**	-25.1292*	136.5593
	jDEsoco	-0.1992457	-2.2365E+01*	1.1083E+02	-0.1992634	-1.9925E-01	2.3453E-05
C13	DE-DPS	**-6.842937E+01**	**-6.842937E+01**	**0.0000000E+00**	**-68.429362**	-66.331412	**2.08493E+00**
	εDEag	**-6.842937E+01**	-6.842936E+01	1.0259600E-06	-6.642473E+01	-6.535310E+01	5.733005E-01
	ECHT-DE	**-68.4294**	-65.1208	2.3750	**-68.4294**	-64.5831	1.6690
	jDEsoco	-68.4293648	-6.8315E+01	5.7018E-01	-68.4293209	**-6.7537E+01**	5.0553E-01
C14	DE-DPS	**0.000000E+00**	**0.000000E+00**	**0.0000000E+00**	8.925796E-21	**5.137406E-14**	**1.31791E-13**
	εDEag	**0.000000E+00**	**0.000000E+00**	**0.0000000E+00**	5.015863E-14	3.089407E-13	5.608409E-13
	ECHT-DE	**0.00000E+00**	7.0242E+05	3.1937E+06	**0.00000E+00**	1.2368E+05	6.7736E+05
	jDEsoco	**0.00000E+00**	9.1221E-01	2.4538	5.7101696E-26	1.5946E-01	7.9732E-01
C15	DE-DPS	**0.000000E+00**	**5.438912E-26**	**2.19329E-25**	**6.336258E-20**	**1.957805E-13**	**5.22687E-13**
	εDEag	**0.000000E+00**	1.798980E-01	8.8131560E-01	2.160345E+01	2.160376E+01	1.104834E-04
	ECHT-DE	**0.00000E+00**	2.3392E+13	5.2988E+13	1.9922E+09	1.9409E+11	4.3524E+11
	jDEsoco	2.0257948E-26	1.2452E+09	3.8127E+09	9.6993452E-16	1.5357E+09	1.6045E+09
C16	DE-DPS	**0.000000E+00**	**0.000000E+00**	**0.0000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.0000000E+00**
	εDEag	**0.000000E+00**	3.702054E-01	3.7104790E-01	**0.000000E+00**	2.168404E-21	1.062297E-20
	ECHT-DE	**0.00000E+00**	3.9327E-02	4.2815E-02	**0.00000E+00**	**0.00000E+00**	**0.00000E+00**
	jDEsoco	**0.000000E+00**	4.1102E-01	3.8359E-01	0.0812907	7.3206E-01	2.9943E-01
C17	DE-DPS	**0.000000E+00**	**1.061273E-24**	**3.88726E-24**	3.236013E-12	**8.766191E-02**	**1.55966E-01**
	εDEag	1.463180E-17	1.249561E-01	1.9371970E-01	2.165719E-01	6.326487E+00	4.986691E+00
	ECHT-DE	**0.00000E+00**	1.1152E-01	3.3152E-01	**0.00000E+00**	2.7496E-01	3.7832E-01
	jDEsoco	0.0302187	8.8958E+01	9.9131E+01	2.9943E-01	5.0398E+02	4.4832E+02
C18	DE-DPS	**0.000000E+00**	3.209082E-23	7.12457E-23	6.747743E-13	9.459914E-08	1.91224E-07
	εDEag	3.731440E-20	9.678765E-19	1.8112340E-18	1.226054E+00	8.754569E+01	1.664753E+02
	ECHT-DE	**0.00000E+00**	**0.00000E+00**	**0.00000E+00**	**0.00000E+00**	**0.00000E+00**	**0.00000E+00**
	jDEsoco	0.1100901	4.0500E+02	7.3762E+02	17.5655328	3.0849E+02	3.0538E+02

rotated unimodal problems, $f06$ to $f12$ are shifted and/or rotated multimodal problems, $f13$ to $f25$ are hybrid composition of different problems with different difficult properties. Results on all of these test problems were derived based on 25 runs, where the maximum fitness evaluations were set to 300K. The values of the parameters are the same as used in the previous sections. We have reported the mean error values from the optimal solutions of our algorithm, as reported by other researchers, along with other algorithms JADE [21], jDE [11], SaDE [12], EPSDE [39], and CoDE [28], in Table XIX.

A comparison summary is shown in Table XVI. From this table, DE-DPS is clearly better for the majority of the test problems. Furthermore, the performance of DE-DPS is statistically better than all other algorithms, except EPSDE, in which there is no significant difference.

The performance profiles are depicted in Fig. 10 with the comparison goal as the average fitness values after 300 000 fitness evaluations. Note that the data are not available to consider other comparison goals. Based on this figure, DE-DPS is the best algorithm, as it is able to reach 1 when $\tau = 6.25$. To add to this, all JADE, SaDE, and CODE are competitive, while both jDE and EPSDE are the worst algorithms.

To this end, a sample of different convergence plots of DE-DPS is presented in Fig. 11.

TABLE XIX

MEAN ERROR TO OPTIMAL SOLUTIONS AND STANDARD DEVIATION RESULTS OF DE-DPS, JADE, jDE, SADE, EPSDE, AND CODE OVER 25 INDEPENDENT RUNS ON 25 TEST FUNCTIONS WITH 30 VARIABLES FOR CEC2005 TEST SET WITH 300 000 FES

Problem (optimal)	Criteria	Algorithms					
		DE-DPS	JADE	jDE	SaDE	EPSDE	CoDE
F1	**Mean Error**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**
(-450)	**(*St. d*)**	**(0.00E+00)**	**(0.00E+00)**	**(0.00E+00)**	**(0.00E+00)**	**(0.00E+00)**	**(0.00E+00)**
F2	**Mean Error**	**0.00E+00**	1.07E-28	1.11E-06	8.26E-06	4.23E-26	1.69E-15
(-450)	**(*St. d*)**	**(0.00E+00)**	(1.00E-28)	(1.96E-06)	(1.65E-05)	(4.07E-26)	(3.95E-15)
F3	**Mean Error**	5.02E+04	**8.42E+03**	1.98E+05	4.27E+05	8.74E+05	1.05E+05
(-450)	**(*St. d*)**	(2.24E+04)	**(7.26E+03)**	(1.10E+05)	(2.08E+05)	(3.28E+06)	(6.25E+04)
F4	**Mean Error**	**0.00E+00**	1.73E-16	4.40E-02	1.77E+02	3.49E+02	5.81E-03
(-310)	**(*St. d*)**	**(0.00E+00)**	(5.43E-16)	(1.26E-01)	(2.67E+02)	(2.23E+03)	(1.38E-02)
F5	**Mean Error**	**5.68E-14**	8.59E-08	5.11E+02	3.25E+03	1.40E+03	3.31E+02
(-310)	**(*St. d*)**	**(2.05E-13)**	(5.23E-07)	(4.40E+02)	(5.90E+02)	(7.12E+02)	(3.44E+02)
F6	**Mean Error**	**0.00E+00**	1.02E+01	2.35E+01	5.31E+01	6.38E-01	1.60E-01
(390)	**(*St. d*)**	**(0.00E+00)**	(2.96E+01)	(2.50E+01)	(3.25E+01)	(1.49E+00)	(7.85E-01)
F7	**Mean Error**	**3.75E-03**	8.07E-03	1.18E-02	1.57E-02	1.77E-02	7.46E-03
(-180)	**(*St. d*)**	**(4.39E-03)**	(7.42E-03)	(7.78E-03)	(1.38E-02)	(1.34E-02)	(8.55E-03)
F8	**Mean Error**	2.09E+01	2.09E+01	2.09E+01	2.09E+01	2.09E+01	**2.01E+01**
(-140)	**(*St. d*)**	**(4.08E-02)**	(1.68E-01)	(4.86E-02)	(4.95E-02)	(5.81E-02)	(1.41E-01)
F9	**Mean Error**	**0.00E+00**	**0.00E+00**	**0.00E+00**	2.39E-01	3.98E-02	**0.00E+00**
(-330)	**(*St. d*)**	**(0.00E+00)**	**(0.00E+00)**	**(0.00E+00)**	(4.33E-01)	(1.99E-01)	**(0.00E+00)**
F10	**Mean Error**	**2.29E+01**	2.41E+01	5.54E+01	4.72E+01	5.36E+01	4.15E+01
(-330)	**(*St. d*)**	(6.14E+00)	(**4.61E+00**)	(8.46E+00)	(1.01E+01)	(3.03E+01)	(1.16E+01)
F11	**Mean Error**	**9.27E+00**	2.53E+01	2.79E+01	1.65E+01	3.56E+01	1.18E+01
(90)	**(*St. d*)**	**(3.24E+00)**	(1.65E+00)	(1.61E+00)	(2.42E+00)	(3.88E+00)	(3.40E+00)
F12	**Mean Error**	**1.01E+03**	6.15E+03	8.63E+03	3.02E+03	3.58E+04	3.05E+03
(-460)	**(*St. d*))**	**(1.40E+03)**	(4.79E+03)	(8.31E+03)	(2.33E+03)	(7.05E+03)	(3.80E+03)
F13	**Mean Error**	1.99E+00	**1.49E+00**	1.66E+00	3.94E+00	1.94E+00	1.57E+00
(-130)	**(*St. d*). *d*)**	(4.13E-01)	**(1.09E-01)**	(1.35E-01)	(2.81E-01)	(1.46E-01)	(3.27E-01)
F14	**Mean Error**	**1.23E+01**	**1.23E+01**	1.30E+01	1.26E+01	1.35E+01	**1.23E+01**
(-300)	**(*St. d*)**	(5.05E-01)	(3.11E-01)	(**2.00E-01**)	(2.83E-01)	(2.09E-01)	(4.81E-01)
F15	**Mean Error**	3.08E+02	3.51E+02	3.77E+02	3.76E+02	**2.12E+02**	3.88E+02
(120)	**(*St. d*)**	(9.092E+01)	(1.28E+02)	(8.02E+01)	(7.83E+01)	**(1.98E+01)**	(6.85E+01)
F16	**Mean Error**	**5.20E+01**	1.01E+02	7.94E+01	8.57E+01	1.22E+02	7.37E+01
(120)	**(*St. d*)**	**(8.80E+00)**	(1.24E+02)	(2.96E+01)	(6.94E+01)	(9.19E+01)	(5.13E+01)
F17	**Mean Error**	**5.35E+01**	1.47E+02	1.37E+02	7.83E+01	1.69E+02	6.67E+01
(120)	**(*St. d*)**	**(2.40E+01)**	(1.33E+02)	(3.80E+01)	(3.76E+01)	(1.02E+02)	(2.12E+01)
F18	**Mean Error**	9.04E+02	9.04E+02	9.04E+02	8.68E+02	**8.20E+02**	9.04E+02
(10)	**(*St. d*)**	**(2.85E-01)**	(1.03E+00)	(1.08E+01)	(6.23E+01)	(3.35E+00)	(1.04E+00)
F19	**Mean Error**	9.04E+02	9.04E+02	9.04E+02	8.74E+02	**8.21E+02**	9.04E+02
(10)	**(*St. d*)**	**(3.64E-01)**	(8.40E-01)	(1.11E+00)	(6.22E+01)	(3.35E+00)	(9.42E-01)
F20	**Mean Error**	9.04E+02	9.04E+02	9.04E+02	8.78E+02	**8.22E+02**	9.04E+02
(10)	**(*St. d*)**	**(3.25E-01)**	(8.47E-01)	(1.10E+00)	(6.03E+01)	(4.17E+00)	(9.01E-01)
F21	**Mean Error**	**5.00E+02**	**5.00E+02**	**5.00E+02**	5.52E+02	8.33E+02	**5.00E+02**
(360)	**(*St. d*)**	**(0.00E+00)**	(4.67E-13)	(4.80E-13)	(1.82E+02)	(1.00E+02)	(4.88E-13)
F22	**Mean Error**	8.60E+02	8.66E+02	8.75E+02	9.36E+02	**5.07E+02**	8.63E+02
(360)	**(*St. d*)**	(3.61E+01)	(1.91E+01)	(1.91E+01)	(1.83E+01)	**(7.26E+00)**	(2.43E+01)
F23	**Mean Error**	**5.34E+02**	5.50E+02	**5.34E+02**	**5.34E+02**	8.58E+02	**5.34E+02**
(360)	**(*St. d*)**	(3.86E-04)	(8.05E+01)	**(2.77E-04)**	(3.57E-03)	(6.82E+01)	(4.12E-04)
F24	**Mean Error**	**2.00E+02**	**2.00E+02**	**2.00E+02**	**2.00E+02**	2.13E+02	**2.00E+02**
(220)	**(*St. d*)**	**(0.00E+00)**	(2.85E-14)	(2.85E-14)	(6.20E-13)	(1.52E+00)	(2.85E-14)
F25	**Mean Error**	**2.10E+02**	2.11E+02	2.11E+02	2.14E+02	2.13E+02	2.11E+02
(220)	**(*St. d*)**	**(4.85E-01)**	(7.92E-01)	(7.32E-01)	(2.00E+00)	(2.55E+00)	(9.02E-01)

J. Contribution of Population Size Switching

The benefit of switching the population size, during the evolution process, is demonstrated using two test problems (F04 and F11). We have solved these two test problems using our algorithm with fixed population sizes of 75 and 100 (i.e., without population adaptation option), and with an option of switching between these two population sizes (as of the proposed DP-DPS algorithm). The convergence plots, comparing these three population size options, are shown in Fig. 12. In the figure, the x-axis represents the number of fitness evaluations and the y-axis represents the deviation from the optimal fitness value (in log scale). In this figure, it shows that the population switching option (with two population sizes, 75 and 100) converges much faster than single population option with size of either 75 or 100. When comparing between two single population options, population size 100 has a slower convergence pattern than population size 75. From this result, it can be concluded that DP-DPS with the population size switching option helps converge quickly to optimality (or near optimality) compared to the fixed population size option.

V. CONCLUSION AND FUTURE WORK

During the last few decades, DE algorithms have shown superior performance over other nature inspired algorithms in solving constrained and unconstrained optimization problems. Selection of appropriate control parameters in any DE is known to be a difficult problem. To overcome this, a considerable number of differential evolution algorithms have been introduced, which use adaptive or self-adaptive mechanisms to adapt the control parameters. Although the self-adaptive variants demonstrated better performances in comparison to DE forms with fixed control parameters, not all the parameters were adapted in one single run. Furthermore, only a handful of such algorithms have been used to solve constrained optimization problems.

In our proposed approach, three sets of parameter values were initialized, one each for the amplification factor, crossover rate, and population size. For a defined number of generations, each individual in the population was assigned to a random combination, and the normalized success for each combination was recorded. Subsequently, the number of combinations was reduced until a restart point, where the success counters were reset.

The performance of the proposed algorithm was analyzed on three well known benchmark problem sets. Two of them are constrained problem sets and the third one is an unconstrained problem set. Based on the obtained results, the best number of combinations were 63 ($F_{sst} = \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}$ and $CR_{set} = \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}$). An interesting conclusion was that the proposed algorithm was able to save the computational time by 14.48% in comparison to a DE algorithm with a single combination of parameters, based on a comparative study of CEC2006 test problems.

Based on the CEC2010 test problems, the proposed algorithm showed outstanding performance on the small scale test problems with several characteristics (rotated and/or shifted and/or multimodal). In addition to this, the algorithm showed superior performance as compared to the state-of-the-art algorithms in the context of problems with large dimensions. As for the CEC2005 test problems, the algorithm confirmed its superiority over the state-of-the-art algorithms.

Finally, our approach started with the parameter settings suggested in the literature and was able to find better control parameter settings. These control parameters helped improve the efficiency of DE when solving both constrained and unconstrained optimization problems considered in this research.

As a future work, we intend to apply this concept with different population based algorithms and use them to solve real-world optimization problems. We also intend to extend this paper by combining operator adaptation (such as mutation and crossover) in DE. Finally, solving other problems, such as multiobjective and dynamic problems, will be an interesting future direction.

Link for supplementary materials: http://seit.unsw.adfa.edu.au/staff/sites/rsarker/SM-TEC-Sarker.pdf.

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